Due Date: Monday, 14 January, 2019

Read before starting: download the Matlab file *hw00Apractice.m* from the Canvas assessments table. Complete this worksheet on a computer version of Matlab (laptop, or lab computer). Only after you have completely understood the coding of the Taylor series should you attempt the *math-apps* online version. You may submit more than once, but only your last score is the one saved by the Canvas gradesheet. Note, there is a daily limit to the number of submissions you can send to the server — you have now been warned. Also, if you are new to Matlab, do NOT leave the submission to the last day!

Begin by completing this worksheet. (Note that if you introduce "unfindable" bugs into your code, you can always start over with a new download.) Answer all of the questions below in the blank spaces. After you have done this correctly, you will be ready to proceed to the online version. If you encounter difficulties, bring your questions and scripts to the Computing Workshop (to be announced in lectures).

A) **Functions and "for" loops**

The following problem will review the basics of using functions, "for" loops, and some basic Matlab arithmetic operations. The code you have been provided computes the first few terms of a Taylor approximation for the function $r(t) = t^2 e^t$, and shows the error from $r(t)$. We will modify it so that it instead addresses the function $r(t) = t^3 e^{at}$ where $a = 2$. You only need to look up the Taylor series for the exponential function. The worksheet below is designed to be a warm-up to basic arithmetic in Matlab.

- Look up the infinite summation for the Taylor series of the exponential function, e^t (index the summation from $k = 0$ to ∞). Give the Taylor series for $r(t)$ using the same indexing, so that the $k = 0$ term is t^2.

$$e^t = \sum_{k=0}^{\infty} \underline{\hspace{4cm}} \quad ; \quad r(t) = \sum_{k=0}^{\infty} \underline{\hspace{4cm}}$$

- Now start the computing part by running the code *hw00Apractice.m* as is. Two plots are generated: the first compares the function $r(t)$ with the series variable s, while the second shows the difference, or error of the Taylor approximation. Although the graphs may look continuous, they actually connect discrete values $\{t_j, r(t_j), s_j\}$. What are the discrete values of t_j used in these initial plots?

- The error, defined by $Err = s - r(t)$, shows the maximum deviations at the endpoints. The numerical values at these locations are the array values ***Err(1)*** and ***Err(end)***. Give these values:

$$t(1) = \underline{\hspace{2cm}} \; ; \; Err(1) = \underline{\hspace{2.5cm}} \; ; \; t(end) = \underline{\hspace{2cm}} \; ; \; Err(end) = \underline{\hspace{2.5cm}} \, .$$

- How many of the Taylor terms are used in the initial code? Rerun the code to show the errors using the first 16 terms. (Note that the plots of $r(t)$ and the $s(t)$ are indistinguishable.)

$$\text{original \# of terms} = \underline{\hspace{2cm}} \; ; \; 16 \text{ terms} \to Err(1) = \underline{\hspace{2.5cm}} \; ; \; Err(end) = \underline{\hspace{2.5cm}} \, .$$

- Introduce a new variable $a = 2$, and then modify the inline function for $r(t)$, so that $r(t) = t^3 e^{at}$. Give the Taylor series for the new $r(t)$:

$$r(t) = \sum_{k=0}^{\infty} \underline{\hspace{4cm}}$$

- As a check, using the 16-term approximation, the errors should be:

$$Err(1) = \_\_\_\_\_\_\_\_\_\_ \; (1.3266e{-}03) \;\; ; \;\; Err(end) = \_\_\_\_\_\_\_\_\_\_ \; ({-}2.1370e{-}03) \; .$$

  But wait, did you get all those digits? The Matlab *format* command sets the numerical screen output mode — in particular, *format long e* gives you values in scientific notation.

- Now we ask that you compute the error for the value $t = t_0$ with $t_0 = 1.25$. Note that this value is not currently in our point list for the t_j. To get this error value, modify the point list to have $\Delta t = 0.05$ — now our list includes the t_0 value. But now you have to figure out the index k_0 so that $t(k_0) = t_0$.

$$t\,(\_\_\_\_) = t_0 = 1.25 \;\; ; \;\; Err0 = Err\,(\_\_\_\_) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \; .$$

  Note that the variable *Err0* will be the graded variable in the online version.

- Lastly, the title on the comparison plot needs to be changed. Edit the string variable to make an appropriate title.
- You are now ready to do the online version of this warm-up. Note that there will be a different value of t_0 that is output to the screen, but the Taylor series ideas are the same as in this worksheet.