

# Causal inference and confounding effects

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## Learning objectives

- ▶ Introduction to causal inference
- ▶ Causal structural model (DAG)
- ▶ Confounding factor adjustment
- ▶ Instrumental Variable
- ▶ Potential outcome model

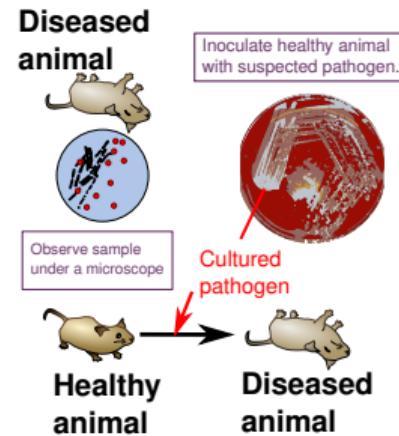
# A century-old question, finding the cause of a disease

## Henle-Koch Postulates

1. The parasite occurs in every case of the disease in question and under circumstances which can account for the pathological changes and clinical course of the disease.
2. It occurs in no other disease as a fortuitous and nonpathogenic parasite.
3. After being fully isolated from the body and repeatedly grown in pure culture, it can induce the disease anew.



Jakob Henle Robert Koch



source: wikipedia

Scientists have been doing causal inference research

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- ▶ **Invariance, stability** arguments in the 1st and 2nd points
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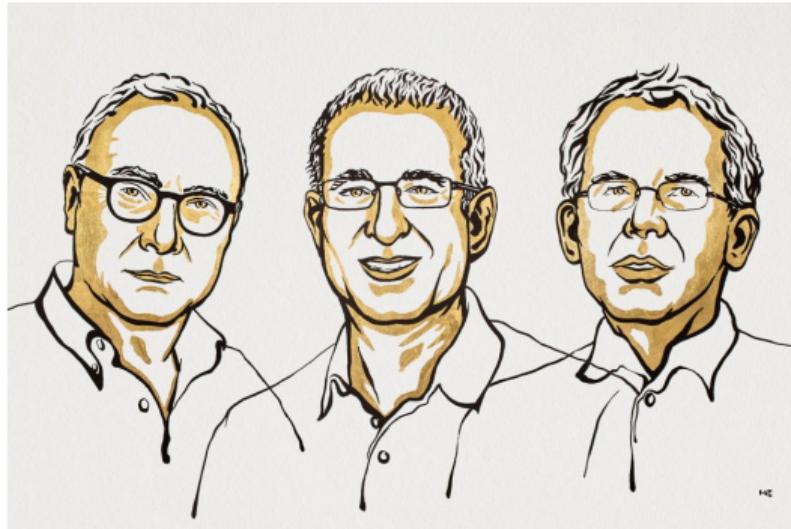
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- ▶ **Invariance, stability** arguments in the 1st and 2nd points
- ▶ The notion of **intervention** in the 3rd point.
- ▶ Just observing a pathogen in the diseased animals isn't credible evidence of the cause of disease

# The Nobel Prize 2021 in Economical Sciences



David Card, Joshua Angrist, Guido Imbens  
instrumental variable      matching, propensity

*“For the methodological contributions to the analysis of causal relationships”*

# Today's lecture

What is causal inference (fundamentals)

Graphical language for causal inference

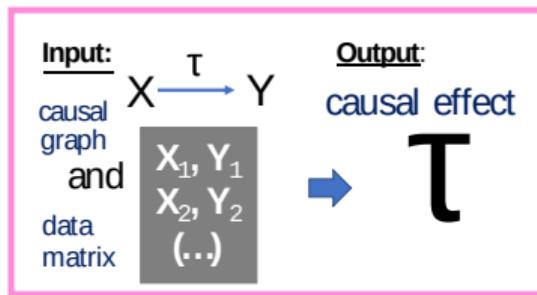
Mendelian Randomization

Potential outcome framework (what if?)

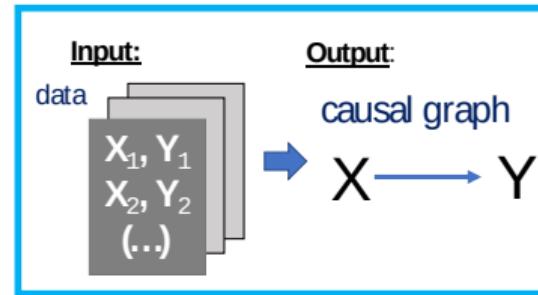
Model-based causal inference

When we don't know an underlying causal graph

## Causal Effect Inference



## Causal structure discovery



Causal “effect” inference in two ways:  $X \rightarrow Y$

## Experiments/interventions

What would be the effect of *doing*  
 $X = x$ ?

$$P(Y|do(X = x))$$

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Sewall Wright



R.A. Fisher

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What if we had  $X = 0$ , not  $X = 1$ , or vice versa?

$$Y_i^{(X=1)} - Y_i^{(X=0)}$$



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Although we only observed “factual”  $Y$ , can we estimate “counterfactual”  $Y$ ?

$$\overset{\text{observed}}{Y}_i = \begin{cases} Y_i^{(1)}, & X = 1 \\ Y_i^{(0)}, & X = 0 \end{cases}$$

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Don Rubin



Jerzy Neyman



David Lewis

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When we don't know an underlying causal graph

## Causal effect inference from an experimentalist's perspective

Goal:

$$P(Y|\text{do}(X = x)) \leftarrow P(Y|X = x)$$

- ▶ What you are seeing is **not** what you are doing!
- ▶ What if we have a nearly complete picture of dependency?

## Common misconception

Isn't causal inference learning a DAG from data?

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<sup>1</sup>A student of Karl Pearson, who also argued the same.

## Common misconception

[On Seawall Wright's graphical model], Henry Niles<sup>1</sup> disparaged,  
“The basic fallacy of the method appears to be **the assumption**  
**that it is possible to set up a priori a comparatively simple**  
**graphical system.**”

Pearl, *The Book of Why*, p.78

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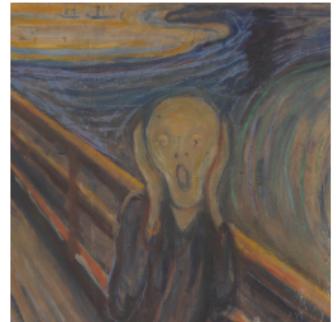
## Common misconception - correlation implies causation

Henry Niles also wrote, “*To contrast ‘causation’ and ‘correlation’ is unwarranted because causation is simply perfect correlation*”

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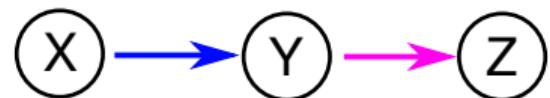
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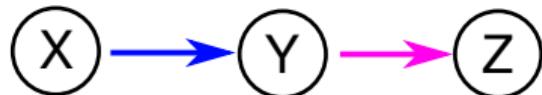
## Causal Directed Acyclic Graph induces joint probability



$$p(X, Y, Z) = p(Y|X) \quad p(Z|Y) \quad p(X)$$

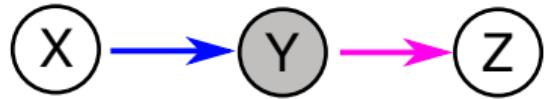
first edge      second edge      first node

## Causal DAG: Causal path/trail and reachability



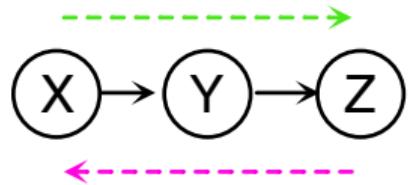
- ▶ There is a path between  $X$  and  $Y$
- ▶ Also between  $Y$  and  $Z$
- ▶ Also from  $X$  to  $Z$

## Causal DAG: Conditioning (and/or adjustment)



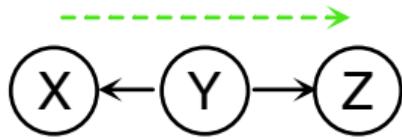
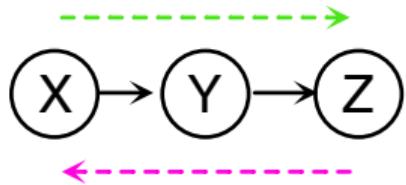
- ▶ A shaded node = conditioning like  $P(Z|Y = y^*)$  and  $P(Y = y^*|X)$
- ▶ There is a path between  $X$  and  $Y$
- ▶ Also between  $Y$  and  $Z$
- ▶ **But** no path from  $X$  to  $Z$

## d-separation: testing conditional independence (flow vs. no flow)



**Flow:** The outcome of  $Z$  depends on the effect of  $X$

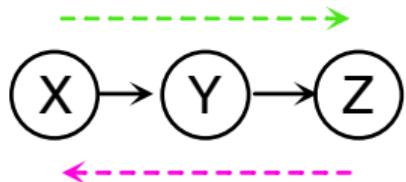
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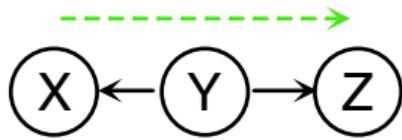
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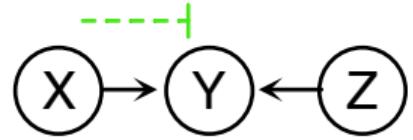
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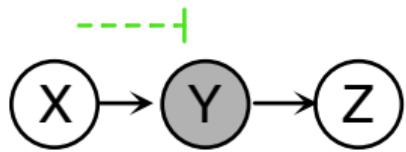
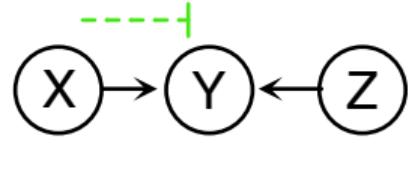
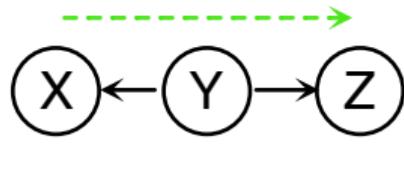
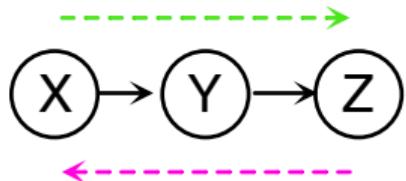


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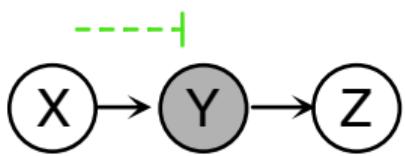
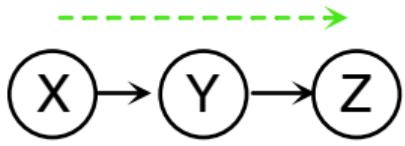
**No Flow:** The outcome of  $Z$  only affects  $Y$ ; do does  $X$

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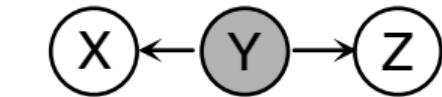
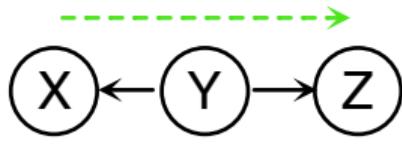


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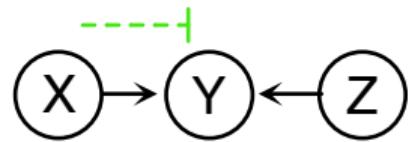
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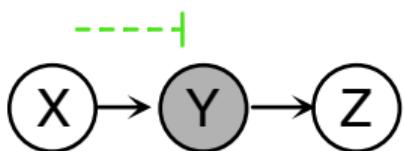
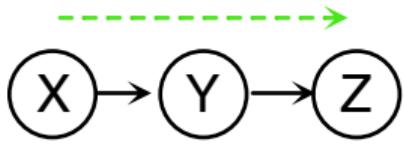
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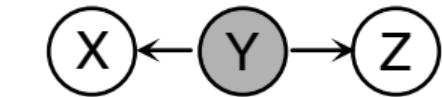
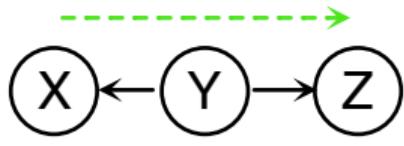
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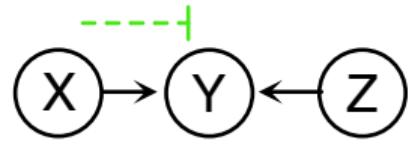
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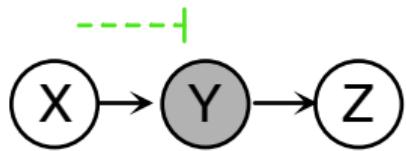
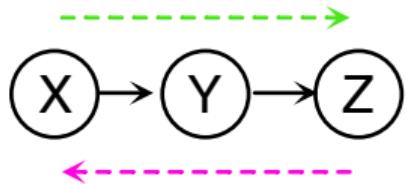


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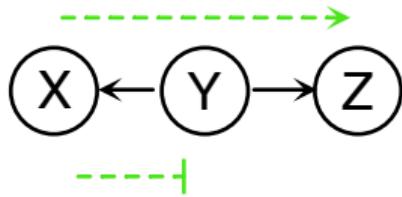


**Flow:** The observed outcome  $Y = y^*$  could stem from  $X$  or/and  $Z$

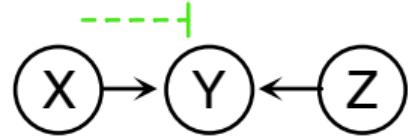
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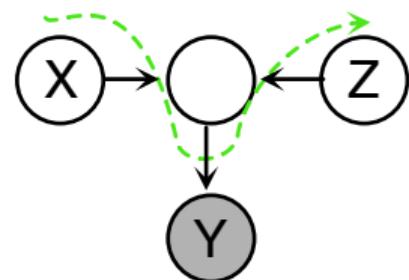
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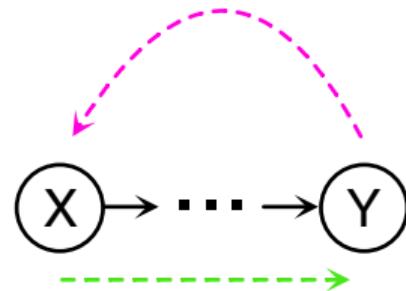


**Flow:** a deeper v-structure



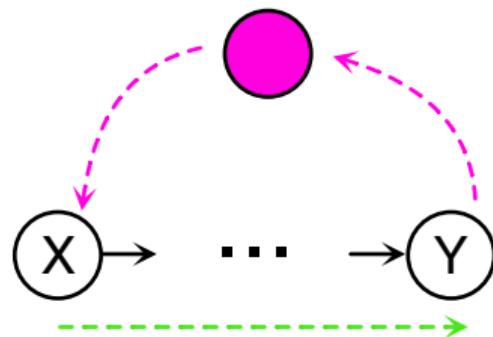
## “Backdoor” exercise

**Question:** Which nodes should be “conditioned” and/or “adjusted” to block a reverse path from  $Y$  to  $X$ ?



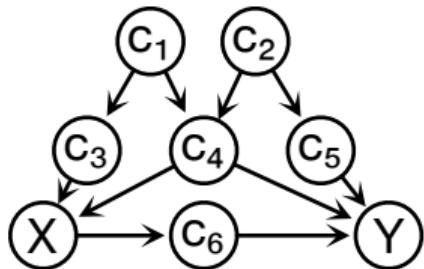
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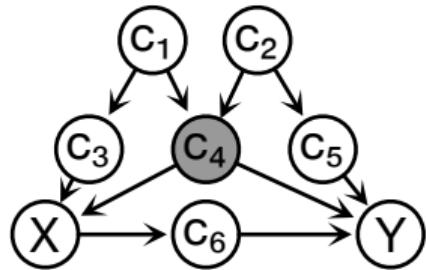
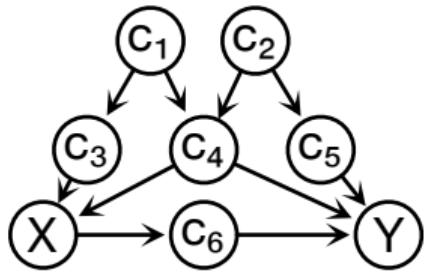
Can you identify and close “backdoor” paths?

