حل تمرین سری 2 سیگنال

Sabject:
Year: Month: Date:

Sa Su Mo Tu We Th

1) a) $x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[n-k]x_2[k] = \sum_{k=-3}^{2} \left(\frac{1}{2}\right)^{n-k} u[n-k]$

$$u[n-k] = \begin{cases} 1 & , n-k \geq 0 \\ 0 & , n-k < 0 \end{cases} = \begin{cases} 1 & , n \geq k \\ 0 & , n < k \end{cases}$$

$$\Rightarrow x_1[n] * x_2[n] = \begin{cases} 0 & , n < -3 \\ \sum_{k=-3}^{n} \left(\frac{1}{2}\right)^{n-k} & , -3 \leq n < 3 \\ \sum_{k=-3}^{2} \left(\frac{1}{2}\right)^{n-k} & , 3 \leq n \end{cases} = \begin{cases} 0 & , n < -3 \\ 2 - \left(\frac{1}{2}\right)^{n+3} & ; -3 \leq n < 3 \\ \left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{2}\right)^{n+3} & , 3 \leq n \end{cases}$$

1) b) $x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[n-k]x_2[k] = \sum_{k=0}^{9} x_1[n-k]$

$$= \sum_{k=0}^{9} \left(\frac{1}{3}\right)^{n-k} \left(u[n-k] - u[n-k-7]\right), \quad u[n-k] - u[n-k-7] = \begin{cases} 1 & , n-6 \leq k \leq n \\ 0 & , \text{o.w.} \end{cases}$$

$$\begin{cases} 0 & , n < 0 \\ \sum_{k=0}^{n} \left(\frac{1}{3}\right)^{n-k} & , 0 \leq n < 6 \\ \sum_{k=n-6}^{n} \left(\frac{1}{3}\right)^{n-k} & , 6 \leq n < 10 \\ \sum_{k=n-6}^{9} \left(\frac{1}{3}\right)^{n-k} & , 10 \leq n < 16 \\ 0 & , 16 \leq n \end{cases} = \begin{cases} 0 & , n < 0 \\ \frac{3 - \left(\frac{1}{3}\right)^{n}}{2} & , 0 \leq n < 6 \\ \frac{3 - \left(\frac{1}{3}\right)^{6}}{2} & , 6 \leq n < 10 \\ \frac{\left(\frac{1}{3}\right)^{n-10} - \left(\frac{1}{3}\right)^{6}}{2} & , 10 \leq n < 16 \\ 0 & , 16 \leq n \end{cases}$$

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1) c) $x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(t-\tau)x_2(\tau)\,d\tau = \int_{0}^{3} e^{2t-2\tau}\,u(t-\tau-2)\,d\tau$

$$u(t-\tau-2) = \begin{cases} 1 & , \tau < t-2 \\ 0 & , \tau > t-2 \end{cases}$$

$$\Rightarrow x_1(t) * x_2(t) = \begin{cases} 0 & , t-2 < 0 \\ \int_0^{t-2} e^{2t-2\tau}\,d\tau & , 0 \leq t-2 < 3 \\ \int_0^{3} e^{2t-2\tau}\,d\tau & , 3 \leq t-2 \end{cases}$$

$$\Rightarrow x_1(t) * x_2(t) = \begin{cases} 0 & , t < 2 \\ \frac{e^{2t}}{2} - \frac{e^{4}}{2} & , 2 \leq t < 5 \\ \frac{e^{2t}}{2} - \frac{e^{2t-6}}{2} & , 5 \leq t \end{cases}$$

1) d) $x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(t-\tau)x_2(\tau)\,d\tau = \int_{0}^{2} x_1(t-\tau)\,d\tau + 2\int_{2}^{\infty} x_1(t-\tau)\,d\tau$

$$= \int_0^2 e^{2t-2\tau}\,u(2-t+\tau)\,d\tau + 2\int_2^{\infty} e^{2t-2\tau}\,u(2-t+\tau)\,d\tau$$

$$u(2-t+\tau) = \begin{cases} 0 & , 2-t+\tau < 0 \\ 1 & , 2-t+\tau \geq 0 \end{cases} \Rightarrow u(2-t+\tau) = 1 , \tau \geq t-2$$

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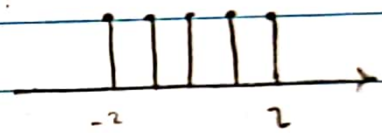
$$\Rightarrow x_1(t) * x_2(t) = \begin{cases} \int_0^2 e^{2t-2\tau}\, d\tau + 2\int_2^{\infty} e^{2t-2\tau}\, d\tau &, t-2<0 \\ \int_{t-2}^{2} e^{2t-2\tau}\, d\tau + 2\int_2^{\infty} e^{2t-2\tau}\, d\tau &, 0 \le t-2 < 2 \\ \int_{t-2}^{\infty} e^{2t-2\tau}\, d\tau &, 2 < t-2 \end{cases}$$

