Q_1 a) $y[n] = x_1[n] * x_2[n] =$

$$= \sum_{-\infty}^{\infty} x_1[n]\, x_2[n] =$$

$$= \sum_{-\infty}^{\infty} \left((\tfrac{1}{2})^n u[n] \right) \left(u[n+3] - u[n-3] \right) =$$

$$= \sum_{0}^{2} (\tfrac{1}{2})^n = 7/4$$

b) $y[n] = x_1[n] * x_2[n] = \sum_{-\infty}^{\infty} x_1[n]\, x_2[n] =$

$$= \sum_{-\infty}^{\infty} \left((\tfrac{1}{3})^n (u[n] - u[n-7]) \right) \left(u[n] - u[n-10] \right)$$

$$= \sum_{0}^{6} (1/3)^n$$

c) $y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt =$

$$= \int_{-\infty}^{\infty} (e^{2t} u(t-2))(u(t) - u(t-3))\, dt =$$

$$= \int_{2}^{3} e^{2t}\, dt = \tfrac{1}{2} e^{2t} \Big]_{2}^{3} = \frac{e^6 - e^4}{2}$$

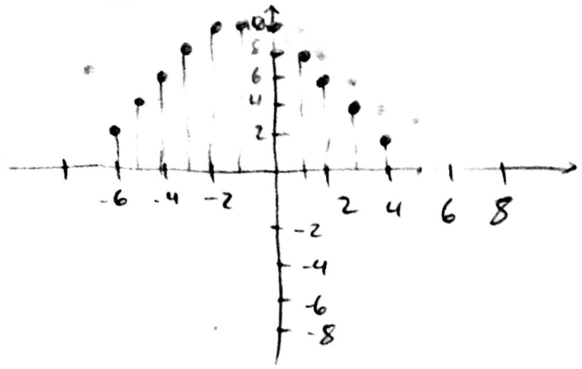
d) $y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt =$

$$= \int_{-\infty}^{\infty} (e^{2t} u(2-t))(u(t) - u(t-3))\, dt =$$

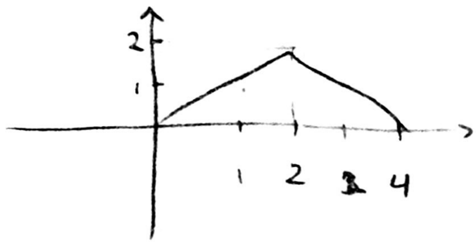
$$= \int_{0}^{2} e^{2t}\, dt = \frac{1}{2} e^{2t} \Big]_{0}^{2} = \frac{e^4 - 1}{2}$$

Q2 $y[n] = \sum_{-\infty}^{\infty} x[k]h[n-k] = \sum_{-\infty}^{\infty} x[n-k]h[k]$

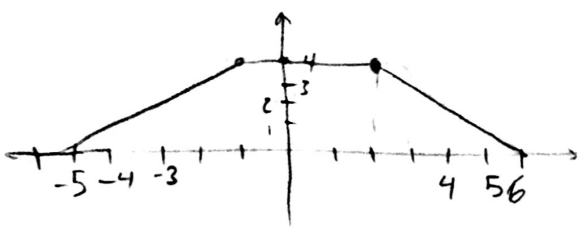
(a)



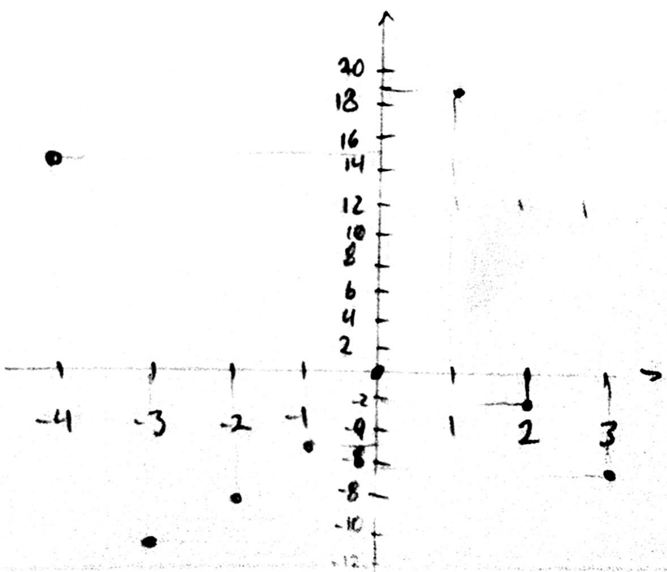
(b) $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$



