# Solutions for Homework #1

1) What are major functions of computer networks?

## **Solution:**

Convenient information sharing and communications

2) What are the major differences between wired and wireless networks?

## **Solution:**

Wired networks transmit data through wired connections such as cable.

Wired channels are usually very reliable.

Wireless networks transmit data by using electromagnetic wave, which does not need physical wired connections. Wireless channels are subject to channel fading, resulting higher error rate as compared to the wired networks

3) Why do we need the wireless communications networks?

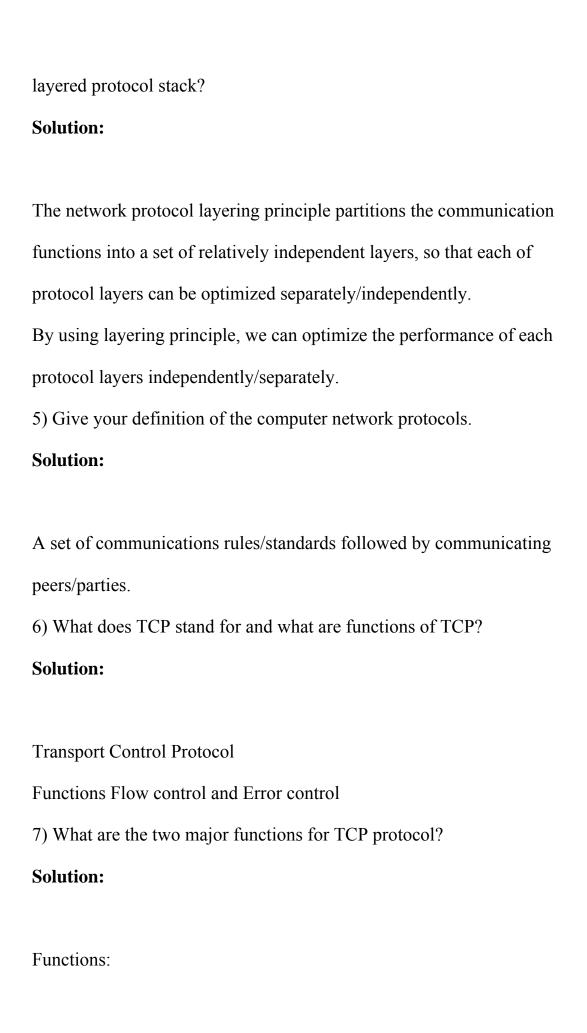
#### **Solution:**

Cable connection is not convenient and inefficient and its costs are high.

Requirements on mobile communications

Requirements on long-distance communications.

4) What's the protocol layering principle and why do we need to use the



Flow control to avoid congestion
Error control for reliable communications.
8) What does MAC stand for and why do we need the MAC protocol?
Solution:
Medium Access Control
We need MAC protocol to share transmission medium among different
links and reduce collisions.
9) What does PHY stand for and why do we need the PHY layer
algorithms?
Solution:
Physical Layer.
We need PHY layer algorithm for efficient resource allocation, error
control, which highly depend on the medium physical properties.
10) What are the major two types of computer network traffics?
Solution:
Non-real-time traffics and real-time traffics.
11) What does QoS stand for and what is QoS for the real-time traffic and

what's QoS requirement for the non-real-time traffic?

## **Solution:**

Quality of Service.

Non-real-time traffics have more stringent requirements on reliability QoS.

Real-time traffics have more stringent requirements on delay QoS.

12) How to classify the flow control mechanisms?

## **Solution:**

o Open-loop control scheme

Flow control function is achieved without using feedback via the closed-loop channel.

o Closed-loop flow control scheme

Flow control adapt its transmission rate to the bottleneck available bandwidth according to the feedback through the closed-loop channel

Window-based scheme vs. Rate-based schemes

Explicit scheme vs. Implicit scheme

End-to-end scheme vs. Hop-by-Hop scheme

o Hybrid schemes

Mixing open-loop flow control with closed-loop scheme

# **Solutions for Review Homework Set**

# **Section 1:**

1) What are the differences between analog and digital signals?

#### **Solution:**

Analog signal:

Signal intensity varies in a smooth fashion over time

No breaks or discontinuities in the signal

Digital signal:

Signal intensity maintains a constant level for some period of time and then changes to another constant level

2) What are three important characteristics of a periodic signal?

## **Solution:**

Frequency (Period), Amplitude, Phase

3) What are the relationships between a signal's spectrum and its bandwidth?

