Sabject:

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پاسخ تمرین سری پنجم

1) a) $\int_{-\infty}^{t} x(t)\,dt \xleftrightarrow{F.T.} \frac{1}{jw} X(jw) + \pi X(0)\,\delta(w)$

nyquist rate $= w_n$

1) b) $x(2t) \xleftrightarrow{F.T.} \frac{1}{2} X\left(\frac{jw}{2}\right) \Rightarrow$ nyquist rate $= 2w_n$

1) c) $x^2(t) \xleftrightarrow{F.T.} \frac{1}{2\pi} X(jw) * X(jw)$: بازه‌ی فرکانسی دو برابر می‌شود

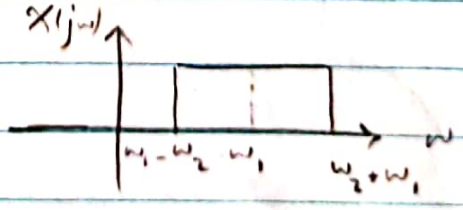
nyquist rate $= 2w_n$: پس

1) d) $x(t)\cos(2\pi t) \xleftrightarrow{F.T.} \frac{1}{2} X[j(w-2\pi)] + \pi X[j(w+2\pi)]$

$\Rightarrow$ nyquist rate $= 2\left(\frac{w_n}{2} + 2\pi\right)$

1) e) $e^{jw_0 t}\, x(t) \xleftrightarrow{F.T.} X[j(w-w_0)] \Rightarrow$ nyquist rate $= 2\left(\frac{w_2}{2} + w_0\right)$

2) a)

$x(t) \xleftrightarrow{F.T.}$ 

$\Rightarrow$ nyquist rate $= 2(w_2 + w_1)$

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2) b) $3te^{-3t}u(t) \xleftrightarrow{F.T.} \frac{3}{(3+j\omega)^2}$ بازه‌ی نرخ نایکوئیست نامحدود است

2) c) $\sin^2\left(\frac{2\pi}{3}t\right) \xleftrightarrow{F.T.} \frac{1}{2\pi} * \frac{\pi}{j}\left[\delta\left(j(\omega-\frac{2\pi}{3})\right) - \delta\left(j(\omega+\frac{2\pi}{3})\right)\right] * \frac{\pi}{j}$

$\left[\delta(\omega-\frac{2\pi}{3}) - \delta\left(j(\omega+\frac{2\pi}{3})\right)\right] \Rightarrow \frac{4\pi}{3}$ = بیشترین نرخ نمونه‌برداری

$\cos(\pi t)\sin(\frac{\pi}{4}t) \xleftrightarrow{F.T.} \pi\left[\delta[j(\omega-\pi)] + \delta[j(\omega+\pi)]\right] * \frac{1}{2\pi} * \frac{\pi}{j}\left[\delta(\omega-\frac{\pi}{4})\right.$

$\left. - \delta\left(j(\omega+\frac{\pi}{4})\right)\right] \Rightarrow \frac{5\pi}{4}$ = بیشترین نرخ نمونه‌برداری

$\Rightarrow \omega_n = 2\times\max\left\{\frac{5\pi}{4}, \frac{4\pi}{3}\right\} = \frac{8\pi}{3}$

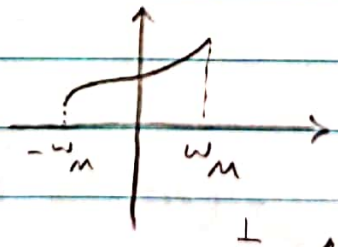
2) d) $\delta(t)+2 \xleftrightarrow{F.T.} 1+4\pi\cdot\delta(\omega)$ بازه‌ی نرخ نایکوئیست نامحدود

2) e) $\frac{\sin^2(\pi t)}{\pi t^2} = \pi\,\frac{\sin(\pi t)}{\pi t} \times \frac{\sin(\pi t)}{\pi t} \xleftrightarrow{F.T.} \frac{1}{2} X(j\omega) * X(j\omega)$

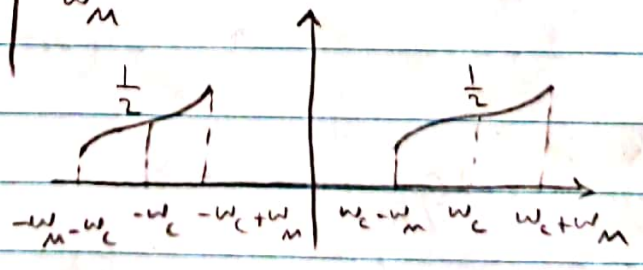
$\Rightarrow \boxed{\omega_n = 4\pi}$

3) a)

