9631407- روز فنی علی

① $T(n) = \begin{cases} \Theta(1) & n = 1 \\ T(n-1) + \Theta(n) & n > 1 \end{cases}$

② ابتدا ورودی را Sort می کنیم ($O(n \lg n)$)؛ سپس برای هر S_i با، روش Binary-Search ($O(\lg n)$) دنبال $S_j = x - S_i$ می گردیم ($O(n \lg n)$).

③ (a) نادرست : $n = O(n^2)$ ولی $n^2 \neq O(n)$

(b) نادرست : $n + n^2 \neq \Theta(\min(n, n^2)) = \Theta(n)$

(c) درست : $f(n) = O(g(n)) \Rightarrow f(n) \leq c \cdot g(n) \Rightarrow \lg(f(n)) \leq \lg(c \cdot g(n)) \leq c' \cdot \lg(g(n))$
$\Rightarrow \lg(f(n)) \leq c' \cdot \lg(g(n)) \Rightarrow \lg(f(n)) = O(\lg(g(n)))$

(d) نادرست : $2n = O(n)$ ولی $2^{2n} = 4^n \neq O(2^n)$

(e) نادرست : $2^{-n} \neq O((2^{-n})^2) = O(4^{-n})$

(f) درست : $f(n) = O(g(n)) \Rightarrow f \leq c \cdot g(n) \Rightarrow \frac{1}{c} \cdot f \leq g \Rightarrow c' \cdot f \leq g$
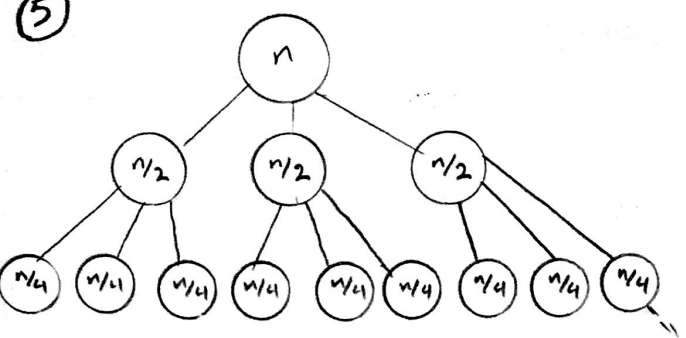$\Rightarrow g(n) = \Omega(f(n))$

(g) نادرست : $4^n \neq \Theta((4)^{n/2}) = \Theta(2^n)$.

(h) درست : $\left.\begin{array}{l} c_1 \cdot f(n) \leq f(n) \leq c_2 \cdot f(n) \\ c_3 \cdot of(n) \leq f(n) \end{array}\right\} \Rightarrow c_1 \cdot f(n) \leq f(n) + of(n) \leq f(n) + c_2 \cdot f(n)$
$\Rightarrow c_1 \cdot f(n) \leq f(n) + of(n) \leq c_3 \cdot f(n) \Rightarrow$
$\Rightarrow f(n) + of(n) = \Theta(f(n))$

④ فرض می کنیم $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n = O(n \lg n)$ اثبات می کنیم : $T(n) \leq c(n-a) \lg(n-a)$

$\Rightarrow T(n) \leq 2c(\lfloor n/2 \rfloor + 17 - a) \lg(\lfloor n/2 \rfloor + 17 - a) + n \leq 2c(n/2 + 17 - a) \lg(n/2 + 17 - a) + n$

$= c(n + 34 - 2a) \lg(\tfrac{1}{2}(n + 34 - 2a)) + n = c(n + 34 - 2a) \lg(n + 34 - 2a) - c(n + 34 - 2a) + n$

$\leq c(n + 36 - 2a) \lg(n + 34 - 2a) \leq c(n-a) \lg(n-a)$

①

⑤

هزینه	سطح
n	1
$\frac{3n}{2}$	2
$\frac{9n}{4}$	3
$\vdots$	$\vdots$
$\left(\frac{3}{2}\right)^{i-1} n$	i
$\vdots$	$\vdots$
$\left(\frac{3}{2}\right)^{\lg n - 1} n$	$\lg n$
$3^{\lg n}$	$\lg n + 1$

★ تعداد برگها $= 3^{\lg n} = n^{\lg 3}$

$$T(n) = \sum_0^{\lg n - 1} \left(\frac{3}{2}\right)^i n + 3^{\lg n} =$$

$$= n \frac{\left(\frac{3}{2}\right)^{\lg n} - 1}{\frac{3}{2} - 1} + \Theta(n^{\lg 3}) =$$

$$= 2n\left[n^{\lg 3/2} - 1\right] + \Theta(n^{\lg 3}) = 2n\left[n^{\lg 3 - \lg 2} - 1\right] + \Theta(n^{\lg 3}) =$$

$$= 2\left[n^{\lg 3} - n\right] + \Theta(n^{\lg 3}) = O(n^{\lg 3})$$

جایگذاری: $\quad T(n) \leqslant c_1 n^{\lg 3} - c_2 n \implies 3T\lfloor n/2 \rfloor + n \leqslant 3\left(c_1 (n/2)^{\lg 3} - c_2 (n/2)\right) + n$

$$= \frac{3}{2^{\lg 3}} c_1 n^{\lg 3} - \left(\frac{3d}{2} - 1\right)n \implies \forall d \geqslant 2 : T(n) \leqslant c_1 n^{\lg 3} - c_2 n$$

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⑥

(a) $T(n) = 4T(n/2) + n \implies a = 4, b = 2, \quad n = O(n^{\lg_b a - \varepsilon}) \implies 0 < \varepsilon \leqslant 1$

$\implies$ حالت اول قضیه اصلی $\implies T(n) = \Theta(n^2)$

(b) $T(n) = 4T(n/2) + n^2 \implies a = 4, b = 2, \quad n^2 = \Theta(n^{\lg_b a}) = \Theta(n^2)$

$\implies$ حالت دوم قضیه اصلی $\implies T(n) = \Theta(n^2 \lg n)$

(c) $T(n) = 4T(n/2) + n^3 \implies a = 4, b = 2, \quad n^3 = \Omega(n^{\lg_b a + \varepsilon}) \implies 0 < \varepsilon \leqslant 1$ $\Bigg\}\implies$

$a \cdot f(n/b) \leqslant c \cdot f(n) \implies 4\left(\frac{n}{2}\right)^3 \leqslant c \cdot n^3 \implies \frac{1}{2} \leqslant c < 1$

$\implies$ حالت سوم قضیه اصلی $\implies T(n) = \Theta(n^3)$

Ⅱ