1 9631407 - علی عزیزی پور

(1) (a) $x_1[n] = 4+\cos(\frac{\pi}{6}n+\frac{\pi}{8})+2\sin(\frac{\pi}{4}n)$

$$= \overset{a_0}{4} + \frac{1}{2}\left(e^{j(\frac{\pi}{6}n+\frac{\pi}{8})} + e^{-j(\frac{\pi}{6}n+\frac{\pi}{8})}\right) + \frac{2}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right) \Rightarrow \omega_0 = \frac{\pi}{12},\ T_0 = 24$$

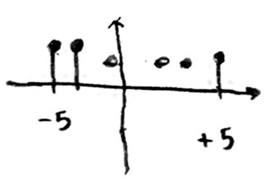
$$\Rightarrow \begin{cases} a_0 = 4 \\ a_2 = \frac{1}{2}e^{j\frac{\pi}{8}}, \quad a_{-2} = \frac{1}{2}e^{-j\frac{\pi}{8}} \\ a_3 = \frac{1}{j} = -a_{-3} \end{cases}$$

$$\hat{x}_1[n] \xleftrightarrow{DTFT} \hat{X}_1(e^{j\omega}) = 2\pi \sum_{0}^{23} a_k \delta(\omega - \frac{2\pi k}{24})$$

$$= 8\pi\delta(\omega) + \pi e^{j\frac{\pi}{8}}\delta(\omega-\frac{\pi}{6}) + \frac{2\pi}{j}\delta(\omega-\frac{\pi}{4}) + \pi e^{-j\frac{\pi}{8}}\delta(\omega+\frac{\pi}{6}) - \frac{2\pi}{j}\delta(\omega+\frac{\pi}{4})$$

(b) $x_2[n] = u[n-2] - u[n-13] =$

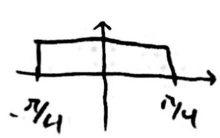

$\xrightarrow[\text{left}]{7\ \text{Shift}}$




$\longleftrightarrow \frac{\sin(\omega(\frac{11}{2}))}{\sin(\omega/2)}$

$$\Rightarrow x_2[n] \longleftrightarrow e^{-j\omega(7)} \frac{\sin(\omega(\frac{11}{2}))}{\sin(\omega/2)}$$

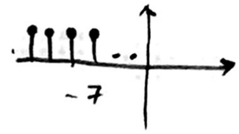
(C) $x_3[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n} \xleftrightarrow{DTFT}$




one period

(d) $x_4[n] = (\frac{1}{4})^{|n|} u[-n-7]$

$u[-n-7]:$




$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} (x[n]) e^{-jn\omega} = \sum_{-\infty}^{\infty} (\frac{1}{4})^{|n|} u[-n-7] e^{-jn\omega} =$$

$$\sum_{-\infty}^{-7} (\frac{1}{4})^{-n} e^{-jn\omega} = \sum_{-\infty}^{-7} (\frac{1}{4}e^{j\omega})^{-n} =$$

$$\sum_{7}^{\infty} (\frac{1}{4}e^{j\omega})^{n} = \sum_{m=0}^{\infty} (\frac{1}{4}e^{j\omega})^{m+7}$$

$$= \frac{1}{16386} e^{j7\omega} \frac{1}{1-\frac{1}{4}e^{j\omega}}$$

(e) $x_5[n] = 2^n \sin(\frac{\pi}{4}n) u[-n]$

$\Rightarrow x_5[-n] = y[n] = 2^{-n}(-\sin(\frac{\pi}{4}n)) u[n]$

$r[n] = -(\frac{1}{2})^n u[n]$, $s[n] = \sin(\frac{\pi}{4}n)$

$$\Rightarrow \begin{cases} R(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \\ \hat{S}(e^{j\omega}) = \frac{\pi}{j}\left(\delta(\omega-\frac{\pi}{4}) - \delta(\omega+\frac{\pi}{4})\right) \end{cases}$$

$$y[n] = r[n]s[n] \longleftrightarrow Y(e^{j\omega}) = \frac{1}{2\pi}\left(R(e^{j\omega}) * \hat{S}(e^{j\omega})\right)$$

$$= \frac{1}{2j}\left(R(e^{j(\omega-\frac{\pi}{4})}) - R(e^{j(\omega+\frac{\pi}{4})})\right) = X(e^{-j\omega})$$

