2.2-1 - Express the function $\frac{n^3}{1000} - 100n^2 - 100n + 3$ in terms of Θ notation.

$\Theta(n^3)$

2.2-2' - Write pseudocode for SELECTION SORT.

<u>SELECTION SORT</u>(A):

```
1   for cur = 0 to A.length:
2        small = A[cur]
3        index = cur
4        for k = cur + 1 to A.length:
5            if A[k] < small:
6                small = A[k]
7                index = k
8        temp = A[cur]              // Swap the values
9        A[cur] = small
10       A[index] = temp
```

2.3-3 - Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrance

$$T(n) = \begin{cases} 2 & \textbf{if } n = 2 \\ 2T\left(\frac{n}{2}\right) + n & \textbf{if } n = 2^k \textbf{ for } k > 1 \end{cases}$$

is $T(n) = n \log_2 n$.

We show that T holds for $n = 2$

$$T(2) = 2 \log_2 2 = 2 * 1 = 2$$

Assuming $T\left(\frac{n}{2}\right) = \frac{n}{2} \log_2 \frac{n}{2}$

$$\begin{aligned} T(n) &= 2\left(\frac{n}{2} \log_2 \frac{n}{2}\right) + n \\ &= 2\left(\frac{n}{2}(\log_2 n - \log_2 2)\right) + n \\ &= 2\left(\frac{n}{2}(\log_2 n - 1)\right) + n \\ &= (n \log_2 n - n) + n \\ &= n \log_2 n \end{aligned}$$

Therefore $T(n) = n \log_2 n$.

2.3-4 - We can express insertion sort as a recursive procedure as follows. In order to sort $A[1..n]$, we recursively sort $A[1..n-1]$ and then insert $A[n]$ into the sorted array $A[1..n-1]$. Write a recurrance for the running time of this recursive version of insertion sort.

Undefined

Induction Theorem:

$$P(n_0) \land (\forall n_i > n + o.P(n_i) \rightarrow P(n_{i+1})) \rightarrow \forall n > n_o P(n)$$

1.1-1 - Use mathematical induction to show that $T(n) = \frac{n^2+n}{2}$ for

$$T(n) = \begin{cases} 1 & \textbf{if } n = 1 \\ T(n-1) + n & \textbf{if } n > 1 \end{cases}$$

We show that T holds for $n_0 = 1$

$$T(1) = \frac{1^2 + 1}{2} = \frac{2}{2} = 1$$

Assuming $T(n-1) = \frac{(n-1)^2 + (n-1)}{2}$ and $n > 1$

$$T(n-1) = \frac{(n-1)^2 + n - 1}{2}$$

$$= \frac{n^2 - 2n + 1 + n - 1}{2}$$

$$= \frac{n^2 - n}{2}$$

By the definition of $T(n)$

$$T(n) = T(n-1) + n$$

$$= \frac{n^2 - n}{2} + n$$

$$= \frac{n^2 - n + 2n}{2}$$

$$= \frac{n^2 + n}{2}$$

Therefore

$$T(n) = \frac{n^2 + n}{2}$$

By the induction theorem:

$$T(n) = \frac{n^2 + n}{2} \quad \forall n \in \mathbb{N}, n \neq 0$$

∎