

PPL - Assignment 4

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Part 1: Theoretical questions:

Q1: Perform typing inference for the expression:

`((lambda (x1 y1) (if (> x1 y1) #t #f)) 8 3)`

Stage I: Rename bound variables:

`((lambda (x y) (if (> x y) #t #f)) 8 3)`

Stage II: Assign type variables for every sub expression:

| Expression | Variable |
|--|-------------------------|
| <code>((lambda (x y) (if (> x y) #t #f)) 8 3)</code> | T₀ |
| <code>(lambda (x y) (if (> x y) #t #f))</code> | T₁ |
| <code>(if (> x y) #t #f)</code> | T₂ |
| <code>(> x y)</code> | T₃ |
| <code>></code> | T_{>} |
| <code>x</code> | T_x |
| <code>y</code> | T_y |
| <code>#t</code> | T_{#t} |
| <code>#f</code> | T_{#f} |
| <code>8</code> | T_{num8} |
| <code>3</code> | T_{num3} |

Stage III: Construct type equations. The equations for the sub-expressions are:

| Expression | Equation |
|---|--|
| <code>((lambda (x y) (if (> x y) #t #f)) 8 3)</code> | $T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$ |
| <code>(lambda (x y) (if (> x y) #t #f))</code> | $T_1 = [T_x * T_y \rightarrow T_2]$ |
| <code>(if (> x y) #t #f)</code> | $T_2 = T_{\#t} \text{ and } T_{\#t} = T_{\#f}$ |
| <code>(> x y)</code> | $T_{>} = [T_x * T_y \rightarrow T_3]$ |

The equations for the primitives are:

| Expression | Equation |
|-----------------|--|
| $>$ | $T_{>} = [number * number \rightarrow number]$ |
| $\#t$ | $T_{\#t} = boolean$ |
| $\#f$ | $T_{\#f} = boolean$ |
| 8 | $T_{num8} = number$ |
| $\underline{3}$ | $T_{num3} = number$ |

Stage IV: Solve the equations.

Step 1:

| Equation | Substitution |
|---|---|
| $T1 = [T_{num8} * T_{num3} \rightarrow T0]$ | $T1 = [T_{num8} * T_{num3} \rightarrow T0]$ |
| $T1 = [T_x * T_y \rightarrow T2]$ | |
| $T2 = T_{\#t}$ | |
| $T_{test} = boolean$ | |
| $T_{\#t} = T_{\#f}$ | |
| $T_{>} = [T_x * T_y \rightarrow T_{test}]$ | |
| $T_{>} = [number * number \rightarrow boolean]$ | |
| $T_{\#t} = boolean$ | |
| $T_{\#f} = boolean$ | |
| $T_{num8} = number$ | |
| $T_{num3} = number$ | |

Step 2:

| Equation | Substitution |
|--|---|
| $T_1 = [T_x * T_y \rightarrow T_{if}]$ | $T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$ |
| $T_{if} = T_{\#t}$ | |
| $T_{test} = \text{boolean}$ | |
| $T_{\#t} = T_{\#f}$ | |
| $T_{>} = [T_x * T_y \rightarrow T_{test}]$ | |
| $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ | |
| $T_{\#t} = \text{boolean}$ | |
| $T_{\#f} = \text{boolean}$ | |
| $T_{num8} = \text{number}$ | |
| $T_{num3} = \text{number}$ | |

Step 3:

| Equation | Substitution |
|--|---|
| $T_{if} = T_{\#t}$ | $T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$ |
| $T_{test} = \text{boolean}$ | |
| $T_{\#t} = T_{\#f}$ | |
| $T_{>} = [T_x * T_y \rightarrow T_{test}]$ | |
| $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ | |
| $T_{\#t} = \text{boolean}$ | |
| $T_{\#f} = \text{boolean}$ | |
| $T_{num8} = \text{number}$ | |
| $T_{num3} = \text{number}$ | |
| $T_{x1} = T_{num8}$ | |
| $T_{y1} = T_{num3}$ | |
| $T_{if} = T_0$ | |

Step 4:

| Equation | Substitution |
|--|--|
| $T_{\#t} = T_{\#f}$ | $T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$ $T_{if} = T_{\#t}$ $T_{test} = \text{boolean}$ |
| $T_{>} = [T_{x1} * T_{y1} \rightarrow T_{test}]$ | |
| $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ | |
| $T_{\#t} = \text{boolean}$ | |
| $T_{\#f} = \text{boolean}$ | |
| $T_{num8} = \text{number}$ | |
| $T_{num3} = \text{number}$ | |
| $T_x = T_{num8}$ | |
| $T_y = T_{num3}$ | |
| $T_{if} = T_0$ | |

