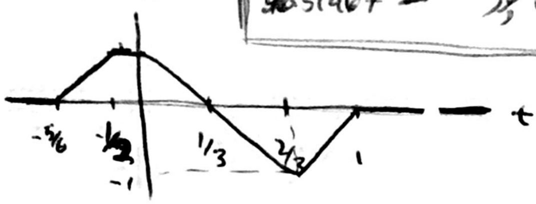
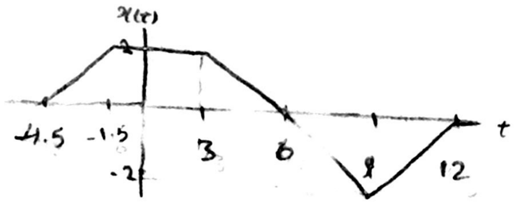
Q1

9631407 — علی خوش پور

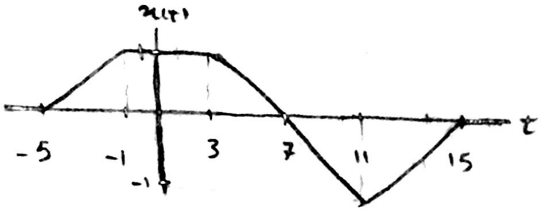
(a) $x(3t+1)$



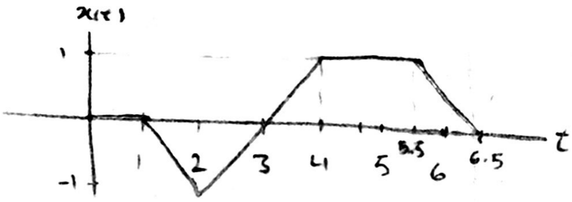
(b) $2x(t/3)$



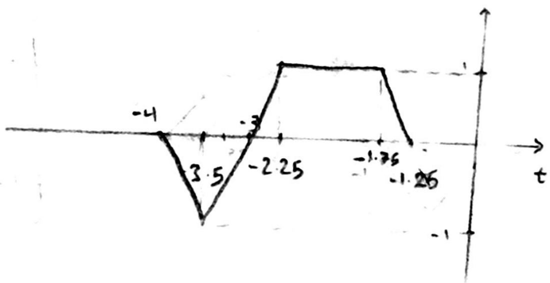
(c) $x\left(\frac{t+1}{4}\right)$



(d) $x(-t+5)$



(e) $x(-2t-4)$



Q2

(a) $x(t) = e^{2t}\sin(t)\,u(-t)$

$$Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[e^{2t}\sin(t)u(-t) + e^{-2t}\sin(-t)\,u(-(-t))]$$

$$\Rightarrow Ev\{x(t)\} = \begin{cases} t<0 & : \frac{1}{2}[e^{2t}\sin(t)] \\ t=0 & : \frac{1}{2}[\sin(t)(e^{2t}-e^{-2t})] \\ t>0 & : \frac{1}{2}[-e^{-2t}\sin(t)] \end{cases}$$

$$Od\{x(t)\} = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[e^{2t}\sin(t)u(-t) - e^{-2t}\sin(-t)\,u(-(-t))]$$

$$\Rightarrow Od\{x(t)\} = \begin{cases} t<0 & : \frac{1}{2}[e^{2t}\sin(t)] \\ t=0 & : \frac{1}{2}\sin(t)[e^{2t}+e^{-2t}] \\ t>0 & : \frac{1}{2}[e^{-2t}\sin(t)] \end{cases}$$

$$x(t) = \begin{cases} t<0 & : e^{2t}\sin(t) \\ t=0 & : e^{2t}\sin(t) \\ t>0 & : 0 \end{cases}$$

$$x(t) = Ev\{x(t)\} + Od\{x(t)\}$$

(b) $x(t) = e^{-|t|}\cos(t)$

$$Ev\{x(t)\} = \frac{1}{2}[e^{-|t|}\cos(t) + e^{-|-t|}\cos(-t)]$$

$$= \frac{1}{2}[2e^{-|t|}\cos(t)] = e^{-|t|}\cos(t)$$

$$Od\{x(t)\} = \frac{1}{2}[e^{-|t|}\cos(t) - e^{-|-t|}\cos(-t)]$$

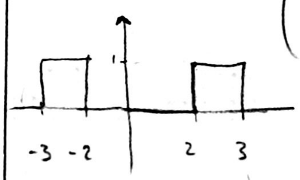
$$= 0$$

(c) $x(t) = 2\Pi(t-2.5)$

$$Ev\{x(t)\} = \frac{1}{2}[2\Pi(t-2.5) + 2\Pi(-t-2.5)] =$$

$$= \frac{1}{2}[2(u(t-2) - u(t-3) + u(-t-2) - u(-t-3)]$$

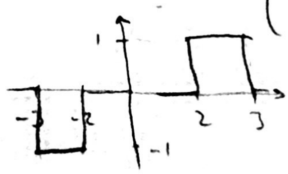
$$\Rightarrow Ev\{x(t)\} = \begin{cases} 2<t<3 \ , \ -3<t<2 & 1 \\ o.w. & 0 \end{cases}$$



$$Od\{x(t)\} = \frac{1}{2}[2\Pi(t-2.5) - 2\Pi(-t-2.5)] =$$

$$= \frac{1}{2}[2(u(t-2) - u(t-3) - u(-t-2) + u(-t-3))]$$

$$\Rightarrow Od\{x(t)\} = \begin{cases} 2<t<3 & 1 \\ -3<t<-2 & -1 \\ o.w. & 0 \end{cases}$$



