①   96314O7 - علی فرخی پور

(a) $x_1(t) = x(5+t) - x(-t+4)$

$x(t) \xleftrightarrow{FT} X(j\omega) \quad \begin{cases} x(-t) \xleftrightarrow{FT} X(-j\omega) \\ x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega) \end{cases}$

$\Rightarrow X_1(j\omega) = e^{-5j\omega} X(j\omega) - e^{-4j\omega} X(-j\omega)$

(b) $x_2(t) = x(3t+1)$

$x(t) \xleftrightarrow{FT} X(j\omega) \; : \; x(\alpha t) \xleftrightarrow{FT} \frac{1}{|\alpha|} X\left(\frac{j\omega}{\alpha}\right)$

$\Rightarrow X_2(j\omega) = \frac{e^{-j\omega}}{3} X\left(\frac{j\omega}{3}\right)$

(c) $x_3(t) = \frac{d^3}{dt^3} x(t-5) \; : \; \frac{d}{dt} x(t) \longleftrightarrow j\omega X(j\omega)$

$\Rightarrow X_3(j\omega) = e^{-5j\omega} \times (j\omega)^3 X(j\omega)$

(d) $x_4(t) = t x(t-1) \; : \; t x(t) \longleftrightarrow j \frac{dX(j\omega)}{d\omega}$

$\Rightarrow X_4(j\omega) = e^{-j\omega} \times j \frac{dX(j\omega)}{d\omega}$

② (a) $x(t) = 2 + \cos(\overset{\text{Period}}{3\pi t} + \pi/4) =$

$= 2 + \frac{1}{2}\left(e^{j(3\pi t + \pi/4)} + e^{-j(3\pi t + \pi/4)}\right) =$

$= \underbrace{2}_{a_0} + \underbrace{\frac{1}{2} e^{j\pi/4}}_{a_1} e^{j(3\pi t)} + \underbrace{\frac{1}{2} e^{-j\pi/4}}_{a_{-1}} e^{-j(3\pi t)} =$

$\Rightarrow X(j\omega) = 2 \times 2\pi \delta(\omega) + 2\pi \frac{1}{2} e^{j\pi/4} \delta(\omega - 6\pi)$
$+ 2\pi \frac{1}{2} e^{-j\pi/8} \delta(\omega + 6\pi)$

(b) $x(t) = t e^{-4t} \cos(2t) u(t) = m(t) n(t)$

$m(t) = t e^{-4t} u(t)$, $n(t) = \cos(2t)$

$x(t) = m(t) n(t) \xleftrightarrow{FT} X(j\omega) = \frac{1}{2\pi}\left(M(j\omega) * N(j\omega)\right)$

$m(t) : \frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \xleftrightarrow{FT} \frac{1}{(a+j\omega)^n}$

$\Rightarrow m(t) \xleftrightarrow{FT} M(j\omega) = \frac{1}{(2+j\omega)^2}$ (I)

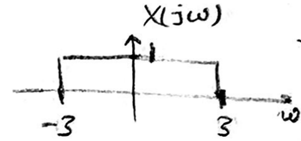
$n(t) : \cos(\omega_0 t) \xleftrightarrow{FT} \pi\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right)$

$\Rightarrow n(t) \xleftrightarrow{FT} N(j\omega) = \pi\left(\delta(\omega+2) + \delta(\omega-2)\right)$ (II)

(I), (II) $\Rightarrow X(j\omega) = \pi\left(M(j(\omega+2)) + M(j(\omega-2))\right)$

(c) $x(t) = t \frac{\sin(3t)}{\pi t}$

$\frac{\sin(3t)}{\pi t} \xleftrightarrow{FT}$



$\Rightarrow$

$t x(t) \xleftrightarrow{FT} j \frac{dX(j\omega)}{d\omega}$
0

$\Rightarrow t \frac{\sin(3t)}{\pi t} \longleftrightarrow 0$

(d) $x(t) = \frac{4t}{(1+t^2)^2}$

$e^{-|t|} \longleftrightarrow \frac{2}{1+\omega^2} \xrightarrow{\text{duality}} \frac{2}{1+t^2} \longleftrightarrow 2\pi e^{-|-\omega|}$

$\xrightarrow{\times(-1)} \frac{-2}{1+t^2} \longleftrightarrow -4\pi e^{-|\omega|} \xleftrightarrow{d/dt}$

