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WEDNESDAY
- quiz on FRIDAY
- matlab warm-up: due MONDAY
- TUTORIALS start TODAY

Tutorials & TAs (starting Wednesday 09 January 2019):
in WMC2830 [math west]
D101: W 14:30-15:30 - Hudson
D102: W 15:30-16:30 - Dan
D103: W 16:30-17:30 - Dan
D104: R 9:30-10:30 - Jackie
D105: R 10:30-11:30 - Jackie
D106: R 11:30-12:30 - Hudson

LAST DAY
- GE, Gaussian Elimination for solving linear systems
- theory & numerical considerations
- Matlab's "\" backslash

BACKSLASH: TEST DRIVE
- purpose: observe impact of - on
- experimental method:

1) for a matrix size , make a matrix

$$[A] = [N \times N \quad] + \frac{1}{\sqrt{\quad}} \cdot [\quad \text{matrix}]$$

(factor for "theoretical scale" matrices)

use "randn" normal dist, mean, var =

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2) make $\vec{x}_0 = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ the vector of all

& set $\vec{b} = [\quad]$

3) use Matlab to solve $\vec{x}_1 = [\quad] \backslash \vec{\ }$

4) because of finite-precision arithmetic,

we do expect. $\vec{\ } = \vec{\ }$

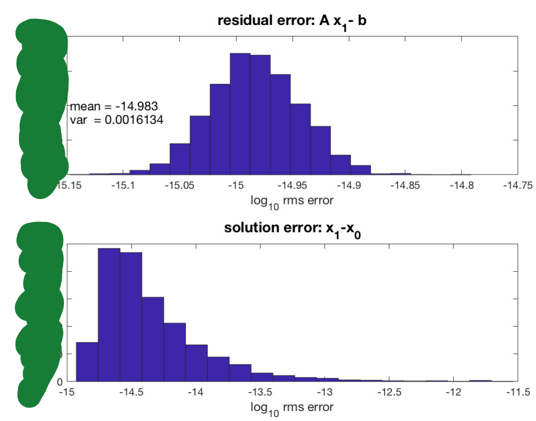
error $\equiv \vec{\ } - \vec{\ }$

error $= [\quad]\vec{\ } - \vec{\ }$

• question: how do the change

as increases ?

• observations:



• conclusions:

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• the test drive will show that error with matrix size
(more $\Rightarrow$ more error)

THE

• a Fourier theorem: every 2π-periodic function $y(t)$
can be uniquely expressed as a trigonometric series

$$y(t) = \underbrace{\sum_{k=0}^{\infty} \cos jt}_{\text{even symmetric part}} + \underbrace{\sum_{k=1}^{\infty} \sin jt}_{\text{odd symmetric part}}$$

where ✓ are Fourier coefficients of $y(t)$
(plus some convergence fine print)

• SAWTOOTH EXAMPLE

23.8 $f(x) = |x| = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}$

$$\frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right)$$

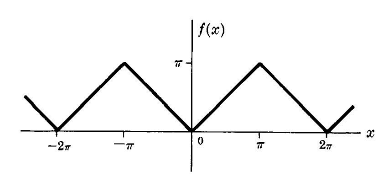

Fig. 23-2

• NUMERICAL QUESTION: given points $(\;, y(\;))$ for $0 \le t \le \pi$
find an -term series that satisfies these values.

• a matrix set of equations

$$\begin{array}{c} j= \\ \\ \\ \\ j= \end{array}\begin{bmatrix} 1 & \cdots & \cos kt_1 & \cdots & \cos(N-1)t_1 \\ \vdots & & \vdots & & \vdots \\ 1 & \cdots & \cos kt_j & \cdots & \cos(N-1)t_j \\ \vdots & & \vdots & & \vdots \\ 1 & \cdots & \cos kt_N & \cdots & \cos(N-1)t_N \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_k \\ \vdots \\ a_{N-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ \vdots \\ y_j \\ \vdots \\ y_{N-1} \end{bmatrix}$$

