SFU
muraki

FRIDAY.

- QUIZ TODAY: quiz sheets near entrances
  have calculators ready
- COMPUTING #1 report: due MONDAY

LAST DAY

- PIVOTING.
  - partial pivoting: makes GE robust to small pivots
  - scaled P. Pivoting: avoid badly scaled equations (WRONG PIVOT)
- OPERATION COUNTS
  - a measure of computing effort → is it exactly the same as COMPUTING TIME? (CA2!).
  - GE without pivoting

back substitution $= \sum_1^N (1+2(j-1)) = N^2$

row reduction $= \sum_1^N (N-j)\{1+2(N-j+1)\}$

$= \frac{2}{3}N^3\left(1+\frac{3}{4N}-\frac{7}{4N^2}\right)$

SFU
muraki

· extra overhead for PPiv $O(N^2)$ ☆
Scaled PPiv $O(N^3)$
Full Piv $O(N^3)$

· GE summarized: ACC, EFF & RoB

CONDITION NUMBER

· estimate of "worst case" error for linear solve

$$err \approx \underbrace{K(A)}_{\sqrt{\frac{\lambda_{max}(A^*A)}{\lambda_{min}(A^*A)}}} \cdot \varepsilon$$

↳ last digit of floating-point precision

we don't
the theory behind this
definition, but we can
calculate this:

& so does matlab: (A)

· condition # is a property of [A], but
whether we are in " case", or ,
often depends on
· this is why Matlab gives
· checking is thing to do

muraki

SECTIONS 6.3 + 6.4

- see textbook review of matrix/vector arithmetic
  - multiplying matrices (+ vectors)

$$[\quad]_{M \times N} = [\quad]_{M \times} * [\quad]_{\times N}$$

  - square matrices : (eye)
    (diag)
    (inv)
    + lower triangular (triu + tril)

  - matrix transpose (transpose)

$$[a_{jk}] = [\quad]$$

- ANY QUESTIONS ? ask in TUTORIALS / WORKSHOPS

IS GE AN ALGORITHM or MATHEMATICS ?

  - most generally, an algorithm is a for
    numerical data.

  - mathematical formulas allow us to do
    analytical OR analysis

SFU muraki

GE is a MATRIX (§6.5)

- fact 1) ALL 3 row operations are on the

- $cE_j \to E_j \quad (c \neq 0)$

$$\to \begin{bmatrix} 1 & & & & \emptyset \\ & \ddots & & & \\ & & 1 & & \\ & & & c & \\ \emptyset & & & & \ddots \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} \cdots R_1 \cdots \\ \\ \cdots R_j \cdots \\ \\ \cdots R_N \cdots \end{bmatrix} = \begin{bmatrix} \cdots R_1 \cdots \\ \\ \cdots \quad \cdots \\ \\ \cdots R_N \cdots \end{bmatrix}$$

- $-cE_j + E_k \to E_k \quad (c \neq 0)$

$$\begin{array}{c c} & \\ \text{row } j & \\ \text{row } k & \end{array} \begin{bmatrix} 1 & & & \emptyset \\ & \ddots & & \\ \emptyset & & \ddots & \\ & & & \ddots \\ & & & & 1 \end{bmatrix} \begin{bmatrix} \cdots R_j \cdots \\ \cdots R_k \cdots \end{bmatrix} = \begin{bmatrix} \cdots \quad \cdots \\ \cdots \quad \cdots \end{bmatrix}$$

col j col k

- $E_j \leftrightarrow E_k$

$$\begin{array}{c} \\ \text{row } j \\ \text{row } k \\ \end{array} \begin{bmatrix} 1 & & & \\ & 1 \ddots & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \cdots R_j \cdots \\ \cdots R_k \cdots \end{bmatrix} = \begin{bmatrix} \cdots \quad \cdots \\ \cdots \quad \cdots \end{bmatrix}$$

col j col k

• all 3 matrices are square $N \times N$ & special

$cE_j \to E_j$ is matrix

$-cE_j + E_k \to E_k$ is

$E_j \leftrightarrow E_k$ is a / MATRIX (p411)

(a single " " in &)

• IDEA #1 : no pivot row-reduction is to -multiplying

$[A]\vec{x} = \vec{b}$ by a string of matrices!

matrix for .

$\underbrace{[T \quad T \quad \ldots \quad T]}[A]\vec{x} = [T_M \quad T_{M-1} \quad \ldots \quad T_1]\vec{b}$

$\underbrace{[\quad]}\vec{x} = \vec{\ } = [T_M \quad T_{M-1} \quad \ldots \quad T_1]\vec{b}$

matrix

effect of on right side.

now solve for $\vec{x}$ by .

SFU

muraki

- fact #2 : each of the 3 row operations are (with $c \neq 0$)

$$\left.\begin{array}{l} E_j \to E_j \\ E_j + E_k \to E_k \\ E_j \leftrightarrow E_k \end{array}\right\}$$

exists! & it is to figure them out!

- IDEA #2 : move all $[T_k]$ to left side

$$[A]\vec{x} = [T^{-1} \ldots T^{-1}][u]\vec{x} = \vec{b}$$

- fact #3 : inverse matrix of $-cE_j + E_k \to E_k$ is also & product of is also

$$[A]\vec{x} = [\quad][u]\vec{x} = \vec{b}$$

LU FACTORIZATION/DECOMPOSITION (p409)

Theorem 6.19 If Gaussian elimination can be performed on the linear system $A\mathbf{x} = \mathbf{b}$ without row interchanges, then the matrix A can be factored into the product of a lower-triangular matrix L and an upper-triangular matrix U, that is, $A = LU$, where $m_{ji} = a_{ji}^{(i)}/a_{ii}^{(i)}$,

$$U = \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & \cdots & a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_{n-1,n}^{(n-1)} \\ 0 & \cdots & 0 & a_{nn}^{(n)} \end{bmatrix}, \quad \text{and} \quad L = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ m_{21} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ m_{n1} & \cdots & m_{n,n-1} & 1 \end{bmatrix}.$$

muraki

• for GE with PARTIAL PIVOTING. results in a permuted (row switched) LU factorization

$$\underbrace{[L^P]}_{\text{lower } \Delta.}[U]\vec{x} = [A]\vec{x} = \vec{b}$$

$$\text{or.}\quad [L][U] = [\quad \overset{\text{matrix}}{}\quad][A]$$

MATLAB does LU.

• $[L,U,P] = lu(A)$ gives partial pivot LU fact.

• $[L^P,U] = lu(A)$ gives $L^P = \underbrace{[P^{-1}][L]}_{\text{not lower } \Delta.}$.

```
>> [mL,mU,mP] = lu(A)

mL =

   1.0000        0.        0
   0.2857   1.0000        0
  -0.5714   0.1803   1.0000


mU =

   7.0000  -1.0000  -4.0000
        0  -8.7143   3.1429
        0       0.  -0.8525


mP =

   0   0   1
   1   0   0
   0   1   0
```

SET YOUR CALCULATOR TO RADIANS

SFU

muraki