GEM2006/GET1028
Logic

8.3 easier quantificational refutations

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how to refute quantificational arguments?

By incorporating the 3 additional sub-steps of quantificational proofs to step 2 of refuting propositional arguments.

recap: refutation /proof of invalidity

- showing ~C together with a premise set leads to a contradiction

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- Showing C cannot be 0 if the premise set is true,
  i.e. valid argument

- showing ~C together with the premise set leads to no contradiction when no inference rules (S- or –I rule and the 4 new inference rules) can be used (and no further assumptions can be made)

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- showing C can be 0 when the premise set =1,
  i.e. invalid argument

members of refutation box

- 3 types of simple wff in quantificational language:

1. P, ~C, Q etc are simple wffs in propositional (and also in quantificational) language

2. Fa, Ba, ~Gb are simple wffs involving specific particular entities

3. Fx, Bx, ~Gx are simple wffs involving generic particular entities.

- Only types 1 and 2 are members of refutation box

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. $(x)(Rx \supset Bx)$
2. $(\exists x)Rx$

$\therefore (x)Bx$

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. (x)(Rx⊃Bx)
2. (∃x)Rx

|∴(x)Bx

3. asm: ~(x)Bx

refutation of quantificational arguments: an example

All ravens are black.
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∴Everything is black.

1. $(x)(Rx \supset Bx)$
2. $(\exists x)Rx$

$| \therefore (x)Bx$

3.* asm: $\sim(x)Bx$
4. $\therefore (\exists x)\sim Bx$ (from 3)

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. (x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4. ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. (x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. a(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7. ∴(Ra⊃Ba)(from 1)

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. ab(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7. ∴(Ra⊃Ba)(from 1)
8. ∴(Rb⊃Bb) (from 1)

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. ab(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7.* ∴(Ra⊃Ba)(from 1)
8. ∴(Rb⊃Bb) (from 1)
9. ∴Ba (from 5 and 7)

refutation of quantificational arguments: an example

All ravens are black.
Something is a raven.
∴Everything is black.

1. ab(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7.* ∴(Ra⊃Ba)(from 1)
8.* ∴(Rb⊃Bb) (from 1)
9. ∴Ba (from 5 and 7)
10. ∴~Rb (from 6 and 8)

refutation of quantificational arguments: an example

1. ab(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7.* ∴(Ra⊃Ba)(from 1)
8.* ∴(Rb⊃Bb) (from 1)
9. ∴Ba (from 5 and 7)
10. ∴~Rb (from 6 and 8)

Refutation box:
Ra, Ba
~Rb, ~Bb

truth-conditions of quantified statements

- An existential wff is true iff the statement prefixed by the existential quantifier is true in at least one case.
  - $(\exists x)Rx$ is true iff there is at least one x that is R.
  - Suppose ~Ra, Rb, ~Rc. $(\exists x)Rx$ is true – true for b
  - Suppose ~Ra, ~Rb, ~Rc. $(\exists x)Rx$ is false –false in all cases

- A universal wff is true iff the statement prefixed by the universal quantifier is true in all cases.
  - $(x)(Rx \supset Bx)$ is true iff $(Rx \supset Bx)$ is true in all cases.
  - Suppose ~Ra, ~Ba, Rb, Bb. $(x)(Rx \supset Bx)$ is true because $(Rx \supset Bx)$ is true in all cases:
    - $(Ra \supset Ba)=(0 \supset 0)=1$
    - $(Rb \supset Bb)=(1 \supset 1)=1$
  - Suppose ~Ra, ~Ba, Rb, ~Bb. $(x)(Rx \supset Bx)$ is false because $(Rx \supset Bx)$ is not true in all cases:
    - $(Ra \supset Ba)=(0 \supset 0)=1$
    - $(Rb \supset Bb)=(1 \supset 0)=0$

refutation of quantificational arguments: an example

1. ab(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7.* ∴(Ra⊃Ba)(from 1)
8.* ∴(Rb⊃Bb) (from 1)
9. ∴Ba (from 5 and 7)
10. ∴~Rb (from 6 and 8)