**Goal:**  $X \rightarrow Y$  (so close other paths  $Y \rightarrow X$ )



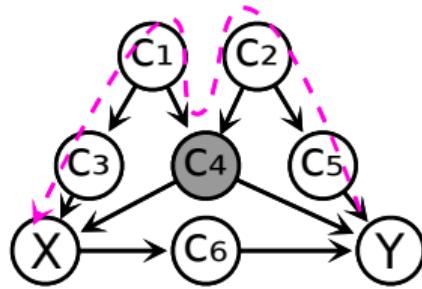
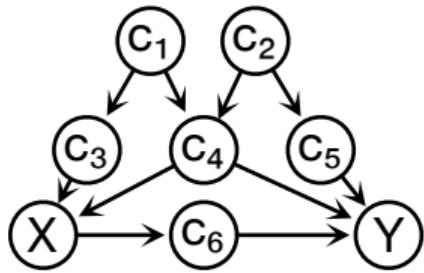
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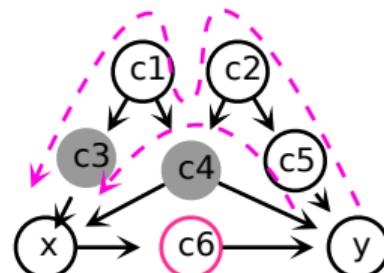
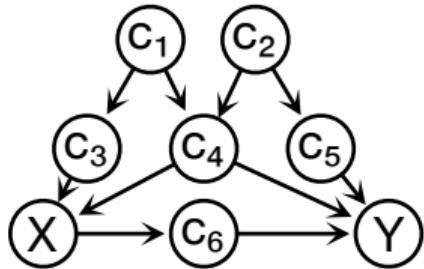
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$$p(Y|do(X)) \neq \int_{C_4} p(Y|X, C_4)p(C_4)dC_4$$

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$$p(Y|do(X)) = \int_{C_2, C_4} p(Y|X, C_2, C_4)p(C_2, C_4)dC_2C_4$$

## Three steps of causal inference using a causal graph

1. Build a causal structural model

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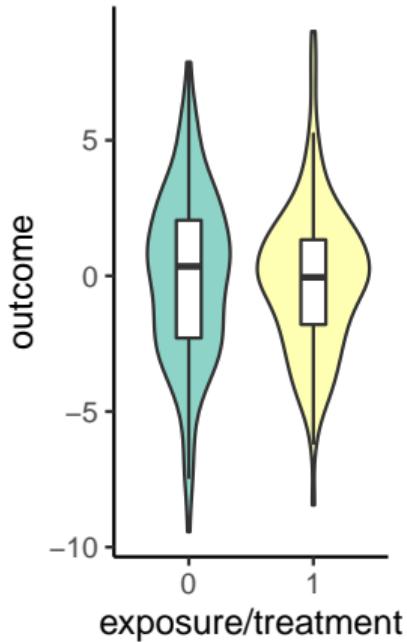
1. Build a causal structural model
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## Three steps of causal inference using a causal graph

1. Build a causal structural model
2. Identify "back-door" variables
3. Adjust "back-door" variables

Estimate causal effects and sensitivity analysis

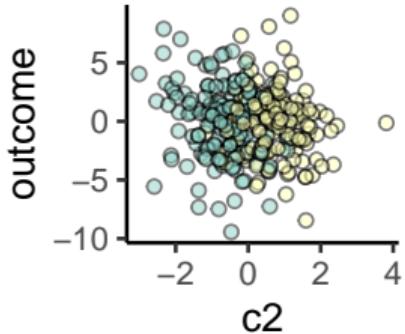
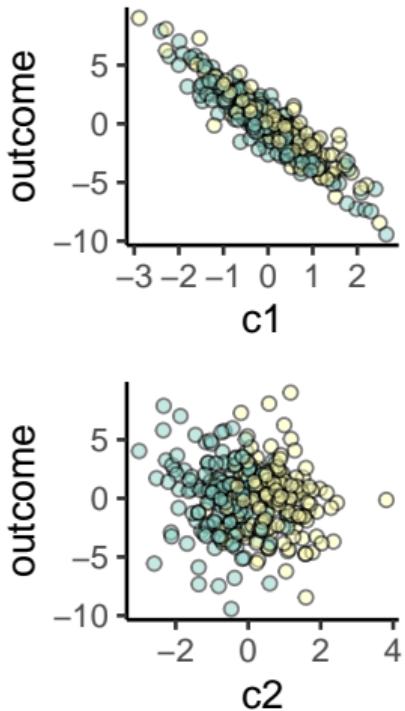
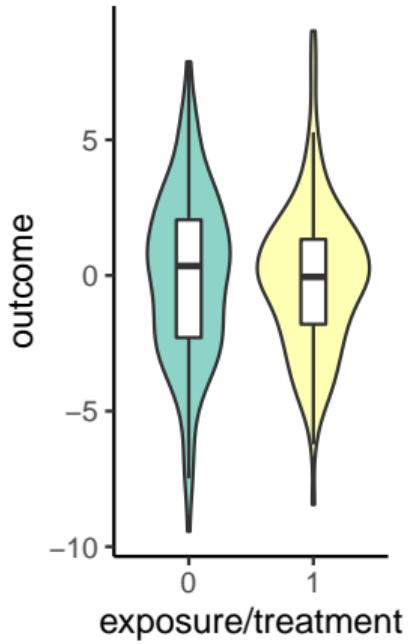
## A working example: confounder adjustment in case-control study



**Key question:**  $X \xrightarrow{?} Y$

**X:** exposure variable  
**Y:** outcome variable

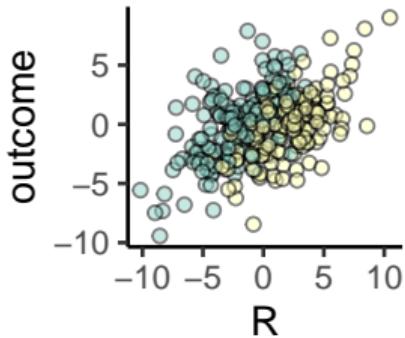
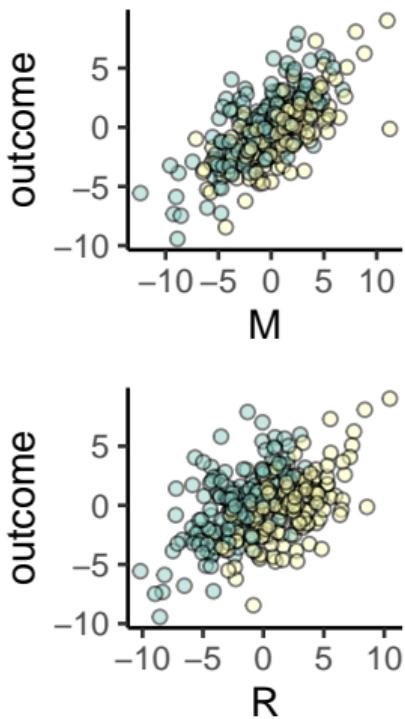
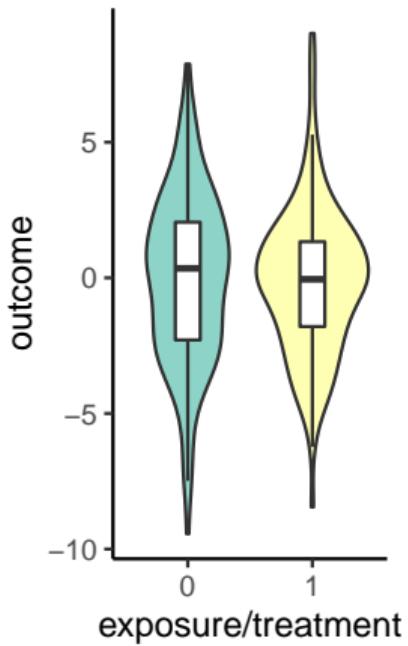
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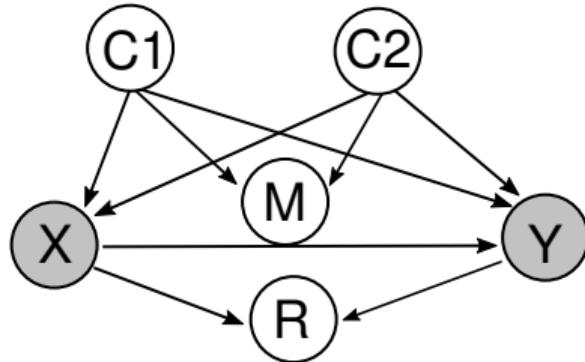
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**M**: other covariate

**R**: other covariate

# Causal inference with a graphical model

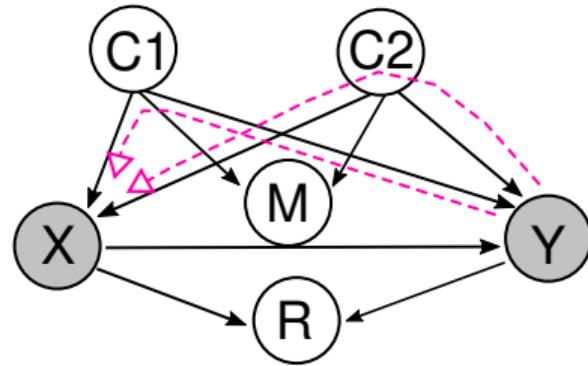
1. Build a causal structural model



What are potential backdoors?

# Causal inference with a graphical model

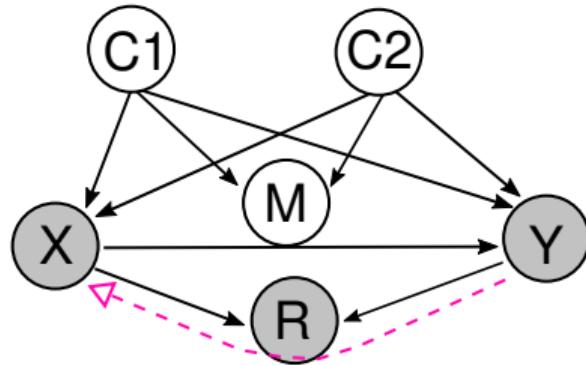
1. Build a causal structural model
2. Identify "back-door" paths/variables (*closing*  $Y \rightarrow X$ , *while opening*  $X \rightarrow Y$ )



How do we close them?

# Causal inference with a graphical model

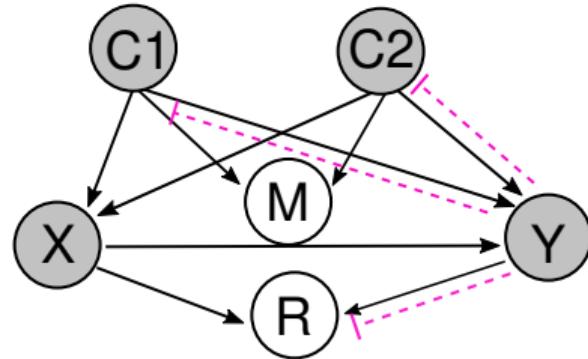
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What about this?

# Causal inference with a graphical model

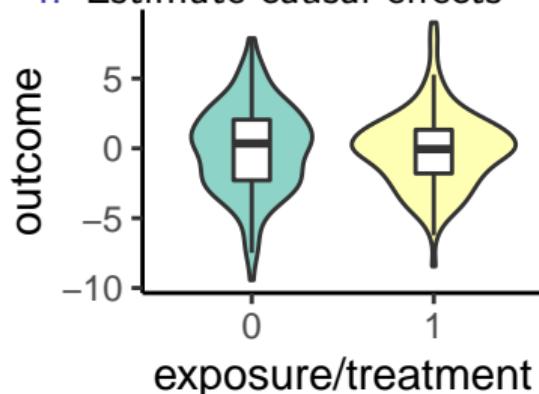
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Is this enough?

## Causal inference with a graphical model

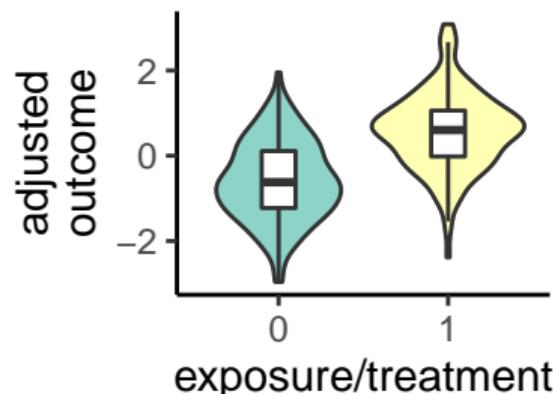
1. Build a causal structural model
2. Identify "back-door" paths/variables (*closing  $Y \rightarrow X$ , while opening  $X \rightarrow Y$* )
3. Adjust "back-door" variables
4. Estimate causal effects



$$Y \leftarrow Y - \sum_{k=1}^2 C_k \hat{\beta}_k$$

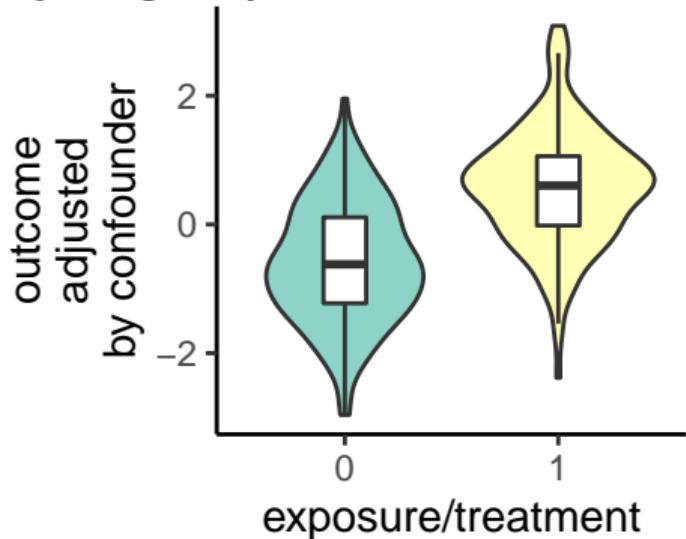
which approximates

$$p(Y|X) = \int_C p(Y|X, C)p(C)dC$$

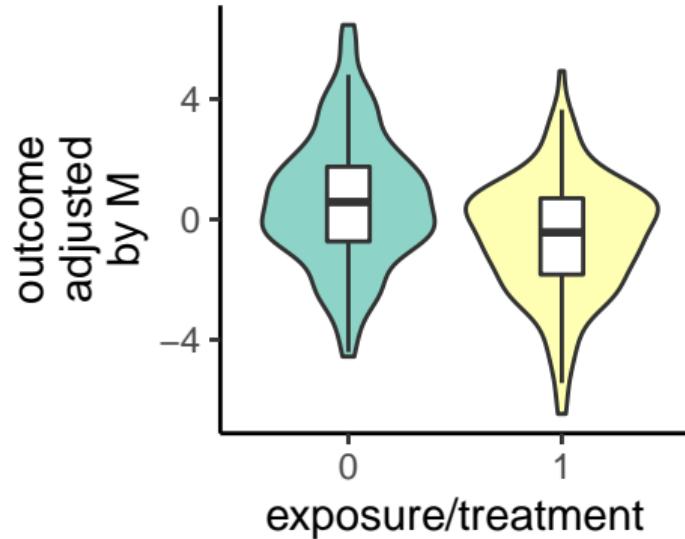


What would happen if we close a wrong “backdoor” variable?

Adjusting  $Y$  by the backdoor  $C$

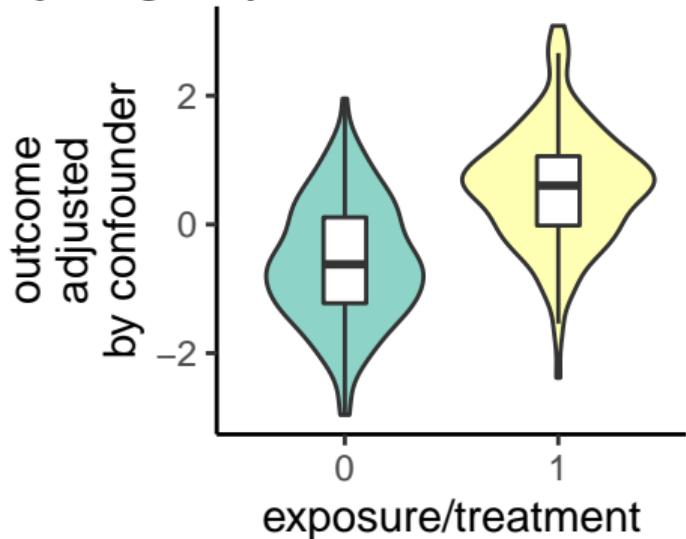


Adjusting  $Y$  by the invalid  $M$

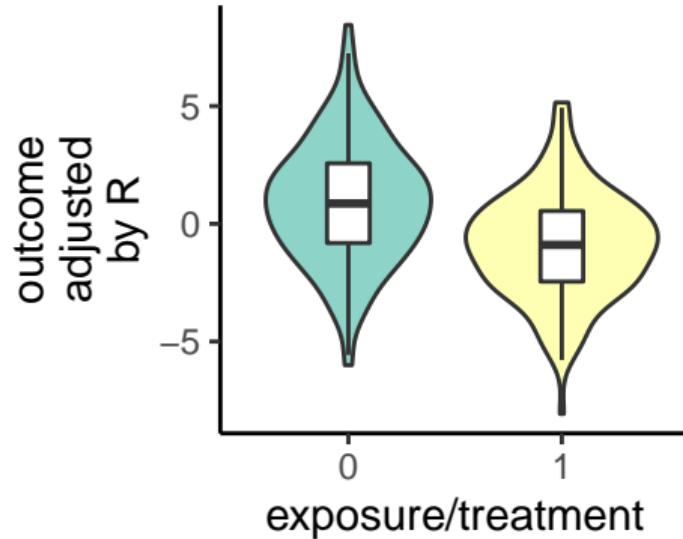


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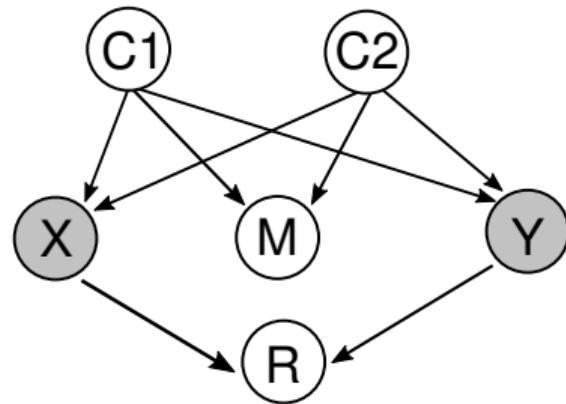


