Midterm Date: Friday, 01 March, 2019

Textbook Reading: Section 8.1 (linear least-squares), 8.2 (Legendre polynomials), lecture notes on the *real* least-squares. Also, you may wish to review the definitions and theorems associated with matrix rank from your favourite linear algebra reference.

This the last set of assigned questions before the midterm (no quiz during reading week). Bring any questions from weeks 01-07 to the week 08 tutorials.

---

0) **Basic Ideas**
Be very familiar with the following:

- squared-error minimization,
- normal equations as derived from a matrix arithmetic perspective,
- polynomial basis sets & least-squares,
- linear independence & matrix (row & column) rank.

1) **Power-Law Least-Squares**

Follow the standard calculus/statistical approach to derive the 2×2 linear system to obtain the best-fit power-law

$$y(x) = A\,x^B$$

where A and B minimize a log-modified residual

$$\mathcal{E}(A,B) = \sum_{k=1}^{N} \left(\log y_k - \log y(x_k)\right)^2 \ .$$

Note that the linear solve is for unknowns, related to, but not exactly A and B.

Choose your favourite best-fit plot from a previous computing assignment, take at least three data points, and verify the slope that you originally calculated.

2) **Normal Equations**

Write down the *perfect-fit* equations for the least-squares quadratic, $y(x) = c_2x^2 + c_1x + c_3$, based on the four points:

$$\{(-3,13), (-1,0), (1,3), (3,2)\} \ .$$

In the lecture, the Normal Equation was derived using a general matrix-arithmetic notation — use this specific least-squares example to verify the key steps of the argument:

(a) $|\vec{e}\,|^2$ as the squared-error quantity;
(b) the use of $\partial\vec{c}/\partial c_j = \hat{e}_j$ for the derivatives; and
(c) the *transpose of a scalar* final step.

Finally, give the equation that follows from setting $\partial\mathcal{E}/\partial c_1 = 0$

3) **Matrix Rank**

If the above quadratic least-squares is done instead for the points:

$$\{(3,13), (1,0), (1,3), (3,2)\}$$

the matrix rank $[A]_{4\times3}$ is now only 2. Because of this, two of the rows can be used in linear combination to express the other two rows — verify this. Then show that two of the columns can be used in a linear combination to express the other column.
