

HW4 Solution

(1) (a) $x(t) \leftrightarrow X(j\omega)$

$x(t+5) \leftrightarrow e^{j\omega 5} X(j\omega)$ → First Part

$x(t+4) \leftrightarrow e^{j\omega 4} X(j\omega)$

$x(-t+4) \leftrightarrow e^{-j\omega 4} X(j\omega)$ → Second Part

$= x(t+5) - x(-t+4) \leftrightarrow e^{j\omega 5} X(j\omega) - e^{-j\omega 4} X(-j\omega)$

(b) $x(t) \leftrightarrow X(j\omega)$

$x(t+1) \leftrightarrow e^{j\omega} X(j\omega)$

$x(3t+1) \leftrightarrow \frac{1}{|3|} e^{j\frac{\omega}{3}} X(j\frac{\omega}{3})$

(c) $x(t) \leftrightarrow X(j\omega)$

$x(t-5) \leftrightarrow e^{-j\omega 5} X(j\omega)$

$\frac{d}{dt} x(t-5) \leftrightarrow j\omega e^{-j\omega 5} X(j\omega)$

$\frac{d^3}{dt^3} x(t-5) \leftrightarrow (j\omega)^3 e^{-j\omega 5} X(j\omega)$

$$(d) \quad x(t) \leftrightarrow X(j\omega)$$

$$p(t) = x(t-1) \leftrightarrow e^{-j\omega} X(j\omega) = P(j\omega)$$

$$tp(t) \leftrightarrow j \frac{d}{d\omega} P(j\omega) = j \frac{d}{d\omega} (e^{-j\omega} X(j\omega)) = \dots$$

قواعد مشتق

$$(2) \quad (a) \quad x(t) = 2 + \cos(3\pi t + \frac{\pi}{4})$$

$$\text{periodic} : \omega_0 = 3\pi \Rightarrow T = \frac{2\pi}{3\pi} = \frac{2}{3}$$

$$x(t) = 2 + \frac{1}{2} \left(e^{j(\omega_0 t + \frac{\pi}{4})} + e^{-j(\omega_0 t + \frac{\pi}{4})} \right)$$

$$= \underbrace{2}_{a_0} + \underbrace{\frac{1}{2} e^{j\frac{\pi}{4}}}_{a_1} e^{j\omega_0 t} + \underbrace{\frac{1}{2} e^{-j\frac{\pi}{4}}}_{a_{-1}} e^{-j\omega_0 t}$$

$$x(t) = \sum_k a_k e^{jk\omega_0 t} \leftrightarrow X(j\omega) = \sum_k 2\pi a_k \delta(\omega - k\omega_0)$$

$$\Rightarrow X(j\omega) = 4\pi \delta(\omega) + \cancel{\pi} \pi e^{j\frac{\pi}{4}} \delta(\omega - \omega_0) + \pi e^{-j\frac{\pi}{4}} \delta(\omega + \omega_0)$$

$$(2) (b) x(t) = t e^{-4t} \cos(2t) u(t)$$

$$= \underbrace{(t e^{-4t} u(t))}_{r(t)} \underbrace{\cos(2t)}_{s(t)}$$

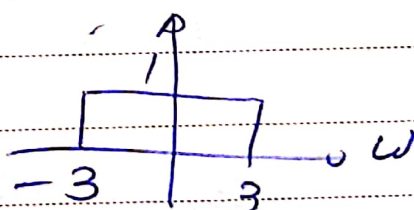
$$r(t) \xleftrightarrow{FT} R(j\omega) = \frac{1}{(4+j\omega)^2}$$

$$s(t) \xleftrightarrow{FT} S(j\omega) = \pi (\delta(\omega-2) + \delta(\omega+2))$$

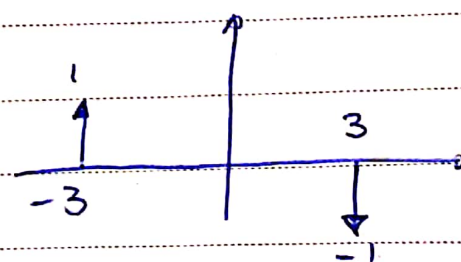
$$x(t) = r(t) s(t) \xleftrightarrow{FT} \frac{1}{2\pi} (R(j\omega) * S(j\omega))$$

$$= \frac{1}{2} (R(j(\omega-2)) + R(j(\omega+2)))$$

$$(c) x(t) = t \underbrace{\frac{\sin(3t)}{\pi t}}_{r(t)}$$

$$r(t) \xleftrightarrow{FT} R(j\omega) =$$


$$x(t) = t r(t) \xleftrightarrow{FT} j \frac{d}{d\omega} R(j\omega)$$

$$= j \times$$


(2) (d)