$X(j\omega)$ =



$x(t)\cos(\omega_c t) \xleftrightarrow{F.T.}$



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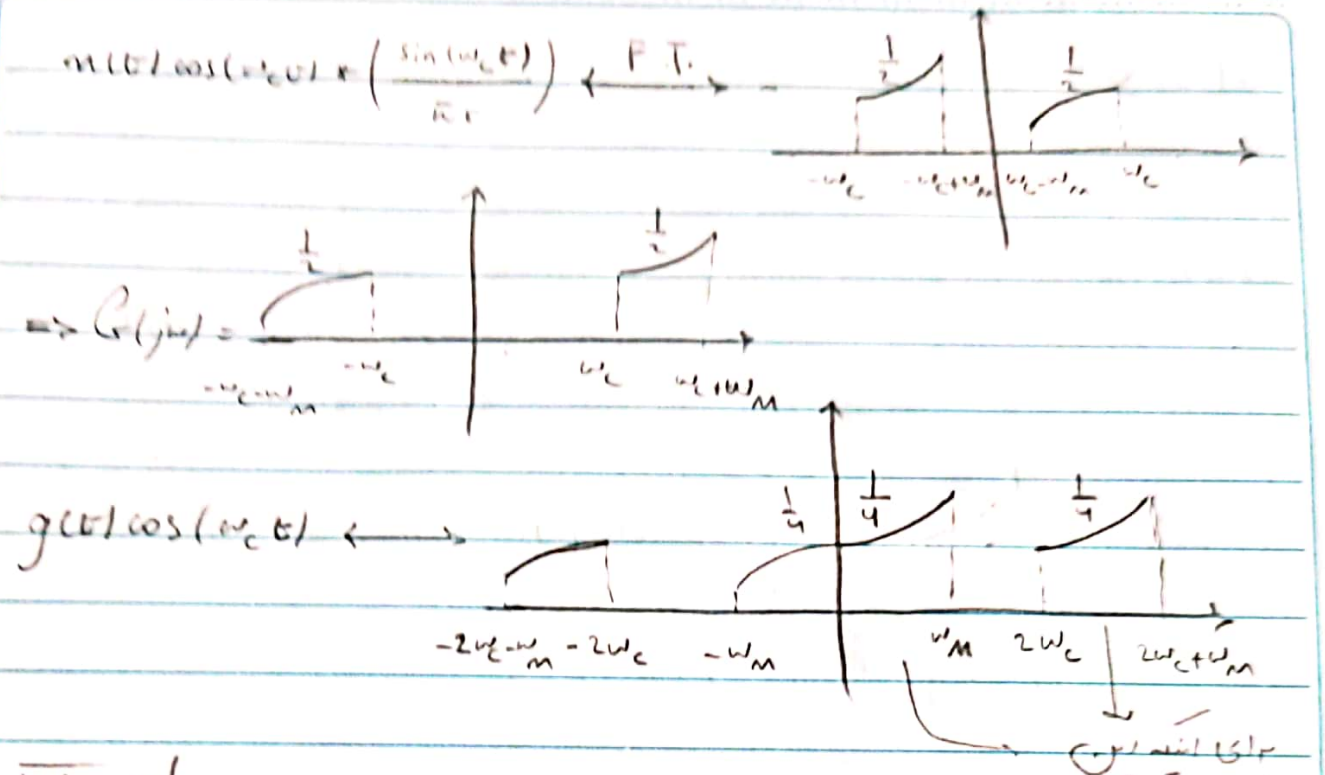

$|A=4|$

$2\omega_c > \omega_M$: برای اینکه این بخش تداخل نکنند

4) a) $n \to n+3$: $a_k e^{jk\frac{2\pi}{N}\times 3}$

$n \to -n$: $a_{-k} e^{-jk\frac{2\pi}{N}\times 3}$

4) b) $\sum_{l=\langle N \rangle} a_l a_{k-l}$

4) c) $N a_k^2 e^{+jk\frac{2\pi}{N}\times 2}$

4) d) a_{k-3}

4) e) $\left(1 - e^{-jk\left(\frac{4\pi}{N}\right)}\right) a_k$

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4) f) a_k^*

5) a) $\sin\left(\frac{2\pi}{N}n\right)+\cos\left(\frac{2\pi}{N}n+\frac{\pi}{4}\right)$