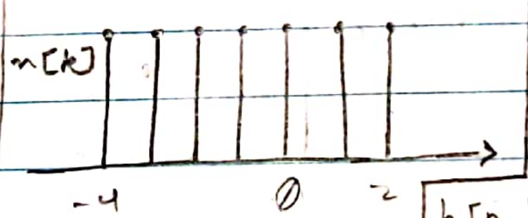
2) a) $x[k]$:

$h[-k]$:



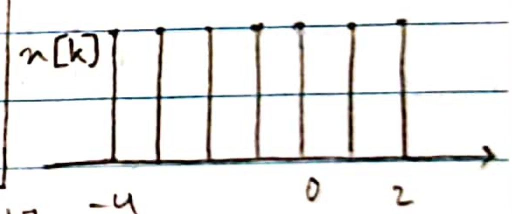
حالا به ازای n های مختلف، h را رسم میکنیم

$n < -6$



$h[n-k] \longrightarrow$

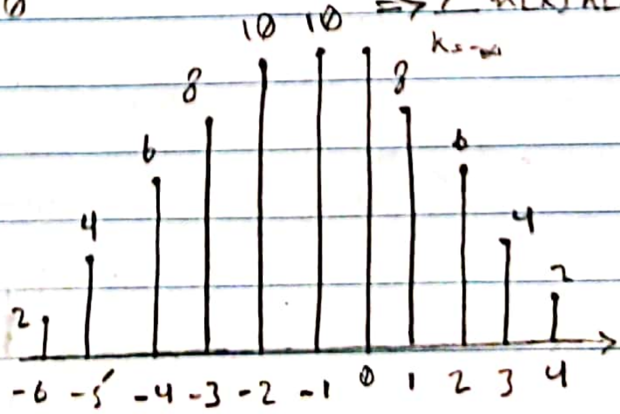
$-6 < n < -2$



$\Rightarrow \forall k: h[n-k] \times x[k] = 0$

$$\Rightarrow \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \begin{cases} 2 &, n=-6 \\ 4 &, n=-5 \\ \vdots \end{cases}$$

$\Rightarrow x[n] * h[n] =$



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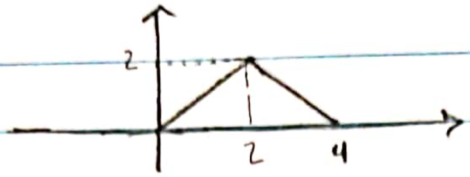
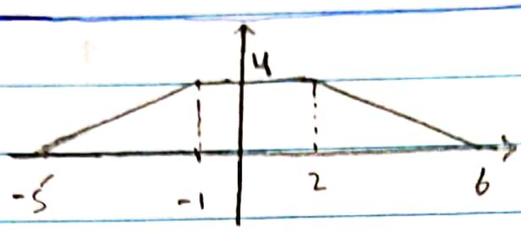
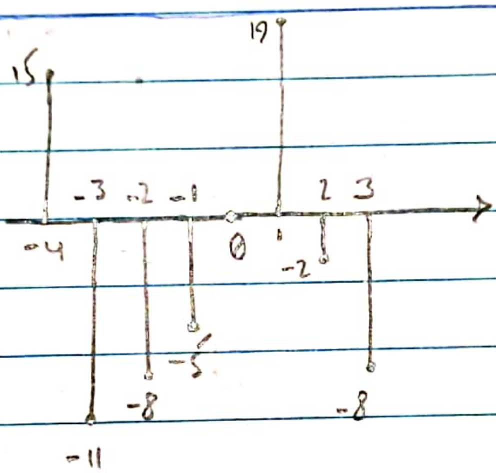
2) b)



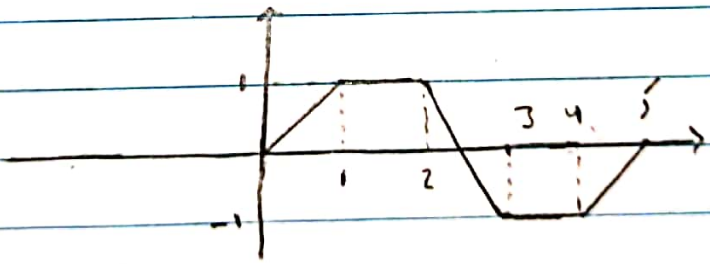
2) c)



