Quiz Date: Friday, 11 January, 2019

Textbook Reading: Section 1.1 (Calc review) and other texts for calculus and linear algebra.

See the Pre-requisites link on the Canvas FAQ for the topics list from calculus and linear algebra.

---

1) **Linear Algebra: Definitions**
Define the following:
   - a linearly independent set of vectors
   - a vector basis for $\mathbb{R}^3$
   - the dimension of a vector space
   - rank of a matrix

2) **Linear Algebra: Gaussian Elimination**
Gaussian elimination, or row reduction, is a systematic way to solve a set of linear equations (Lay, s1.1 & 1.2). Solve the following 3×3 linear system

$$\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 0 \\ & & 2y & - & 8z & = & 8 \\ -4x & + & 5y & + & 9z & = & -9 \end{array}.$$

3) **Linear Algebra: Eigenvalues & Eigenvectors**
Find the eigenvalues $\{\lambda_1, \lambda_2, \lambda_3\}$ and corresponding eigenvectors $\{\vec{w_1}, \vec{w_2}, \vec{w_3}\}$ for the matrix

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$

Give the linear system that determines the linear combination

$$a_1\vec{w}_1 + a_2\vec{w}_2 + a_3\vec{w}_3 = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix},$$

without solving for the a_j coefficients, how can you determine whether this system is solvable?

4) **Linear Algebra: Gram-Schmidt Procedure**
Look up the Gram-Schmidt procedure for producing an orthogonal basis. Apply it to the vector subspace spanned by the vectors

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

---

5) **Calculus: 1D Taylor Series**
Derive the first three terms of the Taylor series for

$$f(x) = \ln x = c_0 + c_1(x-2) + c_2(x-2)^2 + R(x)$$

about the value $x = 2$, where $R(x)$ is the remainder term (Stewart, s11.10). Give the Taylor inequality for the remainder size $|R(x)|$ on the interval $1 \le x \le 3$.

6) **Calculus: 2D Linear Approximation** Derive the best linear approximation (Stewart, s14.4), or linearization, of the function $z(x, y) = xe^{-xy}$ valid near the point $(x, y, z) = (2, 0, 2)$.