$=\frac{1}{2j}\left(e^{j\frac{2\pi}{N}n}-e^{-j\frac{2\pi}{N}n}\right)+\frac{1}{2}\left(e^{j\left(\frac{2\pi}{N}n+\frac{\pi}{4}\right)}+e^{-j\left(\frac{2\pi}{N}n+\frac{\pi}{4}\right)}\right)$

$=\frac{1}{2j}e^{j\frac{2\pi}{N}n}-\frac{1}{2j}e^{-j\frac{2\pi}{N}n}+\frac{1}{2}e^{j\frac{\pi}{4}}e^{j\frac{2\pi}{N}}+\frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j\frac{2\pi}{N}}$

$=\underbrace{\left(\frac{1}{2j}+\frac{1}{2}e^{j\frac{\pi}{4}}\right)}_{a_1}e^{j\frac{2\pi}{N}n}+\underbrace{\left(-\frac{1}{2j}+\frac{1}{2}e^{-j\frac{\pi}{4}}\right)}_{a_{-1}}e^{-j\frac{2\pi}{N}n}$

5) b) $2+3\cos\left(\frac{2\pi}{3}n\right)+\sin\left(\frac{\pi}{3}n\right) \Rightarrow N=\frac{2\pi}{\frac{\pi}{3}}=6$

$=2+\frac{3}{2}\left(e^{2j\frac{2\pi}{6}n}+e^{-2j\frac{2\pi}{6}n}\right)+\frac{1}{2j}\left(e^{j\frac{2\pi}{6}n}-e^{-j\frac{2\pi}{6}n}\right)$

$a_0=2,\ a_2=a_{-2}=\frac{3}{2},\ a_1=-a_{-1}=\frac{1}{2j}$

5) c) $(-1)^n+\frac{1}{2}+\cos\left(\frac{2\pi}{5}n+\frac{\pi}{2}\right)/2 \Rightarrow N=2\times5=10$

$=e^{j\pi n}+\frac{1}{2}+\frac{1}{2}\left(e^{j\left(\frac{2\pi}{5}n+\frac{\pi}{2}\right)}+e^{-j\left(\frac{2\pi}{5}n+\frac{\pi}{2}\right)}\right)$

$=e^{5j\frac{2\pi}{10}n}+\frac{1}{2}+\frac{1}{2}e^{j\frac{\pi}{2}}\times e^{2j\frac{2\pi}{10}n}+\frac{1}{2}e^{-j\frac{\pi}{2}}\times e^{-2j\frac{2\pi}{10}n}$

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$\Rightarrow a_0 = \frac{1}{2}, \; a_5 = 1, \; a_2 = \frac{e^{j\frac{\pi}{2}}}{2}, \; a_{-2} = \frac{e^{-j\frac{\pi}{2}}}{2}$

5)d)

$$a_k = \begin{cases} \dfrac{\sin[(2\pi k/N)(N_1+\frac{1}{2})]}{N\sin[2\pi k/2N]}, & k \neq 0, \pm N, \pm 2N, \ldots \\ \dfrac{2N_1+1}{N}, & k = 0, \pm N, \pm 2N, \ldots \end{cases}$$

5)P) $a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\frac{2\pi}{N}n}, \; N = 5$

$\Rightarrow a_k = \frac{1}{5} \sum_{n=0}^{4} x[n] e^{-jk\frac{2\pi}{5}n}$

$\Rightarrow a_0 = \frac{1}{5}\times1\times1 + \frac{1}{5}\times2\times1 + \frac{1}{5}\times4\times1 + \frac{1}{5}\times0 + \frac{1}{5}\times0 = \frac{7}{5}$

$\Rightarrow a_1 = \frac{1}{5}\times1\times1 + \frac{1}{5}\times2\times e^{-j\frac{2\pi}{5}\times1} + \frac{1}{5}\times4\times e^{-j\frac{2\pi}{5}\times2}$

$\Rightarrow a_2 = \frac{1}{5} + \frac{1}{5}\times2\times e^{-j\times2\times\frac{2\pi}{5}} + \frac{1}{5}\times4\times e^{-j\times2\times\frac{2\pi}{5}\times2}$