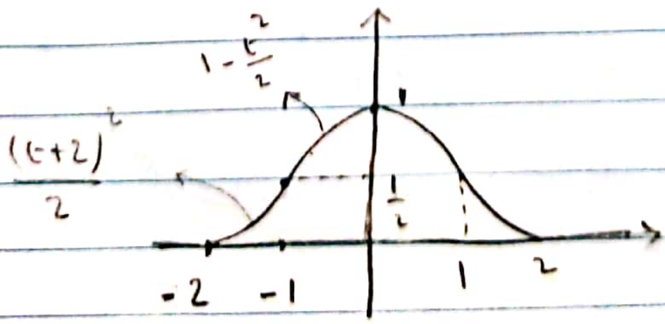
2) d)



2) e)



2) f)



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3) a) $h(t) \neq k\delta(t) \Rightarrow$ memoryless ✗

$\exists t<0 \; , \; h(t) \neq 0 \Rightarrow$ causal ✗

$$\int_{-\infty}^{\infty} \left| e^{-2|t|} \right| dt = \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt = \frac{e^{2t}}{2}\Big|_{-\infty}^{0} - \frac{e^{-2t}}{2}\Big|_{R}^{\infty}$$

stable ✓ $\Leftarrow$ متناهی

3) b) memoryless ✗ causal ✗

$$\int_{-\infty}^{\infty} \left| \sin(t)\, u(1-t) \right| dt = \int_{-\infty}^{1} \left| \sin(t) \right| dt = \infty \Rightarrow \text{stable ✗}$$

3) c) $(t^2+1)\,\delta(t) = \delta(t) \Rightarrow$ memoryless ✓

$\forall t<0 \; , \; h(t)=0 \Rightarrow$ causal ✓

$$\int_{-\infty}^{\infty} |h(t)|\, dt = 1 \Rightarrow \text{stable ✓}$$

3) d) memoryless ✗ $h[-1]=2 \Rightarrow$ causal ✗

$$\sum_{n=-\infty}^{\infty} |h[n]| = 2 \times \frac{1}{1-\frac{1}{2}} = 4 \Rightarrow \text{stable ✓}$$

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3)e) memoryless X causal X

$$\sum_{n=-\infty}^{\infty} |e^{n} u[-n]| = \sum_{n=-\infty}^{0} e^{n} = \frac{e}{e-1} \Rightarrow \text{stable} \checkmark$$

3)f) memoryless X

$$h[n] = \begin{cases} 0 & , n < -2 \\ -1+1 & , n = -2 \\ 0 & , n = -1 \\ \cos\left(\frac{\pi}{2} n\right) & , n \geq 0 \end{cases} \Rightarrow \text{causal} \checkmark$$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left|\cos\left(\frac{\pi}{2} n\right)\right| = \infty \Rightarrow \text{stable} \times$$

4)a) $h_{eq}(t) = h_1(t) + h_2(t) * [h_3(t) + h_4(t)]$

$$= \sin\left(\frac{\pi}{3} t\right) + e^{2t} u(1-t) * [\delta(t+1) + u(t)]$$

$$= \sin\left(\frac{\pi}{3} t\right) + e^{2t+2} u(-t) + \int_{0}^{\infty} e^{2t-2\tau} u(1-t+\tau)\, d\tau$$

$$\int_{0}^{\infty} e^{2t-2\tau} u(1-t+\tau)\, d\tau = \begin{cases} \int_{0}^{\infty} e^{2t-2\tau}\, d\tau & , t < 1 \\ \int_{t-1}^{\infty} e^{2t-2\tau}\, d\tau & , t \geq 1 \end{cases}$$

4) b) $y_1(t) = x(t) * h_1(t) = \int_{-\infty}^{\infty} \sin\left[\frac{\pi}{3}(t-\tau)\right]\left[u(\tau+1)-u(\tau-1)\right] d\tau$

$= \int_{-1}^{1} \sin\left[\frac{\pi}{3}(t-\tau)\right] d\tau = \frac{3}{\pi}\cos\left[\frac{\pi}{3}(t-\tau)\right]\Big|_{-1}^{1} = \frac{3}{\pi}\cos\left(\frac{\pi}{3}t-\frac{\pi}{3}\right) - \frac{3}{\pi}\cos\left(\frac{\pi}{3}t+\frac{\pi}{3}\right)$

