Drill Plan Optimizer

Problem Statement, Mathematical Model,
and MILP Formulation

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1 Problem Statement

The objective is to design a two-dimensional drill plan inside a given bench boundary polygon. A *drill hole* is a point in the plane at which a blast hole will be detonated. Detonation creates a spatially distributed *fragmentation field* whose intensity at any location depends on the distance to, and the geometry of, every hole.

1.1 Inputs

The problem is fully described by a JSON file containing:

`boundary` A closed simple polygon given as an ordered list of N vertices $\{(x_1, y_1), \ldots, (x_N, y_N)\}$ in the plane (metres). The last vertex is implicitly connected back to the first.

`bootlegs` A list of M points $\mathbf{b}_j = (b_j^x, b_j^y)$, $j = 1, \ldots, M$, that represent pre-existing geological failures (*bootlegs*). Each bootleg defines a circular *exclusion zone* of radius $r = 0.75\,\text{m}$ centred at $\mathbf{b}_j$.

1.2 Outputs

A set of drill hole positions $\mathcal{H} = \{(x_1, y_1), \ldots, (x_K, y_K)\}$ that maximises rock fragmentation quality subject to the geometric and physical constraints described below.

2 Physical Parameters and Constants

All constants are fixed by the problem specification and are summarised in Table 1.

Table 1: Fixed model parameters.

Symbol	Value	Interpretation
S	5.0 m	Hole spacing (along a row)
B	2.5 m	Burden (row-to-row distance)
A	0.75	Peak amplitude of the Gaussian damage function
C	0.55	Threshold decay parameter
T	1.0	Fragmentation threshold
T_{over}	1.8	Over-fragmentation threshold
Δ	0.1 m	Evaluation grid resolution
λ	0.5	Penalty weight for over-fragmented area
r	0.75 m	Bootleg exclusion radius

3 Fragmentation Model

3.1 Gaussian Spread Parameters

Each hole produces an anisotropic Gaussian damage pattern. The standard deviations in the x (spacing) and y (burden) directions are derived from the half-power radius of the Gaussian at amplitude C:

$$\sigma_{x_1} = \frac{S}{\sqrt{8 \ln\left(\frac{2A}{C}\right)}} \tag{1}$$

$$\sigma_{y_1} = \frac{B}{\sqrt{6 \ln\left(\frac{2A}{C}\right)}} \tag{2}$$

$$\sigma_{x_2} = \frac{S}{\sqrt{32 \ln\left(\frac{2A}{C}\right)}} \tag{3}$$

$$\sigma_{y_2} = \frac{B}{\sqrt{2 \ln\left(\frac{2A}{C}\right)}} \tag{4}$$

The two components model near-field (σ_1, narrow) and far-field (σ_2, wide) damage zones. Component 2 has the same peak amplitude but a wider x-spread and narrower y-spread than component 1, capturing the anisotropic propagation of shock waves along the free face.

Substituting the numerical values ($A = 0.75$, $C = 0.55$, $S = 5.0\,\mathrm{m}$, $B = 2.5\,\mathrm{m}$):

$$\ln\left(\tfrac{2A}{C}\right) = \ln\left(\tfrac{1.5}{0.55}\right) \approx 1.0033$$

$$\sigma_{x_1} \approx \frac{5.0}{\sqrt{8 \times 1.0033}} \approx 1.764\,\mathrm{m} \tag{5}$$

$$\sigma_{y_1} \approx \frac{2.5}{\sqrt{6 \times 1.0033}} \approx 1.020\,\mathrm{m} \tag{6}$$

$$\sigma_{x_2} \approx \frac{5.0}{\sqrt{32 \times 1.0033}} \approx 0.882\,\mathrm{m} \tag{7}$$

$$\sigma_{y_2} \approx \frac{2.5}{\sqrt{2 \times 1.0033}} \approx 1.766\,\mathrm{m} \tag{8}$$

3.2 Per-Hole Contribution Field

For hole i located at $\mathbf{h}_i = (x_i, y_i)$, the damage contribution at an arbitrary point (x, y) is:

$$f^{(i)}(x, y) = I_1^{(i)}(x, y) + I_2^{(i)}(x, y) \tag{9}$$

where

$$I_1^{(i)}(x, y) = A \exp\left[-\frac{(x - x_i)^2}{2\sigma_{x_1}^2} - \frac{(y - y_i)^2}{2\sigma_{y_1}^2}\right] \tag{10}$$

$$I_2^{(i)}(x, y) = A \exp\left[-\frac{(x - x_i)^2}{2\sigma_{x_2}^2} - \frac{(y - y_i)^2}{2\sigma_{y_2}^2}\right] \tag{11}$$

3.3 Total Fragmentation Field

Given a set $\mathcal{H}$ of K holes, the total fragmentation field is the superposition of all individual contributions:

$$F(x,y) = \sum_{i=1}^{K} f^{(i)}(x,y) \qquad \text{for } (x,y) \in \Omega, \tag{12}$$

where Ω denotes the interior of the boundary polygon. The field is defined to be zero outside Ω.

