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FRIDAY

- quiz today
- NO COMPUTING WORKSHOPS, FRIDAY
- EXTRA MIDTERM TA OFFICE HOURS: MON 25 FEB
  (during comp workshop hours)
  WATCH for CANVAS ANNOUNCEMENT

LAST DAY

- the REAL L-SQ — matrix solve version

- for OVERDETERMINED (   ) linear system

$$[A]_{M\times N}\,\vec{v} = \vec{y}$$

- a L-Sq solution ($\mathcal{E}$ could be zero) exists for

$$M > N \geq \text{rank}\,[A] = \text{rank}\,[A^TA]$$

and

- when $N = \text{rank}\,[A] = \text{rank}\,[A^TA]_{N\times N}$

  then $\vec{v}_{LSQ}$ is

• RANK & LINEAR INDEPENDENCE: definitions

• RANK THEOREM: rank = rank

• L-SQ POLYNOMIALS

use Legendre polynomials on $\leq x \leq$

• shift & rescale domain

$a \leq x \leq b$

means

$-1 \leq \underline{\qquad\qquad} \leq 1$

Rodrigues' formula and other explicit formulas [edit]

An especially compact expression for the Legendre polynomials is given by Rodrigues' formula:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n .$$

This formula enables derivation of a large number of properties of the P_n's. Among these are explicit representations such as

$$P_n(x) = \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k}^2 (x-1)^{n-k}(x+1)^k,$$

$$P_n(x) = \sum_{k=0}^{n} \binom{n}{k} \binom{n+k}{k} \left(\frac{x-1}{2}\right)^k,$$

$$P_n(x) = \frac{1}{2^n} \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} (-1)^k \binom{n}{k} \binom{2n-2k}{n} x^{n-2k},$$

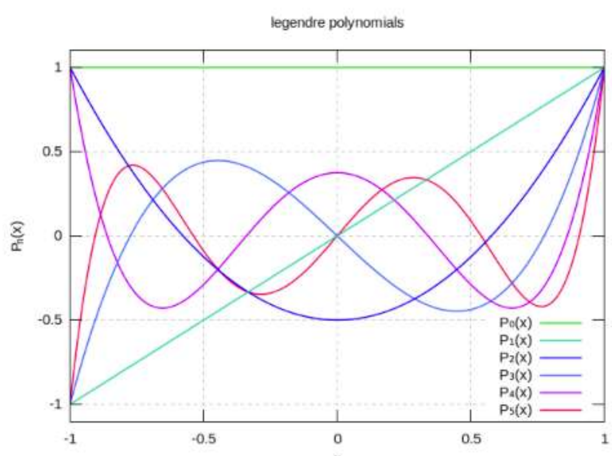
$$P_n(x) = 2^n \sum_{k=0}^{n} x^k \binom{n}{k} \binom{\frac{n+k-1}{2}}{n},$$

where the last, which is also immediate from the recursion formula, expresses the Legendre polynomials by simple monomials and involves the multiplicative formula of the binomial coefficient.

The first few Legendre polynomials are:

n	$P_n(x)$
0	1
1	x
2	$\frac{1}{2}(3x^2-1)$
3	$\frac{1}{2}(5x^3-3x)$
4	$\frac{1}{8}(35x^4-30x^2+3)$
5	$\frac{1}{8}(63x^5-70x^3+15x)$
6	$\frac{1}{16}(231x^6-315x^4+105x^2-5)$
7	$\frac{1}{16}(429x^7-693x^5+315x^3-35x)$
8	$\frac{1}{128}(6435x^8-12012x^6+6930x^4-1260x^2+35)$
9	$\frac{1}{128}(12155x^9-25740x^7+18018x^5-4620x^3+315x)$
10	$\frac{1}{256}(46189x^{10}-109395x^8+90090x^6-30030x^4+3465x^2-63)$

The graphs of these polynomials (up to $n = 5$) are shown below:



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- take points $\{(x_k, y_k)\}_{k=1\to M}$

  + change to $\{(z_k, y_k)\}_{k=1\to M}$

- use $\vec{c} = [A] \backslash \vec{y}$?

  and $y(x) = \sum_{j=1}^{N} P\left(\frac{}{}\right)$?

