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MONDAY

- ASSIGNMENTS PAGE
  - Quiz prep. #1 posted → for FRI QUIZ
    - TUTORIALS start this week
  - matlab warm-up worksheets posted → due MON 14 JAN
    - submission via math-apps queuing server
    - access as Canvas quiz (auto-grades)
    - SPECIAL MATLAB HELP SESSIONS

      [math west] LAB: W $15^{30} - 17^{00}$
      R $10^{30} - 12^{00}$

    - plus COMPUTATIONAL OFFICE HOURS (FRI & MON)

What is the MACM 316 E-policy?

To minimize classroom distractions from electronic devices, the math department standard E-policy will be applied:

1. All course members are expected **to respect the audio & visual environment of their classmates**.
2. **No electronic devices** may be in use (ie **zero tolerance**) in the designated e-free zone (for B9200, this is all seats except for the last two rows) --- except by those students who have a contract for specific educational purposes. Unauthorized use of photographic/recording/transmission technology will be considered a violation of privacy and intellectual property.
3. Students who are permitted the use of electronic devices have agreed **to use them in a manner that minimizes the distraction to others**. Violation of this agreement invalidates permission. E-contracts are to be arranged using the Canvas survey tool.

Use of electronic devices in the classroom are limited to students who have filled out the E-contract (link). You must bring a copy of this E-contract to lecture each time you use your device --- you may keep a snapshot of your completed contract on your device, instead of paper copy.

In tutorials & office hours, E-contracts are not required for electronics running Matlab, or for coursework-related materials. Other uses of electronic devices are prohibited.

LAST DAY

· introduction to floating-point numerical computing

· finite-precision : truncation to 16-digits, rounding

· finite-discretizations : discrete SAMPLING of continuous functions

· also numerically-approximated calculus operations

· finite-operations : controlled stopping of convergent iterative algorithms

· THREE KEY QUESTIONS of COMPUTATIONAL LINEAR ALGEBRA & CALCULUS

· ACCURACY : error from exact calc & lin alg

· EFFICIENCY : cost in time/memory/cpu vs problem size

· ROBUSTNESS : reliability across range of inputs

MATLAB DEMOS

· in-class examples & warm-up for computing reports

· "killer" app → special application examples

1) what & are involved?

2) what are the numerical ?

3) how do the results illuminate upon the ?

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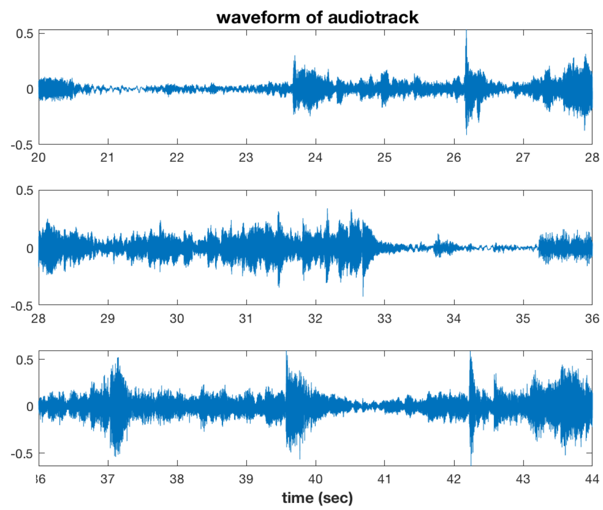
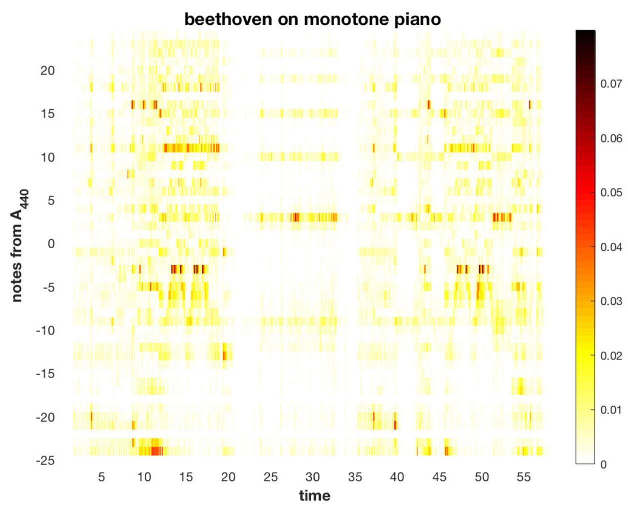
FIRST DEMO

• analysis of piano music

44.1 kHz digital
audio track





SPECTROGRAM

60 sec = M pts

time intervals =

frequencies =

of notes =

• underlying algorithm :