3.4 Area Metrics

The evaluation grid $\mathcal{G}$ is a uniform 2D grid over the bounding box of Ω with spacing $\Delta = 0.1\,\text{m}$; only cells whose centres lie strictly inside Ω are included. Each cell represents an area Δ^2.

Definition 3.1 (Fragmented Area)**.**

$$A_{\text{frag}} = \#\big\{(x,y) \in \mathcal{G} : F(x,y) > T\big\} \cdot \Delta^2 \tag{13}$$

Definition 3.2 (Over-Fragmented Area)**.**

$$A_{\text{over}} = \#\big\{(x,y) \in \mathcal{G} : F(x,y) \geq T_{\text{over}}\big\} \cdot \Delta^2 \tag{14}$$

4 Constraints

Each drill hole $\mathbf{h}_i = (x_i, y_i)$ must satisfy the following constraints.

4.1 Boundary Constraint

Every hole must lie strictly inside the boundary polygon $\partial\Omega$:

$$(x_i, y_i) \in \text{interior}(\Omega) \qquad \forall\, i = 1, \ldots, K. \tag{15}$$

Membership is tested by the *ray-casting algorithm*: a horizontal ray from (x_i, y_i) is cast and the parity of polygon edge crossings is counted.

4.2 Bootleg Exclusion Constraint

Each hole must be at least $r = 0.75\,\text{m}$ away from every bootleg:

$$\|\mathbf{h}_i - \mathbf{b}_j\|_2 \geq r \qquad \forall\, i = 1, \ldots, K, \quad \forall\, j = 1, \ldots, M. \tag{16}$$

4.3 Fixed Hole Parameters

The spacing S and burden B are fixed design parameters, not decision variables. They determine the geometry of the candidate grid (Section 6).

5 Objective Function

The goal is to *maximise* rock fragmentation while penalising over-fragmentation:

$$\boxed{\underset{\mathcal{H}}{\text{maximize}} \quad Z(\mathcal{H}) = A_{\text{frag}} - \lambda\, A_{\text{over}}} \tag{17}$$

with $\lambda = 0.5$. Substituting (13) and (14):

$$Z(\mathcal{H}) = \Delta^2 \Big[\#\{g \in \mathcal{G} : F_g > T\} - 0.5 \cdot \#\{g \in \mathcal{G} : F_g \geq T_{\text{over}}\} \Big], \tag{18}$$

where F_g denotes the value of F at grid cell g.

Interpretation. Adding a hole always increases A_{frag} near that hole, but it also risks raising F above T_{over} in the densely covered central region. The penalty $\lambda = 0.5$ discourages over-packing: each unit of extra over-fragmented area costs half the reward gained from fragmenting a fresh unit of rock.

6 Candidate Hole Generation

Rather than optimising over all of $\mathbb{R}^2$, hole positions are drawn from a finite *candidate set* $\mathcal{C}$ generated by a staggered (hexagonal-like) rectangular grid:

1. Start at $y = y_{\min} + B/2$.
2. For each row $r = 0, 1, 2, \ldots$:
   - Set $x_0 = x_{\min} + S/2 + (r \bmod 2) \cdot S/2$.
   - Place candidates at $x_0,\ x_0 + S,\ x_0 + 2S,\ \ldots$
   - Increment $y \leftarrow y + B$.
3. Discard every candidate that fails constraint (15) or (16).

The row offset of $S/2$ for odd-numbered rows produces an alternating pattern that achieves more uniform spatial coverage than a regular grid, matching standard blasting practice.

7 MILP Formulation

Let $\mathcal{C} = \{c_1, \ldots, c_K\}$ be the candidate set and let $\mathcal{G} = \{g_1, \ldots, g_G\}$ be the optimisation grid cells (resolution $\Delta_{\text{opt}} = 0.5\,\text{m}$ by default; the fine grid $\Delta = 0.1\,\text{m}$ is used only for final metric reporting).

7.1 Decision Variables

$$\begin{aligned} x_k &\in \{0,1\}, \quad k = 1, \ldots, K && \text{(hole selection)} \\ z_g &\in \{0,1\}, \quad g = 1, \ldots, G && \text{(fragmentation indicator)} \\ v_g &\in \{0,1\}, \quad g = 1, \ldots, G && \text{(over-fragmentation indicator)} \end{aligned}$$

$x_k = 1$ means candidate k is selected as a drill hole; $z_g = 1$ means grid cell g is fragmented ($F_g > T$); $v_g = 1$ means cell g is over-fragmented ($F_g \geq T_{\text{over}}$).