Step 5 :

| Equation | Substitution |
|--|---|
| $T_{>} = [T_{x1} * T_{y1} \rightarrow \text{boolean}]$ | $T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$ $T_{if} = T_{\#f}$ $T_{test} = \text{boolean}$ $T_{\#t} = T_{\#f}$ |
| $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ | |
| $T_{\#t} = \text{boolean}$ | |
| $T_{\#f} = \text{boolean}$ | |
| $T_{num8} = \text{number}$ | |
| $T_{num3} = \text{number}$ | |
| $T_x = T_{num8}$ | |
| $T_y = T_{num3}$ | |
| $T_{if} = T_0$ | |

Step 6 :

| Equation | Substitution |
|--|---|
| $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ | $T_1 = [T_{\text{num8}} * T_{\text{num3}} \rightarrow T_0]$ $T_{\text{if}} = T_{\#f}$ $T_{\text{test}} = \text{boolean}$ $T_{\#t} = T_{\#f}$ $T_{>} = [T_{x1} * T_{y1} \rightarrow \text{boolean}]$ |
| $T_{\#t} = \text{boolean}$ | |
| $T_{\#f} = \text{boolean}$ | |
| $T_{\text{num8}} = \text{number}$ | |
| $T_{\text{num3}} = \text{number}$ | |
| $T_x = T_{\text{num8}}$ | |
| $T_y = T_{\text{num3}}$ | |
| $T_{\text{if}} = T_0$ | |

Step 7 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_{\#t} = \text{boolean}$ | $T_1 = [T_{\text{num8}} * T_{\text{num3}} \rightarrow T_0]$ $T_{\text{if}} = T_{\#f}$ $T_{\text{test}} = \text{boolean}$ $T_{\#t} = T_{\#f}$ $T_{>} = [T_x * T_y \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ |
| $T_{\#f} = \text{boolean}$ | |
| $T_{\text{num8}} = \text{number}$ | |
| $T_{\text{num3}} = \text{number}$ | |
| $T_x = T_{\text{num8}}$ | |
| $T_y = T_{\text{num3}}$ | |
| $T_{\text{if}} = T_0$ | |
| $T_x = \text{number}$ | |
| $T_y = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |

Step 8 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_{\#f} = \text{boolean}$ | $T_1 = [T_{\text{num8}} * T_{\text{num3}} \rightarrow T_0]$ $T_{\text{if}} = T_{\#f}$ $T_{\text{test}} = \text{boolean}$ $T_{\#t} = T_{\#f}$ $T_{>} = [T_x * T_y \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ |
| $T_{\text{num8}} = \text{number}$ | |
| $T_{\text{num3}} = \text{number}$ | |
| $T_x = T_{\text{num8}}$ | |
| $T_y = T_{\text{num3}}$ | |
| $T_{\text{if}} = T_0$ | |
| $T_x = \text{number}$ | |
| $T_y = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |

Step 9 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_{\text{num8}} = \text{number}$ | $T_1 = [T_{\text{num8}} * T_{\text{num3}} \rightarrow T_0]$ $T_{\text{if}} = \text{boolean}$ $T_{\text{test}} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [T_x * T_y \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ $T_{\#f} = \text{boolean}$ |
| $T_{\text{num3}} = \text{number}$ | |
| $T_x = T_{\text{num8}}$ | |
| $T_y = T_{\text{num3}}$ | |
| $T_{\text{if}} = T_0$ | |
| $T_x = \text{number}$ | |
| $T_y = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |

Step 10 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_x = T_{\text{num8}}$ | $T_1 = [\text{number} * \text{number} \rightarrow T_0]$ $T_{\text{if}} = \text{boolean}$ $T_{\text{test}} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [T_{x1} * T_{y1} \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ $T_{\#f} = \text{boolean}$ $T_{\text{num8}} = \text{number}$ $T_{\text{num3}} = \text{number}$ |
| $T_y = T_{\text{num3}}$ | |
| $T_{\text{if}} = T_0$ | |
| $T_x = \text{number}$ | |
| $T_y = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |

