Homework 3

Statistical Inference, Spring 2021

1- Determine if the following statements are true or false. If false, explain your reasoning.

a. If a given value (for example, the null hypothesized value of a parameter) is within a 95% confidence interval, it will also be within a 99% confidence interval.

b. Decreasing the significance level (α) will increase the probability of making a Type 1 Error.

c. Suppose the null hypothesis is μ=5 and we fail to reject H_0. Under this scenario, the true population mean is 5.

d. If the alternative hypothesis is true, then the probability of making a Type 2 Error and the power add up to 1.

e. With large sample size, even small differences between the null value and the true value of the parameter, a difference often called the effect size, will be identified as statistically significant.

f. If the sample size is decreased when testing a hypothesis, the power would be expected to increase.

g. If the significance level of a test is decreased, the power would be expected to increase.

h. If a test is rejected at the significance level α, the probability that the null hypothesis is true equals α.

i. The probability that the null hypothesis is falsely rejected is equal to the power of the test.

j. A Type I error occurs when the test statistic falls in the rejection region of the test.

k. In testing a hypothesis, when the difference between the hypothesized mean and the actual mean (shift from μ0 to μa) increases, the power of the test will decrease.

l. In testing a hypothesis, when the standard deviation is decreased, the test's power will decrease.

2- Last month, a large supermarket chain received many consumer complaints about the quantity of chips in 16-ounce bags of a particular brand of potato chips. Suspecting that the complaints were merely the result of the potato chips settling to the bottom of the bags during shipping, but wanting to be able to assure its customers they were getting their money's worth, the chain decided to test the following hypotheses concerning the mean weight (in ounces) of a bag of potato chips in the next shipment of potato chips received from their largest supplier:

$H_0 : \mu = 16$
$H_A : \mu < 16$

If there is evidence that μ < 16, then the shipment would be refused and a complaint would register with the supplier.

a. What is a Type I error, in terms of the problem?

b. What is a Type II error, in terms of the problem?

c. Which type of error would the chain's customers view as more serious?

d. Which type of error would the chain's supplier view as more serious?

3- A manufacturer of batteries claims that the lifetime of their batteries is normally distributed. The manufacturer provides us with the standard deviation $\sigma = 10$ hours. For a random sample of 50 batteries, the sample average was found to be $x = 115$ hours.

a. Construct a 97% confidence interval for the population mean μ.

b. How will you interpret the confidence interval in (a) for a friend of yours who has not taken statistics yet?!

c. A 95% confidence interval for these batteries' average lifetime was found to be 112.23 to 117.77 hours. Determine if each of the following statements is true or false. If false, explain your reasoning.

I. We are 100% confident that batteries in this sample have, on average, 112.23 to 117.77 hours of lifetime.

II. 95% of the factory's batteries have 112.23 to 117.77 hours of lifetime.

III. 95% of the time, the factory's batteries' true average lifetime is between 112.23 to 117.77 hours.

IV. 95% of random samples of any batteries in the market will yield confidence intervals that contain the true average lifetime of this factory's batteries.

4- The Paralyzed Veterans of America is a philanthropic organization that relies on contributions. They send free mailing labels and greeting cards to potential donors on their list and ask for voluntary contribution. To test a new campaign, they recently sent letters to a random sample of 100,000 potential donors and received 4781 donations.

a. Give a 90% confidence interval for the true proportion of those from their entire mailing list who may donate.

b. A staff member thinks that the true rate is 5%. Given the confidence interval you found, do you find that percentage plausible?

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5- (R) The following are the systolic blood pressure for two groups of men and women randomly selected for a heart study:

Men	128.35	160.34	133.74	138.12	91.00	97.43	128.58	148.78	150.65	110.96
135.7	118.77	147.1	107.2	122.46	129.36	158.14	102.72	136.59	146.02	105.88
111.24	131.22	124.6	137.85	136.46	145.31	166.71	158.66	108.63	103.11	149.29

Women	116.62	137.15	106.07	172.58	151.33	98.73	136.11	149.9	140.8	98.58
158.4	97.97	117.99	126.53	128.67	126.57	124.3	120.39	150.08	143.05	130.18
108.04	136.39	124.94	136.86	143.03	128.58	142.51	151.68	120.94		

a. Construct a 95% confidence interval for the difference in mean systolic blood pressure between men and women. Interpret the result.

b. Is there any difference in mean systolic blood pressures between men and women? explain the conclusion.

6- A researcher designs a study to test the hypotheses $H_0: \mu = 28$ versus $H_A: \mu < 28$. A random sample of 50 measurements from the population of interest $\bar{y} = 25.9$ and $s = 5.6$.

a. Using $\alpha = 0.05$, what conclusions can you make about the hypotheses based on the sample information?

b. Calculate the probability of making a Type II error if the actual value of μ is at most 27.

c. Could you have possibly made a Type II error in your decision in part (a)? Explain your answer.

7- (R) Refer to Exercise 6. Sketch the power curve for rejecting $H_0: \mu = 28$ by determining PWR(μ_a) for the following values of μ: 22, 23, 24, 25, 26, and 27.

a. Interpret the power values displayed in your graph.

b. Suppose we keep $n = 50$ but change to $\alpha = 0.01$. Sketch on the same graph as your original power curve the new power curve for $n = 50$ and $\alpha = 0.01$. If α is reduced from 0.05 to 0.01, what would be the effect on the power curve?

c. Suppose we keep $\alpha = 0.05$ but change to $n = 20$. Sketch on the same graph as your original power curve the new power curve for $n = 20$ and $\alpha = 0.05$. If the sample size is reduced from 50 to 20, what would be the effect on the power curve?

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8- To evaluate the success of a 1-year experimental program designed to increase the mathematical achievement of underprivileged high school seniors, a random sample of participants in the program will be selected and their mathematics scores will be compared with the previous year's statewide average of 525 for underprivileged seniors. The researchers want to determine whether the experimental program has increased the mean achievement level over the previous year's statewide average.

   a. If $\alpha = 0.05$, what sample size is needed to have a probability of Type II error at most 0.025 if the actual mean is increased to 550? From previous results, $\sigma = 80$.

   b. Suppose a random sample of 100 students is selected $\bar{y} = 542$ and $s = 76$. Is there sufficient evidence to conclude that the mean mathematics achievement level has been increased? Explain.

9- In a random sample of 744 persons who skipped breakfast, 81 reported that they experienced midmorning fatigue, and in a random sample of 3055 persons who ate breakfast, 298 reported that they experienced midmorning fatigue. Use the 0.05 level of significance to test whether the prevalence of midmorning fatigue is significantly more among persons who skip breakfast than those who do not.