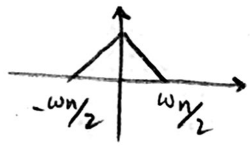
① 9631407 - علی فرخی پور

(a) $\int_{-\infty}^{t} x(t)\,dt$

Nyquist rate $= \omega_n \Rightarrow X(j\omega) =$



$y(t) = \int_{-\infty}^{t} x(t)\,dt \Rightarrow Y(j\omega) = \frac{1}{j\omega} X(j\omega) +$
$\pi X(0)\,\delta(\omega)$

$\Rightarrow$ Nyquist rate $= \omega_n$

(b) $x(2t) \xleftrightarrow{FT} \frac{1}{2} X(\frac{j\omega}{2})$

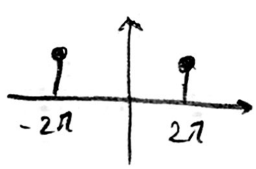
$\Rightarrow$ Nyquist rate $= \omega_n/2$

(c) $x^2(t) \xleftrightarrow{FT} \frac{1}{2\pi}(X(j\omega) * X(j\omega))$

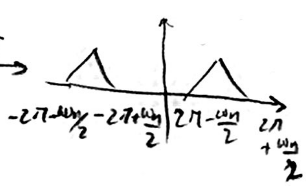
$\Rightarrow$ Nyquist rate $= 2\omega_n$

(d) $x(t)\cos(2\pi t)$

$\cos(2\pi t) \xleftrightarrow{FT}$



$\Rightarrow x(t)\cos(2\pi t) \xleftrightarrow{FT}$



$\Rightarrow$ Nyquist rate $= 2(2\pi + \frac{\omega_n}{2})$

(e) $e^{j\omega_0 t} x(t) \xleftrightarrow{FT}$

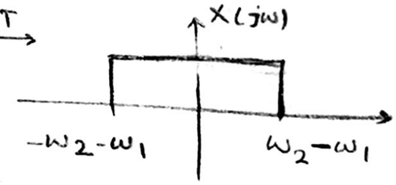
$\frac{1}{2\pi}(2\pi\delta(\omega-\omega_0) * X(j\omega)) =$

$= X(j(\underbrace{\omega-\omega_0}))$ Frequency shift

$\Rightarrow$ Nyquist rate $= \omega_n$

② (a) $x(t) = \underbrace{e^{j\omega_1 t}}_{\text{Frequency shift}} \frac{\sin(\omega_2 t)}{\pi t}$

$\xleftrightarrow{FT}$



$\Rightarrow$ band-limited

Nyquist rate $= 2(\omega_2 - \omega_1)$

(b) $x(t) = 3te^{-3t}u(t) \xleftrightarrow{FT} \frac{3}{(3+j\omega)^2}$

$\longrightarrow$ not band-limited

(c) $x(t) = \underbrace{\sin^2(\frac{2\pi}{3}t)}_{\omega = \pm\frac{4\pi}{3}, 0} + \underbrace{\cos(\pi t)}_{\omega = \pm\pi}\underbrace{\sin(\pi/4\, t)}_{\omega = \pm\frac{\pi}{4}}$

← کانولوشن در فرکانس

$\omega = -\frac{5\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

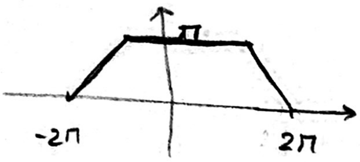
$\max\{\pm\frac{4\pi}{3}, 0, \pm\frac{5\pi}{4}, \pm\frac{3\pi}{4}\} = \frac{4\pi}{3} \Rightarrow \omega_M = \frac{4\pi}{3}$

$\Rightarrow$ Nyquist Rate $= \frac{8\pi}{3}$

(d) $x(t) = \delta(t) + 2 \xleftrightarrow{FT} 1 + 4\pi\delta(\omega)$

(e) $x(t) = \dfrac{\sin^2(\pi t)}{\pi t^2} = \pi \times \dfrac{\sin(\pi t)}{\pi t} \times \dfrac{\sin(\pi t)}{\pi t}$

