$$y_i \approx \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\sum_{i=1}^{3} \left(y_i - \hat{y}_i \right)^2 = KKT$$

k=lmt

$$y = \frac{1}{2} + x$$

min KKT ?
$\hat{\beta}_0, \hat{\beta}_1$

$$(x_1, y_1) = (1,1), \quad (x_2, y_2) = (3,3), \quad (x_3, y_3) = (2,3)$$
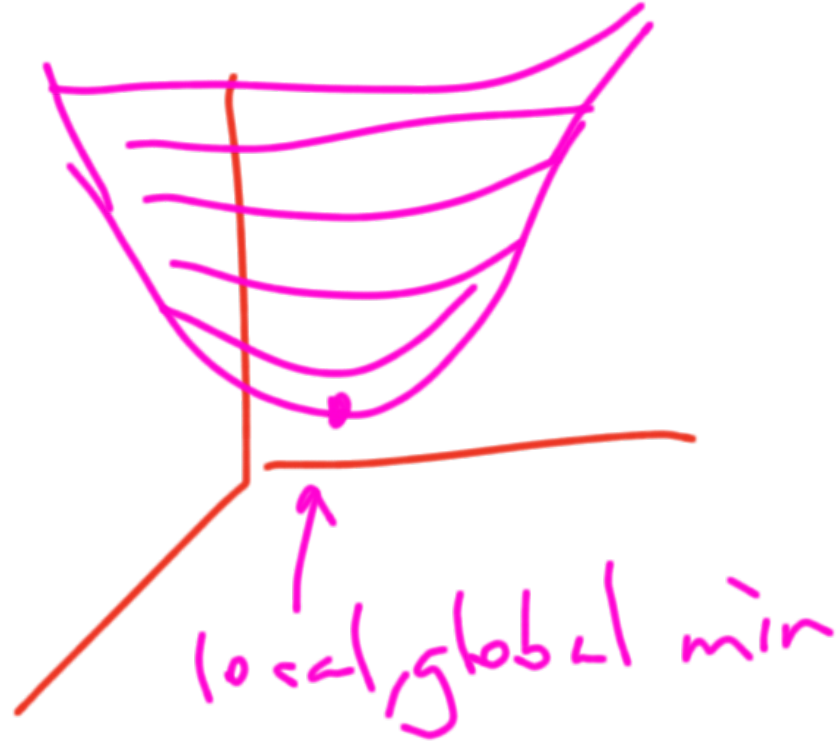
$$KKT = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3)^2$$

$$\min_{\hat{\beta}_0, \hat{\beta}_1} (y_1 - \hat{\beta}_0 - \hat{\beta}_1 x_1)^2 + (y_2 - \hat{\beta}_0 - \hat{\beta}_1 x_2)^2 + (y_3 - \hat{\beta}_0 - \hat{\beta}_1 x_3)^2$$

$$\|$$

$$\min_{\hat{\beta}_0, \hat{\beta}_1} f(\hat{\beta}_0, \hat{\beta}_1)$$

local/global min

$$f_{\hat{\beta}_0} = -2\left(y_1 - \hat{\beta}_0 - \hat{\beta}_1 x_1\right) - 2\left(y_2 - \hat{\beta}_0 - \hat{\beta}_1 x_2\right)$$
$$- 2\left(y_3 - \hat{\beta}_0 - \hat{\beta}_1 x_3\right) = 0$$

$$-2(y_1 + y_2 + y_3) + 6\hat{\beta}_0 + 2\hat{\beta}_1(x_1 + x_2 + x_3) = 0$$

$$f_{\hat{\beta}_1} = -2x_1\left(y_1 - \hat{\beta}_0 - \hat{\beta}_1 x_1\right) - 2x_2\left(y_2 - \hat{\beta}_0 - \hat{\beta}_1 x_2\right)$$
$$- 2x_3\left(y_3 - \hat{\beta}_0 - \hat{\beta}_1 x_3\right) = 0$$
$$- 2(x_1 y_1 + x_2 y_2 + x_3 y_3) + 6\hat{\beta}_0(x_1 + x_2 + x_3) + \ldots = 0$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{3}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{3}(x_i - \bar{x})^2}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\bar{x} = \frac{1}{3}\sum_{i=1}^{3} x_i = \frac{1}{3}(1+2+3) = 2$$

$$\bar{y} = \frac{1}{3}\sum_{i=1}^{3} y_i = \frac{1}{3}(1+3+3) = \frac{7}{3}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{3}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{3}(x_i - \bar{x})^2}$$

$$= \frac{(1-2)(1-7/3) + (2-2)(3-7/3) + (3-2)(3-7/3)}{(1-2)^2 + (2-2)^2 + (3-2)^2}$$

$$= \frac{(-1)(-4/3) + 0 + 1 \cdot (2/3)}{1 + 0 + 1} = \frac{2}{2} = \boxed{1}$$

$$\widehat{\beta}_0 = \overline{y} - \overrightarrow{\beta_1}\overline{x} = \frac{7}{3} - 1 \cdot 2$$

$$= \boxed{1/3}$$

$$\boxed{y \approx \frac{1}{3} + x}$$

$$\min \sum_{i=1}^{3} \left(y_i - \widehat{y}_i\right)^2$$