# GEM2006/GET1028
# Logic

## 8.2 easier quantificational proofs

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# recap: the 3 steps of (easier) propositional proofs

1. Block off the conclusion and assume the negation of the conclusion

2. Apply S- and I- rules with the goal of getting a contradiction

3. Apply the RAA rule: block off the entire section beginning from the last non-blocked off assumption to the occurrence of the contradiction. Infer the conclusion.

# new inference rules 1&2: reverse squiggles

Reverse squiggles for universals:

$\sim(x)Lx \rightarrow (\exists x)\sim Lx$

e.g. not everyone is listening→ someone is not listening

Reverse squiggles for existentials:

$\sim(\exists x)Lx \rightarrow (x)\sim Lx$

e.g. No one is listening → everyone is not listening

# reverse squiggle rules in longer wffs

- Reverse squiggles only when a negated quantified statement is <u>outermost</u> wff

- DON'T reverse squiggles in this case: (P⊃~(x)Qx)

- Reverse squiggles in this case:

~(x)(Px⊃~Qx)

# new inference rule 3: drop existentials

- Drop existentials: $(\exists x)Px \rightarrow Pa$
- Always use a new constant when dropping existential quantifier
  - E.g. 1. $(\exists x)Bx$
    2. $(\exists x)Rx$
    Drop existentials:
    $(\exists x)Bx \rightarrow Ba$
    $(\exists x)Rx \rightarrow Rb$
- Why must a new constant be used?
  - something is B, and something is R – 2 possibilities, *which must be kept open*:

  1. There is an object that is both B and R
  2. An object is B, another object is R.

  - ‘a is B and b is R’ can affirm (1) or (2)
  - ‘a is B and a is R’ can only affirm (1)

# the drop existentials rule in longer wffs

1. The rule can only be applied if (∃x) begins the wff.
   - can't be applied to ((∃x)Rx⊃L)

2. Apply the rule by replacing the variable in a long existential statement with the same new constant.
   - (∃x)(Rx·Bx) → (Ra·Ba)

3. Don't replace the variable of a quantified statement embedded within an existential statement.
   - (∃x)(Fx·(x)Gx) → (Fa·(x)Gx)

# new inference rule 4: drop universals

- Drop universals: (x)Px → Pa, Pb, Pc…
- You can use any constant; but to ensure effective proof all old constants that have appeared so far should be used.
- Don't use a new constant unless it is necessary

# the drop universals rule in longer wffs

1. The rule can only be applied if (x) begins the wff.
   – The rule can't be applied to this case: $((x)Fx \supset (x)Gx))$

2. Apply the rule by replacing the variable in a long universal statement with the same constant.
   – $(x)(Fx \supset Gx) \rightarrow (Fa \supset Ga)$

3. Don't replace the variable of a quantified statement embedded within a universal statement.
   – $(x)(Fx \cdot (\exists x)Gx) \rightarrow (Fa \cdot (\exists x)Gx)$

# an example of quantificational proof

1. All ravens are black
2. Something is a raven

∴Something is black

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1. All ravens are black
2. Something is a raven

∴Something is black

1. $(x)(Rx \supset Bx)$
2. $(\exists x)Rx$

$\therefore (\exists x)Bx$

# an example of quantificational proofs

1. All ravens are black
2. Something is a raven

∴Something is black

1. $(x)(Rx \supset Bx)$
2. $(\exists x)Rx$

$|\therefore(\exists x)Bx$

3. asm: $\sim(\exists x)Bx$

Start: block off conclusion and assume its negation

# an example of quantificational proofs

1. All ravens are black
2. Something is a raven

∴Something is black

1. (x)(Rx⊃Bx)
2. (∃x)Rx

|∴(∃x)Bx

3.* asm: ~(∃x)Bx

4. ∴(x)~Bx (from 3)

Reverse squiggles: line 3

Mark line 3

# an example of quantificational proofs

1. All ravens are black
2. Something is a raven

∴Something is black

1. $(x)(Rx\supset Bx)$
2.* $(\exists x)Rx$

| ∴$(\exists x)Bx$

3.* asm: $\sim(\exists x)Bx$
4. ∴$(x)\sim Bx$ (from 3)
5. ∴$Ra$ (from 2)

Drop existentials (use a new constant): line 2

Mark line 2

# an example of quantificational proofs

1. All ravens are black
2. Something is a raven

∴Something is black

1. a(x)(Rx⊃Bx)

2.* (∃x)Rx

|∴(∃x)Bx

3.* asm: ~(∃x)Bx

4. ∴(x)~Bx (from 3)

