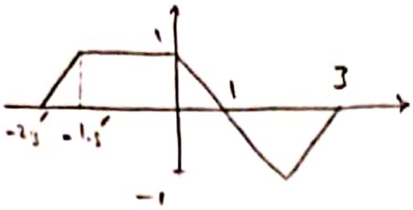
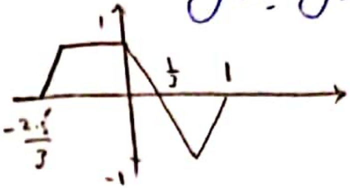
حل تمرین سری اول سیگنال

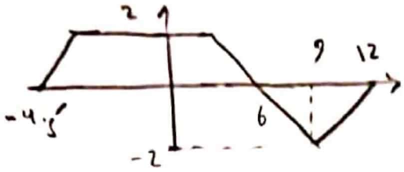
1) a) $x(t+1) =$



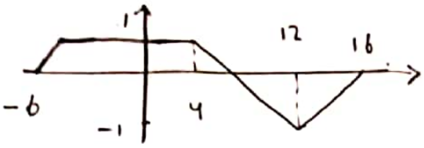
$\Rightarrow x(3t+1) =$



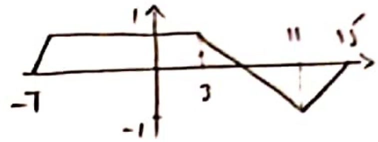
1) b) $2x\left(\frac{t}{3}\right) =$



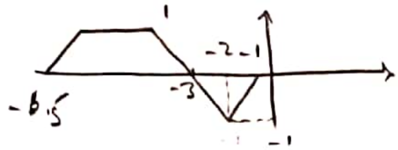
1) c) $x\left(\frac{t}{4}\right) =$



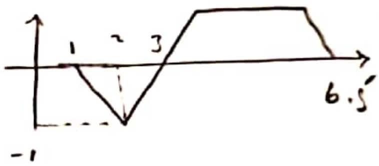
$\Rightarrow x\left(\frac{t+1}{4}\right) =$



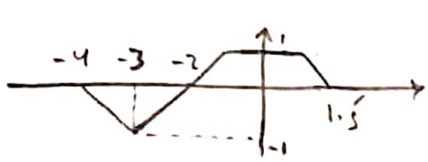
1) d) $x(t+5) =$



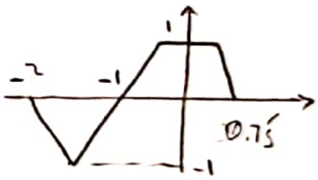
$\Rightarrow x(-t+5) =$



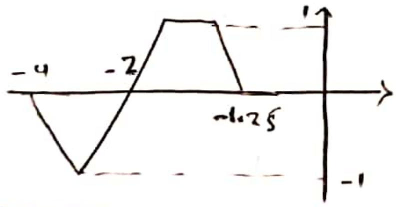
1) e) $x(-t) =$



$\Rightarrow x(-2t) =$



$\Rightarrow x(-2(t+2)) =$



2) a)

$$Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} = \frac{e^{2t}\sin(t)u(-t) + e^{-2t}\sin(-t)u(t)}{2} = \frac{e^{-2|t|}\sin(-|t|)}{2}$$

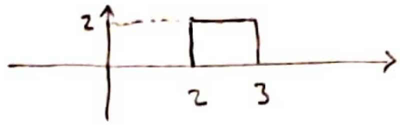
$$Od\{x(t)\} = \frac{x(t)-x(-t)}{2} = \frac{e^{2t}\sin(t)u(-t) - e^{-2t}\sin(-t)u(t)}{2} = \frac{e^{-2|t|}\sin(t)}{2}$$

2) b)

$$Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} = \frac{e^{-|t|}\cos(t) + e^{-|t|}\cos(-t)}{2} = e^{-|t|}\cos(t) = x(t)$$