Adjusting  $Y$  by the collider  $R$



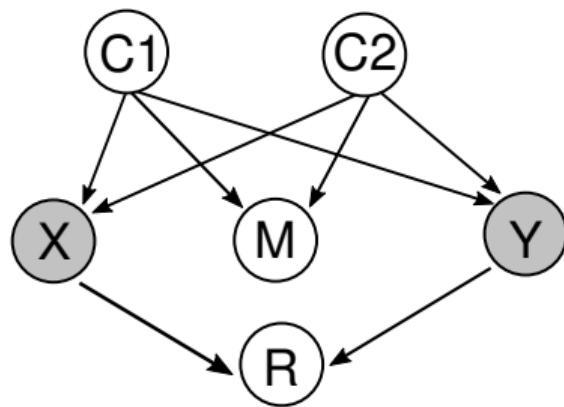
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Take another example:

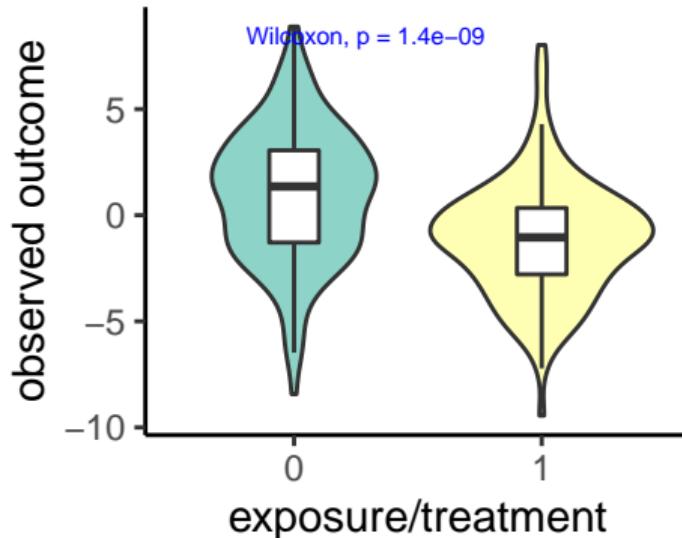


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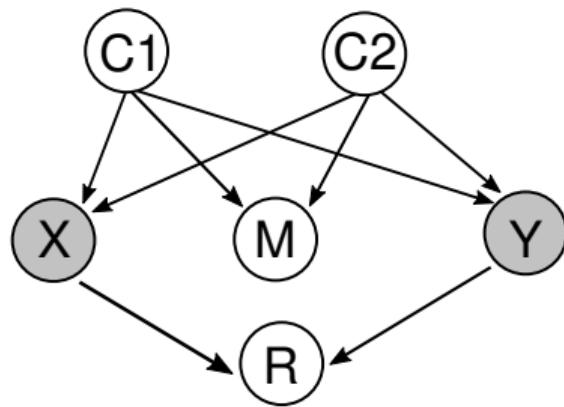


We can see the strong correlation between  $X$  and  $Y$

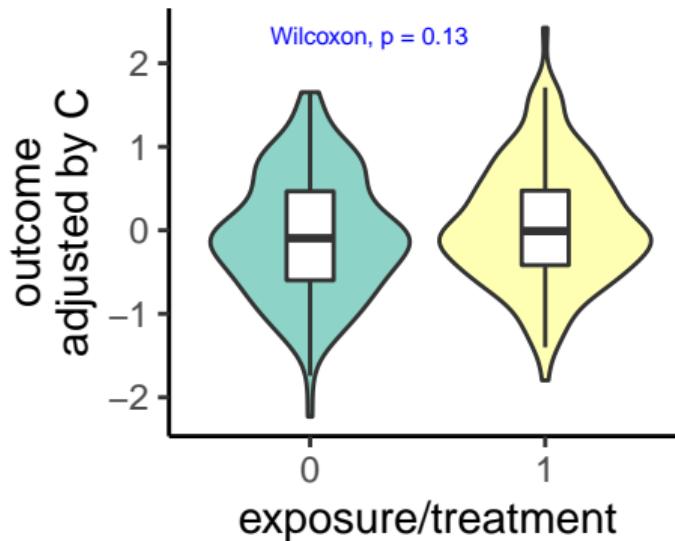


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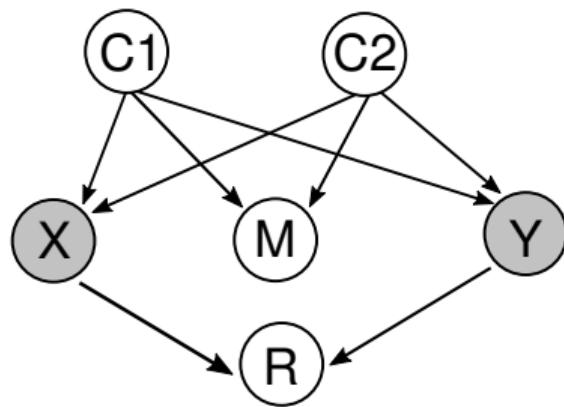


In fact, there is no causal relationship between  $X$  and  $Y$

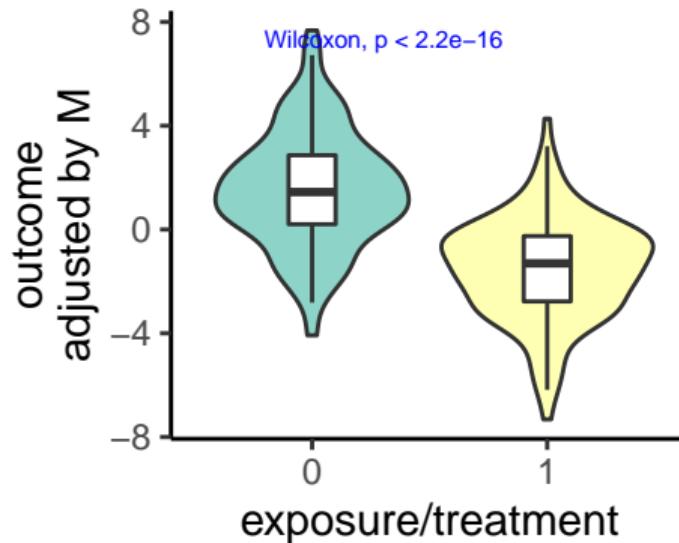


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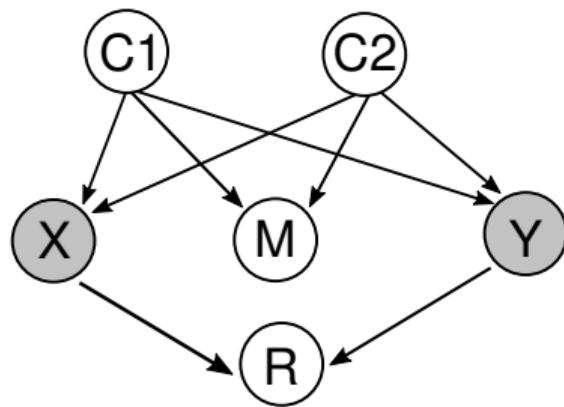


Adjusting  $M$  will not seal the backdoor... reinforce the spurious correlation

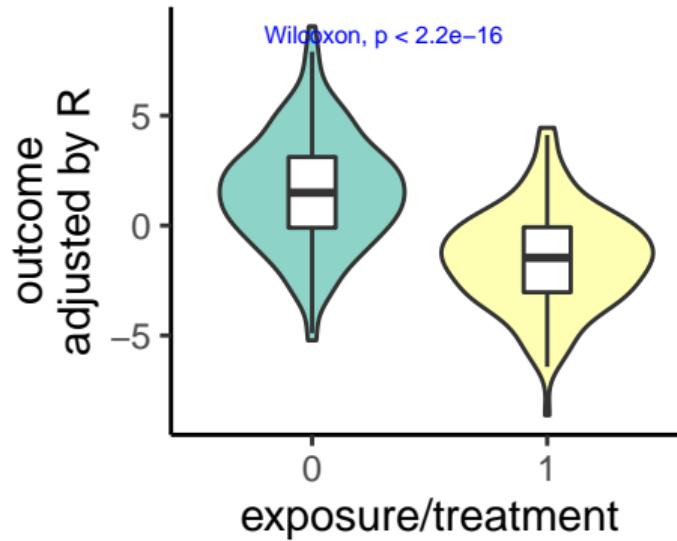


## What would happen if we close a wrong “backdoor” variable?

Take another example:



Adjusting  $R$  will open another backdoor... reinforce the spurious correlation



# Today's lecture

What is causal inference (fundamentals)

Graphical language for causal inference

Mendelian Randomization

Potential outcome framework (what if?)

Model-based causal inference

When we don't know an underlying causal graph

# Mendelian + Randomization

## Randomized experiments - RA Fisher's work at Rothamsted

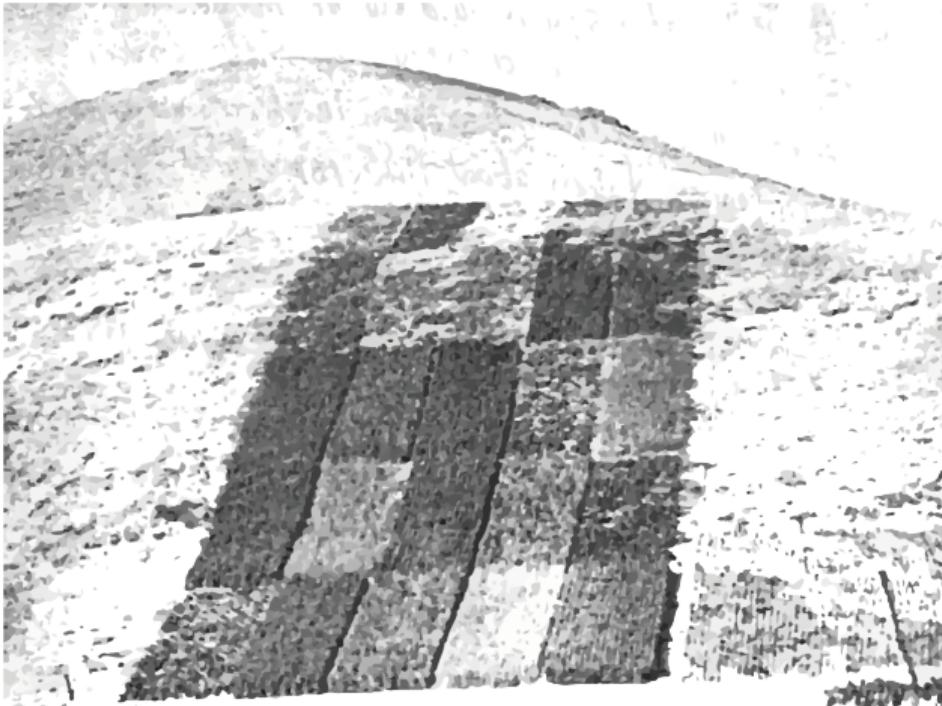
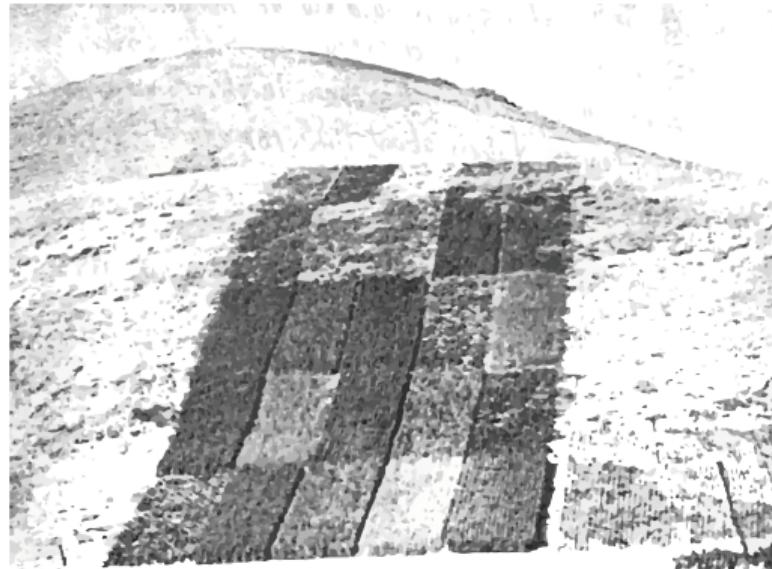


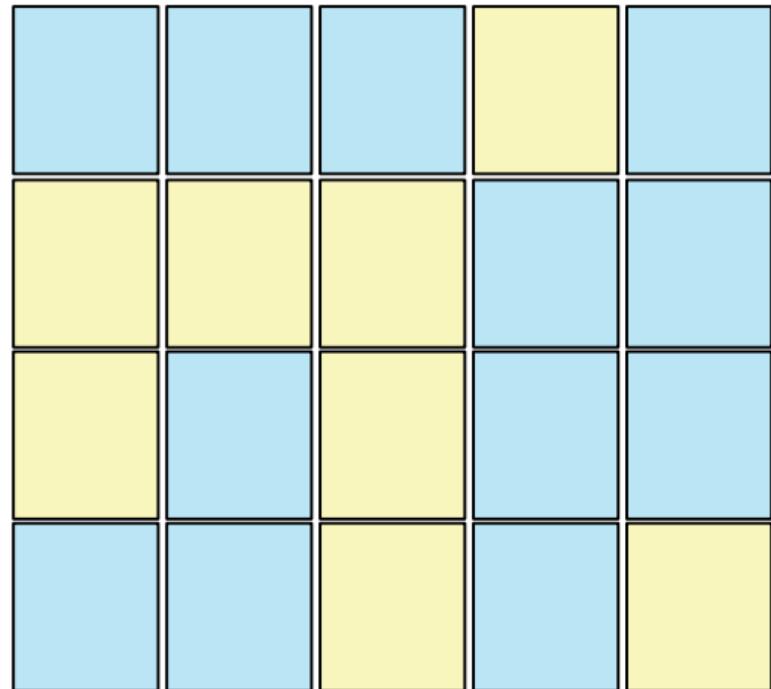
Image source: '[www.adelaide.edu.au](http://www.adelaide.edu.au)'

