1 — 2.1, Proble 10

a) $$\begin{bmatrix} 1 & 2 & 4 \\ -2 & +3 & 1 \\ -4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$$

it is a combination of A's columns

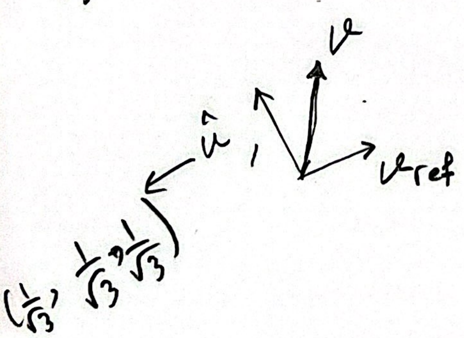
b) $$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+1+0+0 \\ 1+2+1+0 \\ 0+1+2+2 \\ 0+0+1+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 4 \\ 5 \end{bmatrix}$$

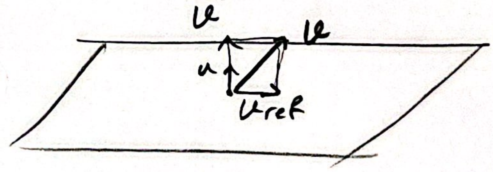
2. a) $$v = \begin{bmatrix} |V|\cos\theta \\ |V|\sin\theta \end{bmatrix} \qquad v'_{30^\circ} = \begin{bmatrix} |V|\cos(\theta-30) \\ |V|\sin(\theta-30) \end{bmatrix} = |V|\begin{bmatrix} \cos\theta\cos 30^\circ + \sin\theta\sin 30 \\ \sin\theta\cos 30 - \sin 30\cos\theta \end{bmatrix}$$

$$Rv = v'_{30^\circ}$$

$$|V|\underbrace{\begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{bmatrix}}_{R}\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = |V|\begin{bmatrix} \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ \\ \sin\theta\cos 30^\circ - \sin 30\cos\theta \end{bmatrix}$$

b)





$$v_{ref} + v'' = v$$

$$v_{ref} = v - v'' = v - (\hat{v}\cdot\hat{u})\,\hat{u}$$

OPS, It is not reflection. it's Projection

$$v_{ref} = v - 2(\hat{v}\cdot\hat{u})\,\hat{u}$$

for $\begin{bmatrix}1\\0\\0\end{bmatrix}$ reflection is $\begin{bmatrix}1\\0\\0\end{bmatrix} - \underbrace{2\left(\frac{1}{\sqrt3}\right)\begin{bmatrix}\frac{1}{\sqrt3}\\\frac{1}{\sqrt3}\\\frac{1}{\sqrt3}\end{bmatrix}}_{\begin{bmatrix}\frac13\\\frac13\\\frac13\end{bmatrix}} = \begin{bmatrix}\frac13\\-\frac23\\-\frac23\end{bmatrix}$

So $B\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}\frac13\\-\frac23\\-\frac23\end{bmatrix}$ so

$$\begin{bmatrix}\frac13 & x & x\\ -\frac23 & x & x\\ -\frac23 & x & x\end{bmatrix} = B$$

for $\begin{bmatrix}0\\1\\0\end{bmatrix} \longrightarrow v_{ref} = \begin{bmatrix}-\frac23\\\frac13\\-\frac23\end{bmatrix} \longrightarrow B = \begin{bmatrix}\frac13 & -\frac23 & -\frac23\\ -\frac23 & \frac13 & -\frac23\\ -\frac23 & -\frac23 & \frac13\end{bmatrix}$

for $\begin{bmatrix}0\\0\\1\end{bmatrix}$

2.2

3-21-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{3}{2} & 0 \\ -\frac{1}{2} & 1 & -\frac{3}{2} & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & -2 & -\frac{3}{2} \\ 0 & 0 & 0 & \frac{5}{2} \end{bmatrix}$$

E A u

OPS, I completly forgot to import negetive signes so do it again

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix} \left[\begin{array}{cccc|c} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{array}\right] = \left[\begin{array}{cccc|c} 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 4 & -3 & 0 \\ 0 & 0 & 0 & 5 & 20 \end{array}\right]$$

E A u

$t = 4 \longrightarrow z = -3 \longrightarrow$ OPS agin I have Problem to write solution corretly there are 22

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & -2 & 3 & 0 \\ 1 & -2 & 3 & -f \end{bmatrix} \left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right] = \left[\begin{array}{cccc|c} \boxed{2} & 1 & 0 & 0 & 0 \\ 0 & \boxed{-3} & -2 & 0 & 0 \\ 0 & 0 & \boxed{f} & 3 & 0 \\ 0 & 0 & 0 & \boxed{-5} & -20 \end{array}\right]$$

E A u

$t = f \rightsquigarrow z = -3 \longrightarrow y = 2 \rightsquigarrow x = -1$

4_4. if $a=2$ it beraek down pormanently

if $a=0$ cause of being pivot zero we need exchang

$3y=-3 \rightarrow y=1 \rightarrow x=3$

5.27

$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} 1 \rightarrow \\ -1 \rightarrow \\ 1 \rightarrow \\ \\ \end{array} & \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 4 \\ 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 8 \\ 0 & 1 & 0 & 1 & 5 \end{array}\right] \end{array}$$

$4-2+8=5$

$5=10$

$a=8$	$a=3$
$b=0$	$b=1$
$c=-2$	$c=1$
$d=10$	$d=9$

it wouldn't psible to element matrix

2.3

6.17

$a+b+c=4$

$a+2b+4c=8$

$a+3b+9c=14$

$$\underset{E_1}{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}} \underset{A}{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 14 \end{array}\right]} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 20 \end{array}\right]$$

$c=1 \rightarrow b=1 \rightarrow a=2$

7.18

$$EF = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \quad F^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2c & 1 \end{bmatrix}$$

$$FE = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ ac+b & c & 1 \end{bmatrix} \quad F^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3c & 1 \end{bmatrix}$$

$$E^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2a & 1 & 0 \\ 2b & 0 & 1 \end{bmatrix}$$

8-25

$$\underset{E}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}} \underset{A}{\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 2 \\ 3 & 5 & 7 & 6 \end{array}\right]} = \underset{U}{\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 3 \end{array}\right]}$$

if we substitut 6 to 3 $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$ is possible

and eqations has alot of answers

2.4

9- 14 a , T | Solution answer

b , T → f $A_{k,m}$ $B_{m \times k}$

C , T

d , T → f if B is complete zero

e

10- 22

$$A = 0.5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^2 = (0.5)^2\, 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

True Answer

$A^2 = 2A$ So $\rightarrow A^n = 2^{n-1}A = 2^{n-1}\,0.5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^3 = 2A^2 = 2(A)A$

$$AB = .5\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow (AB)^2 = (.5)^2\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \text{So } (AB)^n = 0 \quad n \geq 2$$

11- 24

a) $A^2 = 0$ $\quad \cancel{\begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & -2 \\ 1 & -2 & 1 \end{bmatrix}}$ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

A →

b) $A^2 \neq 0$, $A^3 = 0$ $\quad \underset{A}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} \underset{A^2}{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \underset{A^3}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$

2.5 _12.32

$$[A \ I] = \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

13. $$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{B} - \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} = I$$

af bh
ea fc

$$\begin{bmatrix} bg - fc & af + bh - eb - fd \\ & cf - gb \end{bmatrix}$$

So it is imposible