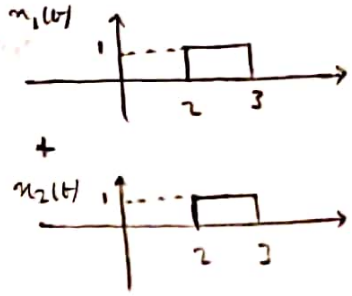
می‌توان نتیجه گرفت که $Od\{x(t)\} = 0$

2) c) $x(t) =$

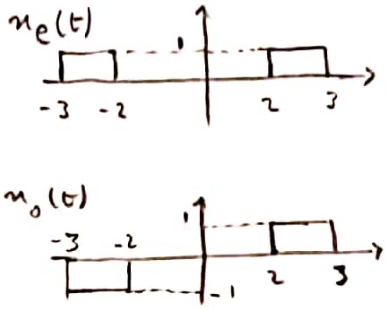


اولین کاری که به ذهن می‌رسد و ساده است این است که سیگنال را به دو قسمت مساوی تقسیم کنیم

: $x(t) =$



$x(t) =$



اما هیچ کدام از این دو قسمت نه فرد هستند نه زوج
باید به ترتیب زوج و فرد بودنشان را تعیین کنیم که :
یکدیگر را خنثی کنند

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3) a) $\omega_0 = \frac{\pi}{3} \Rightarrow T = \frac{2\pi}{\frac{\pi}{3}} = 6$

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3) b) $e^{j\frac{\pi}{3}t} \times e^{-j\frac{\pi}{3}t} = 1 \Rightarrow T = \text{undefined}, \ \omega_0 = 0$

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3) c) $e^{jt+\frac{t}{2}} = \underbrace{e^{\frac{t}{2}}}_{\text{non-periodic}} \times \underbrace{e^{jt}}_{\text{periodic}}$: non periodic

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3) d) $k\,3\pi = m \times 2\pi \Rightarrow k=2, m=3 \Rightarrow$ periodic , $\omega_0 = 3\pi \quad N_0 = \frac{2\pi}{3\pi} \times 3 = 2$

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3) e) e^{j3n} , $\frac{3}{2\pi} \neq \frac{m}{N} \Rightarrow$ متناوب نیست

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3) f) $\underbrace{e^{2|t|}}_{\text{نامتناوب}} \ \underbrace{\cos(t)}_{\text{متناوب}} =$ نامتناوب

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3) g)

$$\left.\begin{array}{l} 2\cos(2\pi n) : \omega_0 = 2\pi , N_0 = 1 \\ \cos\left(\frac{\pi}{3}n\right) : \omega_0 = \frac{\pi}{3} , N_0 = 6 \end{array}\right\} \Rightarrow 2\cos(2\pi n) + \cos\left(\frac{\pi}{3}n\right) : \omega_0 = \frac{\pi}{3} , N_0 = 6$$

3) h) $x[n] =$ 

$$x[n+6] = \sum_{k=-\infty}^{\infty} \delta[n+6-6k] + \delta[n-1+6-6k] = \sum_{k=-\infty}^{\infty} \delta[n-6(k-1)] + \delta[n-1-6(k-1)]$$

$$\xrightarrow{m=k-1} = \sum_{m=-\infty}^{\infty} \delta[n-6m] + \delta[n-1-6m] = x[n] \Rightarrow x[n+6] = x[n] \Rightarrow N_0 = 6,\ \omega_0 = \frac{\pi}{3}$$

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3) i) $\cos^2(3t + \frac{\pi}{6}) = \dfrac{\cos(6t + \frac{\pi}{3}) + 1}{2}$, $\omega_0 = 6$, $T = \frac{\pi}{3}$

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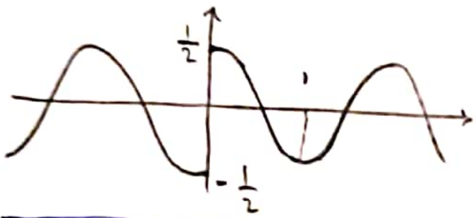
3) j) $Od\{\cos(\pi t)u(t)\} = \dfrac{\cos(\pi t)u(t) - \cos(-\pi t)u(-t)}{2} = \dfrac{\cos(\pi t)u(t) - \cos(\pi t)u(-t)}{2}$