Q3

$x(t) = x(t+T) \Rightarrow$ periodic

(a) $x(t) = e^{j\pi/3\, t} = \cos(\pi/3\, t) + j\sin(\pi/3\, t)$

$\Rightarrow T_0 = \dfrac{2\pi}{\pi/3} = 6$, is Periodic

(b) $x(t) = e^{j\pi/3\, t} \times e^{-j\pi/3\, t} = 1 \Rightarrow$ تابع ثابت

$\Rightarrow T_0 > 0$, is Periodic

(c) $x(t) = e^{jt + t/2} = \cos(3t/2) + j\sin(3t/2)$

$\Rightarrow T_0 = \dfrac{2\pi}{3/2} = \dfrac{4\pi}{3}$, is Periodic

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(d) $x[n] = e^{j3\pi n} = \cos(3\pi n) + j\sin(3\pi n)$

$\Rightarrow T_0 = \dfrac{2\pi}{3\pi} = \dfrac{2}{3}$, is Periodic

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(e) $x[n] = e^{j3n} = \cos(3n) + j\sin(3n)$

$\Rightarrow T_0 = \dfrac{2\pi}{3} \notin \mathbb{Z}$, not periodic

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(f) $x(t) = e^{2(t)} \cos(t)$ $\quad e^{2(t)}$ is not periodic

$\Rightarrow x(t)$ is not periodic

(g) $x[n] = 2\cos(2\pi n) + \cos(\pi/3\, n)$

$x_1[n] = 2\cos(2\pi n) \Rightarrow T_{0_2} = \dfrac{2\pi}{2\pi} = 1$

$x_2[n] = \cos(\pi/3\, n) \Rightarrow T_{0_2} = \dfrac{2\pi}{\pi/3} = 6$

$\Rightarrow T_0 = 6$, is periodic

(h) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-6k] + \delta[n-1-6k]$

$T_0 = 6$, is periodic

(i) $x(t) = \cos^2(3t + \pi/6)$

$T_0 = \dfrac{2\pi}{3} \xrightarrow[\text{رسیده است}]{\text{cos به توان 2}} \pi/3$

is periodic

(j) $x(t) = \mathcal{O}d\{\cos(\pi t)\, u(t)\}$

$= \frac{1}{2}[\cos(\pi t)u(t) - \cos(-\pi t)u(-t)]$

$= \begin{cases} t<0 & -\frac{1}{2}\cos(\pi t) \\ t\geq 0 & \frac{1}{2}\cos(\pi t) \end{cases} \Rightarrow$ not periodic

(k) $x[n] = \cos(\pi/8\, n)$ $\quad T_0 = \dfrac{2\pi}{\pi/8} = 16$

is periodic

Q4

(a) $y(t) = e^{x(t)}$

memory-less: yes, both input and output depend on t

Causal: yes, only depends on $x(t)$

time-invariance: $x(t-t_0) \rightarrow e^{x(t-t_0)}$
$y(t-t_0) \rightarrow e^{x(t-t_0)}$

$\Rightarrow$ time-invariant

stability: $-a < x(t) < a \Rightarrow e^{-a} < y < e^{a}$

linearity: $ax_1(t) + x_2(t) \longrightarrow e^{ax_1(t) + x_2(t)}$

$\Rightarrow$ not linear

(b) $y(t) = \sin^2(t)\, x(t)$

memory-less: yes, input and output depend on t

Causal; only depends on $x(t)$

$\left.\begin{matrix} x(t-t_0) \rightarrow \sin^2(t)\, x(t-t_0) \\ y(t-t_0) \rightarrow \sin^2(t-t_0)\, x(t-t_0) \end{matrix}\right\}$ not time invariant

$\left.\begin{matrix} -a < x(t) < a \\ 0 < \sin(t) < 1 \end{matrix}\right\} \Rightarrow -a < y(t) < a \Rightarrow$ Stable linearity : $a x_1(t) + x_2(t) \rightarrow a y_1(t) + y_2(t)$

$\Rightarrow$ linear

(c) $y(t) = t x^2(t)$

memory-less : yes, input and output depend on t

causal ; yes only depends on $x(t)$

$\begin{cases} x(t-t_0) \rightarrow t\, x^2(t-t_0) \\ y(t-t_0) \rightarrow (t-t_0) x^2(t-t_0) \end{cases}$ not time invariant