## **Solution:**

Usually, wider spectrum can support higher bandwidth, depending on the specific modulation schemes. Spectrum is the width of the physical radio

band. Bandwidth is the achievable data transmission rate.

4) Define the channel capacity.

## **Solution:**

Theoretical maximum that can be achieved through this channel.

5) Differentiate between guided media and unguided media.

#### **Solution:**

Guided Media:

Waves are guided along a solid medium

Unguided Media:

Provides means of transmission but does not guide electromagnetic signals.

Usually referred to as wireless transmission

6) How is interference avoided by using FDM?

#### **Solution:**

Takes advantage of the fact that the useful bandwidth of the medium exceeds the required bandwidth of a given signal. Split the entire spectrum into a number of non-overlapped bands, thus avoiding interferences.

7) What is attenuation?

The variation of the signal power during the transmissions.

8) If a signal has a fundamental frequency of 1000 Hz, then what's its period?

#### **Solution:**

Period = 
$$1 / 1000 \text{ Hz} = 1 \text{ ms}$$

9) What is the channel capacity for a telephone channel with a 300 Hz bandwidth and a signal-to-noise ratio of 3 dB?

## **Solution:**

channel capacity = 
$$B \log_2(1 + SNR)$$
  
=  $300 * \log_2(1 + 10^{\frac{3}{10}}) = 474.8 \ bit/s$ 

- 10) A digital signaling system is required to operate at 9600 bps.
- a. If a signal element encodes a 4-bit word, what is the minimum required bandwidth for the channel?
- b. Repeat a) for the case of 8-bit words.

## **Solution:**

a.

Signal Element Rate (symbol rate): C = 9600/4 = 2400 symbol/sec Required bandwidth B = C/2 = 1200 Hz b.

Signal Element Rate (symbol rate): C = 9600/8 = 1200 symbol/sec Required bandwidth B = C/2 = 600 Hz

11) Given a channel with an intended capacity of 20 Mbps, the bandwidth of the channel is 3 MHz What SNR is required to achieve this capacity?

# **Solution:**

In order to satisfy 
$$B \log_2(1+SNR)=20\times 10^6$$
 bps , we need to have 
$$SNR=2^{\frac{20\times 10^6}{B}}-1=100.6=20~dB$$

## **Section 2:**

1) Differentiate between LAN and WAN.

## **Solution:**

> Scope of a LAN is smaller

LAN interconnects devices within a single building or cluster of buildings

LAN usually owned by organization that owns the attached devices

For WANs, most of network assets are not owned by same organization

- ➤ Internal data rate of LAN is much greater
- 2) What's the principal application that has driven the design of circuit-switching networks?

Real-time services

3) Explain the differences between datagram-based and virtual-circuit based networks.

#### **Solution:**

# Datagram-based:

- ➤ Each packet treated independently, without reference to previous packets
- Each node chooses next node on packet's path
- ➤ Packets don't necessarily follow same route and may arrive out of sequence
- > Exit node restores packets to original order
- Responsibility of exit node or destination to detect loss of packet and how to recover

## Virtual-circuit based

- Preplanned route established before packets sent
- > All packets between source and destination follow this route

- > Routing decision not required by nodes for each packet
- Emulates a circuit in a circuit switching network but is not a dedicated path
- > Packets still buffered at each node and queued for output over a line
- 4) What are the disadvantages of using a circuit-switching based network for data transmissions?

- Utilization not 100%
- > Delay prior to signal transfer for establishment
- 5) Please categorize ATM services.

## **Solution:**

- > Real-time service
  - Constant bit rate (CBR)
  - Real-time variable bit rate (rt-VBR)
- ➤ Non-real-time service
  - Non-real-time variable bit rate (nrt-VBR)
  - Available bit rate (ABR)
  - Unspecified bit rate (UBR)

6) Explain the flaw in the following reasoning: Packet switching requires control and address bits to be added to each packet. This introduces considerable overhead in packet switching. In circuit switching, a transparent circuit is established. No extra bits are needed. Thus, there is no overhead in circuit switching, and, because there is no overhead in circuit switching, line utilization must be more efficient than in packet switching networks.

#### **Solution:**

The circuit switch still needs overhead to build the connections. Also, the bandwidth occupation will not change once the connections have been built, thus often leading to utilization of the assigned bandwidth not equal to 100%, especially for variable rate services. In addition, there is delay prior to signal transfer for establishment. All the above factors may cause inefficiency as compared to packet switching networks.

