Assignment #3: Probability Functions, and Moments (total 10 points), due by 11:59 pm Friday, 27 October 2023

Exercise 1: Probability density function features. Given the triangular distribution defined as follows:

$$p_T(x \mid \alpha, \beta) = \begin{cases} \frac{2(x-\alpha)}{(\beta-\alpha)^2} & \alpha \leq x \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate its first, second, third, and fourth moments. **(0.5 points per correct moment)**

(b) Determine the median of p_T. **(1 point)**

(c) Write a script to plot $p_T(x|\alpha, \beta)$ as a function of x and include vertical lines that mark the mean, the mode, and the median of this probability density function. You can choose the values of α and β. **(1 point)**

You can write your answers for this question using any text editor of your preference, but, please, set the document layout with the following features:

- Page orientation: portrait;
- Page size: 8.5" × 11" (letter);
- Margins: 2.54 cm (top, bottom, left, right);
- Line spacing: between 1.0 and 1.15 pt;
- No double-column format. Write your answers as a 1-column text;
- Paragraph alignment can be 'align left' or 'justify';
- Font type and size: Times New Roman, 12 pt, non-italic, non-bold;
- Use *italic*/**bold**/<u>underline</u> only to emphasize brief pieces of text if needed.
- ONLY `pdf` FILES ARE ACCEPTED. Do not submit editable files such as `.docx`.

Alternatively, you can write your answers on paper and scan your solutions. However, **make sure your answers can be read clearly and the handwriting is legible.** The scanned file to be submitted needs to be in `pdf` format.

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Exercise 2: Thomson's discovery of the electron. A set of groundbreaking experiments by J. J. Thomson in the late nineteenth century aimed to measure the ratio between the mass and charge of a new lightweight particle, which was later named *electron*. The experiment was reported in Thompson, J.: Cathode rays. Philos. Mag. **44**, 293 (1897) and it consists of measuring the deflection of negatively charged cathode rays by a magnetic field H in a tube. Thomson wished to measure the mass m of the charged particles that constituted these cathode rays. The experiment was based on the measurement of the following quantities: W is the kinetic energy of the particles, $Q = Ne$ is the amount of charge with N being the number of particles and e the charge of each particle, and $I = HR$ with R being the radius of curvature of the path of these rays in a magnetic field H. The measurements performed by Thompson were used to infer the ratio m/e and the speed v of the new lightweight particle according to:

$$v = \frac{2W}{QI} \tag{1}$$

$$\frac{m}{e} = \frac{I^2 Q}{2W}. \tag{2}$$

For the purpose of the data analysis of this experiment, it is only necessary to know that W/Q and I are the primary quantities being measured, and inferences on the secondary quantities of interest are based on the equations above. For the proton, the mass-to-charge ratio was known to be approximately 1×10^{-4} g per electromagnetic (EMU) charge unit, where EMU charge unit is equivalent to 10^{-10} electrostatic charge units, or ESU (a more common unit of measure for charge). In Thomson's units, the accepted value of the mass-to-charge ratio of the electron is now 5.7×10^{-8}. Some of the experimental data collected by Thomson are on files `thomson1.dat` (Tube 1) and `thomson2.dat` (Tube 2), in which 'Gas' refers to the gas used in the tubes during the experiments. Note that the measurements marked with a star '*' appear to have values of v or m/e that are inconsistent with the formulas presented above to calculate them from W/Q and I. These may be typographical errors in the original publication or approximations.

(a) Using the datasets provided for Tube 1 and Tube 2 separately, write a script to calculate the sample means and variances of the random variables W/Q and I, and the covariance and correlation coefficients between W/Q and I. **(3 points)**

(b) Using J.J. Thomson's experiment dataset provided, verify the statement that

It will be seen from these tables that the value of m/e is independent of the nature of the gas.

You may do so by calculating the sample mean and standard deviation for the measurements in each gas (air, hydrogen, and carbonic acid), then testing whether the three measurements agree with each other within their standard deviations. **(2 points)**

(c) Explain your code(s) and their outputs using comment lines, markdowns, and/or docstrings – depending on your chosen integrated development environment (IDE) software. In particular, present the results of item (a) in an appropriate table format that your code will output. This item will also evaluate the clarity and readability of your codes. **(1 point)**

IMPORTANT: Whenever applicable, your code needs to output all required numbers of the exercise and they should be printed clearly. The printed values should be properly identified with text saying what is being printed. We cannot debug your code and/or search the answers inside your code for you. This is a graded component of the course, therefore, we can ONLY evaluate what is uploaded and printed on the screen. We will test other parameter variations and settings in your code, but we will not alter its structure and logistics. Interpret your code as any other graded component as 'in paper', meaning what you submitted is your final answer and what is not provided (or printed) in terms of answers, we cannot consider for marks.

Submission Information

Combine your written answers for exercise 1 in a single `pdf` file named `assignment_3_exercise_1.pdf`. Name the code you designed for questions 1 and 2 as `code_assignment_3_exercise_1.ext` and `code_assignment_3_exercise_2.ext`, respectively, in which `ext` is the file extension associated with your coding platform. You will upload the `pdf` file of your answers, your codes, and generated figures to the Gradescope platform. Your codes need to be documented, i.e., introduce comment lines to explain their main command procedures. Pure code lines without explanatory comments will have reduced marks. **It is important that you log in to Gradescope.ca and not Gradescope.com.** You can upload multiple files to the Gradescope platform and you can resubmit your work until the due date. We will test-run your submitted codes for syntax errors and check if they generate the requested answers. We will also check if the results/figures presented in your assignment are 'paper-like' quality and that the quantitative results are scientifically/mathematically founded.

ONLY `pdf` (for the written report), `jpg/jpeg` (for figures), and `.py`, `.ipynb`, or `.m` (for codes) FILES ARE ACCEPTED. This means we are accepting Python, Python Notebook, or Matlab codes. If you are more familiar with another programming language, please, contact the instructor in advance.

IMPORTANT: In your codes include instructions on how to run them and, if applicable, include testing values for the initial conditions or settings that you attempted so we can reproduce the results. We will test other initial conditions and parameter variations to evaluate the robustness of your codes. But we need a place to start. Include also information about the version of your coding platform. If results cannot be reproduced using basic running instructions or clear instructions are not provided, points may be deducted.

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