7.2 Pre-Computed Coefficients

Since the fragmentation field is linear in the hole selection variables, the contribution of candidate k to cell g is computed once before solving:

$$f_{g,k} = I_1(g,k) + I_2(g,k) \qquad g = 1,\ldots,G, \quad k = 1,\ldots,K, \tag{19}$$

where I_1, I_2 are evaluated at grid cell centre g with hole at candidate k. The total field at cell g is then:

$$F_g = \sum_{k=1}^{K} f_{g,k}\, x_k. \tag{20}$$

The maximum possible field value at cell g (tight Big-M constant) is:

$$M_g = \sum_{k=1}^{K} f_{g,k}. \tag{21}$$

7.3 Objective

$$\underset{x,z,v}{\text{maximize}} \quad \sum_{g=1}^{G} z_g \;-\; 0.5 \sum_{g=1}^{G} v_g \tag{22}$$

(The constant factor Δ_{opt}^2 cancels from the maximisation and is applied to recover the physical areas $A_{\text{frag}}, A_{\text{over}}$ after solving.)

7.4 Big-M Linearisation Constraints

Fragmentation indicator (z_g). We require $z_g = 1$ if and only if $F_g > T$.

Upper-bound direction (prevent $z_g = 1$ when $F_g \le T$):

$$F_g \;\le\; T + M_g\, z_g \qquad g = 1,\ldots,G \tag{23}$$

If $z_g = 0$ this forces $F_g \le T$; if $z_g = 1$ the constraint is trivially satisfied for any $F_g \le M_g$.

Lower-bound direction (force $z_g = 1$ when $F_g > T$):

$$F_g \;-\; M_g\, z_g \;\ge\; T + \varepsilon - M_g \qquad g = 1,\ldots,G \tag{24}$$

If $z_g = 0$ this becomes $F_g \ge T + \varepsilon - M_g$, which is non-binding when $M_g \ge T + \varepsilon$ (always true for active cells); if $z_g = 1$ this becomes $F_g \ge T + \varepsilon$, enforcing the strict inequality.

Over-fragmentation indicator (v_g). We require $v_g = 1$ if and only if $F_g \ge T_{\text{over}}$.

Upper-bound direction (prevent $v_g = 1$ when $F_g < T_{\text{over}}$):

$$F_g \;\le\; (T_{\text{over}} - \varepsilon) + M_g\, v_g \qquad g = 1,\ldots,G \tag{25}$$

Lower-bound direction (force $v_g = 1$ when $F_g \ge T_{\text{over}}$):

$$F_g \;-\; M_g\, v_g \;\ge\; T_{\text{over}} - M_g \qquad g = 1,\ldots,G \tag{26}$$

Logical implication. Over-fragmentation implies fragmentation:

$$v_g \;\le\; z_g \qquad g = 1, \ldots, G \tag{27}$$

Pre-fixing inactive cells. Cells for which $M_g \le T + \varepsilon$ can never reach the fragmentation threshold regardless of which holes are selected; they are fixed to $z_g = v_g = 0$ before building the model, reducing the active variable count. Similarly, cells with $M_g < T_{\text{over}}$ have v_g fixed to 0.

7.5 Complete MILP

$$\underset{x_k,\, z_g,\, v_g}{\text{minimize}} \quad -\sum_{g=1}^{G} z_g + 0.5 \sum_{g=1}^{G} v_g \tag{28a}$$

subject to

$$F_g = \sum_{k=1}^{K} f_{g,k}\, x_k, \qquad \forall g, \tag{28b}$$

$$F_g \le T + M_g\, z_g, \qquad \forall g, \tag{28c}$$

$$F_g - M_g z_g \ge T + \varepsilon - M_g, \qquad \forall g, \tag{28d}$$

$$F_g \le (T_{\text{over}} - \varepsilon) + M_g v_g, \qquad \forall g, \tag{28e}$$

$$F_g - M_g v_g \ge T_{\text{over}} - M_g, \qquad \forall g, \tag{28f}$$

$$v_g \le z_g, \qquad \forall g, \tag{28g}$$

$$x_k \in \{0, 1\}, \qquad \forall k, \tag{28h}$$

$$z_g \in \{0, 1\}, \qquad \forall g, \tag{28i}$$

$$v_g \in \{0, 1\}, \qquad \forall g \tag{28j}$$

(Written as a minimisation for standard form; the solver maximises $\sum z_g - 0.5 \sum v_g$.)

7.6 Model Size

Table 2: MILP model dimensions (typical instance, $\Delta_{\text{opt}} = 0.5\,\text{m}$).

