① علی خرمی پور - 9631407

$\omega_{0_1} = \frac{2\pi}{6}$ $\qquad$ $\omega_{0_2} = \frac{2\pi}{12}$

a) $x(t) = 2\sin\left(\frac{2\pi t}{6} + \frac{\pi}{6}\right) + 5\cos\left(\frac{2\pi t}{12}\right)$

$$\left.\begin{aligned} \omega_{0_1} &= \frac{2\pi}{6} \Rightarrow T_{0_1} = 6 \\ \omega_{0_2} &= \frac{2\pi}{12} \Rightarrow T_{0_2} = 12 \end{aligned}\right\} \Rightarrow T_0 = 12 \Rightarrow \omega_0 = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$x(t) = 2 \times \frac{1}{2j}\left(\underbrace{e^{j\left(\overset{2\omega_0}{\frac{2\pi t}{6}} + \frac{\pi}{6}\right)}}_{k=2} - \underbrace{e^{-j\left(\overset{2\omega_0}{\frac{2\pi t}{6}} + \frac{\pi}{6}\right)}}_{k=-2}\right) +$$

$$5 \times \frac{1}{2}\left(\underbrace{e^{j\left(\overset{\omega_0}{\frac{2\pi t}{12}}\right)}}_{k=1} + \underbrace{e^{-j\left(\overset{\omega_0}{\frac{2\pi t}{12}}\right)}}_{k=-1}\right) \Rightarrow$$

$$\Rightarrow \begin{cases} a_2 = \frac{1}{j} e^{\frac{j\pi}{6}} \\ a_{-2} = \frac{-1}{j} e^{\frac{j\pi}{6}} \\ a_1 = \frac{5}{2} = a_{-1} \end{cases}$$

b) $x(t) = 2\cos\left(\overset{\omega_0}{\frac{2\pi t}{3}} + \frac{\pi}{6}\right)$

$$= 2 \times \frac{1}{2}\left(\underbrace{e^{j\left(\frac{2\pi t}{3} + \frac{\pi}{6}\right)}}_{k=1} + \underbrace{e^{-j\left(\frac{2\pi t}{3} + \frac{\pi}{6}\right)}}_{k=-1}\right) \Rightarrow$$

$$\Rightarrow a_1 = e^{\frac{j\pi}{6}}, \quad a_{-1} = e^{-\frac{j\pi}{6}}$$

c) $T_0 = 6 \Rightarrow \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

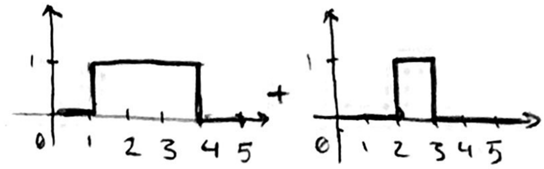
$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\, dt = \frac{1}{6}\int_0^1 (2-2t) e^{-jk\left(\frac{\pi}{3}\right)t}\, dt$$

$$= \frac{1}{3}\left(\int_0^1 e^{-jk\frac{\pi}{3}t}\, dt - \int_0^1 t e^{-jk\frac{\pi}{3}t}\, dt\right) =$$

$$= \frac{1}{3}\left(\frac{j e^{-jk\frac{\pi}{3}t}}{k\frac{\pi}{3}}\Bigg|_0^1 - \frac{e^{-jk\frac{\pi}{3}t}\left(1 + jk\pi/_3\, t\right)}{k\left(\frac{\pi}{3}\right)^2}\Bigg|_0^1\right)$$