بجای $\omega$ باید $-\omega$ بگذاریم تا $X(e^{j\omega})$ به دست بیاید .

② (a) $x[5-n] + x[-2-n]$

$x[5-n] \leftrightarrow e^{-j5\omega} X(e^{-j\omega})$

$x[-2-n] \leftrightarrow e^{-j(-2)\omega} X(e^{-j\omega})$

$\Rightarrow x[5-n]+x[-2-n] \leftrightarrow X(e^{-j\omega})(e^{-j5\omega}+e^{j2\omega})$

(b) $x^*[-n]$

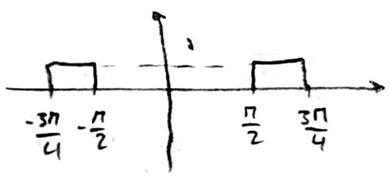
$x^*[n] \leftrightarrow X^*(e^{-j\omega})$

$x^*[-n] \leftrightarrow X^*(e^{j\omega})$

$x[n]$: real $\Rightarrow X^*(e^{j\omega}) = X(e^{-j\omega})$

③ (a) $\hat{X}(e^{j\omega}) = \begin{cases} 1 & \pi/2 < |\omega| < \frac{3\pi}{4} \\ 0 & \text{o.w.} \end{cases}$

$\rightarrow$




$$x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega}) e^{jn\omega}\, d\omega =$$

$$= \frac{1}{2\pi}\left(\int_{-\frac{3\pi}{4}}^{-\pi/2} e^{jn\omega}\, d\omega + \int_{\pi/2}^{3\pi/4} e^{jn\omega}\, d\omega\right) =$$

$$= \frac{1}{2jn\pi}\left(e^{jn\omega}\Big|_{-3\pi/4}^{-\pi/2} + e^{jn\omega}\Big|_{\pi/2}^{3\pi/4}\right) =$$

$$= \frac{1}{2jn\pi}\left(e^{-jn\frac{\pi}{2}} - e^{-jn\frac{3\pi}{4}} + e^{jn\frac{3\pi}{4}} - e^{jn\frac{\pi}{2}}\right)$$

$$= \frac{1}{\pi n}\left(-\sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{3\pi}{4}n\right)\right)$$

(b) $X(e^{j\omega}) = 3 + 3e^{-j2\omega} + 5e^{j144\omega}$

$= 3\delta[n] + 3\delta[n-2] + 5\delta[n+144]$

(c) $$X(e^{j\omega}) = \frac{7\overset{k}{e^{-j\omega}} + 10}{e^{-j2\omega} + 2e^{-j\omega} - 8} =$$

$$= \frac{7k+10}{k^2+2k-8} = \frac{A}{k-2} + \frac{B}{k+4} \Rightarrow A = 4, B = 3$$

$$\Rightarrow X(e^{j\omega}) = \frac{4}{e^{-j\omega}-2} + \frac{3}{e^{-j\omega}+4} =$$

$$= \frac{2}{1/2\, e^{-j\omega} - 1} + \frac{3/4}{1/4\, e^{-j\omega} + 1} =$$

$$= \frac{-2}{1 - 1/2\, e^{-j\omega}} + \frac{3/4}{1 + 1/4\, e^{-j\omega}} \Rightarrow$$

$$\Rightarrow x[n] = -2\left(\tfrac{1}{2}\right)^n u[n] + \tfrac{3}{4}\left(-\tfrac{1}{4}\right)^n u[n]$$