$\xleftrightarrow{FT}$



$\rightarrow$ band-limited

$\Rightarrow$ Nyquist rate $= 4\pi$

(3) (a) $g(t) \xleftrightarrow{F.T.} G(j\omega)$

$x(t)\cos(\omega_c t) \xleftrightarrow{F.T.} \frac{1}{2}\left(X(j(\omega-\omega_c)) + X(j(\omega+\omega_c))\right)$

$x(t)\cos(\omega_c t) * \left(\dfrac{\sin(\omega_c t)}{\pi t}\right) \xleftrightarrow{F.T.}$ // $(-\omega_c < \omega < \omega_c)$

$$\Rightarrow G(j\omega) = \begin{cases} \frac{1}{2}\left(X(j(\omega-\omega_c)) + X(j(\omega+\omega_c))\right) & \omega < |\omega_c| \\ 0 & \omega > |\omega_c| \end{cases}$$

نباید $X(j(\omega-\omega_c))$, $X(j(\omega+\omega_c))$ با هم overlap داشته باشند

پس: $|\omega_M| < |\omega_c|$

(b) $x(t) = (g(t)\cos(\omega_c t)) * \dfrac{A\sin(\omega_M t)}{\pi t}$

از آن‌جایی که $g(t)$ در یک cos ضرب شده، دامنه آن 1/2

می‌شود پس A باید برابر 2 باشد.

(4) (a) $x[3-n]$ $\qquad x[n] \longleftrightarrow a_k$

$\xrightarrow{\text{Shift}} x[n+3] \longleftrightarrow e^{-jk\frac{2\pi}{N}(-3)} a_k$

$\xrightarrow{\text{Reverse}} x[-n+3] \longleftrightarrow e^{-jk\frac{2\pi}{N}(-3)} a_{-k}$

(b) $x^2[n]$. $\qquad x[n] \longleftrightarrow a_k$

$\Rightarrow a_k = \dfrac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\frac{2\pi}{N}n}$

$\Rightarrow x^2[n] \longleftrightarrow a_k * a_k$

(c) $\sum_{r=\langle N\rangle} x[r]\,x[n+2-r]$ $\qquad x[n] \longleftrightarrow a_k$

$\Rightarrow a_k = \dfrac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\frac{2\pi}{N}n}$

(d) $e^{j\frac{6\pi}{N}n}\,x[n]$ $\qquad x[n] \longleftrightarrow a_k$

$\Rightarrow a_k = \dfrac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\frac{2\pi}{N}n}$

(e) $x[n] - x[n-2]$:

$(1 - e^{-2j\omega})\,X(e^{j\omega})$

(f) $x^*[-n]$ $\qquad x[n] \longleftrightarrow a_k$

$\xrightarrow{\text{Reverse}} x[-n] \longleftrightarrow e^{-jk\frac{2\pi}{N}(} a_{-k} = c_k$

$\xrightarrow{\text{conjugate}} x^*[-n] \longleftrightarrow c^*_{-k} = e^{-jk\frac{2\pi}{N}} a^*_k$

(3) (a) $\sin\left(\frac{2\pi}{N}n\right) + \cos\left(\frac{2\pi}{N}n + \frac{\pi}{4}\right)$

$$= \frac{1}{2j}\left(e^{j\frac{2\pi}{N}n} - e^{-j\frac{2\pi}{N}n}\right) + \frac{1}{2}\left(e^{j(\frac{2\pi}{N}n+\frac{\pi}{4})} + e^{-j(\frac{2\pi}{N}n+\frac{\pi}{4})}\right) = e^{j\frac{2\pi}{N}n}\underbrace{\left(\frac{1}{2j} + \frac{1}{2}e^{j\frac{\pi}{4}}\right)}_{a_1} + e^{-j\frac{2\pi}{N}n}\underbrace{\left(\frac{-1}{2j} + \frac{1}{2}e^{-j\frac{\pi}{4}}\right)}_{a_{-1}}$$