- DEMO OBSERVATIONS
  - increasing M (more good data) is better
    - $M \geq N^2/4$ using Legendre poly is ROBUST for large N
  - increasing N leads to large cond # warnings.

- REMAINING question — what is Matlab \ doing ?

MIDTERM REVIEW

• MACM 316 COURSE STRUCTURE

TUTORIALS	LECTURES + TEXTBOOK	COMPUTING WORKSHOPS
problems + QUIZZES		computing + REPORTS
MIDTERMS + EXAMS		

INTRO TO NUMERICAL COMPUTING.

• digital computation with FLOATING POINT algorithms

- •   truncations of real numbers
- •   algorithms (stopping of convergent processes)
- •   of continuous functions (sampling)

• THREE KEYS of ALGORITHM PERFORMANCE

- •
- •
- •

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FLOATING POINT ARITHMETIC (§1.2)

- truncation errors in FP arithmetic
- finite-precision calculations
- absolute vs relative errors
- simple strategies for reducing accumulation of error

Skills:

GAUSSIAN ELIMINATION & SOLVING LINEAR SYSTEMS (ch 6)

- row operations on augmented matrix
- row reduction & back-substitution algorithm steps
- failures of GE: exact & numerical

- partial pivoting & related modifications
- operation counts for RRed (O( )) & BSub (O( ))
- condition number $K(A)$ & solution error

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GE SUMMARY

- ACCURACY → due to accumulated finite-prec truncation
  - minimized by (part., scaled, full)
- EFFICIENCY → op counts are $O(N^3)$
  - only can do better for special matrices (NON-GE algs)
    (, Fourier matrices, etc)
    mostly zeros in matrix
- ROBUSTNESS → pivoting avoids zero/small PIVOTS
  - still need to worry if matrix is near-singular
    OR large CONDITION #

Skills:

LU FACTORIZATION

- row operations as matrix multiplies (+)
- LU solution of linear system
- special matrix solves (diag dom, tri-diagonal, ...)

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Skills:

ZEROS of CONTINUOUS FUNCTIONS (ch 2)

- continuity, intermediate value thm & bracketing
- NM, calc 1 revisited, &
- BiS & SM algorithms
- convergence rate & order of convergence
- error analysis for BiS, SM & NM
  (order of convergence: , ...,)

ROOT FINDING SUMMARY

	ACC / EFF	ROB	
BiS		preserve bracket.	
NM	conv eval / iter	need / |<1 better to start close	need slow/wrong start
SM	conv eval / iter	need / |<1 but No bracket start	
fzero	Bis/SM hybrid.	better for bracket start	

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Skills :

POLYNOMIAL INTERPOLATION (ch 3+)

- polynomial basis sets + linear algebra of interpolation
- monomial basis + poor conditioning of Vandermonde matrix
- Lagrange interpolating polynomial + error term
- NLI as interpolant algorithm of choice, operation counts
- Newton interpolating polynomial
- cubic splines: equations + end conditions (, ,)

Skills :

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POLYNOMIAL INTERPOLATION

	MLI	cubic spline
# polys	ONE of deg	N, each deg
eval	$L_{MLI}(x)$	must find $x_k \le x < x_{k+1}$
continuous derivs	all	, , only
large N	for most $\{x_k\}$	
convergence,	OK, but not guaranteed	$(\max \Delta x)^4$, n-a-k
efficiency	$O(\ )$ pre-comp, $O(\ )$ eval	$O(\ )$

LEAST-SQUARES (ch 8.1-2+)

- linear L-Sq + matrix formulation $[A]\vec{v} = \vec{y}$
- Normal Equation: $[A^T A]\vec{v}_{LSQ} = [A^T]\vec{y}$
- overdetermined + UNIQUE L-Sq: $M > N = \text{rank}(A)$

Skills:

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MIDTERM

- 5-6 quiz-like pages with multiple points
- focus on understanding algorithms + theory
  + implementation trade-offs
- no coding questions, but know implementation issues
  (Matlab input + output details, for instance)
- do familiar questions first, prioritize high-value parts
- BRING CALCULATOR (+ practice using it)
- know ACC/EFF/ROB narratives
- study lecture notes (+ text sections)
  problems, quizzes + computing assignments

  these are all inter-related!

- STUDY QUIZ SKILLS + PROBLEM RECOGNITION

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