5. ∴Ra (from 2)

6. ∴(Ra⊃Ba) (from 1)

Drop universals (use old constant): line 1

DON'T mark line 1 with *

# an example of quantificational proofs

1. All ravens are black
2. Something is a raven

∴Something is black

1. a(x)(Rx⊃Bx)
2.* (∃x)Rx

|∴(∃x)Bx

3.* asm: ~(∃x)Bx
4. a∴(x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴(Ra⊃Ba) (from 1)
7. ∴~Ba (from 4)

Drop universals (use old constant): line 4
DON'T mark line 4 with *

# an example of quantificational proofs

1. All ravens are black
2. Something is a raven

∴Something is black

1. a(x)(Rx⊃Bx)

2.* (∃x)Rx

|∴(∃x)Bx

3.* asm: ~(∃x)Bx

4. a∴(x)~Bx (from 3)

5. ∴Ra (from 2)

6.* ∴(Ra⊃Ba) (from 1)

7. ∴~Ba (from 4)

8. ∴Ba (from 5 and 6)

I-rule: MP (lines 5 & 6)
Mark line 6

# an example of quantificational proofs

1. All ravens are black
2. Something is a raven

∴Something is black

1. a(x)(Rx⊃Bx)
2.* (∃x)Rx

|∴(∃x)Bx

3.* asm: ~(∃x)Bx

4. a∴(x)~Bx (from 3)

5. ∴Ra (from 2)

6.* ∴(Ra⊃Ba) (from 1)

7. ∴~Ba (from 4)

8. ∴Ba (from 5 and 6)

9. ∴(∃x)Bx (from 3; 7 contradicts 8) VALID

RAA: block off the section from asm to 8; infer the negation of the assumption

# additional rules for marking (starring)

- Mark any wff to which

1. a reverse squiggles rule has been applied

2. the drop existentials rule has been applied

- Don’t mark with an asterisk any wff to which the drop universals rule has been applied

# the additional sub-steps of quantificational proofs

- formal method of quantificational proofs =method of propositional proofs + the following additional sub-steps in step 2

1. Reverse squiggles (mark the line)
2. Drop existentials (mark the line)
3. Drop universals (don’t mark the line with *)

*Don’t switch the order of 2 and 3. Otherwise you’ll have to re-apply the drop universal rule.

  – What if there are only universal statements in lines?
    - LogiCola: make a further assumption if possible; don’t apply DU straightaway
    - Wang Yen: LogiCola is unreasonable; go ahead and apply DU (use my rule in MCQs; no such questions will appear in computerised test)

# Exercise 8.2a: 1

1. (x)Fx

∴(x)(GxVFx)

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| ∴(x)(GxVFx)

2. asm: ~(x)(GxVFx)

# Exercise 8.2a: 1

1. (x)Fx

|∴(x)(GxVFx)

2. asm: ~(x)(GxVFx)

3. ∴(∃x)~(GxVFx) (from 2)

# Exercise 8.2a: 1

1. (x)Fx

|∴(x)(GxVFx)

2.* asm: ~(x)(GxVFx)

3. ∴(∃x)~(GxVFx) (from 2)

# Exercise 8.2a: 1

1. (x)Fx
|∴(x)(GxVFx)
2.* asm: ~(x)(GxVFx)
3. ∴(∃x)~(GxVFx) (from 2)
4. ∴~(GaVFa) (from 3)

# Exercise 8.2a: 1

1. (x)Fx

|∴(x)(GxVFx)

2.* asm: ~(x)(GxVFx)

3.* ∴(∃x)~(GxVFx) (from 2)

4. ∴~(GaVFa) (from 3)

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5. ∴Fa (from 1)

# Exercise 8.2a: 1

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|∴(x)(GxVFx)

2.* asm: ~(x)(GxVFx)

3.* ∴(∃x)~(GxVFx) (from 2)

4. ∴~(GaVFa) (from 3)

5. ∴Fa (from 1)

6. ∴~Fa (from 4)

# Exercise 8.2a: 1

1. (x)Fx

| ∴(x)(GxVFx)

<u>2.* asm: ~(x)(GxVFx)</u>

<u>3.* ∴(∃x)~(GxVFx) (from 2)</u>

<u>4. ∴~(GaVFa) (from 3)</u>

<u>5. ∴Fa (from 1)</u>

<u>6. ∴~Fa (from 4)</u>

7. ∴(x)(GxVFx) (from 2; 5 contradicts 6)

Valid

# source

- Gensler (2010)