and it is easy to see from this that

$$(I-Q)^{-1} = \sum_{n=0}^{\infty} Q^n.$$

This covers most of the elementary and usual applications. The rest of this section discusses the uniqueness of the solution to (2.11.4) and the existence of the fundamental matrix in more general contexts. This discussion may contain more detail than is necessary for the beginning student; therefore some readers may wish to skip the rest of this section and continue reading at the beginning of Section 2.12.

When S is infinite, (2.11.4) need not have a unique solution, and this case will now be considered in some detail.

Example 2.11.2. Consider the transient success run chain with

$$P = \begin{pmatrix} q_0 & p_0 & 0 & & \cdots \\ q_1 & 0 & p_1 & 0 & \cdots \\ q_2 & 0 & 0 & p_2 \cdots \\ \vdots & \ddots & & \end{pmatrix}$$

and  $\prod_{i=0}^{\infty} p_i > 0$ ,  $\sum_i (1-p_i) < \infty$ . (Refer to Lemma 2.9.1.) Make 0 absorbing so the matrix becomes

$$P' = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ q_1 & 0 & p_1 & 0 & \cdots \\ q_2 & 0 & 0 & p_2 & \cdots \\ \vdots & \ddots & & & \end{pmatrix}.$$

Ignoring the initial row and column gives

$$Q = \begin{pmatrix} 0 & p_1 & 0 & 0 & \cdots \\ 0 & 0 & p_2 & 0 & \cdots \\ 0 & 0 & 0 & p_3 & \cdots \\ \vdots & \ddots & & & \end{pmatrix}$$

Thus the system (2.11.4) becomes  $(i \ge 1, u_{i0} = u_i)$ 

$$(2.11.12) u_i = p_i u_{i+1} + q_i.$$

Set  $\overline{u_i} = 1 - u_i$  and we get

$$(2.11.13) \overline{u_i} = p_i \overline{u_{i+1}}.$$