7) One key design decision for ATM network was whether to use fixed-or variable-length cells. Let us consider this decision from the point of view of efficiency. We can define transmission efficiency as:

N = (Number of information bytes)/(Number of information bytes + Number of overhead bytes)

a). Consider the use of fixed length packets. In this case, the overhead consists of the header bytes. Define:

L =Data field size of the cell in bytes

H = Header size of the cell in bytes

X = Number of information bytes to be transmitted as a single message

Derive an expression for N. Hint: The expression will need to use the operator of [X] which is taking the smallest integer greater than or equal to X.

b). If cells have variable length, then overhead is determined by the header plus the flags to delimit the cells or an additional length field in the header. Let Hv = additional overhead bytes required to enable the use of variable-length cells. Derive an expression for N in terms of X, H, and Hv.

c) Let L = 48, H = 5, and Hv = 2. Plot N versus message size for fixed and variable-length cells. Comment on the results.

## **Solution:**

a.

We reason as follows. A total of X octets are to be transmitted. This will require a total of  $\left\lceil \frac{X}{L} \right\rceil$  cells. Each cell consists of (L + H) octets, where L is the number of data field octets and H is the number of header octets. Thus

$$N = \frac{X}{\left[\frac{X}{L}\right](L+H)}$$

The efficiency is optimal for all values of X which are integer multiples of the cell information size. In the optimal case, the efficiency becomes

$$N_{\text{opt}} = \frac{X}{\left(\frac{X}{L}\right)(L+H)} = \frac{L}{L+H}$$

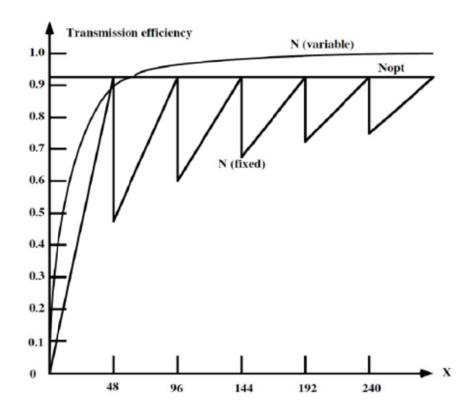
For the case of ATM, with L=48 and H=5, we have  $N_{opt}=0.91$ 

b.

Assume that the entire X octets to be transmitted can fit into a single variable-length cell. Then

$$N = \frac{X}{X + H + H_v}$$

c.



## **Section 3:**

Consider the linear control function:

$$x_i(t+1) = a + bx_i(t),$$

where  $x_i(t)$  is the *i*th users' allocation. The fairness is defined by

$$Fairness \triangleq \frac{(\sum x_i)^2}{n(\sum x_i^2)}$$

Please prove in details the followings:

- 1. Prove the fairness is guaranteed to be non-decreasing, if  $c = \frac{a}{b} \ge 0$
- 2. The fairness is improved if c > 0.

#### **Solution:**

Using F(x(t)) to represent the fairness at time t, we have

$$F(x(t+1)) = \frac{\left(\sum x_i(t+1)\right)^2}{n\left(\sum x_i^2(t+1)\right)}$$

$$= \frac{\left(\sum a + bx_i(t)\right)^2}{n\sum (a + bx_i(t))^2}$$

$$= \frac{\left(\sum c + x_i(t)\right)^2}{n\sum (c + x_i(t))^2}$$

$$= F(x(t)) + (1 - F(x(t)))$$

$$\times \left(1 - \frac{\sum x_i^2(t)}{\sum (c + x_i(t))^2}\right).$$

If  $c=\frac{a}{b}\geq 0$ , we can see that  $\left(1-\frac{\Sigma x_i^2(t)}{\Sigma \left(c+x_i(t)\right)^2}\right)$  is non-negative, and thus  $F\left(x(t)\right)$  is non-decreasing. If c>0, then  $\left(1-\frac{\Sigma x_i^2(t)}{\Sigma \left(c+x_i(t)\right)^2}\right)$  is positive, and thus  $F\left(x(t)\right)$  is an increasing function of t, implying that the fairness is improved.

# **Section 4:**

1) What are the main causes for wireless channel disturbances/corruptions?

**Solution:** The main causes include:

Multipath delay spread

Doppler spread due to motion

Signal fading of frequency-selective and non-frequency-selective variety

2) Define the scatterers in wireless channels?

## **Solution:**

Wireless propagation channel contains objects (particles) randomly scattering the energy of transmitted signals. These objects (particles) are called scatterers

3) What are the consequences for existence of scatterers in wireless channels?