$-a < x(t) < a \Rightarrow -b < t x^2(t) < b \Rightarrow$ stable

$a x_1(t) + x_2(t) \rightarrow a^2 y_1(t) + y_2(t)$

$\Rightarrow$ not linear

(e) $y[n] = x[2n+1]$

memoryless : no, input depends on $2n+1$

not causal ; depends on $n+1$ $(2n+1 = n + (n+1))$

$\begin{cases} x[2n+1-t_0] \rightarrow x[2n-t_0+1] \\ y[n-t_0] \rightarrow x[2n-2t_0+1] \end{cases}$ not time invariant

$-a < x[n] < a \Rightarrow -a < x[2n+1] < a \Rightarrow -b < y[n] < b$
$\Rightarrow$ stable

$a x_1(t) + x_2(t) \rightarrow a y_1(t) + y_2(t) \Rightarrow$ linear

(d) $y[n] = \sum_{k=-\infty}^{n+1} x[k] = x[n+1] + x[n] + \cdots$

memory-less : no, input depends on $n+1, n-1, \cdots$

not causal ; depends on $x[n+1]$

$\begin{cases} x[n-n_0] \rightarrow \sum_{-\infty}^{n+1} x[k] = \sum_{-\infty}^{n-n_0+1} x[k] \\ y[n-n_0] \rightarrow \sum_{-\infty}^{n+1} x[k] = \sum_{-\infty}^{n-n_0+1} x[k] \end{cases} \Rightarrow$ time invariant

$\sum_{-\infty}^{n+1} x[k] = x[n+1] + x[n] + \cdots = \infty \Rightarrow$ not stable

$a x_1[n] + x_2[n] \rightarrow a \sum_{-\infty}^{n+1} x_1[k] + \sum_{-\infty}^{n+1} x_2[k]$

$= a y_1[n] + y_2[n] \Rightarrow$ linear

(f) $y[n] = \sin(x[n])$

memory-less : yes, input and output depend on t

causal : yes, output depends on $x[n]$

$\begin{cases} x[n-n_0] \rightarrow \sin(x[n-n_0]) \\ y[n-n_0] \rightarrow \sin(x[n-n_0]) \end{cases} \Rightarrow$ time invariant

$-1 \leq \sin(x[n]) \leq 1 \Rightarrow -1 \leq y \leq 1 \Rightarrow$ stable

$a x_1[n] + x_2[n] \neq a y_1[n] + y_2[n]$

$\Rightarrow$ not linear

Q5

(a) $y[n] = x[n+1]\, x[n-1] \rightarrow$ not invertible

(b) $y(t) = x(t/2)$ $\quad w(t) = y(2t)$

$\Rightarrow$ invertible

(c) $y(t) = \begin{cases} x(t) & , t \geq 0 \\ x(t-1) & , t < 0 \end{cases}$

$w(t) = \begin{cases} y(t) & t \geq 0 \\ y(t+1) & t < 0 \end{cases} \Rightarrow$ invertible

(d) $y[n] = \begin{cases} x[n] & , n > 0 \\ 1 & , n = 0 \\ -x[n] & , n < 0 \end{cases}$ not invertible

(e) $y(t) = \frac{dx(t)}{dt}$ not invertible

Q6

(a) $x[n] = (\frac{1}{4})^n u[n]$

$P_\infty = \lim_{n\to\infty} \frac{1}{2n+1} \sum_{-n}^{n} |x[t]|^2 = \lim_{n\to\infty} \frac{1}{2n+1} \times$

$\sum_{-\infty}^{\infty} (\frac{1}{4})^{2n} u^2[n] = \lim_{n\to\infty} \frac{1}{2n+1} \sum_{0}^{\infty} (\frac{1}{4})^{2n} =$

$= \lim_{n\to\infty} \frac{1}{2n+1} \frac{1}{1-1/16} = 0$

$$E_\infty = \lim_{n\to\infty} \sum_{-n}^{n} |x[n]|^2 \xrightarrow[\text{قبل}]{\text{طبق تست}} \frac{1}{1-1/16} = 16/15$$

(b) $x(t) = j\cos(t)$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} \cos^2(t) \, dt \; : \; \text{واگرا}$$

$$P_\infty = \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} \cos^2(t) \, dt =$$

$$= \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} \frac{1+\cos(2t)}{2} \, dt =$$

$$= \lim_{t\to\infty} \frac{1}{2t} \left(\frac{t}{2} + \frac{\sin(2t)}{4} \right) = \frac{1}{4}$$

(C) $x(t) = e^{jt+t}$

$$E_\infty = \int_{-\infty}^{\infty} |e^{jt+t}|^2 \, dt \; : \; \text{واگرا}$$

$$P_\infty = \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} |e^{jt+t}|^2 \, dt =$$

$$= \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} |e^{2jt+2t}| \, dt =$$

$$= \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} \cancel{|e^{2jt}|}^{1} |e^{2t}| \, dt =$$

$$= \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} e^{2t} \, dt = \lim_{t\to\infty} \frac{1}{2t} \frac{e^{2t} - e^{-2t}}{2}$$

$$= \infty$$