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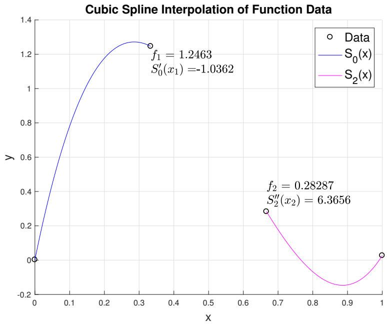

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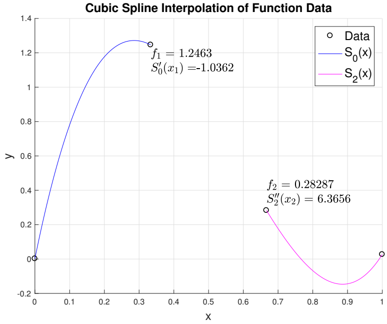

3. **Goal:** Compute the coefficents required to plot the missing $S_1(x)$

Cubic Splines - Conditions

Write down (in words) the set of conditions we want our Cubic Spline $S(x)$ to satisfy, and number them (you should have 5). You can already assume we know $S(x)$ is piece-wise cubic

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5. End point Conditions (e.g Natural, Clamped, Not-a-knot,....)

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- Next to each equation, write the number that corresponds to the condition being enforced.

Cubic Splines - Equations

Note that the condition $S(x_j) = f_j$ is already enforced by our notation. We have 3 unknowns per $S_j(x)$, therefore 9 total unknowns. Our 9 equations are:

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\begin{aligned}
f_0 + b_0 h_0 + c_0 h_0^2 + d_0 h_0^3 &= f_1 \quad ① \\
f_1 + b_1 h_1 + c_1 h_1^2 + d_1 h_1^3 &= f_2 \quad ① \\
f_2 + b_2 h_2 + c_2 h_2^2 + d_2 h_2^3 &= f_3 \quad ① \\
b_0 + 2c_0 h_0 + 3d_0 h_0^2 &= b_1 \quad ③ \\
b_1 + 2c_1 h_1 + 3d_1 h_1^2 &= b_2 \quad ③ \\
2c_0 + 6d_0 h_0 &= 2c_1 \quad ④ \\
2c_1 + 6d_1 h_1 &= 2c_2 \quad ④ \\
6d_0 &= 6d_1 \quad ⑤ \\
6d_1 &= 6d_2 \quad ⑤
\end{aligned}
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f_2 + b_2 h_2 + c_2 h_2^2 + d_2 h_2^3 &= f_3 \quad ① \\
b_0 + 2c_0 h_0 + 3d_0 h_0^2 &= b_1 \quad ③ \\
b_1 + 2c_1 h_1 + 3d_1 h_1^2 &= b_2 \quad ③ \\
2c_0 + 6d_0 h_0 &= 2c_1 \quad ④ \\
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\end{aligned}
$$

The last two equations impose Not-a-knot end point conditions!

Cubic Splines - Solve

In our plot, only b_1, c_1, d_1 are unknown. Write down the 3 equations needed to solve for these coefficents. Use the values given to you on the plot to write down numerical values for all known quantities. Note: The spacing between x values is $1/3$.

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Based on the given information, we can write down the following equations:

$$
\begin{aligned}
S_1(x_2) &= 1.2463 + \frac{1}{3}b_1 + \frac{1}{9}c_1 + \frac{1}{27}d_1 = 0.28287 \\
S_1'(x_1) &= b_1 = -1.0362 \\
S_1''(x_2) &= 2c_1 + 2d_1 = 6.3656
\end{aligned}
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$$2c_1 + 2d_1 = 6.3656$$

Which has the solution:

$$c_1 = -9.9344, \quad d_1 = 13.1175$$

Cubic Splines - Check our answer

Once we have values for b_1, c_1, d_1, fill in the vector *c1* in the code so that it has the form

$$c1 = [d_1\, c_1\, b_1\, f_1]$$

Run the code again and see if you correctly computed $S_1(x)$