Refutation box:
Ra, Ba
~Rb, ~Bb

P1 = (x)(Rx⊃Bx) = 1 because
(Ra⊃Ba)=(1⊃1)=1
(Rb⊃Bb)=(0⊃0)=1

refutation of quantificational arguments: an example

1. ab(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7.* ∴(Ra⊃Ba)(from 1)
8.* ∴(Rb⊃Bb) (from 1)
9. ∴Ba (from 5 and 7)
10. ∴~Rb (from 6 and 8)

Refutation box:
Ra, Ba
~Rb, ~Bb

P1 = (x)(Rx⊃Bx) = 1 because
(Ra⊃Ba)=(1⊃1)=1
(Rb⊃Bb)=(0⊃0)=1

P2 = (∃x)Rx = 1 because
Ra

refutation of quantificational arguments: an example

1. ab(x)(Rx⊃Bx)
2.* (∃x)Rx
|∴(x)Bx
3.* asm: ~(x)Bx
4.* ∴(∃x)~Bx (from 3)
5. ∴Ra (from 2)
6. ∴~Bb (from 4)
7.* ∴(Ra⊃Ba)(from 1)
8.* ∴(Rb⊃Bb) (from 1)
9. ∴Ba (from 5 and 7)
10. ∴~Rb (from 6 and 8)

Refutation box:
Ra, Ba
~Rb, ~Bb

P1 = (x)(Rx⊃Bx) = 1 because
(Ra⊃Ba)=(1⊃1)=1
(Rb⊃Bb)=(0⊃0)=1

P2 = (∃x)Rx = 1 because
Ra

C= (x)Bx = 0 because
~Bb=1, implying that Bx is 0 when x=b

INVALID

exercise 8.3a: 1

1. $(\exists x)Fx$

$\therefore (x)Fx$

exercise 8.3a: 1

1. (∃x)Fx

|∴(x)Fx

2. asm: ~(x)Fx

exercise 8.3a: 1

1. (∃x)Fx

|∴(x)Fx

2.* asm: ~(x)Fx

3. ∴(∃x)~Fx (from 2)

exercise 8.3a: 1

1.* (∃x)Fx

|∴(x)Fx

2.* asm: ~(x)Fx

3. ∴(∃x)~Fx (from 2)

4. ∴Fa (from 1)

exercise 8.3a: 1

1.* (∃x)Fx
|∴(x)Fx
2.* asm: ~(x)Fx
3.* ∴(∃x)~Fx (from 2)
4. ∴Fa (from 1)
5. ∴~Fb (from 3)

exercise 8.3a: 1

1.* (∃x)Fx

|∴(x)Fx

2.* asm: ~(x)Fx

3.* ∴(∃x)~Fx (from 2)

4. ∴Fa (from 1)

5. ∴~Fb (from 3)

Refutation box:

Fa, ~Fb

exercise 8.3a: 1

1.* $(\exists x)Fx$

$|\therefore (x)Fx$

2.* asm: $\sim(x)Fx$

3.* $\therefore(\exists x)\sim Fx$ (from 2)

4. $\therefore Fa$ (from 1)

5. $\therefore \sim Fb$ (from 3)

Refutation box:

Fa, ~Fb

P = $(\exists x)Fx$ = 1 since

Fa

exercise 8.3a: 1

1.* (∃x)Fx
|∴(x)Fx
2.* asm: ~(x)Fx
3.* ∴(∃x)~Fx (from 2)
4. ∴Fa (from 1)
5. ∴~Fb (from 3)

Refutation box:
Fa, ~Fb

P = (∃x)Fx = 1 since
Fa

C = (x)Fx = 0 since
~Fb=1, implying that Fx = 0 when x=b

invalid