# Randomized experiments - RA Fisher's work at Rothamsted

## Crops grown in the plots

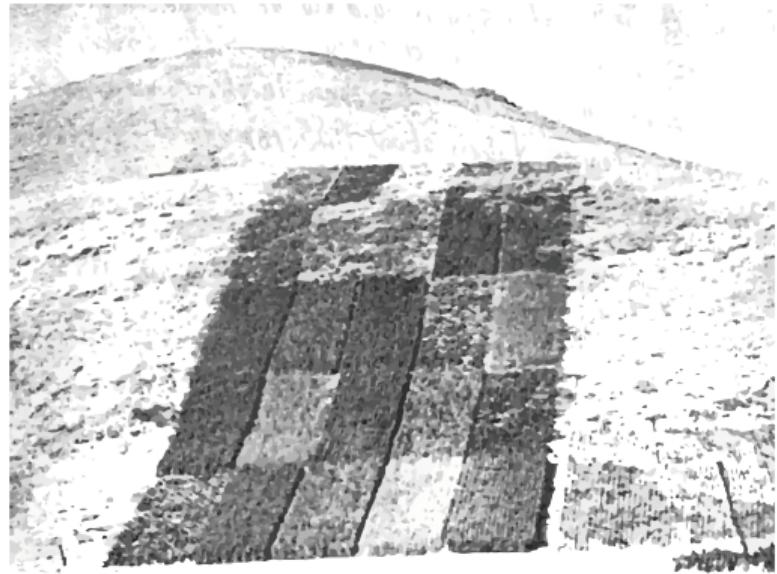


## Experimental design

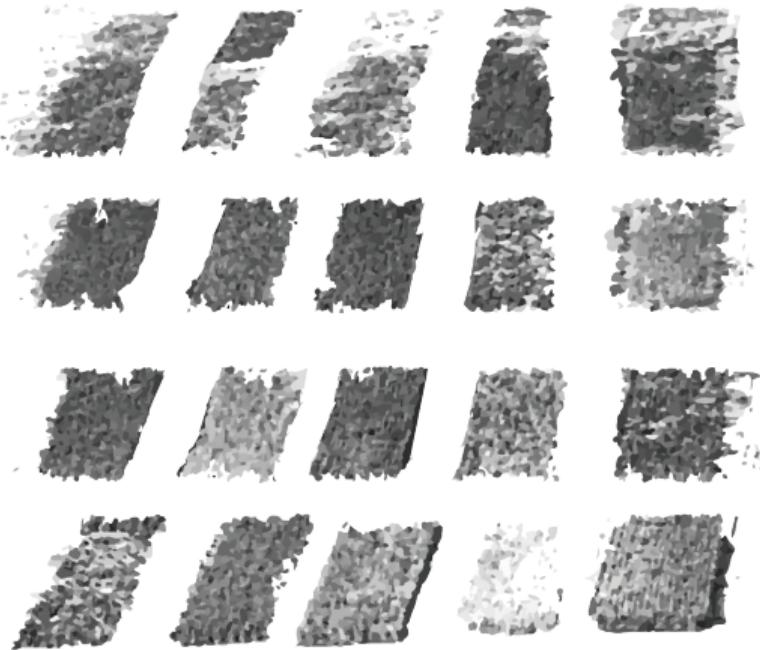


## Randomized experiments - RA Fisher's work at Rothamsted

Crops grown in the plots

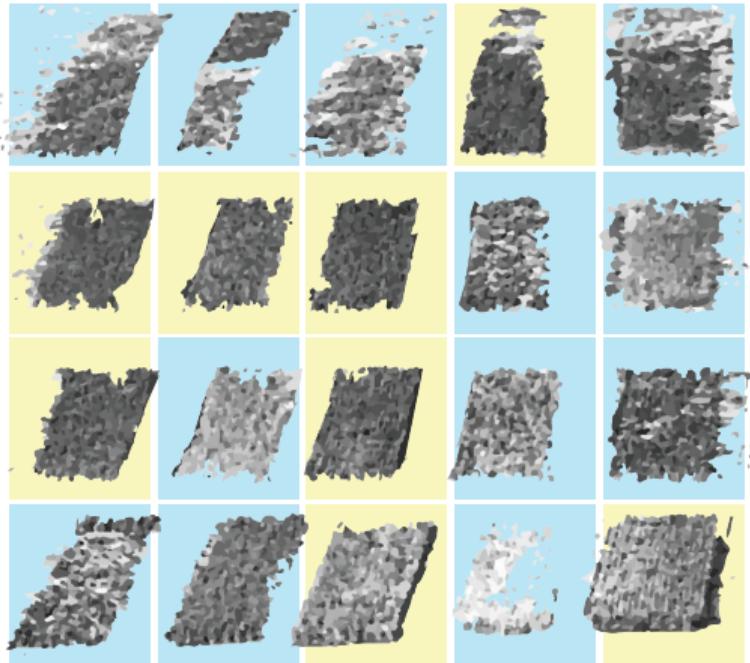
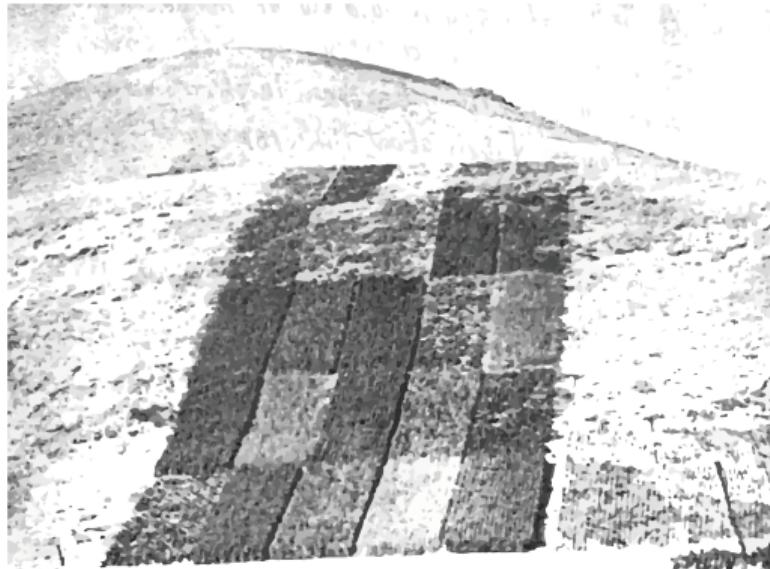


Random assignment of fertilizer



## Randomized experiments - RA Fisher's work at Rothamsted

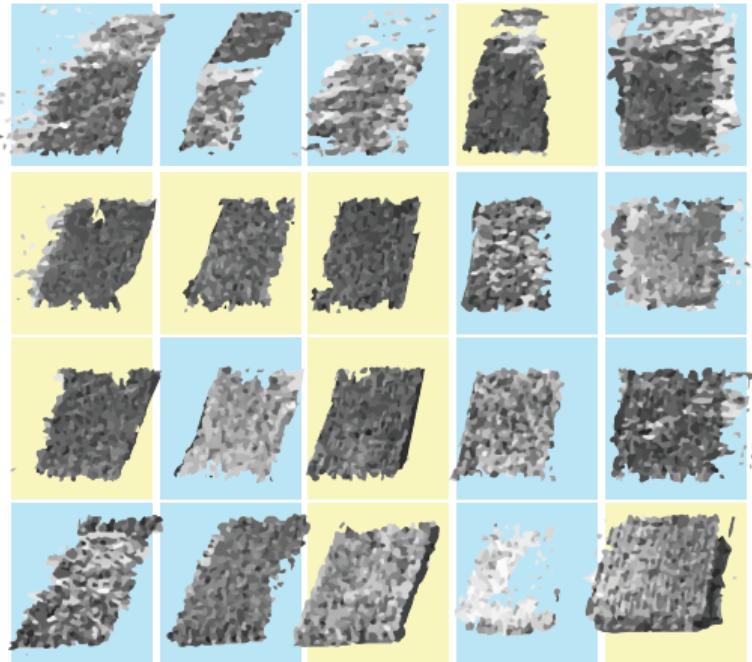
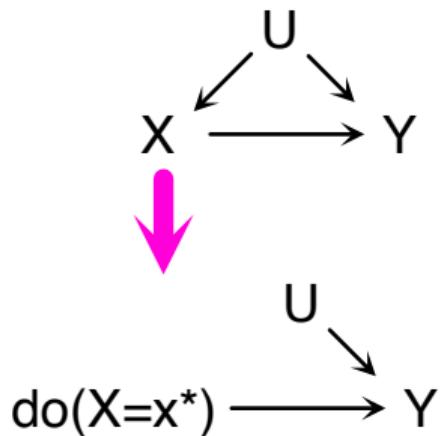
Crops grown in the plots    Compare the yields between the treated vs non-treated



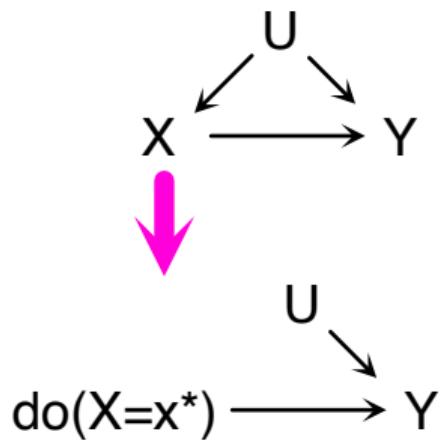
# Randomized experiments - RA Fisher's work at Rothamsted

RCT  $\Rightarrow$  "doing"

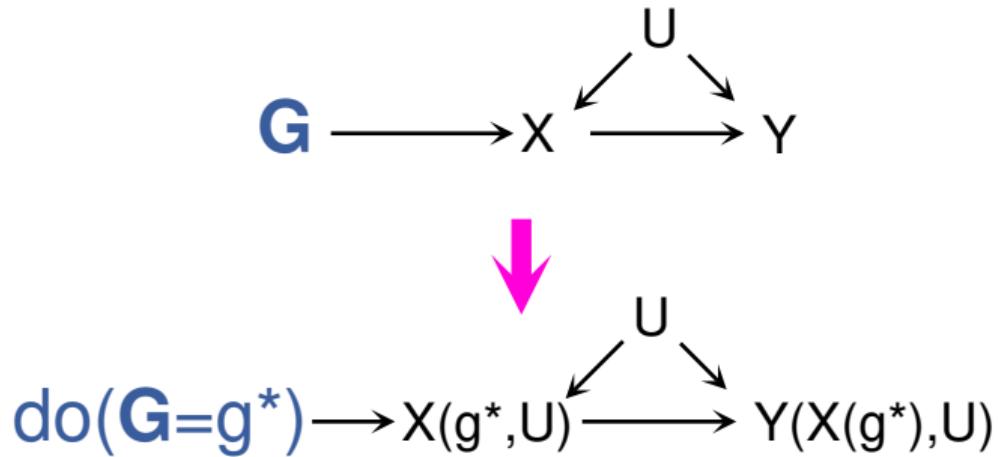
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# Randomization

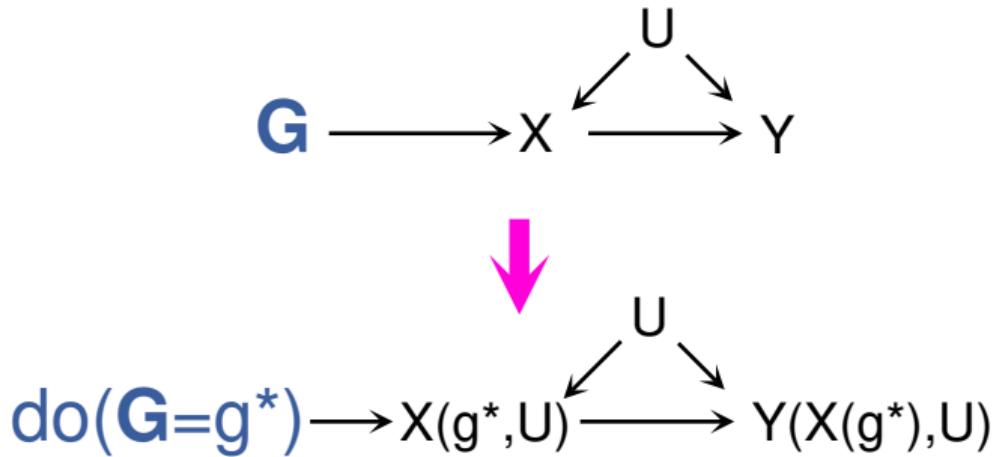


## Mendelian Randomization



We will use a genetic variable  $G$  to mimic RCT

## Mendelian Randomization



If a genetic variable  $G$  is a valid instrumental variable...

# Mendelian Randomization in modern epidemiology studies

- ▶ G: genotype
- ▶ X: APOE protein
- ▶ Y: cancer

$$G \rightarrow X \rightarrow Y$$

## **APOLIPOPROTEIN E ISOFORMS, SERUM CHOLESTEROL, AND CANCER**

SIR,—It is unclear whether the relation between low serum cholesterol levels and cancer<sup>1</sup> is causal. In many studies occult tumour may have depressed cholesterol levels though in others the relation was found when serum cholesterol had been measured many years before the cancer was diagnosed. The relation is probably not explained by diet, because in the Seven Countries Study cohorts with widely different diets and corresponding differences in mean cholesterol levels experienced similar mean cancer rates.<sup>2,3</sup> On the other hand, within each region cancer incidence was higher in men with a serum cholesterol in the lowest part of the cholesterol distribution for that country.<sup>3</sup> Thus, naturally low cholesterol levels are sometimes associated with increased cancer risk.<sup>1,3</sup>

Differences in the aminoacid sequence of apolipoprotein E (apo Katan, *Lancet*, (1986)

# Mendelian Randomization in modern epidemiology studies

- ▶ G: genotype
- ▶ X: APOE protein
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$$G \rightarrow X \rightarrow Y$$



30TH THOMAS FRANCIS JR MEMORIAL LECTURE

'Mendelian randomization': can genetic epidemiology contribute to understanding environmental determinants of disease?\*

George  
Davey Smith

George Davey Smith and Shah Ebrahim

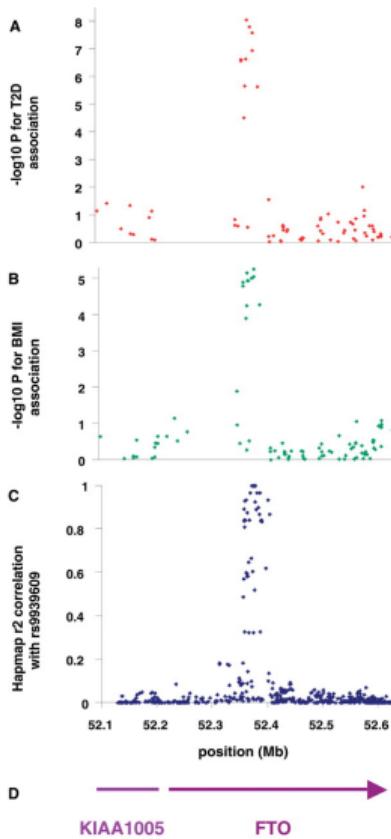
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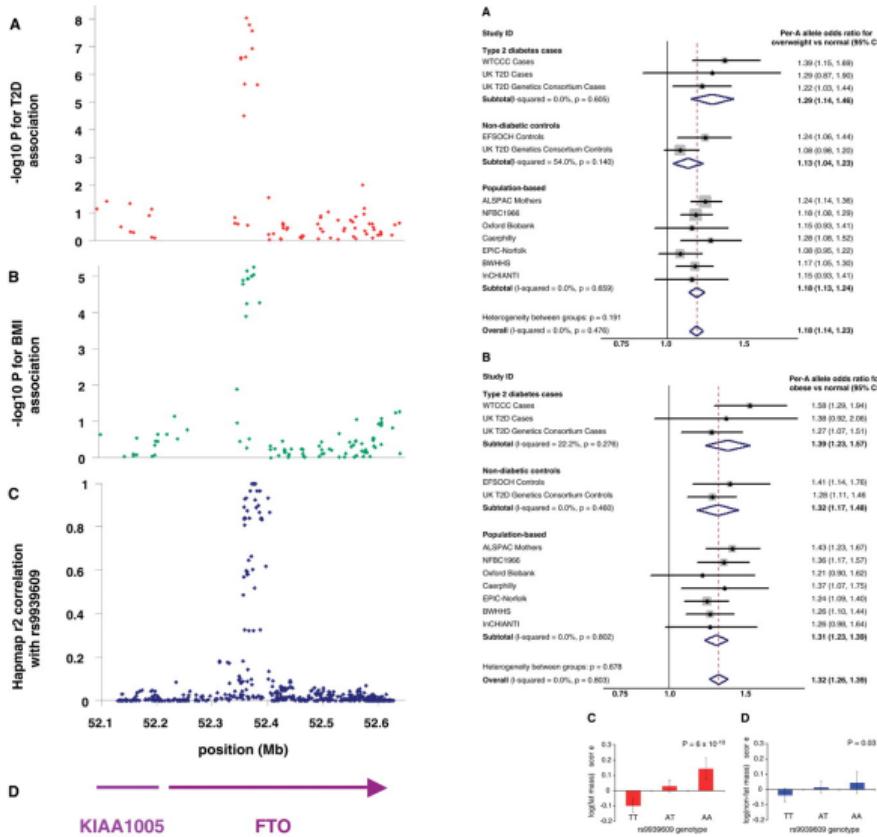
# MR in action: *FTO* → fat mass → obesity, diabetes



Using genotype as an instrumental variable, we test causality between other exposure variables and downstream phenotypes.

► Genotype in *FTO* locus → T2D

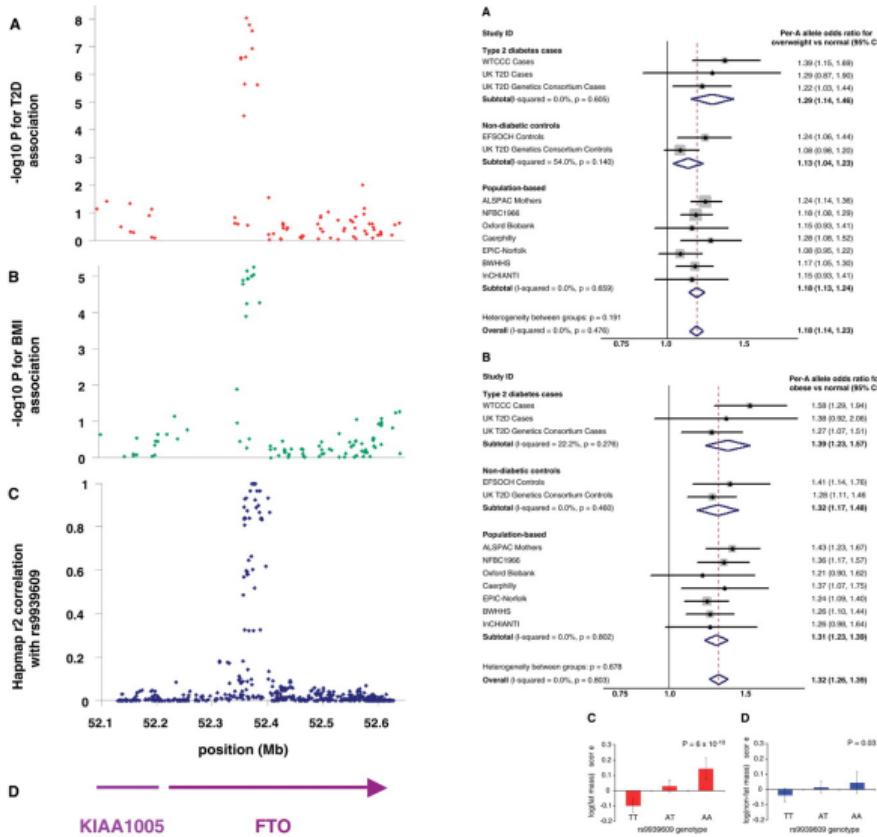
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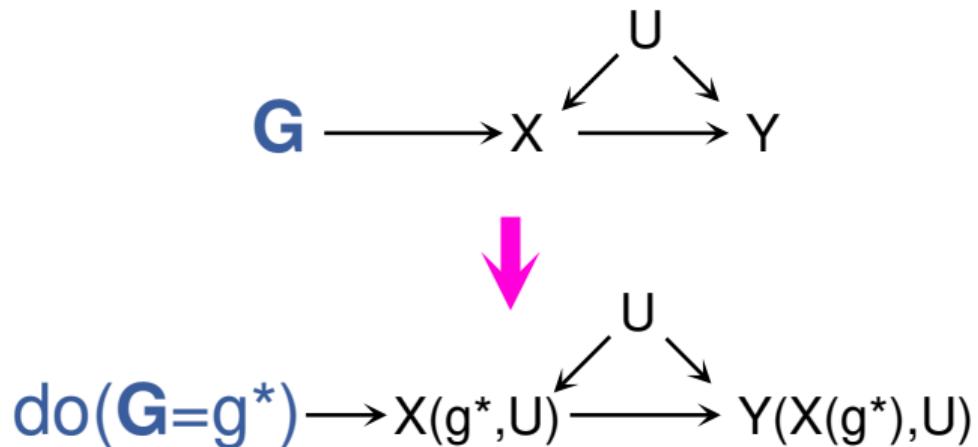
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- ▶ Genotype in *FTO* locus → T2D
  - ▶ *FTO* locus → fat mass
  - ▶ Using *FTO* as "instrumental variable", we can ask other MR questions

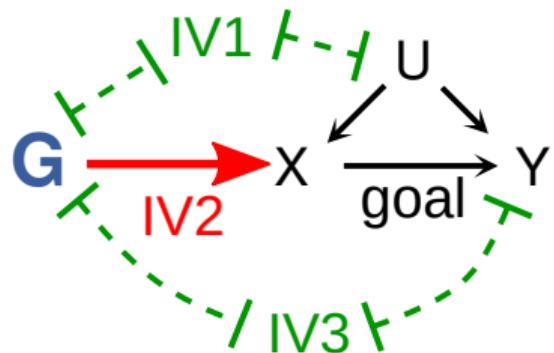
## Why doing Mendelian Randomization?