② $a_k = \dfrac{\sin\left(k\frac{2\pi}{T}T_1\right)}{k\pi}$

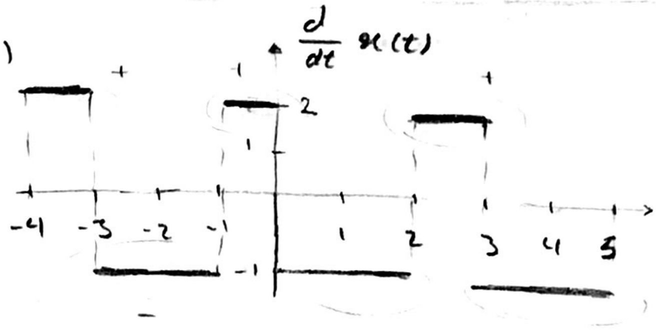
(a) =



| | |
|---|---|
| $T_1 = 1.5$ | $T_1 = 0.5$ |
| $T = 6$ | $T = 6$ |
| 2.5 shift right | 2.5 shift right |

$$\Rightarrow a_k = \frac{e^{-jk\left(\frac{2\pi}{6}\right)(2.5)}\left(\sin\left(k\frac{2\pi}{6}\times\frac{3}{2}\right) + \sin\left(k\frac{2\pi}{6}\times\frac{1}{2}\right)\right)}{k\pi}$$

(b)




| | |
|---|---|
| + قسمت | - قسمت |
| $T_1 = \frac{1}{2}$ | $T_1 = 1$ |
| $T = 3$ | $T = 3$ |
| 0.5 shift left | 2 shift right |

$$\Rightarrow a_k = \frac{e^{-jk\left(\frac{2\pi}{3}\right)(-1/2)}}{jk\left(\frac{2\pi}{3}\right)} \times \frac{\sin\left(k\frac{2\pi}{3}\times\frac{1}{2}\right)}{k\pi}$$

$$- \frac{e^{-jk\left(\frac{2\pi}{3}\right)(1)}}{jk\left(\frac{2\pi}{3}\right)} \times \frac{\sin\left(k\left(\frac{2\pi}{3}\right)\right)}{k\pi}$$

(3) (a) $x(t) \xleftrightarrow{FS} a_k$

$y(t) \xleftrightarrow{FS} \overbrace{H(jk\omega_0)\, a_k}^{b_k}$

$T = 4 \Rightarrow \omega_0 = \frac{\pi}{2}$

$b_{-2} = H(j(-2)\pi/2)\, a_{-2} = H(-\pi j)\, a_{-2} =$

$= |H(-\pi j)|\, e^{\angle H(-\pi j)} \times 1/8 = \frac{\sqrt{8}}{8}\pi e^{-\pi}$

$b_{-1} = H(j(-1)\pi/2)\, a_{-1} = H(-\pi/2\, j)\, a_{-1} =$

$= |H(-\frac{\pi}{2}j)|\, e^{\angle H(-\frac{\pi}{2}j)}\, a_{-1} = \frac{\sqrt{35}}{2}\pi e^{-\frac{\pi}{2}}\, \frac{1}{4} =$

$= \frac{\sqrt{35}}{8}\pi e^{-\pi/2}$

$b_0 = H(0)\, a_0 = 3\pi e^0 \times 1 = 3\pi$

$b_1 = \frac{\sqrt{35}}{8}\pi e^{\pi/2}$, $\quad b_2 = \frac{\sqrt{8}}{8}\pi e^{\pi}$

$\Rightarrow y(t) = b_{-2} e^{j(-2)(\frac{\pi}{2})t} + b_{-1} e^{j(-1)(\frac{\pi}{2})t} +$

$b_0 e^{j(0)(\frac{\pi}{2})t} + b_1 e^{j(1)(\frac{\pi}{2})t} + b_2 e^{j(2)(\frac{\pi}{2})t}$

(b) avg power of $y(t) = \sum_{-2}^{2} |b_k|^2 =$

$= \frac{1}{8}\pi^2 e^{-2\pi} + \frac{35}{64}\pi^2 e^{-\pi} + 9\pi^2 +$

$\frac{35}{64}\pi^2 e^{\pi} + \frac{1}{8}\pi^2 e^{2\pi}$