$w(t) = x(t) * h_2(t) = \int_{-1}^{1} e^{2t-2\tau} u(1-t+\tau) = \begin{cases} \int_{-1}^{1} e^{2t-2\tau} d\tau, & t<0 \\ \int_{t-1}^{1} e^{2t-2\tau} d\tau, & 0 \leq t < 2 \\ 0, & 2 \leq t \end{cases}$

$= \begin{cases} -\frac{e^{2t-2}}{2} + \frac{e^{2t+2}}{2}, & t<0 \\ -\frac{e^{2t-2}}{2} + \frac{e^{2}}{2}, & 0 \leq t < 2 \\ 0, & 2 \leq t \end{cases}$

$y_3(t) = w(t) * h_3(t) = \int_{0}^{\infty} w(t-\tau)\, d\tau = \int_{0}^{2} w(t-\tau)\, d\tau$

$= \int_{0}^{2} -\frac{e^{2t-2\tau-2}}{2} + \frac{e^{2}}{2} d\tau = \frac{e^{2t-2\tau-2}}{4}\Big|_{0}^{2} + e^{2} = \frac{e^{2t-6}}{4} - \frac{e^{2t-2}}{4} + e^{2}$

$y_4(t) = w(t) * h_4(t) = \begin{cases} -\frac{e^{2t}}{2} + \frac{e^{2t+4}}{2}, & t<-1 \\ -\frac{e^{2t}}{2} + \frac{e^{2}}{2}, & -1 \leq t < 1 \\ 0, & 1 \leq t \end{cases}$

5)a) $\left(\frac{1}{5}\right)^n u[n] * \left(\delta[n] - \frac{1}{5}\delta[n]\right) = \left(\frac{1}{5}\right)^n u[n] * \frac{4}{5}\delta[n] \neq \delta[n]$

معکوس پذیر نیست.

5)b) $e^{-t}u(t) * [\delta(t) + \delta'(t)] = e^{-t}u(t) + (e^{-t}u(t))' * \delta(t)$

$= e^{-t}u(t) - e^{-t}u(t) + e^{-t}\delta(t) = \delta(t) \Rightarrow$ معکوس هست

5)c) $u[n] - u[n-1] = \delta[n] \Rightarrow$ معکوس پذیر هست

5)d) $e^{t} * u(t-1) = \int_{1}^{\infty} e^{t-\tau} d\tau \neq \delta(t) \Rightarrow$ معکوس پذیر نیست

6)a)

$$\delta(t) \longrightarrow \boxed{A} \longrightarrow \delta(t+1) + \delta(t-1) \Rightarrow s(t) = u(t+1) - u(t-1)$$

6)b) $s[n] = h[n] * u[n]$

$$= \sum_{k=-\infty}^{\infty} e^{k}\left(\frac{1}{4}\right)^{k} u[k+1]u[n-k] = \sum_{k=-1}^{\infty}\left(\frac{e}{4}\right)^{k} u[n-k]$$

$$= \begin{cases} 0, & n < -1 \\ \sum_{k=-1}^{n}\left(\frac{e}{4}\right)^{k}, & n \geq -1 \end{cases} = \begin{cases} 0, & n < -1 \\ \frac{4}{4-e}\left[1 - \left(\frac{e}{4}\right)^{n+1}\right], & n \geq -1 \end{cases}$$

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6) c) $s(t) = 2u^2(t)$

6) d) $s(t) = h(t) * u(t) = \int_{-\infty}^{\infty} \tau e^{-|\tau|} u(t-\tau)\, d\tau$

$$= \int_{-\infty}^{0} \tau e^{\tau} u(t-\tau)\, d\tau + \int_{0}^{\infty} \tau e^{-\tau} u(t-\tau)\, d\tau$$

$$= \begin{cases} \int_{-\infty}^{t} \tau e^{\tau} d\tau , & t<0 \\ \int_{-\infty}^{0} \tau e^{\tau} d\tau , & t \geq 0 \end{cases} + \begin{cases} 0 , & t<0 \\ \int_{0}^{t} \tau e^{-\tau} d\tau , & t \geq 0 \end{cases}$$

$$= \begin{cases} \left[\tau e^{\tau} - \int e^{\tau} d\tau\right]\Big|_{-\infty}^{t} , & t<0 \\ \left[\tau e^{\tau} - \int e^{\tau} d\tau\right]\Big|_{-\infty}^{0} + \left[-\tau e^{-\tau} + \int e^{-\tau} d\tau\right]_{0}^{t} , & t \geq 0 \end{cases}$$

$$= \begin{cases} t e^{t} - e^{t} , & t<0 \\ -1 - t e^{-t} - e^{-t} - 1 , & t \geq 0 \end{cases}$$

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