1. We do not have enough knowledge of  $U$
2. We do not have a way to make interventions on  $do(X = x)$

*"Genotypes are beautifully randomized" - Fisher (1951)*

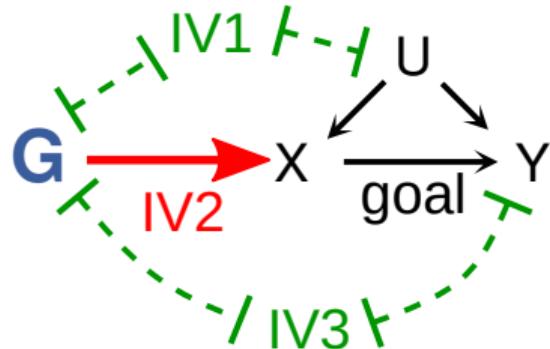
## Definition of instrumental variable in MR study



- ▶ IV1: The genetic variant  $G$  is independent of the potential confounder variable  $U$

Bowden *et al.* (2015)

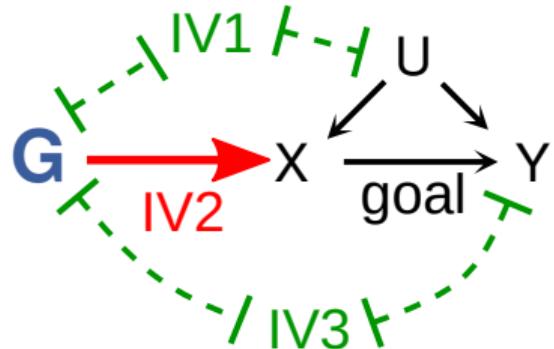
## Definition of instrumental variable in MR study



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- ▶ IV2: The genetic variant is associated with the exposure  $X$

Bowden *et al.* (2015)

## Definition of instrumental variable in MR study



- ▶ IV1: The genetic variant  $G$  is independent of the potential confounder variable  $U$
- ▶ IV2: The genetic variant is associated with the exposure  $X$
- ▶ IV3: The genetic variant is independent of the outcome  $Y$  conditioning on  $X$

Bowden *et al.* (2015)

## MR measures mediation effects of $X$ to $Y$

Given that  $G$  is a valid instrumental variable...

*Example:*

- ▶  $X$ : gene expression
- ▶  $Y$ : disease phenotype

Suppose we have estimated

$$\begin{aligned} G &\xrightarrow{\alpha} X \\ X &= G\hat{\alpha} + \epsilon_X \end{aligned}$$

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and

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**Goal:** What is the causal effect  $\beta$  in  
 $X \xrightarrow{\beta} Y$ ?

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$$Y = X\beta + \epsilon'$$

$$G\hat{\gamma} + \epsilon_Y = (G\hat{\alpha} + \epsilon_X)\beta + \epsilon'$$

and

$$G \xrightarrow{\gamma} Y$$

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$$G\hat{\gamma} = G\hat{\alpha}\beta + \dots$$

and

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$$\begin{aligned} Y &= X\beta + \epsilon' \\ G\hat{\gamma} + \epsilon_Y &= (G\hat{\alpha} + \epsilon_X)\beta + \epsilon' \\ G\hat{\gamma} &= G\hat{\alpha}\beta + \dots \end{aligned}$$

The answer is as simple as

$$\mathbb{E}[\beta] = \frac{\gamma}{\alpha}$$

We will revisit this problem in the  
GWAS lectures

# Today's lecture

What is causal inference (fundamentals)

Graphical language for causal inference

Mendelian Randomization

Potential outcome framework (what if?)

Model-based causal inference

When we don't know an underlying causal graph

## The ultimate goal of causal inference

- ▶ Causality is tied to an action (doing), rather than an observation (seeing).
- ▶ Even the same object/subject/individual at a different time is a different object.
- ▶ **What if** we could have multiverse? **What if** we could turn back in time?

## The ultimate goal of causal inference

- ▶ Causality is tied to an action (doing), rather than an observation (seeing).
- ▶ Even the same object/subject/individual at a different time is a different object.
- ▶ **What if** we could have multiverse? **What if** we could turn back in time?  
*Factual* (observed) vs. *counterfactual* (what could have happened)

## Rubin's causal model ( $X \rightarrow Y$ with confounder $S$ )

Causal assumptions (Rosenbaum & Rubin, 1983)

Unconfoundedness (ignorability)

$$(Y(0), Y(1)) \perp\!\!\!\perp X | S$$

Blocking out sufficient confounder variables, potential outcome  $Y(X)$  is independent of observed exposure  $X = x$ .

E.g., The chance of unvaccinated myself getting COVID in the other universe has nothing to do with the fact that I had been vaccinated in this universe.

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## Overlap (smoothness)

$$0 < P(X = 1|S) < 1$$

There is a positive chance of being exposed and not exposed in all values of  $S$ , e.g., strata.

E.g., There is always some group who get vaccinated and the other group who decide not to get vaccinated across all conditions, e.g., gender, socioeconomic indexes, age, etc.

## SUTVA (Stable Unit Treatment Variable Assumption)

1. The potential outcome of any unit do not vary with the treatment assigned to other units (**No Interference** of causal effects).
2. For each unit, there is no different forms of each treatment level, which lead to different potential outcomes (**No Hidden variation of Treatments**).

## Why is it called a “potential” outcome model?

- ▶ If my causal statement is: “My headache disappeared because I took a pill of aspirin.”
- ▶ Then, we need to investigate counterfactual outcome: “Had I not taken an aspirin, would my headache disappear?”
- ▶ Definition of causal effects: Comparison between two (or multiple) different potential outcomes **on the same unit.**

## Holland (1986)

*The fundamental problem of causal inference is the problem that at most one of the potential outcomes can be realized and thus observed.*

## Causal inference is fundamentally a missing data problem

For  $X_i = 1$ ,

$$Y_i^{\text{obs}} = Y_i(1), Y_i^{\text{mis}} = Y_i(0)$$

For  $X_i = 0$ ,

$$Y_i^{\text{mis}} = Y_i(1), Y_i^{\text{obs}} = Y_i(0)$$

i	X	Y(0)	Y(1)
1	0	Y	?
2	1	?	Y
3	1	?	Y
4	0	Y	?
5	1	?	Y
...	...	...	...

Average causal effect:

$$\frac{1}{N} \sum_{i=1}^N Y_i(1) - Y_i(0)$$

## Statistical aspect of causal inference ( $N > 1$ )

- ▶ Learning about causal effects typically involves multiple units.
- ▶ A fundamental “mistake” of statistical inference: Considering a unit at different time as the same unit.
- ▶ The problem of causal inference is a missing data problem.
- ▶ What was the missing data assignment mechanism?
- ▶ Prediction of assignment mechanisms by pre-treatment variables.

## Causal estimand

- ▶  $Y_i(X_i)$ : Potential outcome of a unit  $i$  with exposure  $X_i$
- ▶  $S_{ik}$ : Covariate  $k$  for a unit  $i$
- ▶ Atomic causal estimand:

$$Y_i(1) - Y_i(0)$$

- ▶ Average causal effect:

$$\frac{1}{N} \sum_{i=1}^N Y_i(1) - Y_i(0)$$

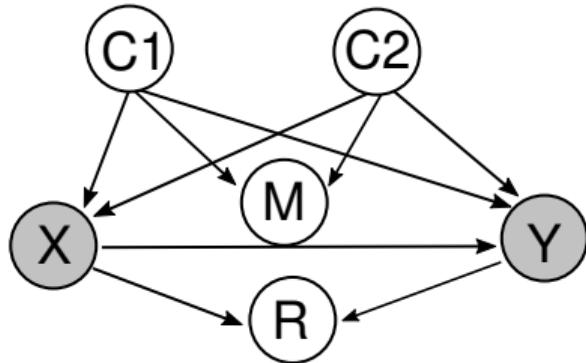
## History of causal inference

- ▶ Potential outcome: Neyman (1923)
- ▶ Randomized assignment: Fisher (1925)
- ▶ Regressing out covariates: Yule (1897)

## A classification of assignment mechanisms

- ▶ Experimental: The assignment mechanism is known and controlled by the researcher.
- ▶ Observational: The assignment mechanism is **not** known to, or **not** under the control of the researcher.

Revisit the same example



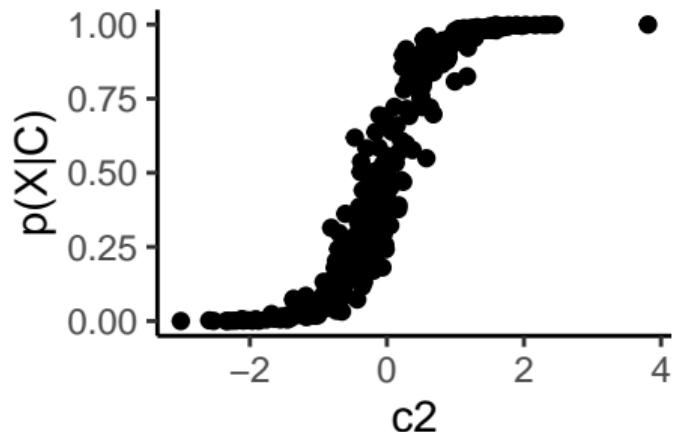
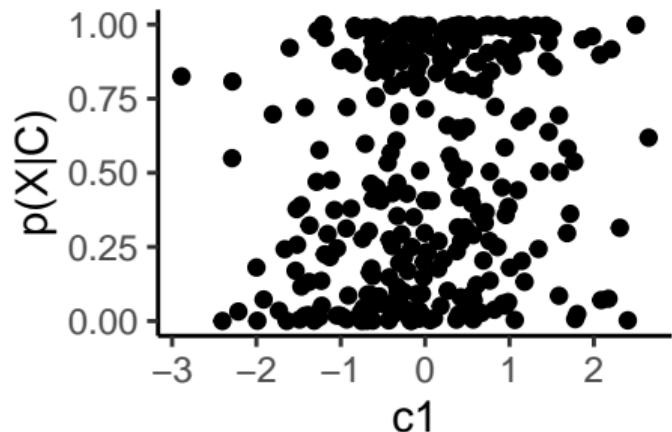
- ▶ What are the backdoor variables?
- ▶ How did we generate  $X$ ?

## Inverse Propensity Weighting

What is the model for  $X$  given confounders (the backdoor variables)?

Propensity = probability of assignment  $X = 1$ :

$$p(X|C_1, C_2) \approx \frac{1}{1 + \exp(-\beta_0 - \beta_1 C_1 - \beta_2 C_2)}$$

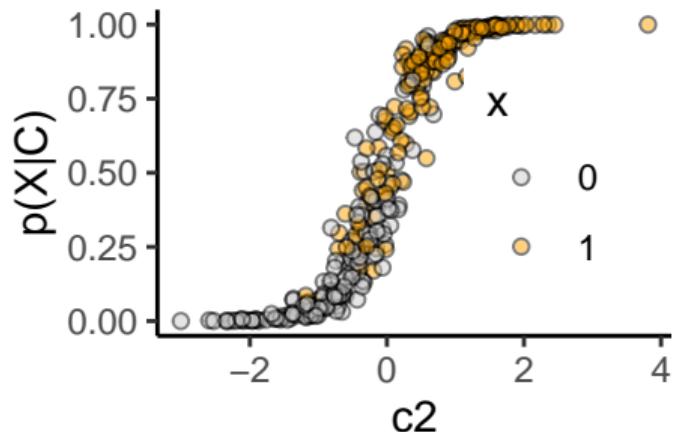
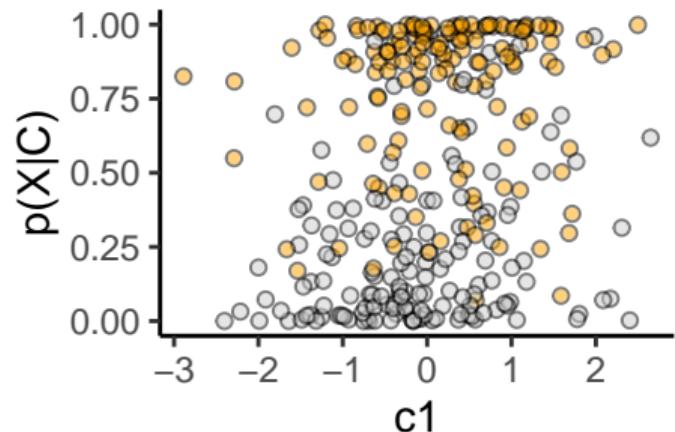


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## Intuition behind IPW

What if we have assigned  $X = 1$  unconfounded by  $C$ ?

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In other words, what if samples could be drawn more from the underrepresented group?

Likewise, what if samples could be dropped in the overrepresented group?

## Inverse Propensity Weighting “inverse” confounded assignment

$$\hat{Y}_i^{(1)} = \frac{X_i Y_i}{\hat{p}(X_i = 1 | C_i)}$$

$$\hat{Y}_i^{(0)} = \frac{(1 - X_i) Y_i}{1 - \hat{p}(X_i = 1 | C_i)}$$

```
p.xc <-  
  glm(x~cc, family="binomial") %>%  
  predict() %>%  
  sigmoid() %>%  
  clamp() # avoid 0 or 1  
  
ww <- x / p.xc + (1-x) / (1-p.xc)
```

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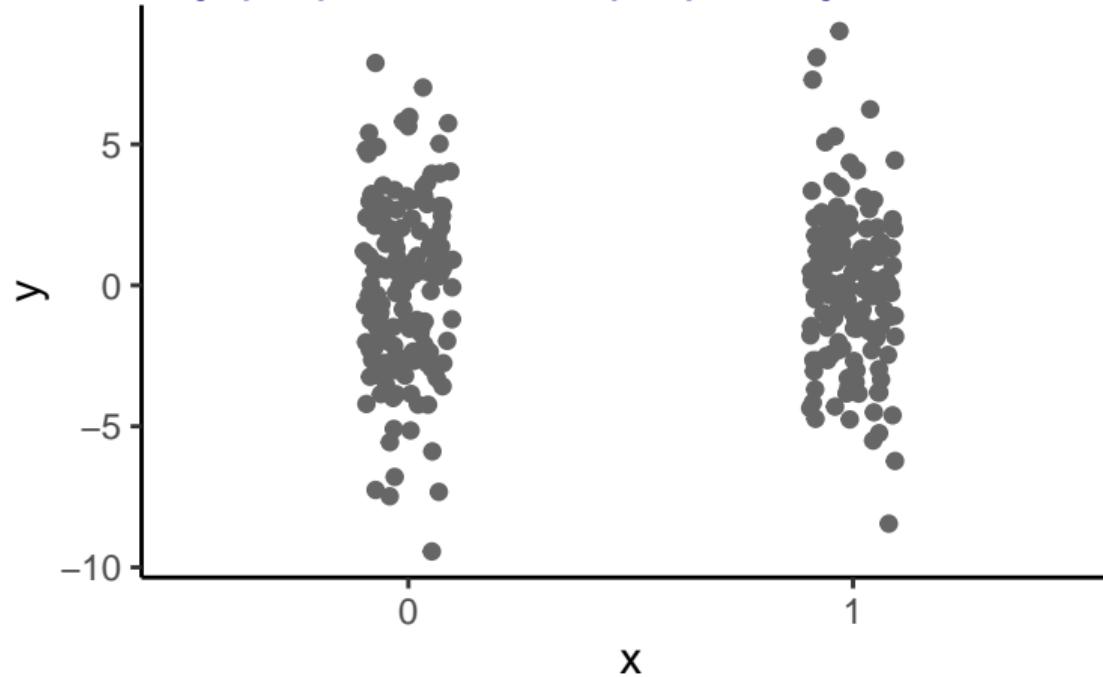
equivalently give weights for  $\forall i$

$$W_i \propto \begin{cases} 1/p(X_i = 1 | C_i) & X_i = 1 \\ 1/p(X_i = 0 | C_i) & X_i = 0 \end{cases}$$