Step 11 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_x = \text{number}$ | $T_1 = [\text{number} * \text{number} \rightarrow T_0]$ $T_{if} = \text{boolean}$ $T_{test} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [T_{x1} * T_{y1} \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ $T_{\#f} = \text{boolean}$ $T_{num8} = \text{number}$ $T_{num3} = \text{number}$ |
| $T_y = T_{num3}$ | |
| $T_{if} = T_0$ | |
| $T_x = \text{number}$ | |
| $T_y = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |

Step 12 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_y = T_{num3}$ | $T_1 = [\text{number} * \text{number} \rightarrow T_0]$ $T_{if} = \text{boolean}$ $T_{test} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [\text{number} * T_{y1} \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ $T_{\#f} = \text{boolean}$ $T_{num8} = \text{number}$ $T_{num3} = \text{number}$ $T_x = \text{number}$ |
| $T_{if} = T_0$ | |
| $T_x = \text{number}$ | |
| $T_y = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |

Step 13 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_{y1} = \text{number}$ | $T_1 = [\text{number} * \text{number} \rightarrow T_0]$ $T_{if} = \text{boolean}$ $T_{test} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [\text{number} * T_{y1} \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ $T_{\#f} = \text{boolean}$ $T_{num8} = \text{number}$ $T_{num3} = \text{number}$ $T_x = \text{number}$ |
| $T_{if} = T_0$ | |
| $T_{x1} = \text{number}$ | |
| $T_{y1} = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |

Step 14 :

| Equation | Substitution |
|-----------------------------------|---|
| $T_{if} = T_0$ | $T_1 = [\text{number} * \text{number} \rightarrow T_0]$ $T_{if} = \text{boolean}$ $T_{test} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ $T_{\#f} = \text{boolean}$ $T_{num8} = \text{number}$ $T_{num3} = \text{number}$ $T_x = \text{number}$ $T_y = \text{number}$ |
| $T_{x1} = \text{number}$ | |
| $T_{y1} = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |
| | |

Step 15 :

| Equation | Substitution |
|-----------------------------------|--|
| $T_{x1} = \text{number}$ | $T_1 = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{if} = \text{boolean}$ $T_{test} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#t} = \text{boolean}$ $T_{\#f} = \text{boolean}$ $T_{num8} = \text{number}$ $T_{num3} = \text{number}$ $T_{x1} = \text{number}$ $T_{y1} = \text{number}$ $T_0 = \text{boolean}$ |
| $T_{y1} = \text{number}$ | |
| $\text{boolean} = \text{boolean}$ | |
| | |

😊😊😊 Step 16 😊😊😊:

| Equation | Substitution |
|----------|--|
| | $T_1 = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{if} = \text{boolean}$ $T_{test} = \text{boolean}$ $T_{\#t} = \text{boolean}$ $T_{>} = [\text{number} * \text{number} \rightarrow \text{boolean}]$ $T_{\#f} = \text{boolean}$ $T_{num8} = \text{number}$ $T_{num3} = \text{number}$ $T_x = \text{number}$ $T_y = \text{number}$ |
| | |
| | |
| | |
| | |

| | |
|--|------------------------|
| | $T_0 = \text{boolean}$ |
|--|------------------------|

Q2: Are these typing statements true? Explain.

a. $\{f:[T1 \rightarrow T2], x: T1\} \vdash (f\ x): T2$

true :

given $f:[T1 \rightarrow T2]$, we can see that $T_f = [T1 \rightarrow T2]$, because the type of a function is it's return type , and so it is $T2$

b. $\{f:[T1 \rightarrow T2], g: [T2 \rightarrow T3]\}, x: T2 \vdash (f\ g\ x): T3$

false :

the types are incompatible, $f:[T1 \rightarrow T2]$, f gets 1 parameter of type $T1$, and we call f on 2 arguments 😞

c. $\{f:[T2 \rightarrow T1], g: [T1 \rightarrow T2], x: T1\} \vdash (f\ (g\ x)): T1$

true.

$g: [T1 \rightarrow T2]$ and $x : T1$ makes g return a $T2$ without error of the type , then f gets a valid argument from $g\ (T2)$ and returns $T1$,which is the return type.

d. $\{f:[T2 \rightarrow \text{Number}], x: \text{Number}\} \vdash (f\ x\ x): \text{Number}$

false

f gets 1 parameter of type $T2$, but it was called with 2 parameters .

