

# Signals and Systems

Assignment 5

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#### Question1

Let x(t) be a signal with Nyquist rate  $\omega_n$ . Determine the Nyquist rate for the following signals:

(a) 
$$\int_{-\infty}^{t} x(t)dt$$

- (b) x(2t)
- (c)  $x^2(t)$
- (d)  $x(t)cos(2\pi t)$
- (e)  $e^{jw_0t}x(t)$

Determine whether or not each of the following signals is band-limited and if it is, determine its Nyquist rate.

(a) 
$$x(t) = e^{j\omega_1 t} \frac{\sin(\omega_2 t)}{\pi t}$$

(b) 
$$x(t) = 3te^{-3t}u(t)$$

(c) 
$$x(t) = \sin^2(\frac{2\pi}{3}t) + \cos(\pi t)\sin(\frac{\pi}{4}t)$$

(d) 
$$x(t) = \delta(t) + 2$$

(e) 
$$x(t) = \frac{\sin^2(\pi t)}{\pi t^2}$$

Consider a band-limited signal x(t) for which  $X(j\omega)$  is zero for  $|\omega| > \omega_M$ . x(t) is modulated using a modulation scheme in which the modulated signal g(t) is obtained as follows:

$$g(t) = x(t)cos(\omega_c t) - x(t)cos(\omega_c t) * \left(\frac{sin(\omega_c t)}{\pi t}\right)$$

- (a) What should be the relation between  $\omega_c$  and  $\omega_M$  so that x(t) is recoverable from g(t)?
- (b) Determine the value of A such that g(t) is demodulated as follows:

$$x(t) = (g(t)cost(\omega_c t)) * \frac{Asin(\omega_M t)}{\pi t}$$

Let x[n] be a periodic signal with fundamental period N and Fourier series coefficients  $a_k$ . Determine the Fourier series coefficients for the following signals:

- (a) x[3-n]
- (b)  $x^2[n]$
- (c)  $\sum_{r=< N>} x[r]x[n+2-r]$
- (d)  $e^{j\frac{6\pi}{N}n}x[n]$
- (e) x[n] x[n-2]
- (f)  $x^*[-n]$

Determine the Fourier series coefficients for the following signals:

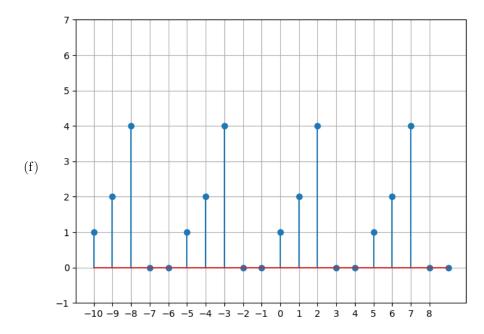
(a) 
$$sin\left(\frac{2\pi}{N}n\right) + cos\left(\frac{2\pi}{N}n + \frac{\pi}{4}\right)$$

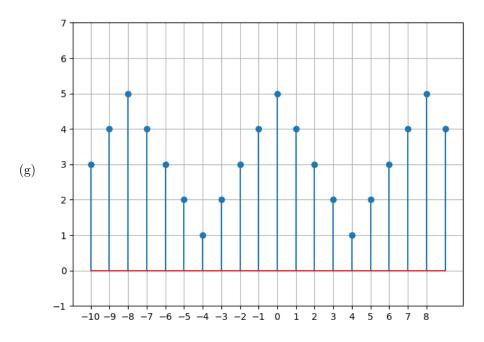
(b) 
$$2 + 3\cos\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{\pi}{3}n\right)$$

(c) 
$$(-1)^n + \cos^2\left(\frac{\pi}{5}n + \frac{\pi}{4}\right)$$

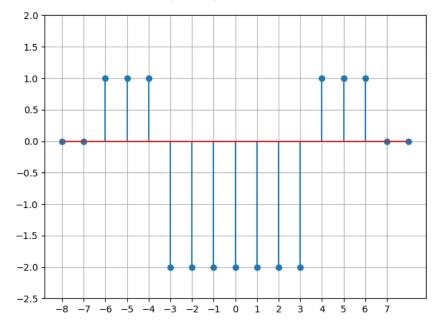
(d) 
$$\sum_{k=-\infty}^{\infty} \delta[n-3k]$$

(e) 
$$\hat{x}[n] = \begin{cases} 1, & |n| \leq N_1 \\ 0, & N_1 < |n| \leq \frac{N}{2} \end{cases}$$





(h) Use the result from part e. (N = 10)



Let x[n] be a real and odd periodic signal with period N=7 and Fourier coefficients  $a_k$ . Given that

$$a_{15} = j, a_{16} = 2j, a_{17} = 3j,$$

determine the values of  $a_0$ ,  $a_{-1}$ ,  $a_{-2}$ , and  $a_{-3}$ .