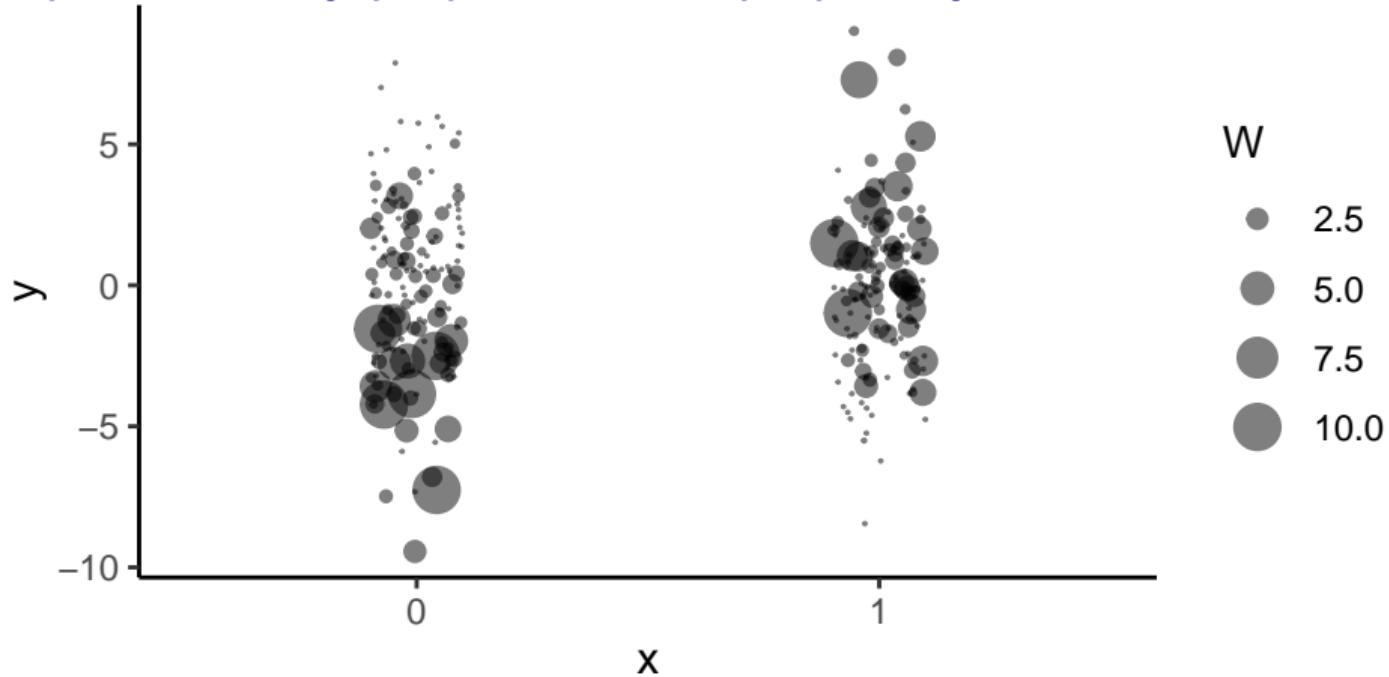
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Take samples inversely proportional to propensity

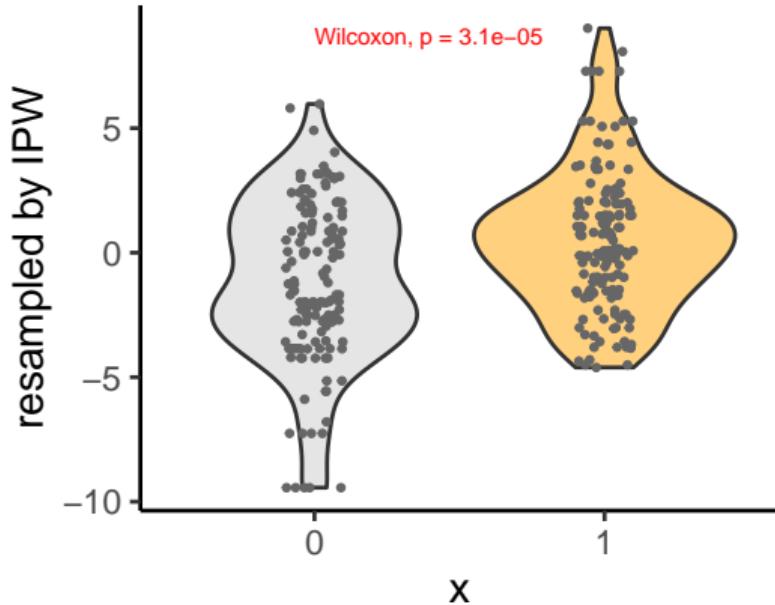
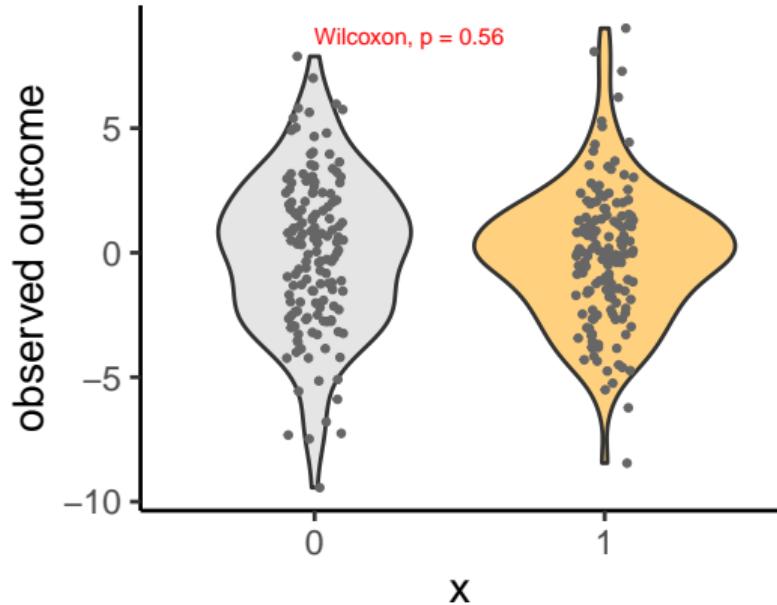


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$$W_i \propto \begin{cases} 1/p(X_i = 1|C_i) & X_i = 1 \\ 1/p(X_i = 0|C_i) & X_i = 0 \end{cases}$$

## Why IPW works? Unbiased estimate potential outcome

Letting  $e(z) = \hat{p}(X = 1|C = z)$ ,

we can prove  $\mathbb{E}[XY/e(X)] \rightarrow \mathbb{E}[Y^{(1)}]$

using

- ▶ Strong ignorability
- ▶ Smoothness
- ▶ Stable Unit Treatment (exposure) Variable

## Why IPW works? Unbiased estimate potential outcome

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$$= \mathbb{E}[Y^{(1)}]$$

When do we use IPW to estimate potential outcomes?

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- ▶ Backdoor variables are sufficiently characterized
- ▶ We have an unbiased way to estimate the propensity model
- ▶ Smooth overlap  $1 > e(X) > 0$

# Today's lecture

What is causal inference (fundamentals)

Graphical language for causal inference

Mendelian Randomization

Potential outcome framework (what if?)

Model-based causal inference

When we don't know an underlying causal graph

## G-formula bridges two different worlds (DAG meets PO)

- ▶ Can we *directly* estimate  $p(Y^{(1)})$  and  $p(Y^{(0)})$ ?
- ▶ In other words,  $p(Y|\text{do}(X = 1))$  and  $p(Y|\text{do}(X = 0))$ .
- ▶ Why is it interesting? Knowing  $p(Y^{(x)})$ , we know  $\mathbb{E}[Y^{(x)}]$  (aka potential outcome).

## The “G”-formula?

“G” for “generalized” method (James Robins, 1986)

Suppose we have characterize potential backdoor variables  $S$  for closing off unwanted paths  $Y \rightarrow X$  to identify a causal path  $X \rightarrow Y$ .

For each condition for exposure variable,  $X = 0$  or  $X = 1$ ,

- ▶  $p(Y^{(1)}) = \int_S p(Y|X = 1, S)p(S)dS$
- ▶  $p(Y^{(0)}) = \int_S p(Y|X = 0, S)p(S)dS$

## How do we derive G-formula?

Bayesian integration results in causal estimand?

$$p(Y^{(1)}) = \int \int p(Y^{(1)}|X, S)p(X|S)p(S)dXdS$$

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## G-formula justifies outcome regression models

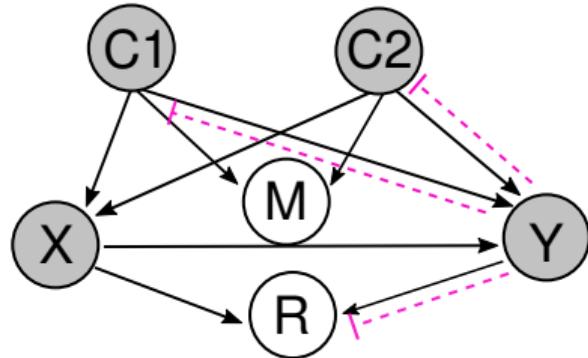
How do we estimate “causal” expectation

$$\begin{aligned}\mathbb{E}[Y^{(1)}] &= \int_y y p(Y^{(1)} = y) dy \\ &= \int_y y \int_S p(Y = y | X = 1, S) p(S) dS dy \\ &= \int_S \mathbb{E}[Y | X = 1, S] dS\end{aligned}$$

Likewise,

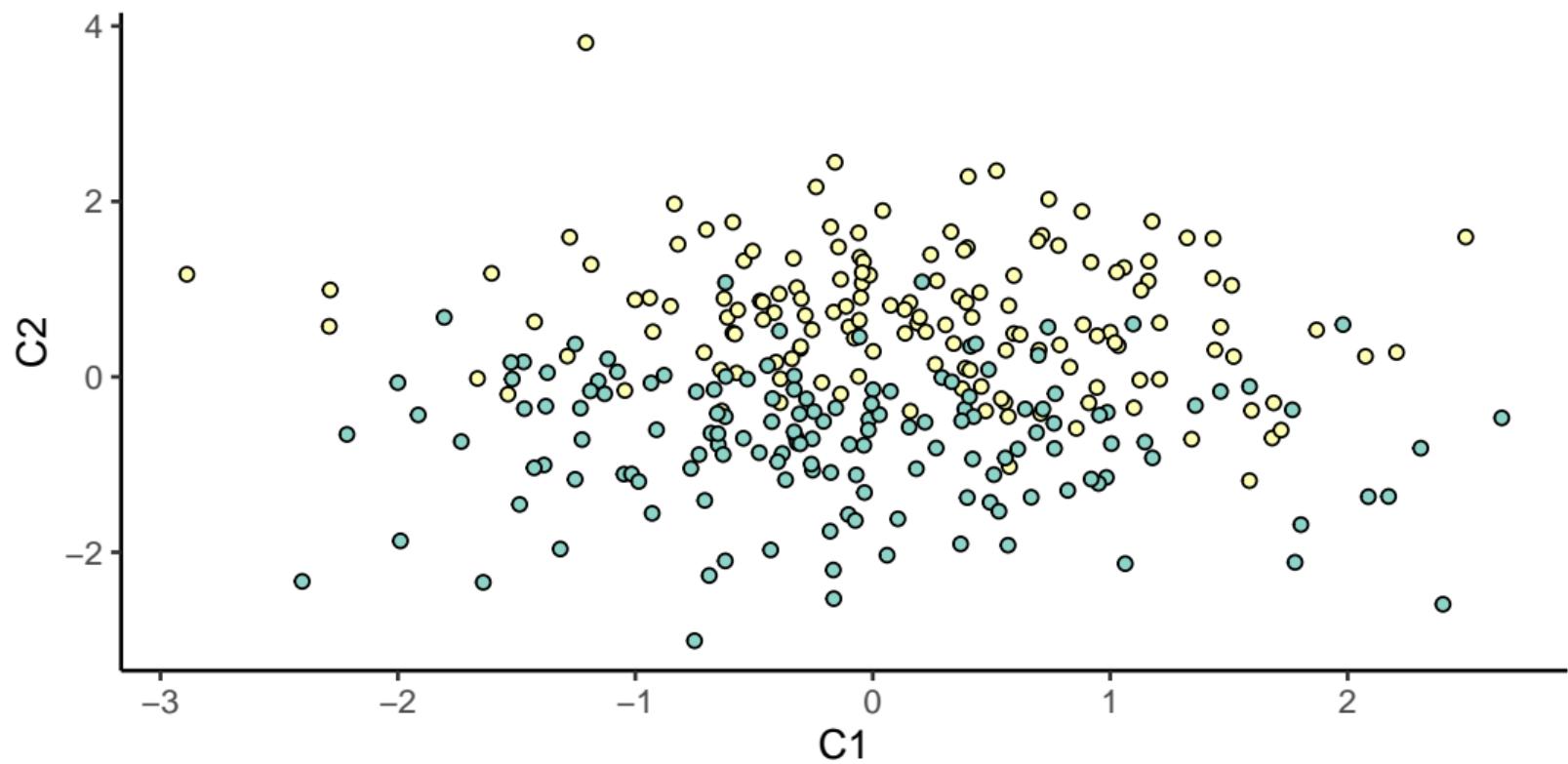
$$\mathbb{E}[Y^{(0)}] = \int_S \mathbb{E}[Y | X = 0, S] dS$$

## Estimating potential outcome by matching

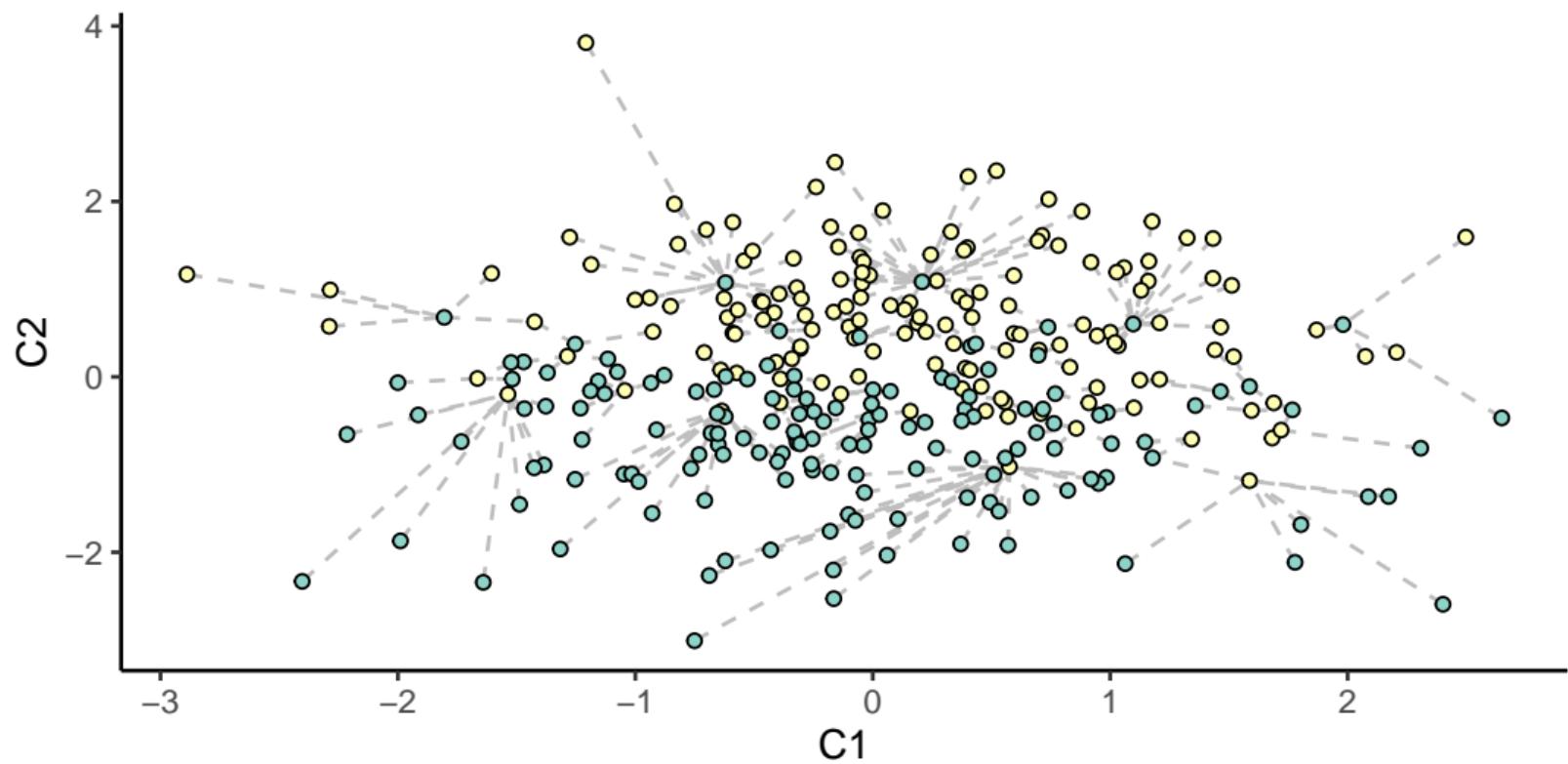


- ▶ Estimate  $\mathbb{E}[Y_i^{(0)}|C_{i1}, C_{i2}]$  for  $X_i = 1$  to compare with  $\mathbb{E}[Y_i|X_i = 1]$
- ▶ Estimate  $\mathbb{E}[Y_i^{(1)}|C_{i1}, C_{i2}]$  for  $X_i = 0$  to compare with  $\mathbb{E}[Y_i|X_i = 0]$

