Computing LU

Compute the P,L,U factorization of the following matrix:

$$A = \begin{bmatrix} 0.055 & 1.41 & 0.159 \\ 1.08 & -1.39 & 0.213 \\ 0.416 & -0.558 & 1.39 \end{bmatrix}$$

and use it to solve the system $Ax = b$ where

$$b = \begin{bmatrix} 1.20 \\ -2.55 \\ 1.60 \end{bmatrix}$$

Operation Counts

This question deals with operation counts for LU factorization

(a) Determine the number of operations needed to compute the LU factorization of an $n \times n$ matrix A.

(b) Using what you know about the operation counts of back- and forward-substitution, what is the leading order operation count for solving the system $Ax = b$ using the LU factorization? How does this compare to GE?

(c) Consider solving the series of equations $Ax^{(k)} = b^{(k)}$ for $k = 1, ..., M$, where A remains constant, and $n \times n$. Consider the leading order cost of all M solves using either:

   (i) GE for each solve

   (ii) First computing the LU factorization of A once, and then solving each equation using L and U.