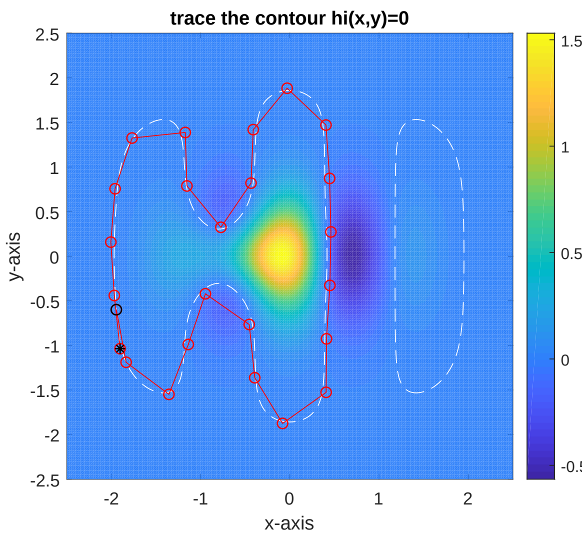
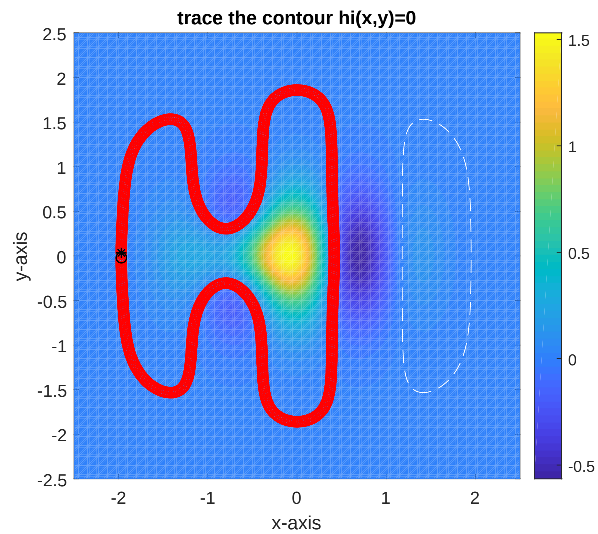
MACM 316 — Computing Assignment #3

David Pham // dhpham@sfu.ca // 301318482

Reproducing the Rough Sketch



(a) Traced by BiS ($\Delta s = 0.6,\ \delta = \frac{3\pi}{4}$)
`Nsteps` = 24



(b) Visually smooth curve traced by BiS
($\Delta s = 0.02,\ \delta = \frac{3\pi}{4}$)
`Nsteps` = 728

How δ and Δs Affect Convergence

Both methods required a $\Delta s = 0.02$ to produce a visually smooth curve. However, the BiS method can work with both large and small values of Δs, whereas SM requires a relatively small Δs value. With regards to δ, the BiS method requires values of δ where 2δ gets close enough to 2π, such that the interval $[\theta - \delta, \theta + \delta]$ represents something close to a closed circle. If δ is too small, then the values searched may be too narrow: maybe only a half-circle in the direction of θ_n is searched and no sign change is found, since the root lies on the other half of the circle. This becomes less of an issue if Δs becomes smaller alongside δ. For example, $\Delta s = 0.6,\ \delta = \pi/2$ makes BiS fail as it rounds the bottom-left bend of the contour, but BiS using $\Delta s = 0.0001,\ \delta = \pi/2$ is able to trace the contour.

The opposite is true for SM: values of $\Delta s \geq 0.1$ practically don't converge at all, and smaller values of δ are better. SM can handle values like $\delta = \pi/10^{10}$, whereas the smallest δ that BiS can handle is $\frac{\pi}{3}$ for $\Delta s = 0.0001$. For any smaller values of δ, BiS fails no matter how small Δs gets. However, for larger δs, it becomes increasingly difficult for SM to converge.

Cost

With my implementation of BiS, larger δ values lead to higher cost. Since the sign of θ may change multiple times over $[\theta - \delta, \theta + \delta]$, I evaluate HI at each point in `[`$\theta - \delta$`:0.1:`$\theta + \delta$`]`. This, in addition with the `avg_evals = 31` it takes for BiS to reach convergence, makes BiS much more costly than SM to run.

For SM, as both Δs and δ get smaller, SM begins to converge faster. At $\Delta s = 0.1,\ \delta = \pi/2$, `avg_evals = 8.34`. For a smaller $\Delta s = 0.001$ and the same δ as before, `avg_evals = 5.99`. With a smaller $\delta = \pi/10^{10}$ and $\Delta s = 0.1$, `avg_evals = 5.86`. With both Δ and δ small at $\Delta s = 0.001,\ \delta = \pi/10^{10}$, `avg_evals = 3.98`. Thus at its fastest, SM is only using two extra function evaluations, aside from the two required evaluations.