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CA4 nm.m -- dhpham -- 12 feb 2019
% MAIN USAGE: Create a 16x16 matrix, each row k containing 16
positive roots
% of the m-th Bessel function, where m = 0,1,...,15, k = 1,2,...,16.
% Find these roots using MATLAB's fzero.
% INPUT: Use an existing 'tolfz' as the convergence tolerance if it
exists,
  otherwise use tol = 1e-6.
% OUTPUT:
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            'Amk': the 16x16 matrix containing roots to the m Bessel
funcs.;
   'mean_Nevals': the mean # of func. evals per iteration
     'bsl reserr': the residual error of J m(x) evaluated at each
root z_{m,k} in Amk.
disp('==== USING FZERO =====')
if (exist('tolfz','var') ~= 1)
 % Default convergence tolerance
    tol=1e-6
else
 % check if existant tol is scalar, set to default if not
    if (sum(size(tolfz)==[1 1])~=2)
        disp('tol is not correct size! setting default tol = 1e-6')
        tol = 1e-6;
    else
        tol = tolfz;
    end
    tol
end
% matrix size (kP = # of zeros, mP-1 = max bessel index)
kP = 16;
mP = kP;
% matrix for zeros
Amk = zeros(mP,kP);
% Count for function evals
Nevals = 0;
% define bessel function
bfunc = @(x,mm) besselj(mm,x);
% fzero options, no output unless non-convergence
opt1 = optimset('TolX',tol,'Display','notify');
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% For Bessel functions J_m(x), 0 <= m <= 'mP-1'
for mm = 0:mP-1
    if mm < 1
          Initial bracket for first root z {0,1}:
        % using the hint given in CA4.pdf where z_{0,1} = 2.5
        ri = [2 3];
    else
        % For Bessel functions m > 0, first root z \{m,1\} bracketed
 by:
        z_{m-1,1} < z_{m,1} < z_{m-1,2}
        ri = [Amk(mm,1) Amk(mm,2)];
    end
    % Find 'kP' zeros, 1 <= 'kk' <= 'kP'</pre>
    for kk = 1:kP
        % The k-th root is approx. at z_{m,k-1} + pi, where k > 1
        if kk > 1
            % left endpoint of interval
            rL = floor(Amk(mm+1,kk-1)+pi);
            % check for sign change at J_m(rL) and J_m(rL+ii)
            ii = [1 2];
            sign changes = diff(sign([bfunc(rL,mm) bfunc(rL+ii,mm)]));
            iix = find(sign_changes,1);
            ri = [rL rL+ii(iix)];
        end
        % Find z {m,k} using fzero
        [Amk(mm+1,kk),err,exitflag,output] = fzero(@(x)
 bfunc(x,mm),ri,opt1);
        Nevals = Nevals + output.funcCount;
    end
end
% Output mean Nevals and take residual error as rms
mean Nevals = Nevals / (kP^2)
% = 1000 residual error as rms when J_m(x) evaluated at roots in Amk
Jmk = zeros(kP);
for mm = 0:mP-1
    Jmk(mm+1,:) = besselj(mm,Amk(mm+1,:));
bsl_reserr = rms(Jmk(:))
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