$k=$ $\qquad$ $k=$ $\qquad$ N Fourier coeffs $\qquad$ N values $= y(t_j)$

• Matlab linear algebra $\quad [\ \ ] = [\cos kt_j]$ $\quad j=1\to N$ rows, $k=1\to N$ cols

$$\vec{\ } = \begin{pmatrix} y_0 \\ \vdots \\ y_{N-1} \end{pmatrix}$$

and $\quad \vec{a} = [\ \ ]\backslash\vec{\ }$

• *experimental method*

1) choose N values t_j, $j=1\to N$
   a) uniformly-spaced
   b) random.
2) solve fn $\vec{a}$
3) check residual error: $[\ \ ]\vec{\ } - \vec{\ }$ $\left(\text{fn perfect world}\right)$

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- observations:

uniform t_j →

random t_j →

- CONCLUSION: there are matrices for which the numerical error is much larger than one might have hoped

EXTREMES of COMPUTER ARITHMETIC (§1.2)

- digital numbers (INT & REAL) are stored in BINARY form (Øs & 1s)

```
>> intmax

ans =

 int32

  2147483647

>> intmin

ans =

 int32

  -2147483648

>> realmax

ans =

 1.7977e+308

>> realmin

ans =

 2.2251e-308

>> eps

ans =

  2.2204e-16
```

"2^16" ″

# of distinct 16-bit binary numbers $= 2^{16}$

" " 32-bit " " $= 2^{32}$

$-32768 \leq n_{16\text{-bit}} \leq 32767$

(Ø breaks symmetry)

- FLOATING-POINT REPRESENTATION is based on notation

(N = 16 significant digits for -bit precision) muraki

MATLAB FLOATING-POINT (different from BF)

$$\pm\,(d_1 . d_2 d_3 \dots d_N) * 10^{p}$$

- largest magnitude $\pm$ realmax $\sim 1.79 \times 10^{+308}$

  violating this is

- smallest magnitude $\pm$ realmin $\sim 2.22 \times 10^{-308}$

  violating this is

- estimate of # in universe $10^{72} - 10^{87}$

UNAVOIDABLE ERRORS IN COMPUTER ARITHMETIC

- addition of floating point

$$a + b = (+D_a * 10^{p_a}) + (+D_b * 10^{p_b})$$

- need to add like powers of 10 (say $p_a > p_b$)
- example, say $p_b = p_a - 5$

$$\begin{array}{rllll} & a.aaa & aaaa & aaaa & aaaa \;\; & & * 10^{p_a} \\ + & \underbrace{. \;\; bbb}_{\text{zeros for } 10^{p_a-p_b}} & & bbbb & bbbb \;\big| & \underbrace{bbbb \;\; b}_{\text{last 5 digits used}} & * 10^{p_a} \end{array}$$

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- can simulate smaller N on calculator
  - add $\left(\frac{1}{3} * 10^5\right) + \left(\frac{1}{7} * 10^3\right)$

```
>> (1/3)*1e5

ans =

     3.333333333333333e+04
```

keeping only 5 digits (IEEE std uses rounding*)

$$
\begin{array}{rll}
 & 3.3333 & \times 10^4 \\
+ & 0.0143 & \times 10^4 \\
\hline
c & 3.3476 & \times 10^4
\end{array}
$$

if carry, drop & increment

(* rounding occurs in binary.)

- multiplication has _ of a digit problem

$$
\begin{array}{rlllll}
 & a.aaa & \ldots & \ldots & aaaa & \times 10^{p_a} \\
\times & b.bbb & \ldots & \ldots & bbbb & \times 10^{p_b} \\
\hline
 & c\,c.ccc & \ldots & \ldots & cccc & \times 10^{p_a+p_b}
\end{array}
$$

if carry $\longrightarrow$ then drop

possible

or

(similar for division)

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- subtraction is interesting for nearly equal numbers

$$
\begin{array}{rl}
 & A.AAa\ aaaa\ \ldots\ aaaa \times 10^{p_a} \\
- & A.AAb\ bbbb\ \ldots\ bbbb \times 10^{p_a} \\
\hline
 & 0.00c\ cccc\ \ldots\ cccc \times 10^{p_a} \\
 & c.ccc\ cccc\ \ldots\ \underbrace{c000}_{\text{"padded" zeros}} \times 10^{p_a-3}
\end{array}
$$

ANSWER contains only 13 sig digits (poss)

- in sequential calculations $(a-b)*d \to$ still only 13 sig digits (even though 16 non-zero digits)

- in sequential calculations, need to round-off at every intermediate step