$$x(t) = \frac{4t}{(1+t^2)^2}$$

$$\text{Duality: } f(t) \leftrightarrow F(j\omega)$$

$$F(t) \leftrightarrow 2\pi f(-\omega)$$

$$\text{pair: } e^{-|t|} \leftrightarrow \frac{2}{1+\omega^2}$$

$$\text{duality: } \frac{2}{1+t^2} \leftrightarrow 2\pi e^{-|\omega|}$$

$$\text{so: } \frac{4t}{(1+t^2)^2} \leftrightarrow j\omega (2\pi e^{-|\omega|})$$

$$(2) (e) \quad x(t) = \underbrace{e^{-3|t|}}_{s(t)} \underbrace{\sin(2t)}_{p(t)}$$

$$s(t) = e^{-3|t|} \leftrightarrow S(j\omega) = \frac{6}{9 + \omega^2}$$

$$p(t) = \sin(2t) \leftrightarrow P(j\omega) = \frac{\pi}{j} (\delta(\omega - 2) - \delta(\omega + 2))$$

$$x(t) = s(t)p(t) \leftrightarrow X(j\omega) = \frac{1}{2\pi} (S(j\omega) * P(j\omega))$$

b. مبر

$$(3) (a) X(j\omega) = 3\delta(\omega+4)$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$e^{-j\omega 4t} \leftrightarrow 2\pi\delta(\omega+4)$$

$$\xrightarrow{\div \pi} \frac{1}{\pi} e^{-j\omega 4t} \leftrightarrow 2\delta(\omega+4)$$

$$\xrightarrow{\times \frac{3}{2\pi}} \frac{3}{2\pi} e^{-j\omega 4t} \leftrightarrow 3\delta(\omega+4)$$

$$(b) X(j\omega) = \frac{-j\omega+5}{(-\omega^2+10j\omega+21)} = \frac{-j\omega+5}{(j\omega+7)(j\omega+3)}$$

$-\omega^2 = (j\omega)^2$

$$= \frac{A}{j\omega+7} + \frac{B}{j\omega+3} = \frac{(A+B)j\omega + (3A+7B)}{(j\omega+7)(j\omega+3)}$$

$$\Rightarrow \begin{cases} A+B = -1 \\ 3A+7B = 5 \end{cases} \quad \begin{matrix} A = \underline{\hspace{1cm}} \\ B = \underline{\hspace{1cm}} \end{matrix}$$

$$\Rightarrow X(j\omega) = \frac{A}{j\omega+7} + \frac{B}{j\omega+3} \leftrightarrow Ae^{-7t}u(t) + Be^{-3t}u(t)$$

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3 (c) $X(j\omega) = \pi e^{-5|\omega|}$

في ديس $e^{-a|t|} \leftrightarrow \frac{2a}{a^2 + \omega^2}$

duality $\frac{2a}{a^2 + t^2} \leftrightarrow 2\pi e^{-a|\omega|} = 2\pi e^{-a|\omega|}$

$a = 5 \rightarrow \frac{10}{25 + t^2} \leftrightarrow 2\pi e^{-5|\omega|}$

$\div 2 \rightarrow \frac{5}{25 + t^2} \leftrightarrow \pi e^{-5|\omega|}$

5 \rightarrow استأجر سؤال 4 في القسم!

$$y''(t) + y'(t) - 12y(t) = 7x(t)$$

(a) تبدیل فوریه زیرین: $(j\omega)^2 Y(j\omega) + j\omega Y(j\omega) - 12Y(j\omega) = 7X(j\omega)$

$$\Rightarrow H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{7}{(j\omega)^2 + j\omega - 12}$$

$$= \frac{7}{(j\omega + 4)(j\omega - 3)} = \frac{A}{j\omega + 4} + \frac{B}{j\omega - 3} \quad \text{حساب A و B با استفاده از مجهول}$$

$$\Rightarrow h(t) = A e^{-4t} u(t) + B e^{3t} u(t)$$

$$(5) (b) x(t) = te^{-4t} u(t) \Rightarrow X(j\omega) = \frac{1}{(4+j\omega)^2}$$

$$Y(j\omega) = X(j\omega) H(j\omega) = \frac{7}{(j\omega+4)^2 (j\omega+4)(j\omega-3)}$$

تبدیل به کسرهای ساده با A, B, C, D و شتاب سوال است

$$(6) a(t) = x(t)p(t) = \frac{\sin(2\pi t)}{\pi t} \times \frac{\sin(2\pi t)}{\pi t}$$

$$\Rightarrow A(j\omega) = \frac{1}{2\pi} (X(j\omega) * P(j\omega))$$

$$= \frac{1}{2\pi} \left(\text{rect}_{[-2\pi, 2\pi]} * \text{rect}_{[-2\pi, 2\pi]} \right)$$

$$= \frac{1}{2\pi} \left(\text{tri}_{[-4\pi, 4\pi]} \right) = \text{tri}_{[-4\pi, 4\pi]}$$