$=$ 

متناوب نیست

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3) k) $x[n] = \cos(\frac{\pi}{8} n) \Rightarrow \omega_0 = \frac{\pi}{8}$, $\frac{\omega_0}{2\pi} = \frac{k}{N} \Rightarrow N_0 = 16$

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4) a) $y(t) = e^{x(t)}$

* خروجی در زمان خاص فقط وابسته به ورودی در همان زمان است : memoryless ✓

* خروجی یک زمان خاص : ورودی در زمان های بیشتر ندارد : Causal ✓

* $y(t-t_0) = e^{x(t-t_0)} \Rightarrow$ time-invariant ✓

* $\underbrace{\exists \alpha,\ |x(t)| \leqslant \alpha}_{\text{bounded input}} \Rightarrow \underbrace{|e^{x(t)}| \leqslant e^{\alpha}}_{\text{bounded output}} \Rightarrow$ stable ✓

* $e^{x_1(t)+x_2(t)} \neq e^{x_1(t)} + e^{x_2(t)} \Rightarrow$ linear ✗

4) b) $y(t) = \sin^2(t)\, x(t)$ * memoryless ✓ * Causal ✓

* $x(t) \to y(t)$
$x(t-t_0) \to \sin^2(t)\, x(t-t_0) \neq y(t-t_0)$ $\Rightarrow$ time-invariant ✗

* $\underbrace{\sin^2(t)}_{\text{bounded}} \times \underbrace{x(t)}_{\text{bounded}} = y(t) \Rightarrow y(t)$: bounded $\Rightarrow$ stable ✓

* $\sin^2(t)\left[x_1(t) + x_2(t)\right] = \sin^2(t)\, x_1(t) + \sin^2(t)\, x_2(t) = y_1(t) + y_2(t) \Rightarrow$ linear ✓

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4) c) $y(t) = t\, x^2(t)$ * memoryless ✓ * Causal ✓

* $x(t) \to y(t)$
$x(t-t_0) \to t\, x^2(t-t_0) \neq y(t-t_0)$ $\Rightarrow$ time-invariant ✗

* $\underbrace{x(t) = 1}_{\text{bounded input}} \longrightarrow \underbrace{y(t) = t}_{\text{unbounded output}} \Rightarrow$ stable ✗

* $t\left[x_1(t) + x_2(t)\right]^2 \neq y_1(t) + y_2(t) \Rightarrow$ linear ✗

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4) d) $y[n] = \sum_{k=-\infty}^{n+1} x[k]$ * memoryless ✗ * Causal ✗

* $\underbrace{x[n] = 1}_{\text{bounded input}} \longrightarrow \underbrace{y[n] = \infty}_{\text{unbounded output}} \Rightarrow$ stable ✗

* $x[n] \to y[n]$

$$x[n-n_0] \to \left.\sum_{k=-\infty}^{n+1} x[k-n_0]\right\}_{m = k-n_0} = \sum_{m=-\infty}^{n+1-n_0} x[m] = y[n-n_0] \Rightarrow \text{time-invariant ✓}$$

* $\sum_{k=-\infty}^{n+1} x_1[k] + x_2[k] = \sum_{k=-\infty}^{\infty} x_1[k] + \sum_{k=-\infty}^{\infty} x_2[k] = y_1[n] + y_2[n] \Rightarrow$ linear ✓

4) e) $y[n] = x[2n+1]$ * memoryless ✗ * causal ✗

$$\left.\begin{array}{l} x[n] \longrightarrow y[n] \\ x[n-n_0] \longrightarrow y[n-n_0] \end{array}\right\} \Rightarrow \text{time-invariant} \checkmark$$

* stable ✓ * linear ✓

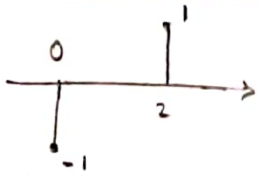
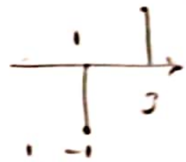
4) f) $y[n] = \sin(x[n])$ * memoryless ✓ * causal ✓

