Formulas of Integration

Trigonometric Substitutions:

What you See	Use the Substitution	Derivative Substitution	Trig Identity	Result
$\sqrt{a^2 - x^2}$	$x = a \sin\theta$	$dx = a\cos\theta d\theta$	$\cos^2\theta = 1 - \sin^2\theta$	$a\cos\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan\theta$	$dx = a\sec^2\theta\, d\theta$	$1 + \tan^2\theta = \sec^2\theta$	$a\sec\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec\theta$	$dx = a\sec\theta\tan\theta d\theta$	$\tan^2\theta = \sec^2\theta - 1$	$a\tan\theta$

1)

$$\int 1dx = x + c$$

2)

$$\int adx = ax + c$$

3)

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

4)

$$\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

5)

$$\int \frac{1}{x}dx = \ln x + c$$

6)

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

7)

$$\int a^x\,dx = \frac{a^x}{\ln a} + c$$

8)

$$\int a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

9)

$$\int e^x\,dx = e^x + c$$

10)

$$\int e^{f(x)}dx = e^{f(x)} + c$$

11)

$$\int af(x)dx = a\int f(x)$$

12)

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

13)

$$\int f(x)\cdot g(x)dx = f(x)\left(\int g(x)dx\right) - \int\left[f'(x)\left(\int g(x)dx\right)\right]dx$$

14)

$$\int \ln x dx = x(\ln x - 1) + c$$

15)

$$\int \sin x dx = -\cos x + c$$

16)

$$\int \cos x dx = \sin x + c$$

17)

$$\int \tan x dx = \ln \sec x + c$$

or

$$-\ln \cos x + c$$

18)

$$\int \cot x dx = \ln \sin x + c$$

19)

$$\int \sec x dx = \ln(\sec x + \tan x) + c$$

or

$$\ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$$

20)

$$\int \csc x dx = \ln(\csc x - \cot x) + c$$

or

$$\ln \tan \frac{x}{2} + c$$

21)

$$\int \sec^2 x dx = \tan x + c$$

22)

$$\int \csc^2 x dx = -\cot x + c$$

23)

$$\int \sec x \tan x dx = \sec x + c$$

24)

$$\int \csc x \cot x dx = -\csc x + c$$

25)

$$\int \sinh x dx = \cosh x + c$$

26)

$$\int \cosh x dx = \sinh x + c$$

27)

$$\int \tanh x dx = \ln \cosh x + c$$

28)

$$\int \coth x dx = \ln \sinh x + c$$

29)

$$\int \sec \mathrm{h} x dx = \tan^{-1}(\sinh x) + c$$

30)

$$\int \csc \mathrm{h} x dx = -\coth^{-1}(\cosh x)$$

31)

$$\int \sec \mathrm{h}^2 x dx = \tanh x + c$$

32)

$$\int \operatorname{csch}^2 x dx = -\coth x + c$$

33)

$$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + c$$

34)

$$\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + c$$

35)

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c$$

or

$$\cos^{-1} \frac{x}{a} + c$$

36)

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1} \frac{x}{a} + c$$

or

$$\ln(x + \sqrt{x^2 - a^2}) + c$$

37)

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1} \frac{x}{a} + c$$

or

$$\ln(x + \sqrt{x^2 + a^2}) + c$$

38)

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \tanh^{-1} \frac{x}{a} + c$$

or

$$\frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)+c$$

39)

$$\int\frac{1}{x^2-a^2}dx=-\frac{1}{a}\coth^{-1}\frac{x}{a}+c$$

or

$$\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)+c$$

40)

$$\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

41)

$$\int\frac{1}{x\sqrt{a^2-x^2}}dx=-\frac{1}{a}\operatorname{sech}^{-1}\frac{x}{a}+c$$

or

$$-\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)+c$$

42)

$$\int\frac{1}{x\sqrt{x^2-a^2}}dx=\frac{1}{a}\sec^{-1}\frac{x}{a}+c$$

43)

$$\int\frac{1}{x\sqrt{x^2+a^2}}dx=-\frac{1}{a}\operatorname{csch}^{-1}\frac{x}{a}+c$$

or

$$-\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)+c$$

44)

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

45)

$$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}x\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a} + c$$

or

$$\frac{1}{2}x\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right) + c$$

46)

$$\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}x\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + c$$

or

$$\frac{1}{2}x\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right) + c$$

47)

$$\int e^{ax}\sin(bx+c)dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c) - b\cos(bx+c)\right]$$

48)

$$\int e^{ax}\cos(bx+c)dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos(bx+c) + b\sin(bx+c)\right]$$

49)

$$\int \sin mx\cos nx dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + c$$

50)

$$\int \sin mx\sin nx dx = -\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + c$$

51)

$$\int \cos mx \cos nx dx = \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + c$$

52)

$$\int \sin^{-1} x dx = x\sin^{-1} x + \sqrt{1-x^2} + c$$

53)

$$\int \cos^{-1} x dx = x\cos^{-1} x - \sqrt{1-x^2} + c$$

54)

$$\int \tan^{-1} x dx = x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + c$$

55)

$$\int \cot^{-1} x dx = x\cot^{-1} x + \frac{1}{2}\ln(1+x^2) + c$$

56)

$$\int \sec^{-1} x dx = x\sec^{-1} x - \ln\left(x + \sqrt{x^2-1}\right) + c$$

57)

$$\int \csc^{-1} x dx = x\csc^{-1} x + \ln\left(x + \sqrt{x^2-1}\right) + c$$

58)

$$\int \frac{1}{a + b\sin x} dx = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1}\left(\frac{a\tan^{-1}\frac{x}{2} + b}{\sqrt{a^2-b^2}}\right) + c$$

if $a^2 > b^2$

59)

$$\int \frac{1}{a+b\sin x}dx = \frac{1}{\sqrt{a^2-b^2}}\ln\left(\frac{a\tan\frac{x}{a}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{a}+b+\sqrt{b^2-a^2}}\right)+c$$

if $a^2 < b^2$

60)

$$\int \frac{1}{a+b\cos x}dx = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)+c$$

if $a^2 > b^2$

61)

$$\int \frac{1}{a+b\cos x}dx = \frac{1}{\sqrt{a^2-b^2}}\ln\left(\frac{\sqrt{b+a}+\tan\frac{x}{2}\sqrt{b-a}}{\sqrt{b+a}-\tan\frac{x}{2}\sqrt{b-a}}\right)+c$$

if $a^2 < b^2$

62)

$$\int \frac{1}{a+b\sinh x}dx = \frac{1}{\sqrt{a^2+b^2}}\ln\left(\frac{\sqrt{a^2+b^2}+a\tanh\frac{x}{2}-b}{\sqrt{a^2+b^2}-a\tanh\frac{x}{2}+b}\right)+c$$

63)

$$\int \frac{1}{a+b\cosh x}dx = \frac{\sqrt{a+b}+\sqrt{a-b}\tanh\frac{x}{2}}{\sqrt{a+b}-\sqrt{a-b}\tanh\frac{x}{2}}+c$$

if $a > b$

64)

$$\int \frac{1}{a+b\cosh x}dx = \frac{2}{\sqrt{b^2-a^2}}\tan^{-1}\sqrt{\frac{b-a}{b+a}}\tanh^{-1}\frac{x}{2}+c$$

if $a < b$