$\frac{4t}{(1+t^2)^2} \longleftrightarrow (j\omega)\left(-4\pi e^{-|\omega|}\right)$

(e) $x(t) = e^{-3|t|} \sin(2t)$

$$e^{-3|t|} = (e^{-|t|})^3 \xleftrightarrow{FT} \left(\frac{1}{1+t^2} * \frac{1}{1+t^2} * \frac{1}{1+t^2} \right) \quad \mathrm{I}$$

$$\sin(2t) \longleftrightarrow j\pi \left(\delta(\omega+2) - \delta(\omega-2) \right) \quad \mathrm{II}$$

$$\Rightarrow X(j\omega) = \mathrm{I} * \mathrm{II}$$

3 (a) $X(j\omega) = 3\delta(\omega+4)$

$$2\pi\delta(\omega-(-4)) \longleftrightarrow e^{-4jt} \xrightarrow{\times 3/2\pi}$$

$$\frac{3}{2\pi} e^{-4jt} \longleftrightarrow 3\delta(\omega+4)$$

(b)
$$\frac{-j\omega+5}{-\omega^2+10j\omega+21} = \frac{-j\omega+5}{(j\omega)^2+10j\omega+21} =$$

$$= \frac{-j\omega+5}{(j\omega+3)(j\omega+7)} = \frac{A}{j\omega+3} + \frac{B}{j\omega+7} \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=-1 \\ 7A+3B=5 \end{cases} \Rightarrow A=2,\ B=-3$$

$$\frac{2}{j\omega+3} \longleftrightarrow 2e^{-3t}u(t), \quad \frac{-3}{j\omega+7} \longleftrightarrow -3e^{-7t}u(t)$$

$$\Rightarrow X(j\omega) = \frac{2}{j\omega+3} + \frac{-3}{j\omega+7} \longleftrightarrow$$

$$2e^{-3t}u(t) - 3e^{-7t}u(t)$$

(C) $X(j\omega) = \pi e^{-5|\omega|}$

$$e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+\omega^2} \xrightarrow{\text{duality}} \frac{2a}{a^2+t^2} \longleftrightarrow$$

$$2\pi e^{-a|-\omega|} \xrightarrow{a=5} 2\pi e^{-5|\omega|} \xrightarrow{\div 2}$$

$$\pi e^{-5|\omega|} \longleftrightarrow \frac{5}{25+t^2}$$

5 (a) $\xrightarrow{FT} (j\omega)^2 Y(j\omega) + (j\omega)Y(j\omega)$

$$-12Y(j\omega) = 7X(j\omega), \quad H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} =$$

$$= \frac{7}{(j\omega)^2+(j\omega)-12} = \frac{7}{(j\omega+4)(j\omega-3)}$$

$$= \frac{A}{j\omega+4} + \frac{B}{j\omega-3} \Rightarrow A=-1,\ B=1$$

$$\Rightarrow H(j\omega) = \frac{-1}{j\omega+4} + \frac{1}{j\omega-3} \Rightarrow$$

$$\Rightarrow h(t) = -e^{-4t}u(t) + e^{3t}u(t)$$

(b) $x(t) = te^{-4t}u(t) \Rightarrow X(j\omega) =$

$$= \frac{1}{(4+j\omega)^2}, \quad H(j\omega)X(j\omega) = Y(j\omega) =$$

$$= \frac{1}{(4+j\omega)^2} \cdot \frac{7}{(j\omega+4)(j\omega-3)} =$$

$$= \frac{7}{(j\omega+4)^3(j\omega-3)} = \frac{A}{(j\omega+4)^3} + \frac{B}{(j\omega+4)^2}$$

$$+ \frac{C}{(j\omega+4)} + \frac{D}{(j\omega-3)} \Rightarrow$$

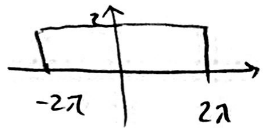
(برای شلوغ نشدن، فقط جواب آورده شده است)

$$\Rightarrow D = \frac{7}{199},\ C = \frac{-7}{199},\ B = \frac{-49}{199},\ A = \frac{-231}{199}$$

$$\Rightarrow y(t) = \frac{-231}{199} \frac{t^2}{2} e^{-4t}u(t) - \frac{49}{199} te^{-4t}u(t)$$