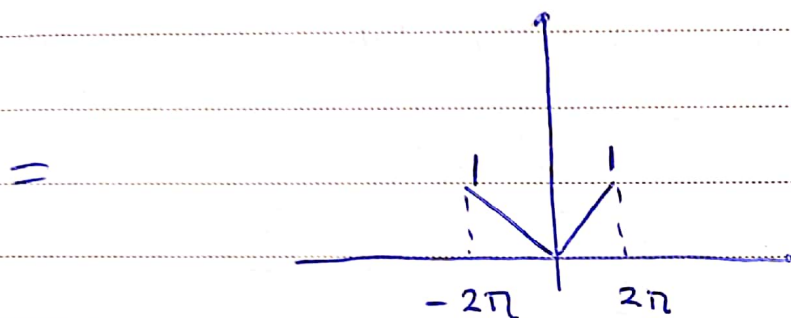
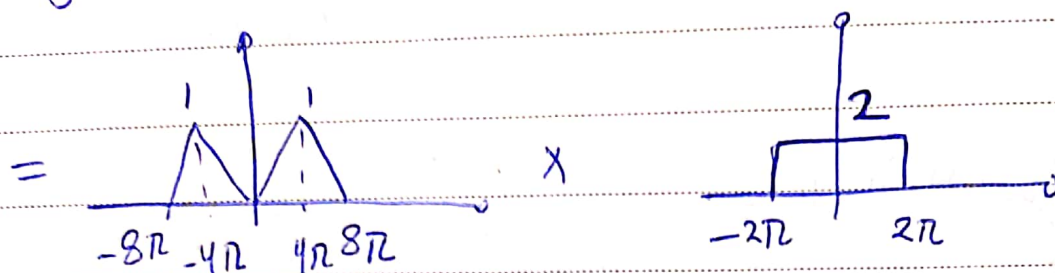
$$b(t) = a(t)q(t) \Rightarrow B(j\omega) = \frac{1}{2\pi} (A(j\omega) * Q(j\omega))$$

$$= \frac{1}{2\pi} (A(j\omega) * \pi(\delta(\omega-4\pi) + \delta(\omega+4\pi)))$$

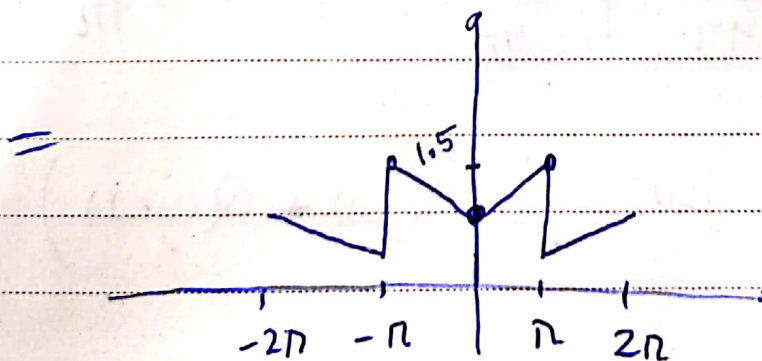
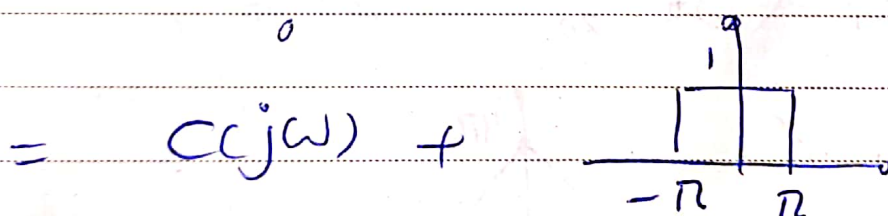
$$= \frac{1}{2} (A(j(\omega-4\pi)) + A(j(\omega+4\pi))) = \frac{1}{2} \left(\text{tri}_{[-4\pi, 4\pi]} + \text{tri}_{[-4\pi, 4\pi]} \right)$$

$$= \text{tri}_{[-8\pi, 8\pi]}$$

$$C(j\omega) = B(j\omega) H(j\omega)$$



$$D(j\omega) = C(j\omega) + R(j\omega)$$



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$$(7) H(j\omega) = \frac{j\omega + 10}{(j\omega)^2 + 21j\omega + 98} = \frac{Y(j\omega)}{X(j\omega)}$$

$$\Rightarrow (j\omega)^2 Y(j\omega) + 21j\omega Y(j\omega) + 98 Y(j\omega) = j\omega X(j\omega) + 10 X(j\omega)$$

$$\Rightarrow \frac{d^2}{dt^2} y(t) + 21 \frac{d}{dt} y(t) + 98 y(t) = \frac{d}{dt} x(t) + 10 x(t)$$