

Mathematical Analysis I

Make-Up Quiz on Chapter: *Numerical Sequences and Series*

Time allowed: 1 hour

December 31, 2025

Questions:

1. Suppose that the coefficients of the power series

$$\sum_{n=0}^{\infty} a_n z^n$$

are integers, infinitely many of which are nonzero. Prove that the radius of convergence is at most 1.

2. Consider the series

$$\sum_{n=1}^{\infty} (\sinh n) x^n.$$

Show that it converges absolutely for $|x| < e^{-1}$ and diverges for $|x| > e^{-1}$.