Calculus Review

Recall that the linear approximation for a function of 1 variable, $f(x)$ aout $x = x_0$ is

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

and that the linear approximation for a function of 2 variables, $f(x, y)$ about the point (x_0, y_0) is (note: this is the one your homework asks you to do)

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Based on these equations for linear appromxation, propose an equation for the linear approximation of a function of 3 variables, $f(x, y, z)$. Then use your proposed equation to approximate the function

$$f(x, y, z) = e^{x-z}\cos(x)\sin(y^2)$$

about the point $(x, y, z) = (0, \sqrt{\pi/2}, 0)$ (why is this a good point to approximate about?).

Linear Algebra Review

An important topic in linear algebra is solving linear systems, i.e given a matrix A and a vector $\vec{b}$, solve the matrix equation $A\vec{x} = \vec{b}$. One such way to solve this system is to compute the inverse of A, called A^{-1}. Then we can assert $x = A^{-1}b$.

Consider the following matricies and vector:

$$A = \begin{bmatrix} 1 & 1/2 & -1/3 \\ 5/2 & 2 & 1 \\ 2 & 3/2 & 2/3 \end{bmatrix} \quad V = \begin{bmatrix} -2 & -10 & 14 \\ 4 & 16 & -22 \\ -3 & -6 & 9 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ -1/2 \\ 4/3 \end{bmatrix}$$

Complete the following problems:

(a) Solve the system $Ax = b$ using Gaussian elimination.

(b) Set $y = Vb$. What do you notice about y compared to x? What does this tell us about the relationship between A and V?

(c) **Bonus**: Solve for A^{-1} explicitly using row reduction on $[A|I]$. What do you expect the answer to be?