where ε_{ij} are independent with mean 0 and variance σ^2 . This model arises when we want to compare k treatments. We have n_i observations on the i-th treatment. The parameter μ is interpreted as the "general effect," and α_i is the "effect due to the i-th treatment". We wish to find the RSS. Instead of writing the model in standard form we follow a different approach. The RSS is the minimum value of

$$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \mu - \alpha_i)^2.$$
 (7.8)

We use the fact that if u_1, \ldots, u_m are real numbers then

$$\sum_{i=1}^{m} (u_i - \theta)^2$$

is minimized when $\theta = \overline{u}$, the mean of u_1, \ldots, u_n . This is easily proved using calculus. The sum (7.8) is minimized when $\mu + \alpha_i = \overline{y}_i$, $i = 1, \ldots, k$; and therefore

RSS =
$$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \overline{y}_{i.})^2$$
.

Now suppose we wish to find the RSS subject to the constraints $\alpha_i - \alpha_j = 0$ for all i, j. Since $\alpha_i - \alpha_j$ is estimable, we may proceed to apply 7.6. Thus we must calculate $\tilde{\alpha}$ using the formula immediately preceding 7.6. However, again there is a more elementary way. Let α denote the common value of $\alpha_1, \ldots, \alpha_k$. Then we must minimize

$$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \mu - \alpha)^2$$

and this is achieved by setting

$$\mu + \alpha = \overline{y}_{..} = \frac{1}{n} \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij},$$

where $n = \sum_{i=1}^{k} n_i$. Thus the RSS now is

$$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \overline{y}_{..})^2.$$

The computation of RSS subject to linear restrictions will be useful in deriving a test of the hypothesis that the restrictions are indeed valid. This will be achieved in the next chapter.