Q3: What is the type of the following primitive operators:

a. cons :

$[x: T1, y: T2] \rightarrow \text{Pair}(T1, T2)$

Cons receives two variables , x of type $T1$, y of type $T2$,as parameters and returns a pair which has the values inside it (x as first one , and y as the second)

b. car

$[Pair (T1, T2)] \rightarrow T1$

Car receives a tuple as parameter , and returns the first variable of the parameter in tuple , which type is T1

c. cdr

$[Pair (T1, T2)] \rightarrow T2$

Cdr receives a tuple as parameter , and returns the second variable of the parameter in tuple , which type is T2

Q4: Write the type of the following function: (Define f (lambda (x) (values x x x)))

$[T1 \rightarrow (T1 * T1 * T1)]$

Q5. Write the MGU of the following expressions, or state that there is no such MGU.

a. T1 , T2 =>

MGU is $\{T1 = T2\}$ – since this unifier does a substitution only for the critical types

b. Number , Number =>

MGU is $\{ \}$ (empty unifier) - because numbers stays numbers 😊

c. $[T1 * [T1 \rightarrow T2] \rightarrow Number]$, $[[T3 \rightarrow Number] * [T4 \rightarrow Number] \rightarrow N]$

MGU is $\{T1 = [T3 \rightarrow Number], T4 = [T3 \rightarrow Number], T2 = Number\}$

d. $[T1 \rightarrow T1]$, $[T1 \rightarrow [Number \rightarrow Number]]$

MGU is $\{T1 = [Number \rightarrow Number]\}$

PART 2 .3:

```
(define f: [number -> Tuple (number , number)]  
  (lambda (x: number): (number , number) (  
    values x (+ x 1))))
```

```
(define g: [T1 -> Tuple (string , T1)]  
  (lambda (x: T1): (string , T1)  
    (values "x" x)))
```

PART 3 :

```
28 }  
29  
30  
31  
32 export function* braid(generator1: () => Generator, generator2: () => Generator) :Generator{  
33   const g1 = generator1();  
34   const g2 = generator2();  
35  
36   let gens = [g1, g2];  
37  
38   let nxt1 = g1.next();  
39   let nxt2 = g2.next();  
40  
41   while(!nxt1.done || !nxt2.done){  
42     if(!nxt1.done){  
43       yield nxt1.value;  
44     }  
45     if(!nxt2.done){  
46       yield nxt2.value;  
47     }  
48  
49     nxt1 = g1.next();  
50     nxt2 = g2.next();  
51  
52   }  
53 }
```

```
64
65 function* biased(g1: () => Generator, g2: () => Generator): Generator {
66     let gen1 = g1();
67     let gen2 = g2();
68     let nxt1=gen1.next().value;
69     let nxt2=gen2.next().value;
70
71     while (nxt1!=undefined || nxt2!=undefined)
72     {
73         if (nxt1!=undefined)
74         {
75             yield nxt1
76             nxt1=gen1.next().value
77         }
78         if (nxt1!=undefined)
79         {
80             yield nxt1
81             nxt1=gen1.next().value
82         }
83         if (nxt2!=undefined)
84         {
85             yield nxt2
86             nxt2=gen2.next().value
87         }
88     }
89 }
90
91
```

PART 4:

```
//Question 1
export function f(x: number): Promise<number> {
    return new Promise<number>((resolve : any, reject: any) => {
        if (x === 0) {
            reject("ERROR in function f : division by 0 !");
            return;
        }
        resolve(1 / x);
    });
}

export function g(x: number): Promise<number> {
    return new Promise<number>((resolve, reject) => {
        resolve(x * x);
        return;
    });
}

export function h(x: number): Promise<number> {
    return new Promise<number>((resolve, reject) => {
        g(x)
            .then((res) => f(res) )
            .then((res) => resolve(res) )
            .catch((error) => reject(error) );
    });
}
```

Code also in the next page , and Part 4 .b :

```

41
42 //Question2
43 export const slower = <T>(proms :[Promise<any>, Promise<any>]):Promise <any> => {
44     return new Promise(resolve => {
45         const wrap_promise = (index: number) => (value: T) => {
46             if(race.add(index).size === 2){
47                 resolve(`${index}, ${value}`)
48             }
49         }
50         const race=new Set
51         proms.map((prom,index)=>{
52             prom.then(wrap_promise(index))
53         })
54     })
55 }
56

```

Part 4: What are the benefits of the promise interface compared to the callback interface?

- Functions in promise are very close to the synchronous interface (except that it returns result of Promise<T>), while Callbacks are more complicated .
- Error handling is much easier in the Promise interface , we can use catch and reject , while in the CallBack , handling error require more work (for each error).
- We can use the .then() calls in sequential order , which help us build compound functions , and it assure that we have a synchronization .