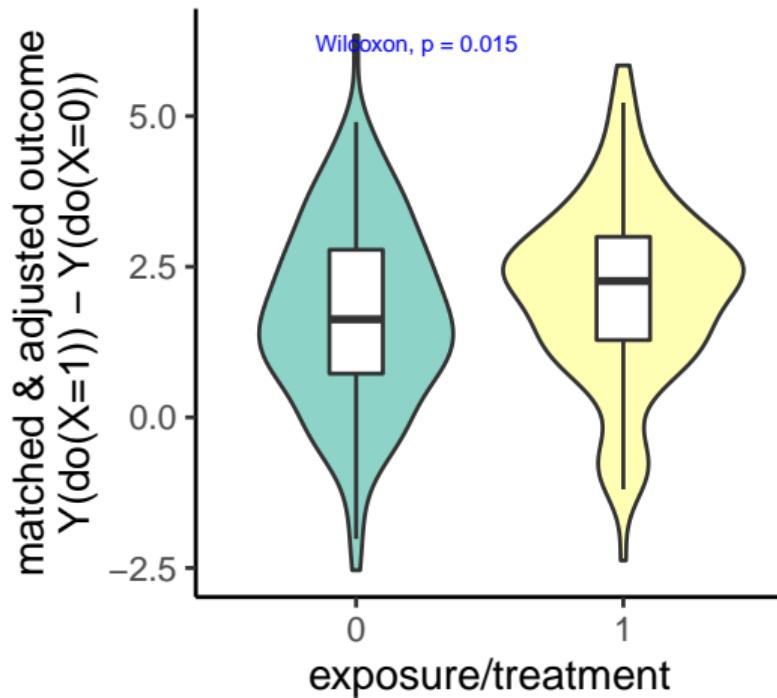
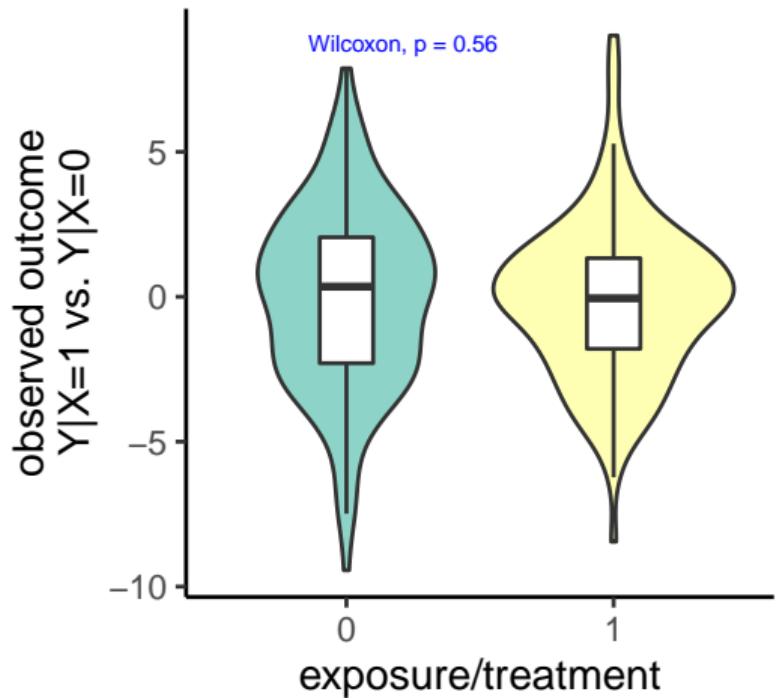
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## Bayesian Additive Regression Tree (BART) approach

- ▶ G-formula → outcome regression models
- ▶ Regression model for  $Y \sim S$  for each  $X = 1$  and  $X = 0$  using BART

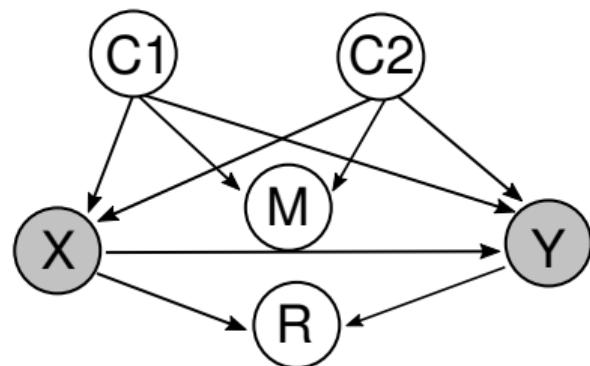
$$\mathbb{E}[Y^{(1)}] = \int_S \mathbb{E}[Y|X=1, S] dS$$

$$\mathbb{E}[Y^{(0)}] = \int_S \mathbb{E}[Y|X=0, S] dS$$

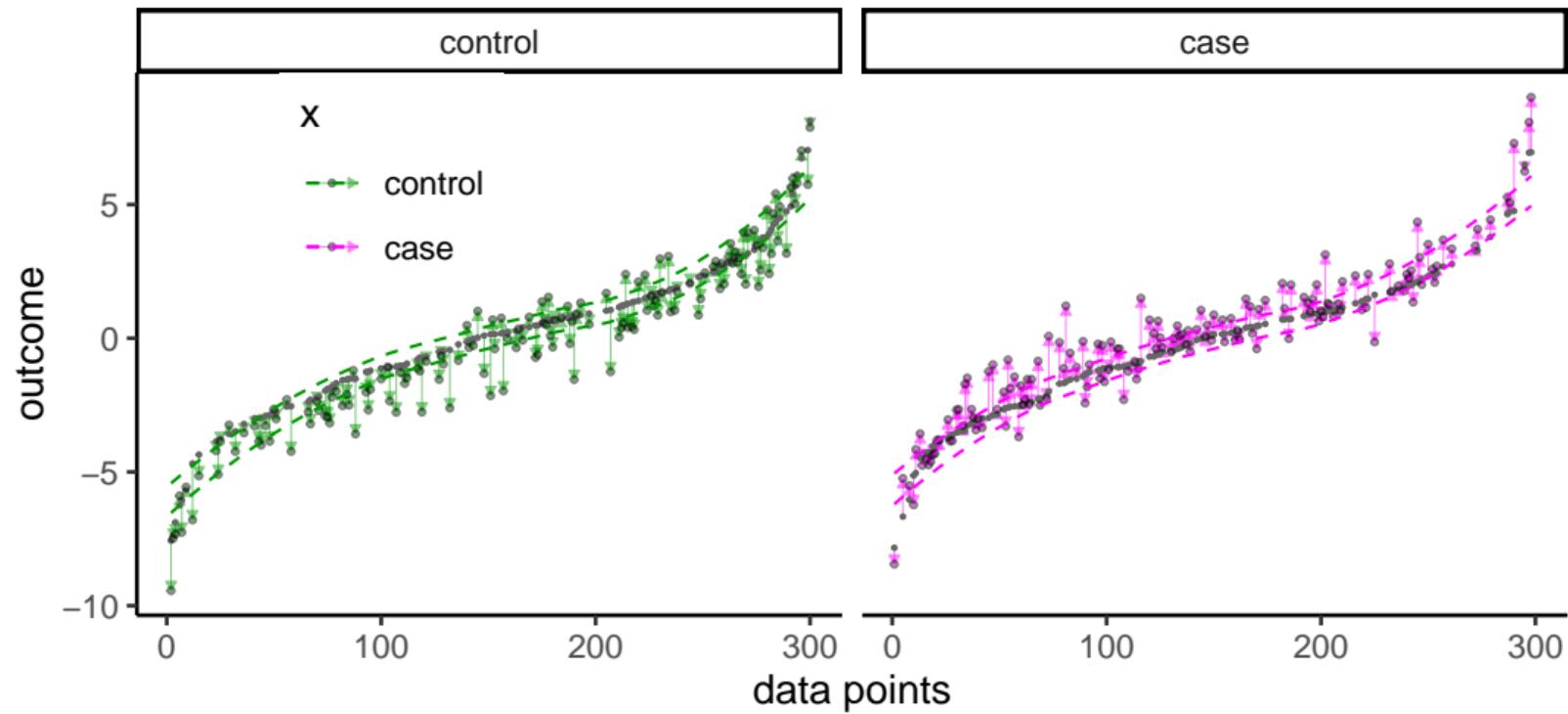
- ▶ Estimate causal effect:  $\mathbb{E}[Y^{(1)}] - \mathbb{E}[Y^{(0)}]$

Hill, *Bayesian Nonparametric Modeling for Causal Inference* (2011)

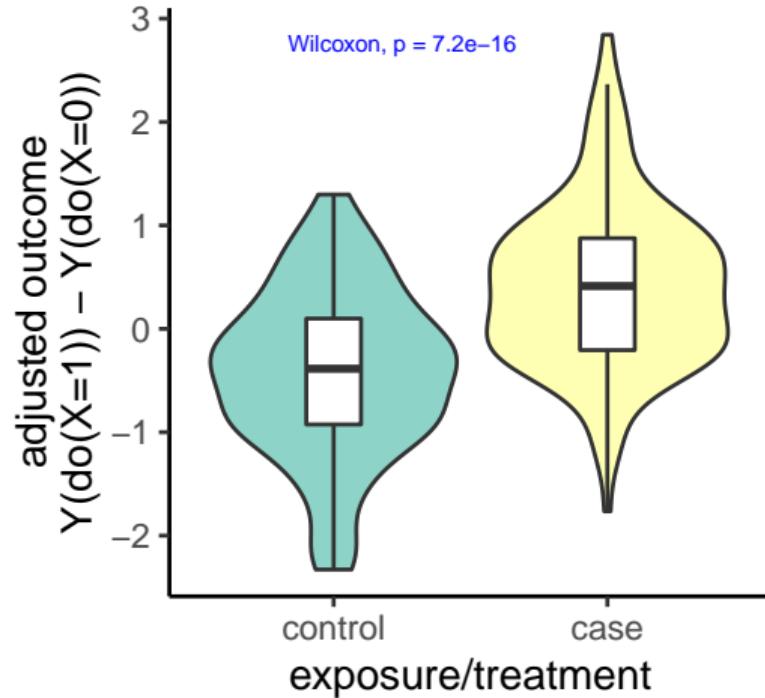
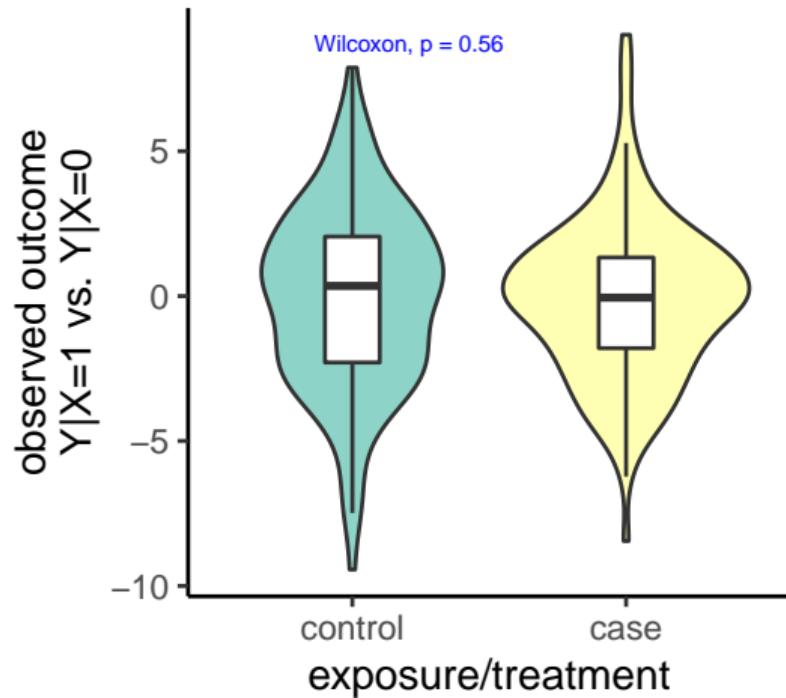
The same example with causal relationship from  $X$  to  $Y$



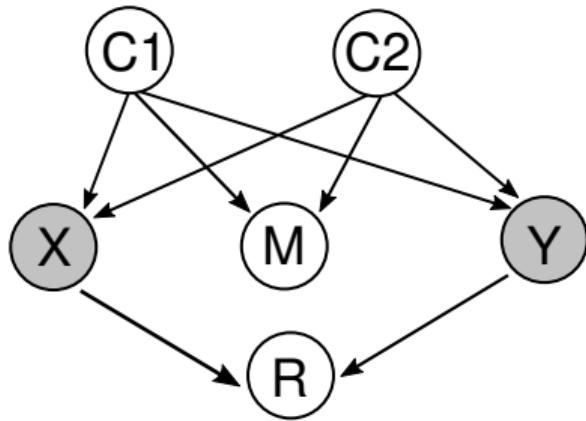
## BART: a regression model to impute potential outcomes



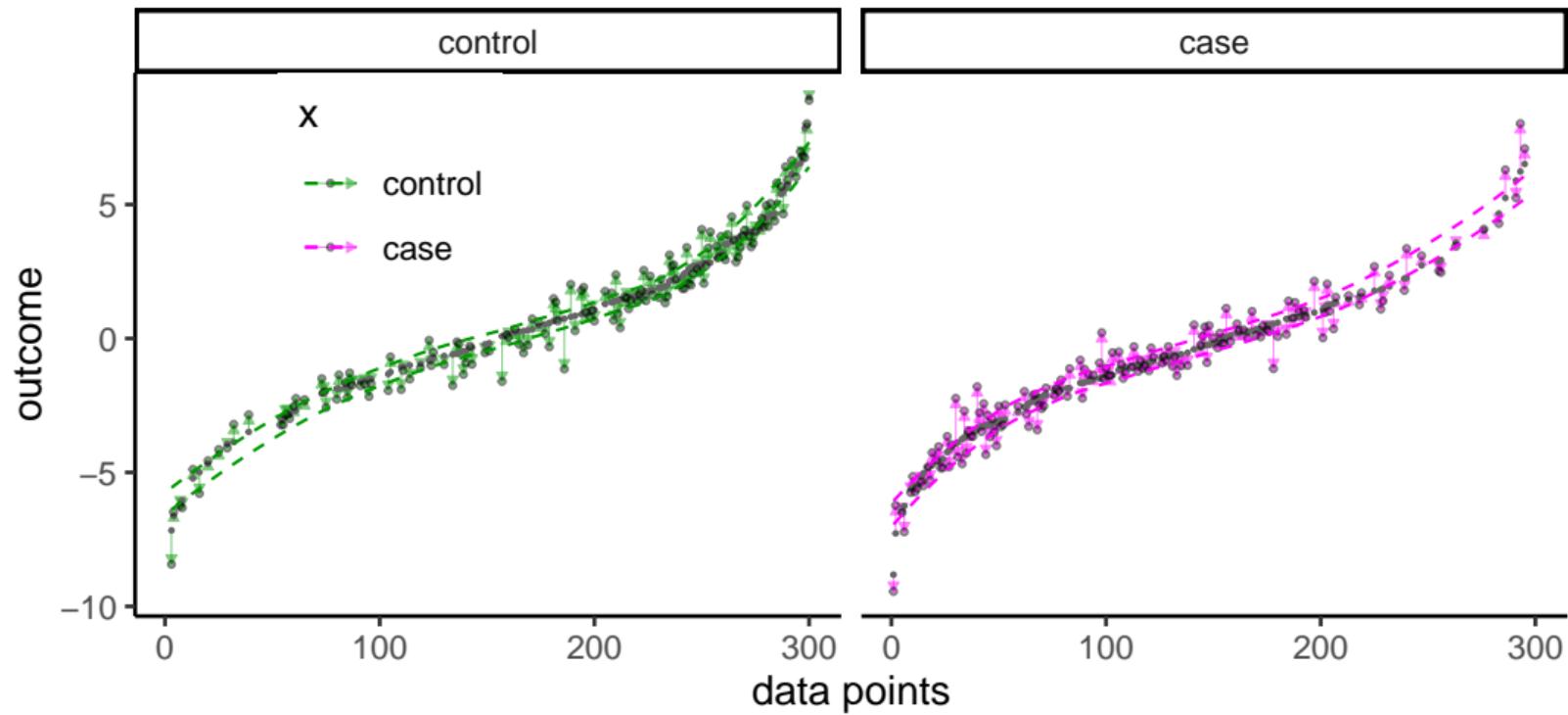
# BART: a regression model to impute potential outcomes



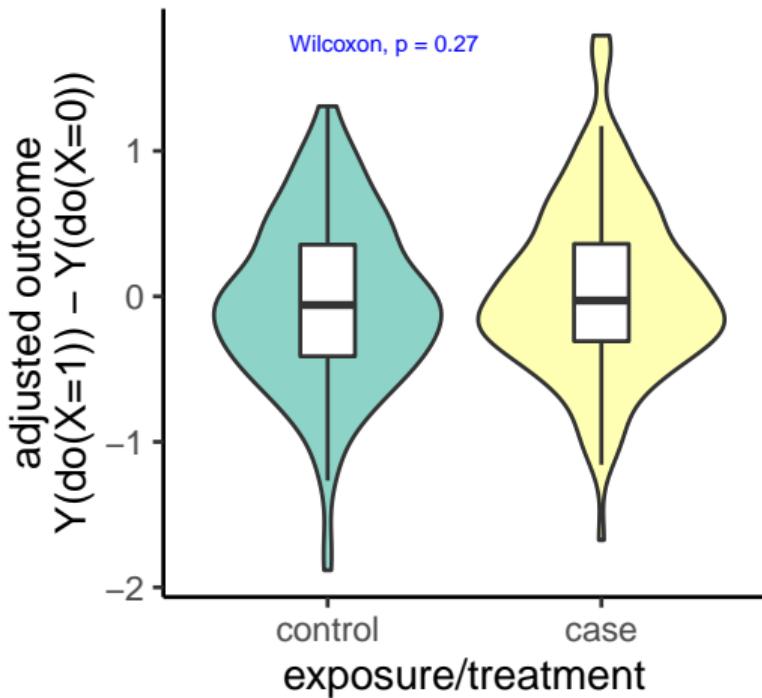
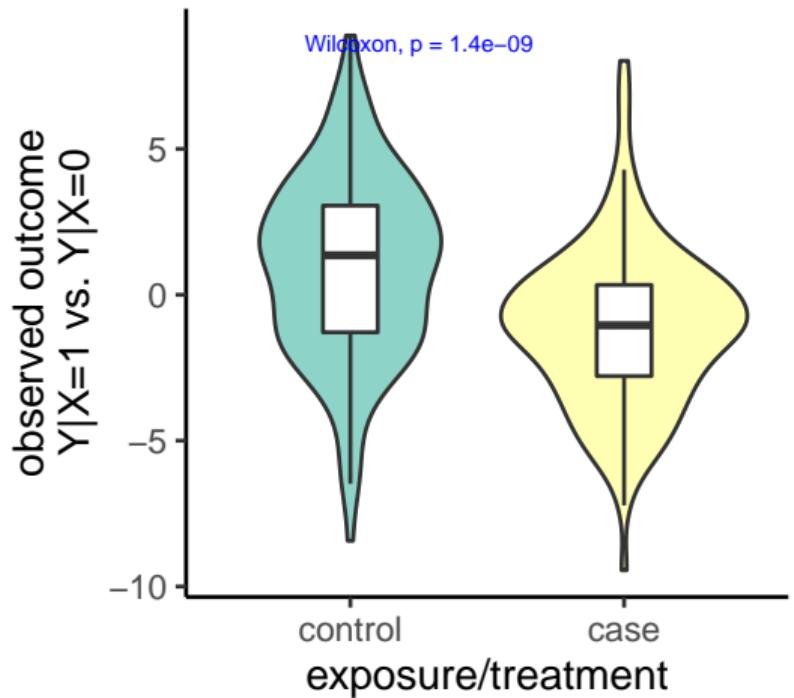
Revisit the example without causal relationship from  $X$  to  $Y$



## BART correctly adjusts spurious correlation



## BART correctly adjusts spurious correlation



# Today's lecture

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Graphical language for causal inference

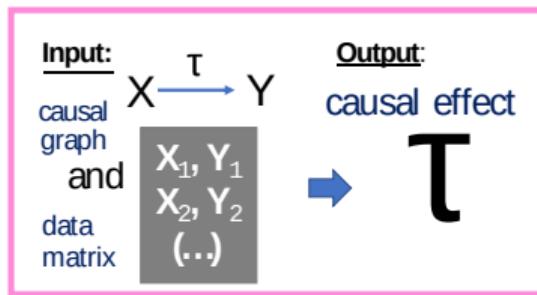
Mendelian Randomization

Potential outcome framework (what if?)

