GET1028/GEX1014 Final Letter

21 APR 2023

Dear Students,

The Semester is almost over! Many congratulations, for almost completing the course! I am very proud of all of you and here is why. You are very very intelligent. You demonstrated patience in learning and proficiency in understanding the ideas of logic. These ideas, like your rules of inferences, are invisible and elusive—but you are able to see, grasp, and follow them. Sometimes, when contradictory ideas conflict, you are minded to hold them together and seek a resolution. This skill of yours to see what is abstract but powerful and fundamental to human thought impresses me thoroughly.

As students who will eventually graduate from this logic course, you may now accept that you have been given great gifts of logical reasoning and effective reasoning. Please do not squander them, but recognize how they may be used to make this unjust world a better place—planetary restoration, cultural redemption, resisting the hoarding of wealth and power, etc., you name and go for it. Bite like a Socratic gadfly against evil and argue your case for good with the logical clarity and sharpness you have demonstrated in my classes. Whatever you decide to do, I hope you will never stop being a good person. Pursue what's meaningful and be the best you can be!

Here is my summary of the course as promised in the tutorials. We began tutorials with the question "what is a good argument?". One proposal is that a good argument is valid; such an argument's conclusion cannot be false if its premises are true. But, we queried deeper by asking, what does "cannot" mean? Surely, it does not mean that the conclusion lacks a physical ability to be false. We settled on one way of understanding "cannot" in propositional logic—in terms of the possible truth value combinations of the argument's premises and conclusion.

We then learnt the truth conditions of the different logical operators and some ways of ascertaining validity, such as the truth-table and truth-assignment tests, and Gensler's indirect method of proof. To perform Gensler's indirect method of proof, we also learnt how to rely on rules of inferences (such as S- or I- rules) to create "mini" valid arguments to see whether it is possible for the premises to be true and conclusion to be false. We also learnt about making further assumptions because we cannot learn infinitely many S- or I- rules, so we need to settle with being content with a fixed number of helpful rules.

The idea of making further assumptions follows from Gensler's indirect method of proof. As we are interested in seeing whether the premise set can be true while the conclusion is false, we make further assumptions to consider possible ways some unbroken complex wffs can be true; so long as we find one way where it's possible for some unbroken complex wffs to be true together with the truth of the remaining premise set while the conclusion is false, we would have proven the argument to be invalid.

The study of quantificational logic is premised on the insufficiency of propositional logic in giving us the correct verdicts on validity in some cases. While quantificational logic has all the components of propositional logic, it also has additional components, such as quantifiers, variables and constants. It also has more rules of inferences, such as the drop existential and universal rules and reverse squiggle rules. With these comes the need to re-learn Gensler's indirect method proof. Much of the proof method remains the same. We need only be mindful of how to apply the new S- and I- rules here. This takes us to the end of the course.

For your final exams, here is my only advice. Look at the practice paper from Dr. Lee and categorize the questions. Ask yourselves what you need to know and understand to prepare for questions as such. Be thorough in your preparations. Do not leave everything to the last minute. Also, please know what are S- or I- ++ rules and how they differ from your S- and I- rules. I'm available for consultations so if you need help, feel free to reach out. As I have also promised some puzzles for fun (in case you are bored from exam revision), I leave them on page 3.

Once again, thank you for being wonderful and kind to me this semester. I would greatly appreciate it if you could submit a favorable feedback on my behalf as I intend to apply for a teaching award next semester. Thank you very much.

Best wishes,
YT

## Puzzles

*Note: these puzzles are NOT RELEVANT to your final exam. Ignore them if you wish. They are for interests' sake. If you have philosophy seniors, you may gently ask them to help you… :D*

Problem 1. Let σ1, σ2, ... , σn be sentence letters. Prove that the number of assignments of truth values, to these sentence letters, is 2n.

Hint: use mathematical induction.

Before presenting the next problem, here are three definitions. Let φ and ψ be sentences of P. Suppose that φ contains sentences letters σ1, σ2, ... , σn and ψ contains sentences letters ρ1, ρ2, ... , ρm, where of course, n and m are non-zero natural numbers. Say that φ and ψ are 'logically equivalent' just in case for every assignment of truth values to all of these sentence letters—that is, to all of σ1, σ2, ... , σn, ρ1, ρ2, ... , ρm—the truth value of φ under that assignment is the same as the truth value of ψ under that assignment. Say that φ and ψ are 'logically inequivalent' just in case φ and ψ are not logically equivalent. Finally, say that the 'relation of logical equivalence' is the relation which obtains between any propositional logic sentences φ and ψ just in case φ and ψ are logically equivalent.

Problem 2. Prove that the relation of logical equivalence satisfies the following three conditions.

1. For all sentences φ of P, φ is logically equivalent to φ.
2. For all sentences φ and ψ of P, if φ is logically equivalent to ψ then ψ is logically equivalent to φ.
3. For all sentences φ, ψ, and χ of P, if φ is logically equivalent to ψ and ψ is logically equivalent to χ then φ is logically equivalent to χ.

Quick aside: since the relation of logical equivalence satisfies these three conditions, it is called an 'equivalence relation'; these sorts of relations are studied throughout different areas of mathematics.

Hint: look up soundness/completeness theorems on youtube.