exist, and we shall introduce and investigate below the basic one, the Hodgkin-Huxley model. Notwithstanding its lack of biophysical realism, however, Poisson type models are very important in the neurosciences because, on one hand, they relate well to the experimental practice of recording spikes, and on the other hand, they can be the basis for models of information transmission in neural systems.

## 3.4 Branching Processes

References for this section are [80, 56].

We start with the simplest branching process, the Galton-Watson process. Here, each individual lives in a fixed generation n and independently of all other individuals produces a random number of offspring that become members of generation n+1. This random variable, the number of offspring, is the same for all individuals in all generations. Thus, the numbers of offspring for the individuals are independent and identically distributed random variables. We denote their common generating function by G(s). We also assume that there is a positive probability for having more than one offspring. If the probability of having m offspring is p(m), this means that p(0) + p(1) < 1.

Let the random variable  $Z_n$  denote the size of generation n. One usually assumes that the process starts with a single individual in generation 0, that is,  $Z_0 = 1$ . Let  $G_n(s) = E(s^{Z_n})$  be the generating function of  $Z_n$ .

**Lemma 3.4.1.**  $G_n$  is the n-th fold iterate of G,

$$G_n(s) = G \circ \cdots \circ G(s),$$
 (3.4.1)

and thus also for  $m, n \in \mathbb{N}$ 

$$G_{m+n}(s) = G_m(G_n(s)).$$
 (3.4.2)

*Proof.* We shall show (3.4.2) which easily implies (3.4.1) by iteration. Let the random variable  $Y_i$  denote the number of members of the (m + n)th generation that derive from member i of the mth one. We then have

$$Z_{m+n} = Y_1 + \dots + Y_{Z_m}. (3.4.3)$$

By our assumptions, the  $Y_i$  are independent and identically distributed, in fact identical to  $Z_n$ , the number of offspring deriving from an individual n generations ago. Lemma 3.1.3 c) then yields the claim.