(b) $2 + 3\cos\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{\pi}{3}n\right) =$

$$= 2 + \frac{3}{2}\left(e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right) + \frac{1}{2j}\left(e^{j\frac{\pi}{3}n} - e^{-j\frac{\pi}{3}n}\right) \Longrightarrow \omega_0 = \frac{\pi}{3}$$

$$\Longrightarrow \begin{cases} a_0 = 2 \\ a_2 = a_{-2} = \frac{3}{2} \\ a_1 = a_{-1} = \frac{1}{2j} \end{cases}$$

(C) $(-1)^n + \cos^2\left(\frac{\pi}{5}n + \frac{\pi}{4}\right) = (-1)^n + \frac{1}{2}\left(\cos\left(\frac{2\pi}{5}n + \frac{\pi}{2}\right)\right) =$

$$\begin{cases} -1 + \frac{1}{2}\left(e^{j\frac{2\pi}{5}n}e^{j\frac{\pi}{2}} + e^{-j\frac{2\pi}{5}n}e^{-j\frac{\pi}{2}}\right) & n = 2m+1 \\ 1 + \frac{1}{2}\left(e^{j\frac{2\pi}{5}n}e^{j\frac{\pi}{2}} + e^{-j\frac{2\pi}{5}n}e^{-j\frac{\pi}{2}}\right) & n = 2m \end{cases}$$

$$\Longrightarrow \begin{cases} a_0 = 1 + -1 + \cdots = 0 \\ a_1 = \frac{1}{2}e^{j\frac{\pi}{2}}, \quad a_{-1} = \frac{1}{2}e^{-j\frac{\pi}{2}} \end{cases}$$

(d) $\sum_{K=-\infty}^{\infty} \delta[n-3K]$

$$a_K = \frac{1}{N} \overset{N=3}{=\!=} \frac{1}{3} \text{ for all } K$$

(e) $\hat{x}[n] = \begin{cases} 1 & |n| \leq N_1 \\ 0 & N_1 < |n| \leq \frac{N}{2} \end{cases}$

(f) $\omega_0 = \frac{2\pi}{5}$, $a_K = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\omega_0 n}$

$$\Longrightarrow a_K = \frac{1}{5}\sum_{0}^{4} x[n]e^{-jk\frac{2\pi}{5}n} \Longrightarrow$$

$$\Longrightarrow a_K = \frac{1}{5}\left[e^{-jk\frac{2\pi}{5}(1)} + 2e^{-jk\frac{2\pi}{5}(2)} + 4e^{-jk\frac{2\pi}{5}(3)}\right]$$

(g) $\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$, $a_K = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\omega_0 n}$

$$\Longrightarrow a_K = \frac{1}{8}\sum_{0}^{7} x[n]e^{-jk\frac{2\pi}{5}n} \Longrightarrow$$

$$\Longrightarrow a_K = \frac{1}{7}\left[5\cancel{e^{-jk\frac{\pi}{4}(0)}} + 4e^{-jk\frac{\pi}{4}(1)} + 3e^{-jk\frac{\pi}{4}(2)} + 2e^{-jk\frac{\pi}{4}(3)} + e^{-jk\frac{\pi}{4}(4)} + 2e^{-jk\frac{\pi}{4}(5)} + 3e^{-jk\frac{\pi}{4}(6)}\right]$$

(h)

6 $x[n]$: real & odd $\Rightarrow$

$$\left\{\begin{array}{l} x[n] : \text{real} \Rightarrow x[n] = x^*[n] \Rightarrow a_k = a^*_{-k} \\ x[n] : \text{odd} \Rightarrow x[n] = -x[-n] \Rightarrow a_k = -a_{-k} \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} a^*_{-k} = -a_{-k} \\ a^*_k = -a_k \end{array}\right\} \Rightarrow a_k : \text{Im } \& \text{ odd}$$

$$\Rightarrow a_0 = 0$$

$N = 7$

$$\left\{\begin{array}{l} a_{15} = a_8 = a_1 = j \\ a_{16} = a_9 = a_2 = 2j \\ a_{17} = a_{10} = a_3 = 3j \end{array}\right\} \xrightarrow{a_k \text{ odd}} \left\{\begin{array}{l} a_{-1} = -j \\ a_{-2} = -2j \\ a_{-3} = -3j \end{array}\right.$$