Computing Modified Lagrange Interpolant

Compute the interpolating polynomial P(x) through the data

x	0	1/3	2/3	1
f(x)	0	0.866	-0.866	0

using the Modified Lagrange interpolating formula.

Optional 1: Repeat the above, but using the Barycentric interpolating formula

Optional 2: Imagine someone now gives you the additional data point (0.25, 1). Using either formula, update your interpolant. Try to minimize the extra work required for this update.

Newton's Method

Use Newton's method to approximate a root of P(x) from the first question, using $x_0 = 1/3$. Do enough iterations such that $|x_n - x_{n-1}| < 10^{-2}$.

Optional: As a comparison, how many iterations of bisection or secant method would be needed to ensure a tolerance of 10^{-2} , using suitable starting conditions?