Quantity	Count
Binary hole variables (x_k)	K
Binary fragmentation indicators (z_g)	G
Binary over-frag. indicators (v_g)	G
Total binary variables	$K + 2G$
Fragmentation constraints (28c+28d)	$2G_{\text{active}}$
Over-frag. constraints (28e+28f)	$2G_{v\text{-active}}$
Implication constraints (28g)	G_{active}

Since the Gaussian contribution $f_{g,k}$ decays rapidly with distance, a sparsity threshold $f_{g,k} < 10^{-6}$ is used to drop negligible terms from the F_g expression, reducing constraint density.

8 Solution Procedure

8.1 MILP (Pyomo + Gurobi)

The MILP (28) is built using **Pyomo** and solved with **Gurobi**. Key solver settings:

- Time limit: 300 s (configurable).
- MIP gap tolerance: 1% (configurable).
- Pre-solve level 2; cut generation level 2; MIP focus mode 1 (optimise for best bound).

After solving, the selected holes are evaluated on the fine grid ($\Delta = 0.1\,\mathrm{m}$) to report the definitive A_{frag}, A_{over}, and objective.

8.2 Greedy Heuristic (Baseline)

A greedy forward-selection algorithm serves as a fast baseline:

Algorithm 1 Greedy Forward Selection

$\mathcal{H} \leftarrow \emptyset, \quad F \leftarrow \mathbf{0}$
repeat
  $c^* \leftarrow \arg\max_{c \in \mathcal{C} \setminus \mathcal{H}} Z(F + f_c) - Z(F)$
  if $Z(F + f_{c^*}) - Z(F) > 0$ **then**
    $\mathcal{H} \leftarrow \mathcal{H} \cup \{c^*\}, \quad F \leftarrow F + f_{c^*}$
  else
    break
  end if
until $\mathcal{C} \setminus \mathcal{H} = \emptyset$
return $\mathcal{H}$

The greedy heuristic exploits the near-submodular structure of the objective: the marginal gain of each additional hole is non-increasing, making greedy selection near-optimal.

9 Summary of Notation

Table 3: Complete notation reference.

Symbol	Description	Domain
Ω	Boundary polygon interior	$\mathbb{R}^2$
N	Number of boundary vertices	$\mathbb{Z}_{>0}$
M	Number of bootlegs	$\mathbb{Z}_{\geq 0}$
K	Number of candidate holes	$\mathbb{Z}_{>0}$
G	Number of grid cells	$\mathbb{Z}_{>0}$
$\mathbf{h}_i$	Hole position i	$\mathbb{R}^2$
$\mathbf{b}_j$	Bootleg position j	$\mathbb{R}^2$
S	Spacing (horizontal)	5.0 m
B	Burden (vertical)	2.5 m
r	Bootleg exclusion radius	0.75 m
A	Gaussian peak amplitude	0.75
C	Decay parameter	0.55
$\sigma_{x_1}, \sigma_{y_1}$	Narrow Gaussian spreads	m
$\sigma_{x_2}, \sigma_{y_2}$	Wide Gaussian spreads	m
F_g	Fragmentation field at cell g	$\mathbb{R}_{\geq 0}$
T	Fragmentation threshold	1.0
T_{over}	Over-fragmentation threshold	1.8
Δ	Fine evaluation grid resolution	0.1 m
Δ_{opt}	MILP optimisation grid resolution	0.5 m
λ	Over-fragmentation penalty weight	0.5
ε	Strict-inequality offset	10^{-4}
$f_{g,k}$	Pre-computed contribution of hole k at cell g	$\mathbb{R}_{\geq 0}$
M_g	Per-cell Big-M constant	$\mathbb{R}_{>0}$
x_k	Binary hole-selection variable	$\{0, 1\}$
z_g	Binary fragmentation indicator	$\{0, 1\}$
v_g	Binary over-fragmentation indicator	$\{0, 1\}$

10 Implementation Notes

Grid resolution trade-off. Using $\Delta_{\text{opt}} = 0.5\,\text{m}$ for optimisation rather than 0.1 m reduces the number of grid cells by a factor of 25, making the MILP tractable. The coarser grid introduces a small approximation error; the final metrics are always computed on the fine $\Delta = 0.1\,\text{m}$ grid.

Sparsity exploitation. The coefficient matrix $f_{g,k}$ is sparse: each hole has negligible influence beyond a few metres. Entries below 10^{-6} are dropped from the F_g expressions, significantly reducing the number of non-zeros in the constraint matrix.

Per-cell Big-M tightness. Using the cell-specific bound $M_g = \sum_k f_{g,k}$ rather than a global $M = \max_g M_g$ produces much tighter constraints, improving LP relaxation quality and solver performance.

Variable pre-fixing. Cells with $M_g \leq T + \varepsilon$ are pre-fixed to $z_g = v_g = 0$; cells with $M_g < T_{\text{over}}$ have v_g pre-fixed to 0. In practice, cells far from all candidates account for the majority of the grid and are pre-fixed, dramatically reducing the active model size.