The probability density that the momentum assumes a value in the vicinity of p is, then, given by

$$P(p) = \frac{2\pi^{\frac{1}{2}}\delta}{h} \exp\left\{-\left(\frac{2\pi}{h}\right)^2 \delta^2(p-p^0)^2\right\}, \tag{59.18}$$

which implies that the values of p are essentially within the domain of width $h/2\pi\delta$ around p^0.

The probability density of the coordinate q for ψ of Eq. (59.12), by the way, is given by

$$P(q) = \frac{1}{\pi^{\frac{1}{2}}\delta} \exp\left\{-\frac{1}{\delta^2}(q-q^0)^2\right\}. \tag{59.19}$$

(iv) THE UNCERTAINTY RELATION

The two uncertainties (Δq and Δp) obtained in the last subsection seem to show a reciprocal relation. Namely, according to Eqs. (59.14) and (59.17), respectively,

$$\begin{aligned} \Delta q &= \frac{1}{\sqrt{2}}\,\delta \\ \Delta p &= \left(\frac{h}{2\pi}\right)\frac{1}{\delta\sqrt{2}}\,. \end{aligned} \tag{59.20}$$

That is, Δq is proportional to δ while Δp is to $1/\delta$. Therefore, an effort to reduce Δq inevitably causes an increase in Δp and vice versa.

From Eq. (59.20) we find that

$$\Delta p \cdot \Delta q = \tfrac{1}{2}\,\frac{h}{2\pi}\,. \tag{59.21}$$

It says, for instance, that Δp approaches infinity if Δq tends to 0 and Δq approaches infinity it Δp tends to 0. In other words, in a state in which the coordinate has a definite value, the value of momentum is completely indefinite and in a state in which the momentum has a definite value, the value of coordinate is completely indefinite.

The relation (59.21) has been derived from the consideration of a specific wave packet of the form of Eq. (59.12). The question is then, what happens if we take a state other than that chosen here. We give the answer first. For a general ψ, the relation given in Eq. (59.21) does not necessarily hold, but the product $\Delta p\Delta q$ never becomes smaller than the value given by Eq. (59.21). We have namely