④ $$H(j\omega) = H_1(e^{j\omega}) H_2(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$= \left(\frac{3 - e^{-j8\omega}}{1 + \underset{k}{e^{-j\omega}}}\right)\left(\frac{1}{1 - \frac{1}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}\right) =$$

$$= \frac{3 - k^8}{1 - \frac{1}{4}k + \frac{1}{8}k^2 + k - \frac{1}{4}k^2 + \frac{1}{8}k^3} =$$

$$= \frac{3 - k^8}{1 + \frac{3}{4}k - \frac{1}{8}k^2 + \frac{1}{8}k^3} \Rightarrow$$

$$3X(e^{j\omega}) - e^{-j8\omega}X(e^{j\omega}) = Y(e^{j\omega}) +$$

$\frac{5}{4} e^{-j\omega} Y(e^{j\omega}) - \frac{1}{8} e^{-2j\omega} Y(e^{j\omega}) +$

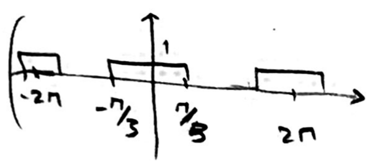
$\frac{1}{8} e^{-3j\omega} Y(e^{j\omega}) \Rightarrow 3x[n] - x[n-8] =$

$y[n] + \frac{3}{4} y[n-1] - \frac{1}{8} y[n-2] + \frac{1}{8} y[n-3$

(5) $h[n] = \frac{\sin(\frac{\pi}{3}n)}{\pi n} \frac{\sin(\frac{\pi}{4}n)}{\pi n} = h_1[n] h_2[n]$

$\Rightarrow H(e^{j\omega}) = \frac{1}{2\pi}(H_1(e^{j\omega}) * \hat{H}_2(e^{j\omega})) =$

$= \frac{1}{2\pi}($

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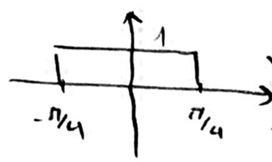


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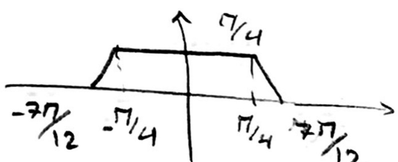
$= \frac{1}{2\pi}($

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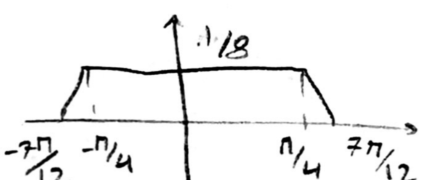



$= \hat{H}(e^{j\omega})$

$\hat{X}(e^{j\omega}) = \frac{\pi}{j}(\delta(\omega - \frac{\pi}{12}) - \delta(\omega + \frac{\pi}{12})) -$

$12\pi(\delta(\omega - \frac{2\pi}{3}) + \delta(\omega + \frac{2\pi}{3})) \Rightarrow$

$\Rightarrow \hat{Y}(e^{j\omega}) = \underbrace{\hat{H}(e^{j\frac{\pi}{12}})}_{1/8} \sin(\frac{\pi}{8}n)$

$-2\underbrace{\hat{H}(e^{j\frac{2\pi}{3}})}_{0} \cos(\frac{2\pi}{3}n) = \frac{1}{8}\sin(\frac{\pi}{8}n)$

(6)(a) $W(e^{j\omega}) = 3X(e^{j\omega})$

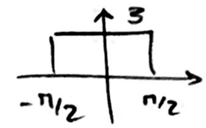
(b) $H_1(e^{j\omega}) = \frac{W(e^{j\omega})}{X(e^{j\omega})} = \frac{3X(e^{j\omega})}{X(e^{j\omega})} = 3$