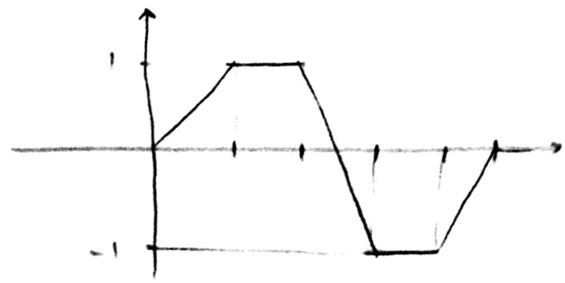
(c) $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$



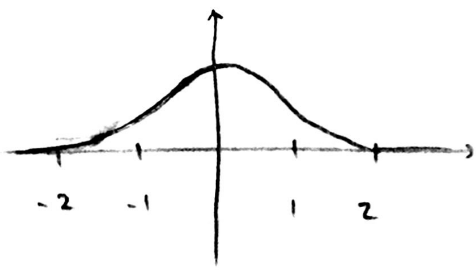
(d) $y[n] = \sum_{-\infty}^{\infty} x[k]h[n-k]$



(e) $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$



(f) $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$



Q3

a) $h(t) = e^{-2|t|}$

memoryless: yes, output depends on t

causal: no, $h(\tau) \neq 0$ $(\tau < 0)$ $h(-1) = e^{-2} \neq 0$

stable: $\int_{-\infty}^{\infty} |h(t)|\, dt = \infty \Rightarrow$ not stable

b) $h(t) = \sin(t)\, u(1-t)$

memoryless: no, output depends on $(1-t)$

causal: no, $h(t) \neq 0$ $(\forall t < 0)$ $h(-\pi/2) = -1 \neq 0$

stable: $-1 \leq h(t) \leq 1 \Rightarrow$ stable

c) $h(t) = (t^2+1)\,\delta(t)$

memoryless: yes, output only depends on t

causal: yes, $h(t) = 0$ $(\forall t < 0)$ $(\delta(t) = 0)$

stable: $\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |(t^2+1)\,\delta(t)|\, dt = 1 \Rightarrow$ stable

d) $h[n] = (1/2)^n u[n+1]$

memoryless: no, output depends on $n+1$

causal: no, $h(-1) = 2$

stable: $\sum_{-\infty}^{\infty} |h[k]| = \sum_{-\infty}^{-1} \cancel{|h[k]|}^{0} + \sum_{-1}^{\infty} |h[k]| =$

$= 0 + 2 + 1 + 1/2 + \cdots = 4 \Rightarrow$ stable

e) $h[n] = e^n u[-n]$

memoryless: no, output depends on $-n$

Causal: no, $h[-1] = e^{-1} \neq 0$

Stable: $\sum_{-\infty}^{\infty} |h[k]| = \sum_{1}^{\infty} \cancel{|h[k]|}^{0} + \sum_{-\infty}^{0} |h[k]|$

$= 1 + e^{-1} + e^{-2} + \cdots = 1 + \frac{1}{e-1} = \frac{e}{e-1} < \infty$

$\Rightarrow$ stable

(f) $h[n] = \cos(\pi/2\, n)\, u[n+2] + \delta[n+2]$

memoryless: no, output depends on $n+2$

causal: yes, $h[n] = 0$ $(\forall n < 0)$

stable: $\sum_{-\infty}^{\infty} |h[k]| = \sum_{-\infty}^{-1} \cancel{|h[k]|}^{0} +$

$|h[0]| + \sum_{1}^{\infty} \cancel{|h[k]|}^{0} = 1$

$\sum_{1}^{\infty} |h[k]| = \cos(\pi/2) + \cos(\pi) + \cos(3\pi/2)$

$+ \cos(2\pi) + \cdots = 0$

Q4

a) $y_1(t) = x(t) * h_1(t)$ $\quad w(t) = x(t) * h_2(t)$

$y_3(t) = w(t) * h_3(t) = (x(t) * h_2(t)) * h_3(t)$

$y_4(t) = w(t) * h_4(t) = (x(t) * h_2(t)) * h_4(t)$

$\Rightarrow y(t) = y_1(t) + y_3(t) + y_4(t) =$

$= x(t) * \underbrace{(h_1(t) + h_2(t) * h_3(t) + h_2(t) * h_4(t))}_{h_{eq}}$

b) $w(t) = x(t) * h_2(t) = (u(t+1) - u(t-1)) *$

$e^{2t} u(1-t) = u(t+1) * e^{2t} u(1-t) - u(t-1) * e^{2t} u(1-t)$

$= \int_{-\infty}^{\infty} u(t+1) e^{2t} u(1-t) dt - \int_{-\infty}^{+\infty} u(t-1) e^{2t} u(1-t) dt =$

