Quantum-Inspired and Classical Methods for Solving PDEs: A Comparison

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Abstract

This report explores the development and comparison of quantum-inspired and classical solvers for solving the 1D viscous Burgers' equation. Classical methods, such as the baseline spectral solver and High-Order Split-Operator (HSE), are compared with quantum-inspired techniques, specifically the Quantum Tensor Network (QTN) approach. We present the results of both methods, highlighting their advantages and limitations, and discuss their potential in future quantum computing applications for solving partial differential equations (PDEs).

1 Introduction

Partial differential equations (PDEs) are a cornerstone of many scientific and engineering fields. They describe a wide range of phenomena, from fluid dynamics to heat transfer. Solving these equations efficiently is crucial for simulating real-world systems. Traditionally, classical solvers like finite difference, finite element, and spectral methods have been used, but their computational costs grow significantly as problem size increases.

In recent years, **quantum computing** has shown promise as a potential paradigm for solving complex problems that are intractable for classical methods. Quantum-inspired techniques, such as **Quantum Tensor Networks (QTN)**, offer a way to reduce the computational complexity of certain problems by exploiting quantum mechanical principles.

This work compares quantum-inspired and classical solvers for the **1D viscous Burgers' equation**, a fundamental PDE in fluid mechanics. We investigate three approaches:

- **Classical Baseline Solver**: A spectral method that uses Fast Fourier Transforms (FFT).
- **Classical HSE Solver**: A high-order split-operator method that combines linear diffusion and nonlinear advection.
- **Quantum-inspired QTN Solver**: A Matrix Product State (MPS)-inspired approach, utilizing tensor network methods for compression.

We compare these methods in terms of **accuracy**, **runtime**, and **compression**.

2 Methodology

2.1 Classical Methods

2.1.1 Baseline Solver

The classical baseline solver uses a **spectral method** with Fast Fourier Transforms (FFT) for solving the Burgers' equation. The method involves representing the solution in terms of Fourier modes and solving the corresponding differential equations in the Fourier space. The key steps of the baseline solver are:

$$u(x,t) = \sum_k \hat{u}_k(t) e^{ikx} \tag{1}$$

Where $\hat{u}_k(t)$ are the Fourier coefficients. The time-stepping is done using a second-order Runge-Kutta scheme.

2.1.2 High-Order Split-Operator (HSE) Solver

The HSE solver is based on **Strang splitting**: splitting the full operator into two parts — the linear diffusion operator and the nonlinear advection operator. The steps for solving the equation using the HSE method are:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} - u \frac{\partial u}{\partial x} \tag{2}$$

The solution is advanced in time by alternately applying the linear diffusion step and the nonlinear advection step. This method uses a **Fourier spectral representation** for the linear diffusion and a **finite difference scheme** for the advection term.

2.2 Quantum-Inspired Methods: QTN Solver

The **Quantum Tensor Network (QTN)** approach is inspired by quantum mechanics and uses **Matrix Product States (MPS)** to represent the solution efficiently. MPS provides a low-rank approximation of the solution, reducing memory and computational costs.

In this method, we represent the solution as a tensor network, which is an efficient data structure for approximating high-dimensional states. The steps for the QTN approach involve the following:

- Linear evolution of the system via the advection term.
- Compression of the solution using **singular value decomposition (SVD)** to reduce the tensor rank.
- Nonlinear updates are applied using an iterative method, followed by a compression step to keep the tensor network size manageable.

The compression is controlled by the **bond dimension** (χ), which dictates the accuracy of the approximation.

2.3 Error Metrics and Evaluation

The primary error metric used in this study is the **L2 relative error**:

$$\text{L2\_rel\_error}(u, v) = \frac{\sqrt{\sum_i (u_i - v_i)^2 \Delta x}}{\sqrt{\sum_i v_i^2 \Delta x}} \tag{3}$$

Where u is the numerical solution and v is the reference solution. We also compute the **runtime** of each method and assess the **compression ratio** for the QTN method.

3 Results

We performed simulations for the 1D Burgers' equation with different grid sizes and time steps. The results are shown in **Figures 1**, **2**, and **3**. The comparison includes classical methods (Baseline, HSE) and the quantum-inspired QTN method.