( )

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SOLVING LINEAR SYSTEMS (BF, ch 6)

- Gaussian elimination (GE) for a 2×2 system

$$ax + by = f_1$$

$$cx + dy = f_2$$

$$-- (ax + by) = -- f_2 \quad \Big\} \; \text{add} \rightarrow (--) y = (--)$$

row operation

- solve for y:

$$y = \frac{f_2 - \frac{c}{a} f_1}{d - \frac{c}{a} b} = \frac{-}{-}$$

RULE 's

and $x = -(-)$

- in summary, solution of y by ( ) $\{+, -, \times, / \}$
  " of x by (1-var solve using)

DISCUSSION

1) the GE solution for y involves by = -

→ GE algorithm when det =

- but a theorem of lin alg says solution
  ONLY when det
  → GE theory when det (# of solutions κ or)

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2) the GE solution for x involves by

· what happens for $a =$? $y = /$ $\left.\begin{array}{l} \\ \end{array}\right\}$ ok since b, c

$x = \frac{1}{\ }(\ -\ )$

→ GE as a formula or algorithm is formulated

2') GE does NOT lead to a algorithm - can
equations & order of eliminating variables

(choices for GE)

→ requires choices/decisions to avoid

... and we have addressed finite-prec arithmetic

3) formulas may not give
results under finite-prec arithmetic

→ an issue for "scientific method" & reproducability

4) how do we test for det = case ?

$$\begin{bmatrix} 1 & 0 \\ 0 & \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ \ \end{pmatrix}$$

has the unique solution $(1,1)$ — it is -

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MATLAB & SOLVING NxN LINEAR SYSTEMS.

- there are MANY numerical approaches for solving

$$[A]\vec{x} = \vec{b}$$

- Matlab's simplest is "backslash"

$$\vec{x} = [A] \quad \vec{b}$$

a highly-evolved GE routine

- WHY alternative approaches?

- : what if [A] is a matrix?

matrix (zeros)

- : some mathematical problems involve large matrices

for which GE works

- matrices : solved by algorithms than GE

- solves : many different $\vec{\ }$ for same [ ]

- WHY NOT use matrix ? (or CRAMER's RULE?)

$$\vec{x} = [\quad]\vec{b}$$

(A) in Matlab has extra overhead

→ numer meth. for solving linear systems do produce the inverse matrix as an

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'SIZE MATTERS' DEMO

- a Fourier theorem: every 2π-periodic function $y(t)$

  can be uniquely expressed as a trigonometric series

$$y(t) = \underbrace{\sum_{j=0}^{\infty} a_j \cos jt}_{\text{even symmetric part}} + \underbrace{\sum_{j=1}^{\infty} b_j \sin jt}_{\text{odd symmetric part}}$$

  where a_j & b_j are Fourier coefficients of $y(t)$

  (plus some convergence fine print)

- SAWTOOTH EXAMPLE

23.8 $f(x) = |x| = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}$

$$\frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right)$$

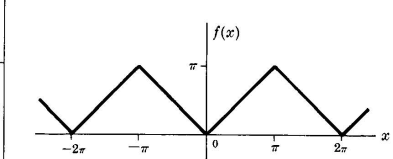

Fig. 23-2

- NUMERICAL QUESTION: given N points $(t_j, y(t_j))$ for $0 \le t \le \pi$

  find an N-term cosine series that satisfies these values.

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- a matrix set of equations

$$\begin{bmatrix} 1 & \cdots & \cos kt_1 & \cdots & \cos(N-1)t_1 \\ \vdots & & \vdots & & \vdots \\ 1 & \cdots & \cos kt_j & \cdots & \cos(N-1)t_j \\ \vdots & & \vdots & & \vdots \\ 1 & \cdots & \cos kt_N & \cdots & \cos(N-1)t_N \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_k \\ \vdots \\ a_{N-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ \vdots \\ y_j \\ \vdots \\ y_{N-1} \end{bmatrix}$$

$k=0$ col

$k=N-1$ col

N Fourier coeffs

N values $y_j = y(t_j)$

- Matlab linear algebra $[M] = [\cos kt_j]_{\substack{j=1\to N \\ k=1\to N}}$ rows cols

$$\vec{b} = \begin{pmatrix} y_0 \\ \vdots \\ y_{N-1} \end{pmatrix}$$

and $\vec{a} = [M] \backslash \vec{b}$

- numerical experiment

1) choose N values t_j, $j=1\to N$
   a) uniformly-spaced
   b) random.
2) solve for $\vec{a}$
3) check residual error: $[M]\vec{a} - \vec{b}$ (zero for perfect world)

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