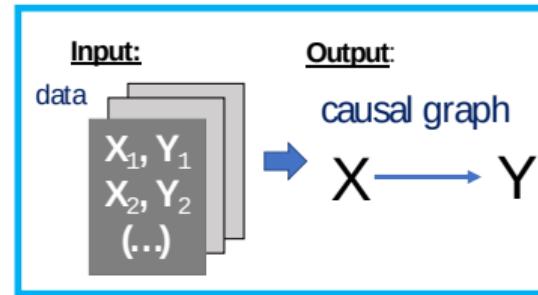
Model-based causal inference

When we don't know an underlying causal graph

## Causal Effect Inference



## Causal structure discovery



# Causal DAG from data: an intractable, yet very tempting problem

## Intractability

Large-sample learning learning of a Bayesian Network is NP-hard.

Chickering and Heckerman, *UAI*, (2002)

# Causal DAG from data: an intractable, yet very tempting problem

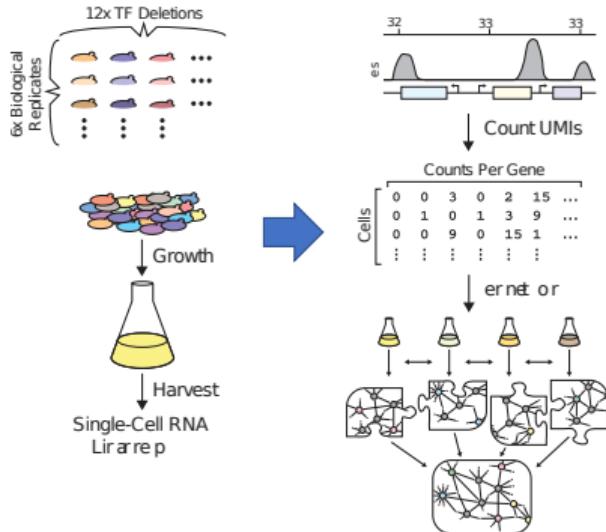
## Intractability

Large-sample learning learning of a Bayesian Network is NP-hard.

Chickering and Heckerman, *UAI*, (2002)

But there is hope if we have "intervention" data

Can we borrow the awesome power of genetics?



## Early 2000's

$X_{11}, X_{12}, X_{13}$   
 $X_{21}, X_{22}, X_{23}$   
(...)

A single or only a handful of conditions

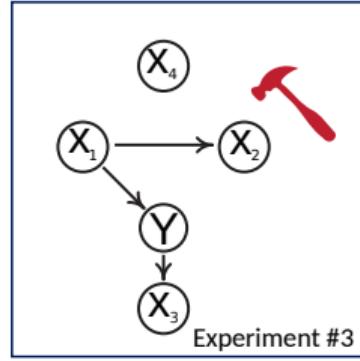
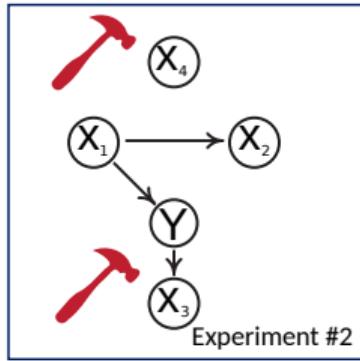
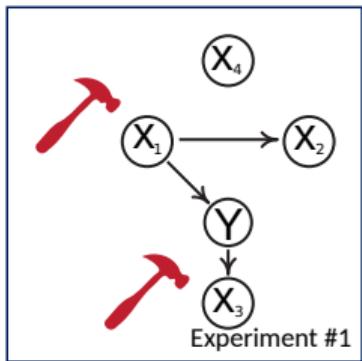
## After perturb-seq

X<sub>11</sub>, X<sub>12</sub>  
X<sub>21</sub>, X<sub>22</sub>  
(...)

Massive (random)  
perturbation assays à  
hundreds of data  
matrices

Jackson et al. (2020)

# Causal Structure Discovery by Invariance

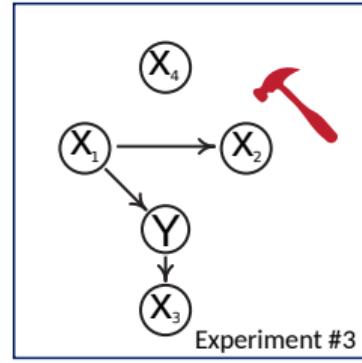
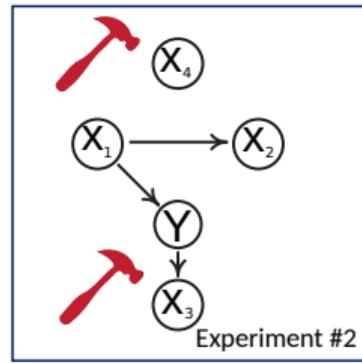
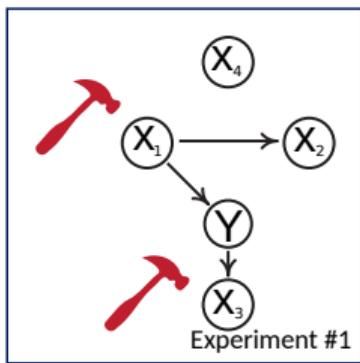


Knock-out ↗  $\text{do}(X=0)$

**The premise:** There is a causal structure that remain **invariant** across multiple experimental conditions.

# Causal Structure Discovery by Invariance

Testing hypothesis (model):  $Y \sim X_1 + \epsilon$



**Knock-out ↗ do( $X=0$ )**

Exp #1:

$$Y = \epsilon_Y \text{ and } X_1 = 0$$

Exp #2:

$$Y = X_1 + \epsilon_Y \text{ and } X_1 = \epsilon_1$$

Exp #3:

$$Y = \epsilon_Y \text{ and } X_1 = 0$$

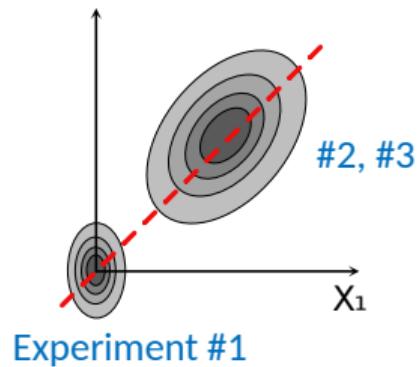
# Does this model make sense in all the experiments?

## Observation:

$$Y = \varepsilon_y \quad \text{Experiment #1}$$

$$Y = X_1 + \varepsilon_y \quad \text{Experiment #2}$$

$$Y = X_1 + \varepsilon_y \quad \text{Experiment #3}$$



## Regression Residuals:

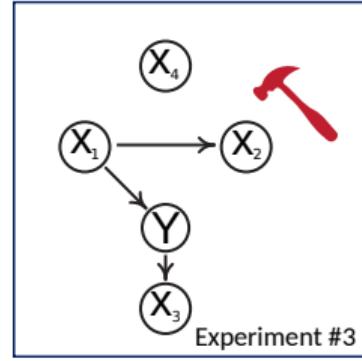
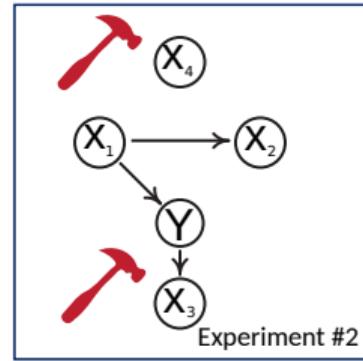
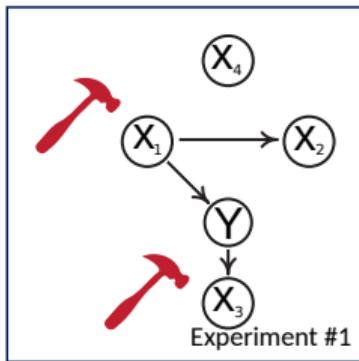
$$\begin{aligned} \varepsilon_y &\sim 0 \quad \#1 \\ X_1 + \varepsilon_y &\sim X_1 \quad \#2 \\ X_1 + \varepsilon_y &\sim X_1 \quad \#3 \end{aligned}$$

The same distribution  
of the residuals

Cannot reject any  
experiments

## Causal Structure Discovery by Invariance - 2

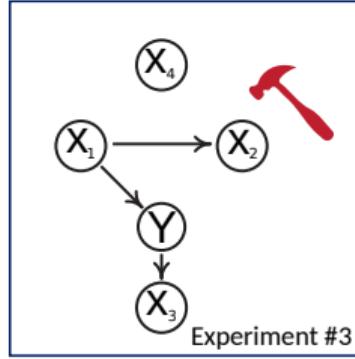
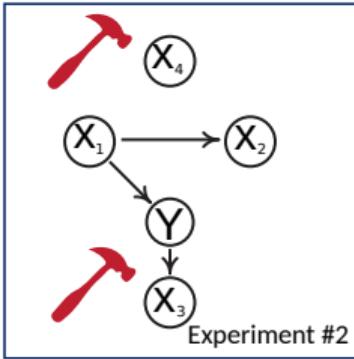
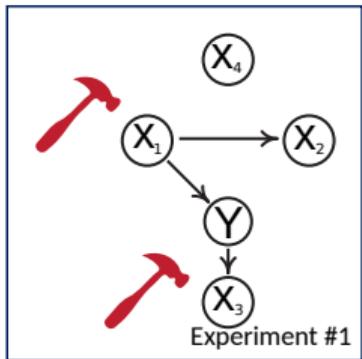
Testing hypothesis (model):  $Y \sim X_1 + \textcolor{teal}{X}_3 + \epsilon$



**Knock-out ↗ do( $X=0$ )**

## Causal Structure Discovery by Invariance - 2

Testing hypothesis (model):  $Y \sim X_1 + \textcolor{teal}{X}_3 + \epsilon$



Exp #1:

$$Y = \epsilon_Y \text{ and } X_1 = 0, X_3 = 0$$

Exp #2:

$$Y = X_1 + \epsilon_Y \text{ and } X_1 = \epsilon_1, X_3 = 0$$

**Knock-out**  $\text{do}(X=0)$

Exp #3:

$$Y = \epsilon_Y \text{ and } X_1 = 0, X_3 = Y + \epsilon_3$$

# Does this model make sense in all the experiments?

Observation:

$$\#1 \quad Y = \varepsilon_y \quad X_3 = 0$$

$$\#2 \quad Y = X_1 + \varepsilon_y \quad X_3 = 0$$

$$\#3 \quad Y = X_1 + \varepsilon_y \quad X_3 = \varepsilon_3$$

Regression Residuals:

$$\begin{aligned} & \varepsilon_y \sim 0 & \#1 \\ \rightarrow & X_1 + \varepsilon_y \sim X_1 + 0 & \#2 \\ & X_1 + \cancel{\varepsilon_y} \sim X_1 + Y + \varepsilon_3 & \#3 \\ & \sim X_1 + (X_1 + \cancel{\varepsilon_y}) + \varepsilon_3 \end{aligned}$$

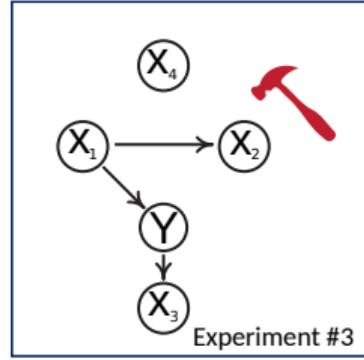
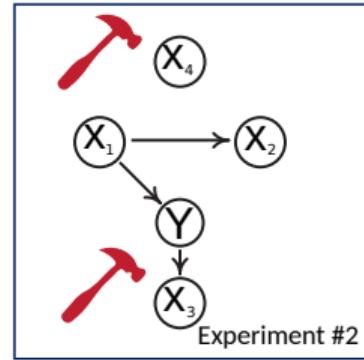
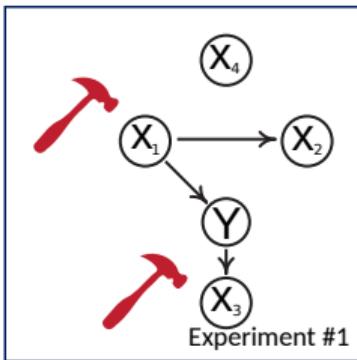
Invariance violated

Reject the experiment #3

→ Reject the invariance of  $X_3$

## Causal Structure Discovery by Invariance - 3

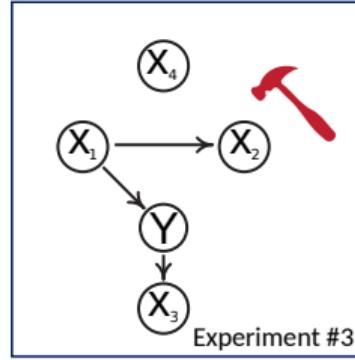
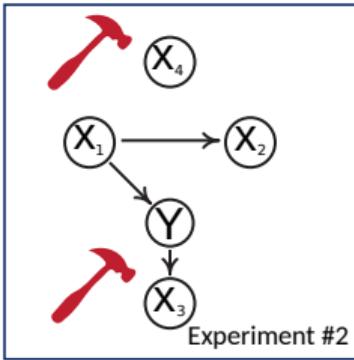
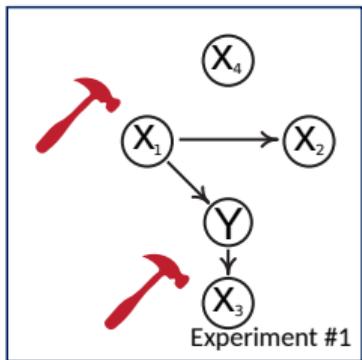
Testing hypothesis (model):  $Y \sim X_2 + \epsilon$



**Knock-out ↗ do( $X=0$ )**

# Causal Structure Discovery by Invariance - 3

Testing hypothesis (model):  $Y \sim X_2 + \epsilon$



Exp #1:

$$Y = \epsilon_Y \text{ and } X_2 = \epsilon_2$$

Exp #2:

$$Y = X_1 + \epsilon_Y \text{ and } X_2 = X_1 + \epsilon_2$$

Knock-out do( $X=0$ )

Exp #3:

$$Y = \epsilon_Y \text{ and } X_2 = 0$$

# Does this model make sense in all the experiments?

## Observation:

$$\#1 \quad Y = \varepsilon_y \quad X_2 = \varepsilon_2$$

$$\#2 \quad Y = X_1 + \varepsilon_y \quad X_2 = X_1 + \varepsilon_2$$

$$\#3 \quad Y = X_1 + \varepsilon_y \quad X_2 = 0$$



## Regression Residuals:

$$\boxed{\varepsilon_y} \sim \varepsilon_2$$

$$\cancel{X_1} + \varepsilon_y \sim \cancel{X_1} + \varepsilon_2$$

$$\boxed{X_1} + \varepsilon_y \sim \varepsilon_1$$

#1

#2

#3

Invariance violated

Reject the experiment #3

→ Reject the invariance of  $X_2$

## Summary

- ▶ Introduction to causal inference
- ▶ Causal structural model (DAG)
- ▶ Confounding factor adjustment
- ▶ Potential outcome model
- ▶ Causal DAG learning