$\Rightarrow a_3 = \frac{1}{5} + \frac{1}{5}\times2\times e^{-j\times3\times\frac{2\pi}{5}} + \frac{1}{5}\times4\times e^{-j\times3\times\frac{2\pi}{5}\times2}$

$\Rightarrow a_4 = \frac{1}{5} + \frac{1}{5}\times2\times e^{-j\times4\times\frac{2\pi}{5}} + \frac{1}{5}\times4\times e^{-j\times4\times\frac{2\pi}{5}\times2}$

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5) g) $N=8 \Rightarrow a_k = \frac{1}{8}\sum_{n=0}^{7} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$

$a_0 = \frac{1}{8}\times 5 + \frac{1}{8}\times 4 + \frac{1}{8}\times 3 + \frac{1}{8}\times 2 + \frac{1}{8}\times 1 + \frac{1}{8}\times 2 + \frac{1}{8}\times 3 + \frac{1}{8}\times 4$

$= 3$

$a_1 = \frac{5}{8} + \frac{4}{8} e^{-j\times\frac{2\pi}{8}} + \frac{3}{8} e^{-j\times\frac{2\pi}{8}\times 2} + \frac{2}{8} e^{-j\times\frac{2\pi}{8}\times 3} + \frac{1}{8} e^{-j\times\frac{2\pi}{8}\times 4}$

$+ \frac{2}{8} e^{-j\frac{2\pi}{8}\times 5} + \frac{3}{8} e^{-j\frac{2\pi}{8}\times 6} + \frac{4}{8} e^{-j\frac{2\pi}{8}\times 7}$

$a_2 = \frac{5}{8} + \frac{4}{8} e^{-j\times 2\times\frac{2\pi}{8}} + \frac{3}{8} e^{-j\times 2\times\frac{2\pi}{8}\times 2} + \frac{2}{8} e^{-j\times 2\times\frac{2\pi}{8}\times 3}$

$+ \frac{1}{8} e^{-j\times 2\times\frac{2\pi}{8}\times 4} + \frac{2}{8} e^{-j\times 2\times\frac{2\pi}{8}\times 5} + \frac{3}{8} e^{-j\times 2\times\frac{2\pi}{8}\times 6}$

$+ \frac{4}{8} e^{-j\times 2\times\frac{2\pi}{8}\times 7}$

a_3, a_4, a_5, a_6, a_7 هم به همین شکل به دست می‌آیند

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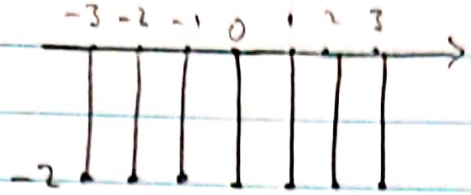
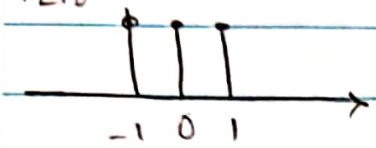
5) h)



$$a_k = \begin{cases} \frac{-2\sin\left[\frac{7}{2}\times\frac{2k\pi}{10}\right]}{10\sin[2k\pi/10]}, & k \neq 10k' \\ -\frac{14}{10}, & k = 10k' \end{cases}$$

$N_1 = 3$, $N = 10$

f[n]



$\Rightarrow N_1 = 1$, $N = 10 \rightarrow b_k = \begin{cases} \sin\left[\frac{3}{2}\times\frac{2k\pi}{10}\right], & k = 10k' \\ \frac{3}{10}, & k = 10k' \end{cases}$

$$\Rightarrow c_k = b_k e^{jk\frac{2\pi}{10}\times 5}$$

$$\Rightarrow \boxed{d_k = a_k + c_k}$$

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6) $x[n] \longleftrightarrow a_k$

x[n] real and odd $\Rightarrow$ a_k purely imaginary and odd

$\Rightarrow a_0 = 0$

$a_{15} = a_{2\times7+1} = a_1$, $a_1 = -a_{-1} = -j$

$a_{16} = a_{2\times7+2} = a_2$, $a_2 = -a_{-2} = -7j$

$a_{17} = a_{2\times7+3} = a_3$, $a_3 = -a_{-3} = -3j$

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