**Quiz Date:** Friday, 15 February, 2019

**Textbook Reading:** Section 3.1 (Lagrange interpolation), lecture notes on *Modified Lagrange interpolant*, 3.3 (Newton basis polynomials), 3.5 (cubic splines).

Reminder: solutions will not be posted, but the TAs are expecting you to bring your questions to the tutorials. You may also bring questions to the Wednesday afternoon office hours.

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0) **Basic Ideas**
Be very familiar with the following:

- performance characteristics of MLI (acc, eff, rob),
- flop count scalings & operational advantages of MLI,
- Newton basis polynomials & its triangular linear solve,
- cubic spline equations & endpoint choices,
- performance characteristics for cubic splines,
- MLI versus splines.

1) **Modified Lagrange & Newton's Interpolations**
Textbook problem from Section 3.1:

#5 (a) Last week, you did this problem using the MLI. You will need to reuse some of that calculation this week, as you will interpolate for the derivative $f'(8.4)$. The best way to do this is to start from the derivative of the log of $L_{MLI}(x)$:

$$\frac{d}{dx}\log\{L_{MLI}(x)\} = \frac{d}{dx}\log\left(L(x)\right) + \frac{d}{dx}\log\left(\sum f_k \frac{w_k}{x-x_k}\right)$$

and evaluate from the chain rule form of the above. Note that many of the terms/factors are reused values from the evaluation of $f(8.4)$. Indicate what new values need to be computed in getting $f'(8.4)$, and explain why it is an $O(N)$ flop count operation.

#5 (a) For the same data, construct the lower triangular linear system that results from a Newton basis that takes $\{x_k\}$ in the order $\{8.1, 8.2, 8.6, 8.7\}$. Note that if one takes the order $\{8.7, 8.6, 8.2, 8.1\}$ one gets a different matrix (although the final interpolant will be the same).

2) **Cubic Spline**
Textbook problems from Section 3.5:

#3 (c) Use the equation (3.21) for the $\{c_k\}$-coefficients to get the $4 \times 4$ linear system that determines the spline with natural end conditions. Use Matlab to obtain the values for the $c_k$, and then choose, from the lecture notes or textbook, the equations that determine the $b_k$ and $d_k$. Interpolate the value of $f(-0.1)$.

#11 Start by writing the complete set of equations for the $N = 1$ spline. Using these equations, find the missing coefficients. (Expect this type of question on the exams.)

3) **Not-a-knot**
From the lecture notes:

For the case of not-a-knot, list the four equations that involve the coefficients from the $S_0$ spline. By eliminating $d_k$-coefficients, derive $c_0 = 2c_1 - c_2$ which gives the extra end condition for not-a-knot. What is then the corresponding condition for $c_N$? (~~You have just verified diagonal-dominance for the not-a-knot!~~) Finally, confirm that these end conditions hold for the numbers shown in the Friday lecture demo.

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