$$+ \int_{E} Y_{s}(\mathrm{d}x) \int_{M(E)^{\circ}} \left[G(\langle Y_{s}, f \rangle + \langle \nu, f \rangle) - G(\langle Y_{s}, f \rangle) - \langle \nu, f \rangle G'(\langle Y_{s}, f \rangle) \right] H(x, \mathrm{d}\nu) + G''(\langle Y_{s}, f \rangle) \langle Y_{s}, cf^{2} \rangle$$

$$+ G'(\langle Y_{s}, f \rangle) \langle Y_{s}, \kappa_{0}(Y_{s}, \cdot, f) \rangle \left\{ \mathrm{d}s + (\bar{\mathcal{G}}_{t+}) \text{-local mart.} \right\}$$

for $G \in C^2(\mathbb{R})$ and $f \in D(A)$, where

$$\kappa_0(\nu, x, f) = \int_F r(\nu, y) p(x, y) \kappa f(y) \lambda(\mathrm{d}y). \tag{10.27}$$

We can interpret $\{Y_t : t \ge 0\}$ as a superprocess with an extra interactive non-local branching mechanism given by (10.27).

10.4 General Interactive Immigration

In this section, we give some generalizations of the immigration models considered in the previous sections. Suppose that F_0 and F_1 are Lusin topological spaces. Let $\lambda_0(\mathrm{d}y)$ and $\lambda_1(\mathrm{d}y)$ be σ -finite Borel measures on F_0 and F_1 , respectively. Let $\kappa(y,\mathrm{d}x)$ be a bounded kernel from F_0 to E and let $K(y,\mathrm{d}\nu)$ be a kernel from F_1 to $M(E)^\circ$ satisfying

$$\sup_{y \in F_1} \int_{M(E)^{\circ}} \langle \nu, 1 \rangle K(y, d\nu) < \infty. \tag{10.28}$$

Suppose that $(D_0, \mathscr{A}^0, \mathscr{A}^0_t, \mathbf{Q}_{\nu})$ is the canonical càdlàg realization of the (ξ, ϕ) -superprocess. Let $\{(X_t, \mathscr{F}_t) : t \geq 0\}$ be a càdlàg (ξ, ϕ) -superprocess with deterministic initial state $X_0 = \mu \in M(E)$. For i = 0, 1 let $\{N_i(\mathrm{d}s, \mathrm{d}y, \mathrm{d}u, \mathrm{d}w)\}$ be a Poisson random measure on $(0, \infty) \times F_i \times (0, \infty) \times D_0$ with intensity $\mathrm{d}s\lambda_i(\mathrm{d}y)\mathrm{d}u\mathbf{Q}_i(y,\mathrm{d}w)$, where

$$\mathbf{Q}_0(y, \mathrm{d}w) = \int_E \kappa(y, \mathrm{d}x) \mathbf{Q}_{L(x)}(\mathrm{d}w), \quad y \in F_0, w \in D_0,$$

and

$$\mathbf{Q}_1(y, \mathrm{d}w) = \int_{M(E)^{\circ}} K(y, \mathrm{d}\nu) \mathbf{Q}_{\nu}(\mathrm{d}w), \quad y \in F_1, w \in D_0.$$

We assume that the process $\{(X_t, \mathscr{F}_t) : t \geq 0\}$ and the Poisson random measures $\{N_0(\mathrm{d} s, \mathrm{d} y, \mathrm{d} u, \mathrm{d} w)\}$ and $\{N_1(\mathrm{d} s, \mathrm{d} y, \mathrm{d} u, \mathrm{d} w)\}$ are defined on a complete probability space $(\Omega, \mathscr{G}, \mathbf{P})$ and are independent of each other. For $t \geq 0$ let \mathscr{G}_t be the σ -algebra generated by \mathscr{F}_t and the collection of random variables