#### **Solution:**

Cause fading, multipath delay spread, Doppler spread, and attenuation.

4) Define the Multipath delay spread.

Scattering by randomly located scatterers gives rise to different paths with different path-lengths/propagation-delays, resulting in multipath delay spread

5) What is the scatterer ellipse and explain its properties.

# **Solution:**

All scatterers are located on ellipses with transmitter (Tx) and receiver (Rx) as the foci. One ellipse is associated with one path length/delay.

6) Define the Flat fading and frequency-selective fading and how to differentiate them?

## **Solution:**

If max difference in delay spread is small compared with symbol duration of transmitted signal, channel is said to exhibit flat fading. If difference in delay spread is large compared with the symbol duration of transmitted signal, the channel exhibits frequency-selective fading.

7) Define inter-symbol interference (ISI) and what is the parameter affect ISI? How to overcome ISI?

In time domain, received signals corresponding to successive transmitted symbols through frequency-selective fading channel will overlap, giving rise to a phenomenon called inter-symbol interference (ISI). ISI degrades transmission performance, which can be overcome by the channel equalization techniques

8) What is the Additive White Gaussian (AWGN)?

#### **Solution**

Inherent background noise can be approximated as thermal noise and treated as Additive White Gaussian (AWGN)

9) Why can the multipath components affect the received signal strength constructively or destructively?

# **Solution:**

The multipath components can affect the received signal strength constructively or destructively, depending on carrier frequency and delay differences among the multi-paths. If the phases of signals from different paths are the same, the signal is strengthened; if the phases of signals from different paths are opposite, the signal is weakened.

10) Precisely define the wireless channel fading and what are main

causes for the wireless channel fading?

## **Solution:**

As a mobile station moves, the position of each scatterer w.r.t. transmitter and receiver may change. The overall effect caused by multipath delay spread, Doppler spread, attenuation, thermal noise, etc. is that the received signal level fluctuates with time, which the phenomenon is called fading

11) Define LOS and NLOS and what are the differences between them? **Solution:** 

The delay of Line-of-Sight (LOS) or direct path is the shortest path among the multi paths, having smallest propagation delay (often assumed to be zero); the delay of non-line-of-sight (NLOS) or reflected path has longer propagation delay.

12) (i) Prove that the following statement holds true. The received signal, in the absence of noise, given as follows:

$$r(t) = \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos(2\pi f_c (t - \tau)),$$

where  $\alpha_1$  and  $\alpha_2$  are the amplitudes of the signal components from the two paths respectively. The received signal can also be represented as

$$r(t) = \alpha \cos(2\pi f_c t + \emptyset),$$

where

$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2\cos(2\pi f_c\tau)}$$

and

$$\emptyset = -\tan^{-1} \left[ \frac{\alpha_2 \sin(2\pi f_c \tau)}{\alpha_1 + \alpha_2 \cos(2\pi f_c \tau)} \right]$$

(ii) Plot  $\alpha$  against  $f_c\tau$  (assuming  $\alpha_1=\alpha_2=2$ ) by using Matlab and discuss what you obtain about the wireless channel deep fading.

## **Solution:**

(i) 
$$\tau(t)$$

$$= \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos(2\pi f_c (t - \tau))$$

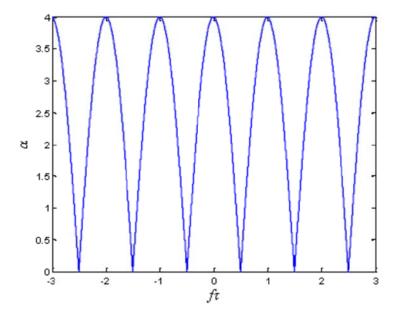
$$= \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos(2\pi f_c t) \cos(2\pi f_c \tau) + \alpha_2 \sin(2\pi f_c t) \sin(2\pi f_c \tau)$$

$$= (\alpha_1 + \alpha_2 \cos(2\pi f_c \tau)) \cos(2\pi f_c t) + \alpha_2 \sin(2\pi f_c t) \sin(2\pi f_c \tau)$$

$$= (\alpha_1 + \alpha_2 \cos(2\pi f_c \tau)) \cos(2\pi f_c t) + \alpha_2 \sin(2\pi f_c t) \sin(2\pi f_c \tau)$$

$$= (\tan \theta) + (\tan \theta) +$$

(ii)



13) Define the wireless channel long-term fading or short-term fading and what are their causes?