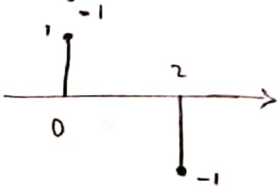
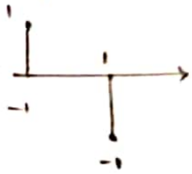
$$\left.\begin{array}{l} x[n] \longrightarrow y[n] \\ x[n-n_0] \longrightarrow \sin(x[n-n_0]) = y[n-n_0] \end{array}\right\} \Rightarrow \text{time-invariant} \checkmark$$

* $|\sin(x[n])| \leqslant 1 \Rightarrow$ stable ✓

linear ✗

* $\sin(x_1[n]+x_2[n]) \neq \sin(x_1[n]) + \sin(x_2[n])$

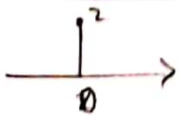
5) a) $x_1[n] =$  $\Rightarrow x_1[n+1]\,x_1[n-1] =$  $\times$  $=$ 

$x_2[n] =$  $\Rightarrow x_2[n+1]\,x_2[n-1] =$  $\times$  $=$ 

$\Rightarrow y_1[n] = y_2[n] \Rightarrow$ معکوس پذیر نیست

5) b) $y^{-1}(t) = x(2t)$ $\quad x(t) \xrightarrow{y} x(\frac{t}{2}) \xrightarrow{y^{-1}} x(t)$

5) c) $x(t) \xrightarrow{y} \begin{cases} x(t), & t \geqslant 0 \\ x(t-1), & t<0 \end{cases} \xrightarrow{y^{-1}} \begin{cases} x(t), & t \geqslant 0 \\ x(t), & t<0 \end{cases} \Rightarrow y^{-1} = \begin{cases} x(t), & t \geqslant 0 \\ x(t+1), & t<0 \end{cases}$

5) d) $x_1[n] =$  $\Rightarrow y_1[n] =$ 

$x_2[n] =$  $\Rightarrow y_2[n] =$ 

$\Rightarrow y_1[n] = y_2[n] \Rightarrow$ معکوس پذیر نیست

5) e) $x_1(t) = t \Rightarrow y_1(t) = 1$, $x_2(t) = t+1 \Rightarrow y_2(t) = 1$ $\Rightarrow y_1(t) = y_2(t) \Rightarrow$ معکوس پذیر نیست

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6) a) $E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} \left(\left(\frac{1}{4}\right)^n u[n]\right)^2 = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \frac{1}{1-\frac{1}{16}} = \frac{16}{15}$

$$P_\infty = \lim_{N\to\infty} \frac{E_\infty}{2N+1} = \lim_{N\to\infty} \frac{\frac{16}{15}}{2N+1} = 0$$

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6) b) $E_\infty = \int_{-\infty}^{\infty} |j\cos(t)|^2 dt = \int_{-\infty}^{\infty} \cos^2(t) dt = \infty$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \cos^2(t)\,dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{\cos(2t)+1}{2}\,dt = \lim_{T\to\infty} \frac{1}{4T}\left(\int_{-T}^{T}\cos(2t) + 2T\right)$$

$$= \lim_{T\to\infty} \frac{1}{4T}\left(\underbrace{\frac{1}{2}\sin(2t)\Big|_{-T}^{T}}_{\text{finite}} + 2T\right) = \lim_{T\to\infty} \frac{1}{2} = \boxed{\frac{1}{2}}$$

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6) c) $E_\infty = \int_{-\infty}^{\infty} |e^{jt+t}|^2 dt = \int_{-\infty}^{\infty} e^{2t} dt = \infty$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} e^{2t}\,dt = \lim_{T\to\infty} \frac{1}{2T} \times \frac{1}{2}\left(e^{2T} - e^{-2T}\right) = \infty$$