(c) $H_{eq}(e^{j\omega}) = H_1(e^{j\omega}) H_2(e^{j\omega})$

$H_2(e^{j\omega}) =$


$\Rightarrow$

$\Rightarrow H_{eq}(e^{j\omega}) =$




(d) $\hat{X}(e^{j\omega}) = \pi(\delta(\omega-0.4\pi)+\delta(\omega+0.4\pi))$

$+ \frac{\pi}{j}(\delta(\omega-0.6\pi)-\delta(\omega+0.6\pi)) + 4\pi\delta(\omega)$

(e) $Y(e^{j\omega}) = X(e^{j\omega})H_{eq}(e^{j\omega}) =$

$6\pi + \frac{\pi}{j}(0-0) + 12\pi = 18\pi$

$y[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} 18\pi e^{jn\omega} d\omega =$

$= \frac{9}{jn}(e^{jn\omega}\Big|_{-\pi/2}^{\pi/2}) = \frac{9}{jn}(e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}})$

$= \frac{9}{2n}(\sin\frac{\pi}{2}n)$

7 (a) $Y(e^{j\omega}) - \frac{1}{3}e^{-j\omega}Y(e^{j\omega}) = 2X(e^{j\omega})$

$$\Rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1-\frac{1}{3}e^{-j\omega}}$$

(b) $X(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \left(\frac{1}{1-\frac{1}{3}e^{-j\omega}}\right)\left(\frac{2}{1-\frac{1}{3}e^{-j\omega}}\right) = \frac{2}{1-\frac{1}{3}e^{-j\omega}-\frac{1}{3}\underbrace{e^{-j\omega}}_{K}+\frac{1}{9}e^{-2j\omega}}$$

$$= \frac{2}{1-\frac{2}{3}K+\frac{1}{9}K^2} = \frac{2}{(\frac{1}{3}K-1)^2} =$$

$$= \frac{A}{(K-r_1)} + \frac{B}{(K-r_2)} \Rightarrow \begin{cases} r_1 = 3-\sqrt{10} \\ r_2 = 3+\sqrt{10} \\ A = \frac{1}{\sqrt{10}},\ B = \frac{-1}{\sqrt{10}} \end{cases}$$

(c) $Y(e^{j\omega}) = \left(\frac{1}{1+\frac{1}{3}e^{-j\omega}}\right)\left(\frac{2}{1-\frac{1}{3}e^{-j\omega}}\right) =$

$$= \frac{2}{1-\frac{1}{9}e^{-2j\omega}} = \frac{A}{1-\frac{1}{3}e^{-j\omega}} + \frac{B}{1+\frac{1}{3}e^{-j\omega}}$$

$\Rightarrow A = B = 1 \Rightarrow y[n] = (\frac{1}{3})^n u[n] +$

$(-\frac{1}{3})^n u[n]$

(d) $Y(e^{j\omega}) = \left(\frac{1-\frac{1}{4}K}{1+\frac{1}{2}K}\right)\left(\frac{2}{1-\frac{1}{3}K}\right)$

$$= \frac{2-\frac{1}{2}K}{1-\frac{1}{3}K+\frac{1}{2}K-\frac{1}{6}K^2} \stackrel{\times 6/6}{=\!=} \frac{12-3K}{6+K-K^2} =$$

$$\frac{A}{(3-K)} + \frac{B}{(2+K)} \Rightarrow A = 3/5,\ B = 18/5$$

$$\Rightarrow Y(e^{j\omega}) = \frac{3/5}{3-e^{-j\omega}} + \frac{18/5}{2+e^{-j\omega}} =$$

$$= \frac{3/15}{1-\frac{1}{3}e^{-j\omega}} + \frac{18/10}{1+\frac{1}{2}e^{-j\omega}} \Rightarrow$$

$$\Rightarrow y[n] = \frac{3}{15}\left(\frac{1}{3}\right)^n u[n] + \frac{9}{5}\left(-\frac{1}{2}\right)^n u[n]$$