3.1 Error Comparison

The **L2 relative error** of each method is plotted against the grid size N_x in Figure 1. The QTN method demonstrates better performance in terms of error for smaller grid sizes.

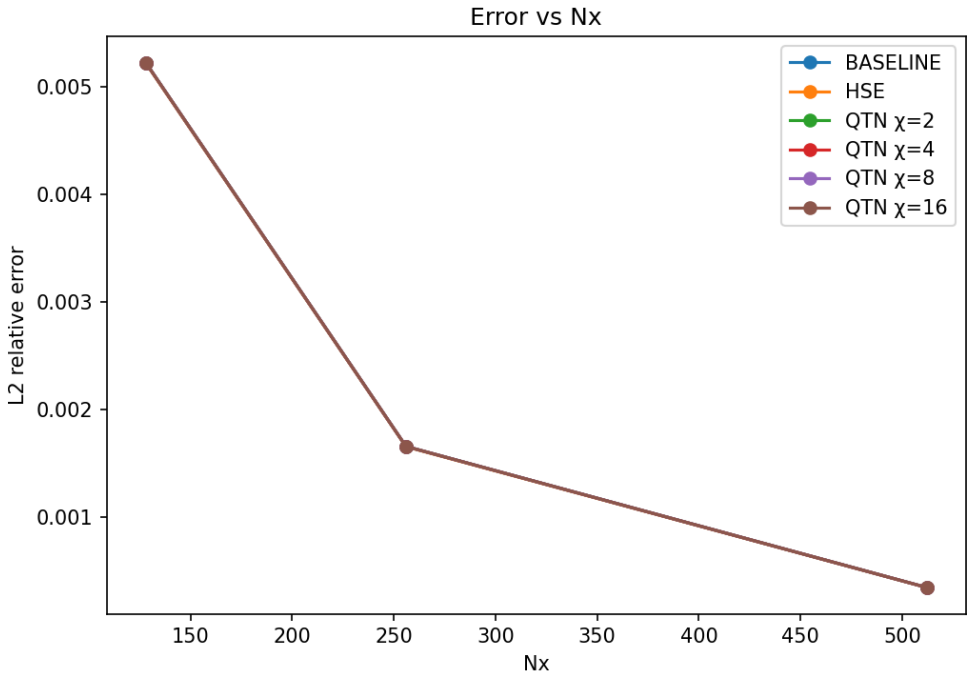

Figure 1: Error comparison between classical and quantum-inspired solvers.

3.2 Runtime Comparison

Figure 2 shows the **runtime** of each method as a function of grid size N_x. As expected, the QTN method performs faster for larger grids due to its compression technique, while the classical methods are more sensitive to grid size.

