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WEDNESDAY
- TUTORIALS this week
- QUIZ on friday

LAST DAY
- LU factorization is the mathematical formula representing a GE algorithm
- Matlab's lu command uses partial pivoting

$$[L][U] = [\quad][A].$$

- structure of $[L]$: s on , on -diag
- LU allows linear solve by 2 . + O( ) flops.

$$[L]\vec{\ } = [P]\vec{b} \quad + \quad [U]\vec{x} = \vec{\ }$$

- applications of LU: multiple solves, pre-computing & det
- special matrices & non-GE linear solves
  - DIAGONALLY-DOMINANT matrix: robust, no-piv GE
  - CHOLESKY factorization
  - SPARSE & tridiagonal matrices.

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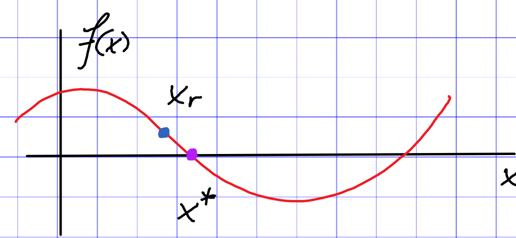
SOLUTION of NONLINEAR EQUATIONS (ch 2)

- ______ function of one or more variables with an

we want to find x^* with $f(x^*) = 0$,



involves

:

$$\lim_{x_r \to x^*} f(x_r) =$$

→ when x_r is to,

$f(x_r)$ is to

- additional advantages from functions & functions.

REVISIT NEWTON'S METHOD from CALC 1.

- fn $f(x)$ near

lin approx of $f(x)$ $=$ $) + ( )(-)$

graph is to $f(x)$

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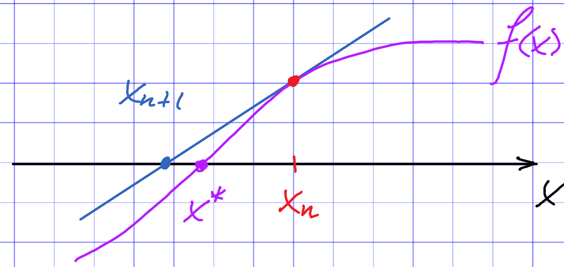
• IDEA for Newton's meth.



• if x_n is a for x^*, then

x_{n+1} is a potentially

since $(x_{n+1}, 0)$ is on

$$f(x_n) + f'(x_n)\,(x_{n+1} - x_n) =$$

• Newton's update: solve for x_n.

If we keep repeating this process, we obtain a sequence of approximations $x_1, x_2, x_3, x_4, \ldots$ as shown in Figure 3. In general, if the nth approximation is x_n and $f'(x_n) \neq 0$, then the next approximation is given by

2
$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

If the numbers x_n become closer and closer to r as n becomes large, then we say that the sequence *converges* to r and we write

?

$$\lim_{n \to \infty} x_n = r$$

• $\sqrt{\ }$ example, $f(x) = x^2 - a = 0$

```
>> NMsqrt

a =

     3
```

$$x_{n+1} = x_n - (\quad)/(\quad)$$

0	1.000000000000000e+00	-0.732050807568877
1	2.000000000000000e+00	+0.267949192431123
2	1.750000000000000e+00	+0.017949192431123
3	1.732142857142857e+00	+0.000092049573980
4	1.732050810014728e+00	+0.000000002445850
5	1.732050807568877e+00	+0.000000000000000

zeros after :

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- two observations from this demo
  - need a
  - when x_n is , NM is

this is tricky

CONVERGENT

$error_{n+1} = O(\quad)$

" of # of in error"

- NM is NOT a ALGORITHM

$| \quad | = |f(x_n)| <$ STOP

?

parameter ( )

- introductory comments & questions
  - NM is NOT a - routine (need)
  - buggy NM code can still ( )
  - NM can completely
    - can give /
    - might converge
    - can converge (-order zero)

  } choose better

KEY IDEA is solving of

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MORE THAN ONE VARIABLE

$$\vec{F}(\vec{x}) = \begin{pmatrix} f_1(x_1, x_2 \dots x_N) \\ \\ f_N(x_1, x_2 \dots x_N) \end{pmatrix} = \begin{pmatrix} f_1(\vec{x}) \\ \vdots \\ f_N(\vec{x}) \end{pmatrix}$$

- N-variable linear approximation at $\vec{y}_n$

$$\text{linear approx of } F(\vec{y}) = \vec{F}(\vec{y}_n) + \underbrace{\big[\, J \,\big]}_{} (\vec{y} - \vec{y}_n)$$

Jacobian matrix

$$[J] = \begin{array}{c} j=1 \\ \\ \\ j=N \end{array} \begin{bmatrix} \partial f_1/\partial x_1 & \dots & \partial f_1/\partial x_N \\ \vdots & & \vdots \\ \partial f_N/\partial x_1 & \dots & \partial f_N/\partial x_N \end{bmatrix} = \left[\, \partial f_j / \partial x_k \,\right]$$
$$\qquad\qquad k=1 \qquad\qquad\qquad k=N$$

- N-variable vector update: solve for y_{n+1} when LIN APPROX $= \vec{0}$

$$\vec{y}_{n+1} = \vec{y}_n - [J] \backslash \vec{F}(\vec{y}_n)$$

backslash!

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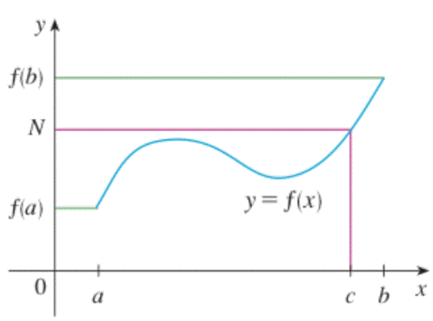
FINDING ZEROS

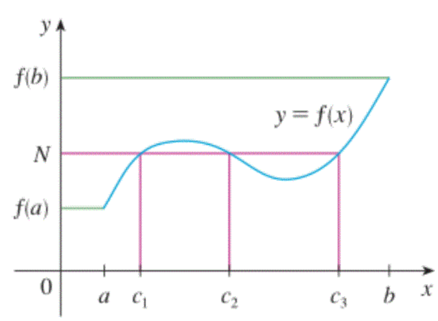
- key question: for what x does $f(x) = 0$?
- also: are there any zeros at all ?
  if more than ONE, how many ?
  or which one might we want ?

- there is NO GENERAL ALGORITHM
  - need some human thought: THINK
    PLOT
    KNOW your function
  - special case: N-degree polynomials
    # of zeros $= N$
    (including multiples)

10 The Intermediate Value Theorem Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

The Intermediate Value Theorem states that a continuous function takes on every intermediate value between the function values $f(a)$ and $f(b)$. It is illustrated by Figure 8. Note that the value N can be taken on once [as in part (a)] or more than once [as in part (b)].





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