(III, 41) which here is

(2) 
$$\mathscr{G}(v) = \frac{\partial^2 v}{\partial x^2} + \frac{\partial v}{\partial y} = 0.$$

These two equations have a single family of characteristics, which consists of parallels to the x-axis. We shall say that one of these functions, u(x, y) or v(x, y), is regular in a domain D if it is continuous and has continuous first partial derivatives in this domain. It will even be sufficient to say that the derivative with respect to y is continuous; for if  $\partial u/\partial y$ , for example, is a continuous function, equation (1) proves that the same is true of  $\partial^2 u/\partial x^2$  and, consequently, of  $\partial u/\partial x$ .

Since equation (1) has constant coefficients, it possesses particular integrals of the form  $e^{ax+by}$ , (III, 27); the relation between a and b is, in this case,  $b=a^2.2$  From the integral  $e^{ax+a^2y}$  thus obtained we can derive any infinity of others by taking its successive derivatives with respect to the parameter a, or, what amounts to the same thing, by taking the successive coefficients of the expansion of this integral, in powers of a. Let us write this expansion in the form

(3) 
$$e^{ax+a^2y} = 1 + \sum_{n=1}^{+\infty} \frac{n^n}{n!} V_n(x, y);$$

 $V_n(x, y)$  is a polynomial of degree n in x, y, homogeneous in x and  $\sqrt{y}$ ,

(4) 
$$\begin{cases} V_n(x,y) = x^n + n(n-1)x^{n-2}y + \dots \\ + \frac{n(n-1)\dots(n-2p+1)}{p!}x^{n-2p}y^p + \dots \end{cases}$$

which is terminated by a term in  $y^{\frac{1}{2}n}$  if n is even and by a term in  $xy^{\frac{1}{2}(n-1)}$ , if n is odd. These polynomials  $V_n$  are integrals of equation (1), from their very definition. We can easily verify this by observing that equation (3), differentiated with respect to x and to y, gives the relations

(5) 
$$\frac{\partial V_n}{\partial x} = n V_{n-1}, \qquad \frac{\partial V_n}{\partial y} = n(n-1) V_{n-2},$$

<sup>2</sup> Replacing a by  $\alpha i$ , we again find the integrals  $e^{-\alpha^2 y}$  cos  $\alpha x$ ,  $e^{-\alpha^2 y}$  sin  $\alpha x$  (III, 31).