## **Solution:**

Short-term fading is rapid fluctuations caused by the local multipath (e.g., Rayleigh fading).

Long-term fading is long-term slow variation in the mean level of received signal strength (e.g., Lognormal fading) caused by movement over large enough distance.

14) Why do we need to model the wireless channel as a linear time-variant (LTV) system and how to do so?

## **Solution:**

As a mobile station moves, the position of each scatterer w.r.t. transmitter

and receiver may change. So, we need to use linear time-variant system. In particular, we use time-varying channel impulse response to characterize the LTV, which includes two important independent variables, one is the time t, the other is the path delay  $\tau$ .

# **Section 6:**

## Problem 2-1:

For a linear time-invariant channel with impulse response h(t), given the channel input x(t), the channel output is r(t) = x(t) \* h(t). Verify that this channel input-output relation is a special case of Eq. (2.2.6).

Eq. (2.2.6) is given as follows:

$$r(t) = \int_{-\infty}^{\infty} h(\tau, t) x(t - \tau) d\tau.$$

# **Solution:**

For an LTI with impulse response h(t), the impulse response  $h(\tau, t)$  in Eq. (2.2.6) can be simplified to  $h(\tau)$ . From Eq. (2.2.6), we have

$$r(t) = \int_{-\infty}^{\infty} h(\tau, t) x(t - \tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau = x(t) * h(t)$$

# Problem 2-2:

A wireless channel is specified by the time-variant channel impulse response

$$h(\tau, t) = \left(1 - \frac{\tau}{T}\right) \cos(\Omega t + \theta_0), \quad 0 \le \tau \le T,$$

where T=0.05 ms,  $\Omega = 10\pi$ , and  $\theta_0 \in [-\pi, +\pi]$  is a constant.

- a. Determine the channel time-variant transfer function.
- b. Given that the channel input signal is

$$x(t) = \begin{cases} 1, & 0 \le t \le T_s \\ 0, & \text{otherwise} \end{cases},$$

where  $0 < T_s < T$ , determine the channel output signal.

- c. For continuous digital transmission with symbol interval  $T_s$ , if the channel fading is frequency selective, specify the relation between  $T_s$  and T.
- d. Plot the received signal at the wireless channel output as a function of time *t* using e.g., Matlabs.

## **Solution:**

(a) 
$$H(f,t) = \mathcal{F}[h(\tau,t)]$$

$$= \int_{-\infty}^{\infty} h(\tau,t)e^{-j2\pi f\tau}d\tau$$

$$= \int_{-\infty}^{\infty} \left(1 - \frac{\tau}{T}\right)\cos(\Omega t + \theta_0)e^{-j2\pi f\tau}d\tau$$

$$= \cos(\Omega t + \theta_0)\left[\int_0^T e^{-j2\pi f\tau}d\tau - \frac{1}{T}\int_0^T \tau e^{-j2\pi f\tau}d\tau\right]$$

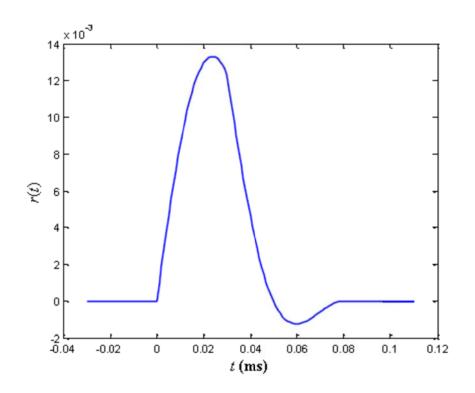
$$= \cos(\Omega t + \theta_0)\left[\frac{1}{j2\pi f} + \frac{1 - e^{-j2\pi fT}}{(2\pi f)^2 T}\right]$$

(b) 
$$r(t) = \int_{-\infty}^{\infty} h(\tau, t) x(t - \tau)$$

$$= \begin{cases} 0, & t < 0; \\ \int_0^t h(\tau, t) d\tau, & 0 \le t \le T_s; \\ \int_{t-T_s}^t h(\tau, t) d\tau, & T_s \le t \le T; \\ \int_{t-T_s}^T h(\tau, t) d\tau, & T < t \le T + T_s; \\ 0, & t > T + T_s \end{cases}$$

$$= \begin{cases} 0, & t < 0; \\ \cos(\Omega t + \theta_0) \left(t - \frac{t^2}{2T}\right), & 0 \le t \le T_s; \\ \cos(\Omega t + \theta_0) \left(T_s + \frac{T_s^2}{2T} - \frac{T_s t}{T}\right) & T_s \le t \le T; \\ \frac{\cos(\Omega t + \theta_0)(t - T - T_s)^2}{2T}, & T < t \le T + T_s; \\ 0, & t > T + T_s \end{cases}$$

- (c) From r(t) given in (b), for each transmitted symbol, the received symbol has a duration of  $T+T_s$ . That is, the channel introduces a delay spread of duration T. If the channel fading is frequency selective, then T should not be much smaller than  $T_s$ .
- (d) The figure for r(t) vs. t is as follows, where  $T_s = 0.03$  ms and  $\theta_0 = 0$ . Note that different values of the parameters will cause plots with different shapes.