$= \int_{-1}^{1} e^{2t} dt = \frac{e^2 - e^{-2}}{2}$

$y_1(t) = x(t) * h_1(t) = (u(t+1) - u(t-1)) * \sin(\pi/3 t)$

$= \int_{-\infty}^{\infty} u(t+1) \sin(\pi/3 t) dt - \int_{-\infty}^{\infty} u(t-1) \sin(\pi/3 t) dt =$

$= \int_{-1}^{\infty} \sin(\pi/3 t) dt - \int_{1}^{\infty} \sin(\pi/3 t) dt =$

$= \int_{-1}^{1} \sin(\pi/3 t) dt + \int_{1}^{\infty} \sin(\pi/3 t) dt - \int_{1}^{\infty} \sin(\pi/3 t) dt = 0$

$y_3(t) = w(t) * h_3(t) = \left(\frac{e^2 - e^{-2}}{2}\right) * u(t)$

$= \frac{1}{2}\left(\int_{-\infty}^{\infty} e^2 u(t) dt - \int_{-\infty}^{\infty} e^{-2} u(t) dt\right) = \infty$

$y_4(t) = w(t) * h_4(t) = \left(\frac{e^2 - e^{-2}}{2}\right) *$

$\delta(t+1) = \frac{1}{2}\left(e^2 \int_{-\infty}^{\infty} \delta(t+1) dt - e^{-2} \int_{-\infty}^{\infty} \delta(t+1) dt\right)$

$= \frac{1}{2}(e^2 - e^{-2})$

Q5

a) $h_1[n] * h_2[n] = ((\frac{1}{5})^n u[n]) * (\delta[n] - \frac{1}{5}\delta[n])$

$= \sum_{-\infty}^{\infty} (\frac{1}{5})^n u[n]\delta[n] - \sum_{-\infty}^{\infty} (\frac{1}{5})^n u[n] \frac{1}{5}\delta[n] = 0$

$\Rightarrow$ not inverses

b) $h_1(t) * h_2(t) = (e^{-t}u(t)) * (\delta(t) + \delta'(t))$

$= \int_{-\infty}^{\infty} e^{-t}u(t)\delta(t)\,dt + \int_{-\infty}^{\infty} e^{-t}u(t)\delta'(t)\,dt =$

$= 1$ $\Rightarrow$ inverses

c) $h_1[n] * h_2[n] = u[n] * (\delta[n] - \delta[n-1])$

$= \sum_{-\infty}^{\infty} u[n]\delta[n] - \sum_{-\infty}^{\infty} u[n]\delta[n-1] = 0$

$\Rightarrow$ not inverses

d) $h_1(t) * h_2(t) = u(t-1) * e^t = \int_{-\infty}^{\infty} u(t-1)e^t\,dt = \infty$

$\Rightarrow$ not inverses

Q6

a) $h(t) = \delta(t+1) - \delta(t-1)$ پاسخ پله $= g(t)$

$g(t) = \int_{-\infty}^{t} h(\tau)\, d\tau =$

$= \int_{-\infty}^{t} (\delta(\tau+1) - \delta(\tau-1))\, d\tau =$

$= \int_{-\infty}^{t} \delta(\tau+1)\, d\tau - \int_{-\infty}^{t} \delta(\tau-1)\, d\tau = u(t+1) - u(t-1)$

b) $h[n] = e^n (1/4)^n u[n+1]$ پاسخ پله $= g[n]$

$g[n] = \sum_{-\infty}^{n} h[k] = \sum_{-\infty}^{n} e^n (1/4)^n u[n+1] =$

$= \sum_{-1}^{n} (e/4)^n = 4/e + \frac{1-(e/4)^{n+1}}{1-e/4}$

c) $h(t) = 2\delta^2(t)$ پاسخ پله $= g(t)$

$g(t) = \int_{-\infty}^{t} h(\tau)\, d\tau = \int_{-\infty}^{t} 2\delta^2(\tau)\, d\tau =$

$=$

d) $h(t) = te^{-|t|}$ پاسخ پله $= g(t)$

$g(t) = \int_{-\infty}^{t} \tau e^{-|\tau|}\, d\tau$

I $t<0$: $g(t) = \int_{-\infty}^{t} \tau e^{\tau}\, d\tau =$

$te^t - e^t \Big|_{-\infty}^{t} = te^t - e^t$

II $t>0$, $g(t) = \int_{-\infty}^{0} \tau e^{\tau}\, d\tau + \int_{0}^{\infty} \tau e^{-\tau}\, d\tau$

$= 0 + [-te^{-t} - e^{-t}]_{0}^{\infty} = 0 - (-1) = 1$