

$$\int (3x^2 + 4x + 5) \ dx \ = \int 3x^2 \ dx + \int 4x \ dx + \int 5 \ dx$$
 (4.0.11)

$$= 3 \int x^2 dx + 4 \int x dx + \int 5 dx$$
 (4.0.12)

$$=3\cdot\frac{1}{3}x^3+4\cdot\frac{1}{2}x^2+5x+C \tag{4.0.13}$$

$$= x^3 + 2x^2 + 5x + C (4.0.14)$$

In practice we generally do not write out all these steps, but we demonstrate them here for completeness.

- Rule #5 is the Power Rule of indefinite integration. There are two important things to keep in mind:
  - 1. Notice the restriction that  $n \neq -1$ . This is important:  $\int \frac{1}{x} dx \neq \frac{1}{0}x^0 + C$ ; rather, see Rule #14.
  - 2. We are presenting antidifferentiation as the "inverse operation" of differentiation. Here is a useful quote to remember: "Inverse operations do the opposite things in the opposite order."
    - When taking a derivative using the Power Rule, we first multiply by the power, then second subtract 1 from the power. To find the antiderivative, do the opposite things in the opposite order: **first** *add* one to the power, then **second** *divide* by the power.
- Note that Rule #14 incorporates the absolute value of x. The exercises will work the reader through why this is the case; for now, know the absolute value is important and cannot be ignored.

## Initial Value Problems

In Section 2.3 we saw that the derivative of a position function gave a velocity function, and the derivative of a velocity function describes acceleration. We can now go "the other way:" the antiderivative of an acceleration function gives a velocity function, etc. While there is just one derivative of a given function, there are infinite antiderivatives. Therefore we cannot ask "What is the velocity of an object whose acceleration is -32ft/s<sup>2</sup>?", since there is more than one answer.

We can find the answer if we provide more information with the question, as done in the following example. Often the additional information comes in the form of an initial value, a value of the function that one knows beforehand.

## Example 4.0.3: Solving initial value problems

The acceleration due to gravity of a falling object is -32 ft/s<sup>2</sup>. At time t=3, a falling object had a velocity of -10 ft/s. Find the equation of the object's velocity.

## **Solution**

We want to know a velocity function, v(t). We know two things:

- 1. The acceleration, i.e.,  $v^{\prime}(t)=-32$  , and
- 2. the velocity at a specific time, i.e., v(3) = -10.

Using the first piece of information, we know that v(t) is an antiderivative of v'(t)=-32. So we begin by finding the indefinite integral of -32:

$$\int (-32) \ dt = -32t + C = v(t). \tag{4.0.15}$$

Now we use the fact that v(3) = -10 to find C:

$$v(t) = -32t + C (4.0.16)$$

$$v(3) = -10 \tag{4.0.17}$$

$$v(3) = -10$$
 (4.0.17)  
 $-32(3) + C = -10$  (4.0.18)

$$C = 86 (4.0.19)$$

Thus v(t)=-32t+86 . We can use this equation to understand the motion of the object: when t=0 , the object had a velocity of v(0) = 86 ft/s. Since the velocity is positive, the object was moving upward.

When did the object begin moving down? Immediately after v(t) = 0: