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FRIDAY
- QUIZ TODAY
- COMPUTING WORKSHOP @ 2^{30}

LAST DAY.
- continuity + numerical solution of nonlinear equations
- REVISIT NEWTON'S METHOD (NM)
  - iterative algorithm, based on SOLVING a LINEAR APPROXIMATION
  - ISSUES: choosing a STOPPING CONDITION
    RATE of CONVERGENCE
    choosing a FIRST GUESS
    FAILURE MODES / LIMITATIONS
- "killer app": solving hanging chain equations in HIGH DIMENSIONS.

FINDING ZEROS / ROOTS
- key question: for what x does $f(x) = 0$?
- also: are there ?
  if then , how ?
  or which might we ?

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- there is ALGORITHM
  - need some thought:

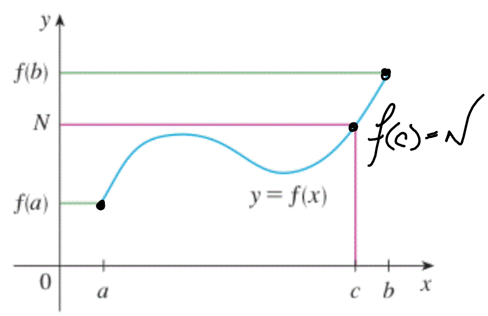
your function

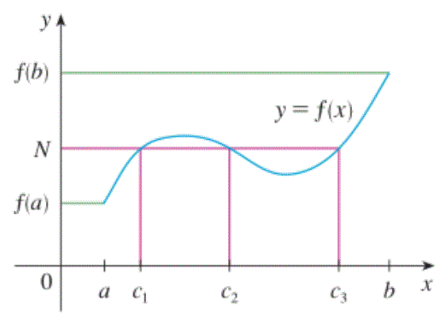
- special case: -

# of zeros = (including multiples)

> **10 The Intermediate Value Theorem** Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

The Intermediate Value Theorem states that a continuous function takes on every intermediate value between the function values $f(a)$ and $f(b)$. It is illustrated by Figure 8. Note that the value N can be taken on once [as in part (a)] or more than once [as in part (b)].





- for $N = 0$, this translates to:

for $f(x)$ on $a \le x \le b$, and $f(a), f(b)$

are with , then there

one value $a <$ $< b$

with $f(x_0) =$

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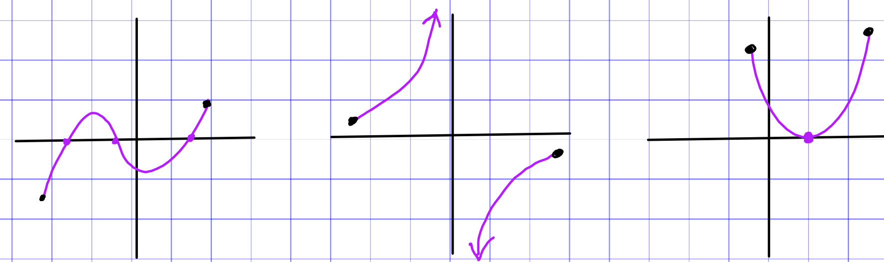
• alternative realities: can be more than
can be if
can be an x_0

such
CONTINUOUS
SIGN CHANGE

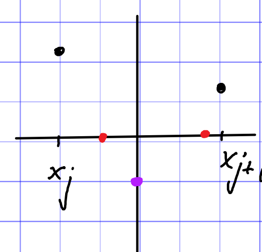


"BRACKETTING" – a search strategy where $f(x)$ is evaluated at a of points $\{a < x_1 < x_2 \ldots < x_n < b\}$ to

$$\{f(a), f(x_1), f(x_2), \ldots f(x_n), f(b)\}$$
$$+ \quad + \quad - \qquad\qquad + \quad -$$

(x_1, x_2) $(?,?)$ (x_n, b) → intervals with zeros!

• NOTE: you can still !



CALCULATING ROOTS as a CONVERGENT PROCESS

• bisection (§6.1) – applies once a (BiS) interval is identified

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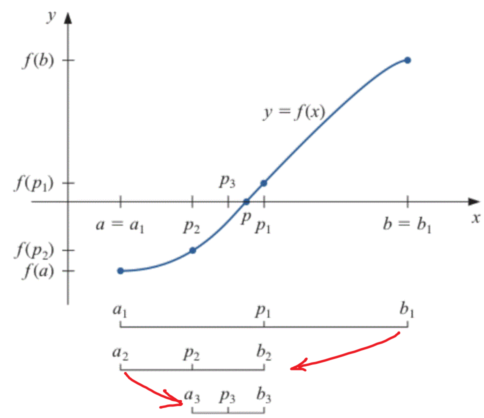
0) start with $x_L < x_R$

& $f(x_L), f(x_R)$ have signs



1) compute

$x_B = x_L + \frac{\quad}{2}$

2) function evaluation $f(\quad)$

3) decision:

if $f(x_B) = \quad$ OR $|\quad| < \epsilon_{TOL}$

accept $x_{final} =$

else update

if $f(x_L), f(x_B)$ same sign $\rightarrow \quad = x_B$

else $\rightarrow \quad = x_B$

4) repeat from step 1

• in the end, we are guaranteed $|x_B - \quad | < \epsilon_{TOL}$

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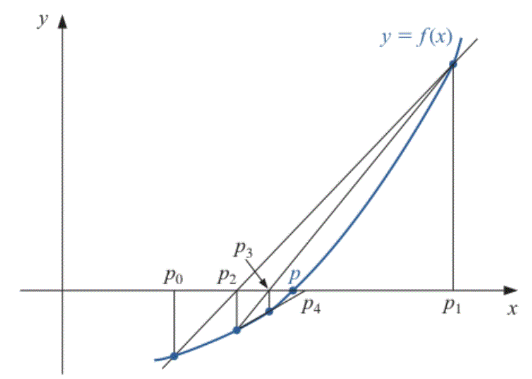
SECANT METHOD (SM). (§6.3)

• another -point method → resembles , as it solves a : approximation:

• line joining $(x_0, f(x_0))$ + $(x_1, f(x_1))$



is

$$y - f(\ ) = \frac{f(\ ) - f(\ )}{\ } \cdot (x - \ )$$

• now find x_2 such that $(\ ,\ )$ is on

$$x_2 = x_1 - f(x_1) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

looks like $(\qquad)^{-1}$

$$\lim_{x_0 \to x_1} \frac{f(x_1) - f(x_0)}{x_1 - x_0} =$$

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0) start with 2 guesses $(x_0, f(x_0))$ + x_1

1) function evaluation : $f(\ \ )$

2) compute secant update, x_{n+1}

$$x_{n+1} = x_n - f(x_n) \cdot \frac{-}{f(\ \ ) - f(\ \ )}$$

3) if $|x_{n+1} - x_n| < \varepsilon_{TOL}$

accept. x_{n+1}

or if. $n =$,

else repeat from step 1.

NOTE: there is no guaranteed preservation of property (p 72)

or even that the interval gets !?

- SM can to converge, but.

→ nonetheless, WHEN IT

SM convergence. than BiS.

(note: the STOPPING CONDITION only checks that the change in x is small + does guarantee $|x_{n+1} - x^*| < \varepsilon_{TOL}$)

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SQUARE ROOT DEMOS

- YOUR notes on bisection DEMO : BiSsqrt.m

- YOUR notes on secant method DEMO : SMsqrt.m

ACCURACY.

- depends on
- relative errors to 10 -10 in $|x_{comp} - x \quad |$

or residual $= f(x_{comp})$ generally

unless function evals to finite-precision

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CONVERGENCE RATE & EFFICIENCY

- HOW FAST DOES ITERATION CONVERGE?
- BISECTION (p 51-52)
  - bracketted root. $x_L^0 < x_e < x_R^0$ exact zero

$$|x_B^1 - x_T| < \frac{x_R^0 - x_L^0}{2} \quad \text{after } 1^{st} \text{ bisection,}$$

$$|x_B^k - x_T| < \frac{x_R^{k-1} - x_L^{k-1}}{2} \quad \text{" } k^{th} \text{ "}$$

and since $x_R - x_L$ decreases by $1/2$ each iteration

$$|x_B^k - x_T| < \underbrace{\left(-\right)^k}_{\text{exponential decay of error}} (x_R^0 - x_L^0) = (\Delta x)^0\, 2$$

$$= (\Delta x^0) \cdot 10^{-(\quad)}$$

.3010

- an additional decimal place of accuracy for every $1/.301 \simeq$. iterations

CONVERGENCE ORDER (p78)

Order of Convergence

Suppose $\{p_n\}_{n=0}^{\infty}$ is a sequence that converges to p, with $p_n \neq p$ for all n. If positive constants λ and α exist with

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^{\alpha}} = \lambda,$$

then $\{p_n\}_{n=0}^{\infty}$ **converges to p of order α, with asymptotic error constant λ.** ■

An iterative technique of the form $p_n = g(p_{n-1})$ is said to be of *order* α if the sequence $\{p_n\}_{n=0}^{\infty}$ converges to the solution $p = g(p)$ of order α.

In general, a sequence with a high order of convergence converges more rapidly than a sequence with a lower order. The asymptotic constant affects the speed of convergence but not to the extent of the order. Two cases of order are given special attention.

(i) If $\alpha = 1$ (and $\lambda < 1$), the sequence is **linearly convergent**. BiS

(ii) If $\alpha = 2$, the sequence is **quadratically convergent**. NM

→ basically, $\dfrac{|\text{error}_{n+1}|}{|\text{error}_n|^{\alpha}} \to$ constant