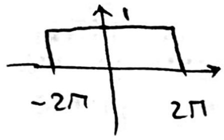
$$- \frac{7}{199} e^{-4t}u(t) + \frac{7}{199} e^{3t}u(t)$$

(6) $H(j\omega) =$



$$A(j\omega) = \frac{1}{2\pi}\left(X(j\omega) * P(j\omega)\right) =$$

$= \frac{1}{2\pi}$



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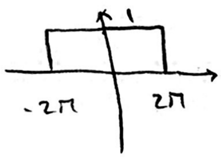
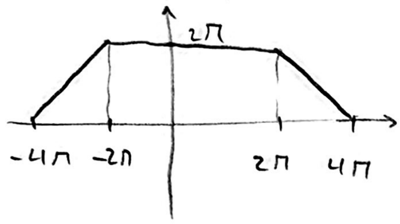


$= \frac{1}{2\pi}$



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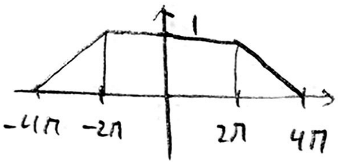
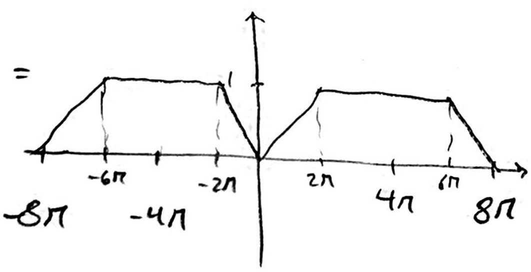


$$B(j\omega) = \frac{1}{2\pi}\left(A(j\omega) * Q(j\omega)\right) =$$

$$= \frac{1}{2\pi}\left(A(j\omega) * \pi\left(\delta(\omega+4\pi) + \delta(\omega-4\pi)\right)\right) =$$

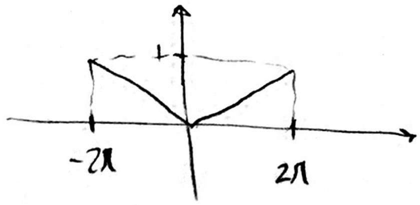
$$= \frac{1}{2}\left(A(j(\omega+4\pi)) + A(j(\omega-4\pi))\right)$$

$\Rightarrow B(j\omega) =$



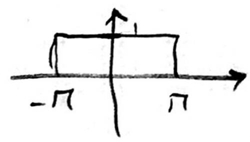
$C(j\omega) = B(j\omega)\, H(j\omega) \Rightarrow$

$\Rightarrow C(j\omega) =$

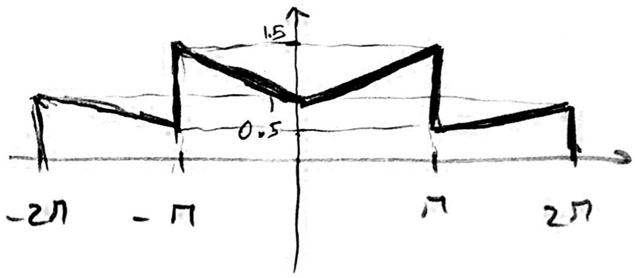


$D(j\omega) = C(j\omega) + R(j\omega)$

$R(j\omega) =$



$\Rightarrow D(j\omega) =$



(7) $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)} = \dfrac{j\omega + 10}{98 - \omega^2 + 21\omega}$

$$= \frac{j\omega + 10}{(j\omega)^2 + 21\omega + 98} =$$

$$= \frac{j\omega + 10}{(j\omega + 14)(j\omega + 7)} =$$

$$= \frac{A}{j\omega + 14} + \frac{B}{j\omega + 7} \Rightarrow$$

$$\Rightarrow A = 4/7 , B = 3/7$$

$$\Rightarrow h(t) = \left(\frac{4}{7} e^{-14t} u(t) + \frac{3}{7} e^{-7t} u(t)\right)$$

$$y(t) = x(t) * h(t)$$

$$\Rightarrow y(t) = x(t) * \left(4/7\, e^{-14t} u(t) + \frac{3}{7} e^{-7t} u(t)\right)$$