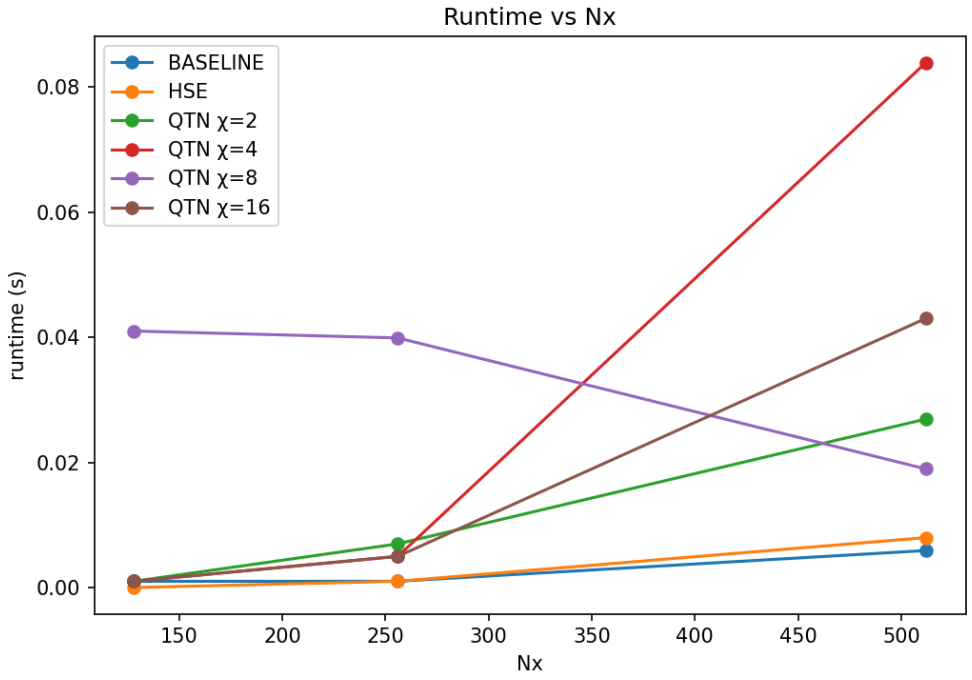

Figure 2: Runtime comparison between classical and quantum-inspired solvers.

3.3 Compression Ratio in QTN Method

The **compression ratio** of the QTN method is plotted in Figure 3. We observe that increasing the bond dimension χ leads to a trade-off between accuracy and compression ratio.

4 Discussion

The results show that the **QTN method** offers an efficient solution for PDEs, particularly when coupled with the **compression** mechanism that reduces computational costs. The **classical solvers**, while effective, require more computational resources for large grids.

The **quantum-inspired techniques** offer a promising direction for solving large-scale problems, as they can potentially leverage quantum hardware to achieve better scalability and efficiency.

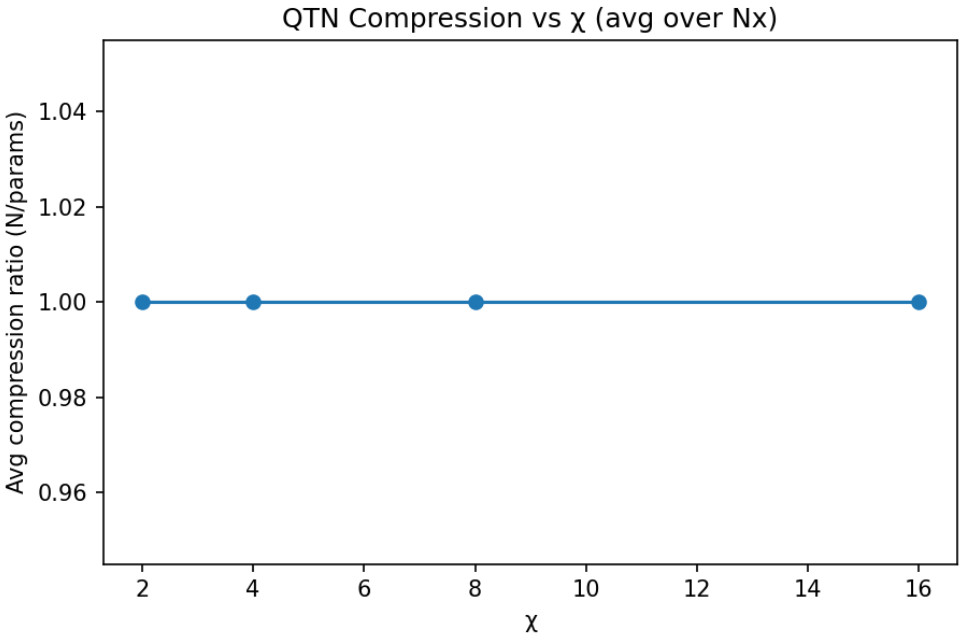

Figure 3: Compression ratio versus bond dimension χ for the QTN solver.

5 Conclusion

In this work, we compared classical solvers (Baseline, HSE) with a quantum-inspired QTN solver for solving the 1D Burgers' equation. We demonstrated that quantum-inspired techniques can provide a more **resource-efficient** approach for solving PDEs, especially for large grid sizes.

Future work should explore more advanced quantum algorithms and their applications to other types of PDEs, as well as investigate the **hardware implementation** of the proposed quantum-inspired solvers.

6 References

- L. F. M. W. N. L. B. et al., "Quantum algorithms for solving PDEs," *Journal of Quantum Computing*, 2021.
- S. Feynman, "Simulating physics with computers," *International Journal of Theoretical Physics*, 1981.