Page 29

2.2-1 - Express the function $\frac{n^3}{1000} - 100n^2 - 100n + 3$ in terms of Θ notation.

$\Theta(n^3)$

2.2-2' - Write pseudocode for SELECTION SORT.

```
SELECTION SORT(A):
for cur = 0 to A.length:
    small = A[cur]
    index = cur
    for k = cur + 1 to A.length:
        if A[k] < small:
            small = A[k]
            index = k
    temp = A[cur]                // Swap the values
    A[cur] = small
    A[index] = temp
```

Page 39

2.3-3 - Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrance

$$T(n) = \begin{cases} 2 & \textbf{if } n = 2 \\ 2T(\frac{n}{2}) + n & \textbf{if } n = 2^k \textbf{ for } k > 1 \end{cases}$$

is $T(n) = n \log n$.

Undefined

Induction Theorem:

$$P(n_0) \land (\forall n_i > n + o. P(n_i) \to P(n_{i+1})) \to \forall n > n_o P(n)$$

1.1-1 - Use mathematical induction to show that $T(n) = \frac{n^2+n}{2}$ for

$$T(n) = \begin{cases} 1 & \textbf{if } n = 1 \\ T(n-1) + n & \textbf{if } n > 1 \end{cases}$$

We show that T holds for $n_0 = 1$

$$T(1) = \frac{1^2 + 1}{2} = \frac{2}{2} = 1$$

Assuming $T(n-1) = \frac{(n-1)^2+(n-1)}{2}$ and $n > 1$

$$\begin{aligned} T(n-1) &= \frac{(n-1)^2 + n - 1}{2} \\ &= \frac{n^2 - 2n + 1 + n - 1}{2} \\ &= \frac{n^2 - n}{2} \end{aligned}$$

By the definition of $T(n)$

$$\begin{aligned} T(n) &= T(n-1) + n \\ &= \frac{n^2 - n}{2} + n \\ &= \frac{n^2 - n + 2n}{2} \\ &= \frac{n^2 + n}{2} \end{aligned}$$

Therefore

$$T(n) = \frac{n^2 + n}{2}$$

By the induction theorem:

$$T(n) = \frac{n^2 + n}{2} \; \forall n \in \mathbb{N}, n \neq 0$$

■