# Problem 2-3:

Derive the relation given in Eq. (2.2.8).

Eq. (2.2.8) is given as follows:

$$r(t) = \int_{-\infty}^{\infty} R(f, t) e^{j2\pi f t} df$$

# **Solutions:**

$$r(t) = \int_{-\infty}^{\infty} h(\tau, t)x(t - \tau)d\tau$$

$$= \int_{-\infty}^{\infty} x(t - \tau) \left[ \int_{-\infty}^{\infty} H(f, t) \exp(j2\pi f \tau) df \right] d\tau$$

$$= \int_{-\infty}^{\infty} H(f, t) \left\{ \int_{-\infty}^{\infty} x(t - \tau) \exp[-j2\pi f(t - \tau)] d\tau \exp(j2\pi f t) df \right\}$$

$$= \int_{-\infty}^{\infty} H(f, t) \left[ \int_{-\infty}^{\infty} x(\xi) \exp(-j2\pi f \xi) (-d\xi) \right] \exp(j2\pi f t) df$$

$$= \int_{-\infty}^{\infty} H(f, t)X(f) \exp(j2\pi f t) df$$

$$\triangleq \int_{-\infty}^{\infty} R(f,t) \exp(j2\pi f t) df$$

# Problem 2-4:

For the channel specified in Example 2.1 and the transmitted signal  $x_1(t)$ , calculate the received signal by using the relation derived in Problem 2-3.

Plot the received signal at the wireless channel output as a function of time t using e.g., Matlabs.

## **Solution:**

From Example 2.1, we have

$$H(f,t) = \frac{4T\cos(\Omega t)}{1 + j2\pi f\tau}$$

For the transmitted signal  $x_1(t)$ ,

$$X_1(f) = \mathcal{F}_t[x_1(t)] = \int_{-T_0}^{T_0} e^{-j2\pi ft} dt = 2T_0 \operatorname{sinc}(2fT_0)$$

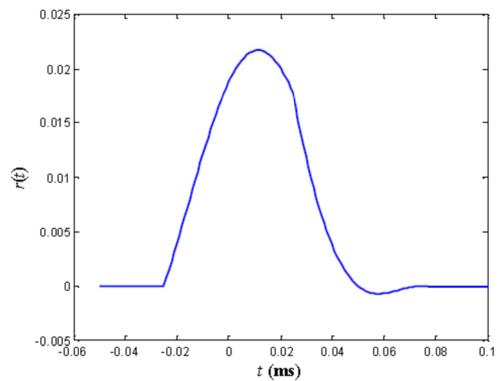
where  $sinc(x) \triangleq \frac{\sin(\pi x)}{\pi x}$ . Then, we have

$$R(f,t) = H(f,t)X_1(f) = \frac{4T\cos(\Omega t)}{1 + j2\pi fT} \times 2T_0 sinc(2fT_0)$$

$$r(t) = \int_{-\infty}^{\infty} R(f, t) e^{j2\pi f t} d$$

$$= 4T\cos(\Omega t)\mathcal{F}_f^{-1}\left[\frac{1}{1+j2\pi fT} \times 2T_0 sinc(2fT_0)\right]$$

$$= \begin{cases} 0, & t < -T_0 \\ \cos(\Omega t + \theta_0) \left(t + T_0 - \frac{(t + T_0)^2}{2T}\right), & -T_0 \le t \le T_0 \\ \cos(\Omega t + \theta_0) \left(2T_0 - \frac{2tT_0}{T}\right), & T_0 \le t \le T - T_0 \\ \frac{\cos(\Omega t + \theta_0)(t - T - T_s)^2}{2T}, & T - T_0 \le t \le T + T_0 \\ 0, & t > T + T_0 \end{cases}$$



Compared to Problem 2-2, we can see that the received signal in this problem is just a time-shift of the received signal in problem 2-2.