First recall that the bare and renormalized field are related by

$$\phi(p) = Z(M)^{-1/2}\phi_0(p). \tag{12.62}$$

This equation expresses the dependence of the field rescaling on M. If M is increased by $\delta M,$ the renormalized field is shifted by

$$\delta \eta = \frac{Z(M + \delta M)^{-1/2}}{Z(M)^{-1/2}} - 1.$$

Hence our original definition (12.39) of γ gives us immediately

$$\gamma(\lambda) = \frac{1}{2} \frac{M}{Z} \frac{\partial}{\partial M} Z. \tag{12.63}$$

Since $\delta_Z = Z - 1$ (Eq. (10.17)), this formula is in agreement with (12.50) to leading order. Formula (12.63), however, is an exact relation. This expression clarifies the relation of γ to the field strength rescaling. However, it obscures the fact that γ is independent of the cutoff Λ . To understand this aspect of γ , we have to go back to the original definition of this function in terms of renormalized Green's functions, whose cutoff independence follows from the renormalizability of the theory.

Similarly, we can find an instructive expression for β in terms of the parameters of bare perturbation theory. Our original definition of β in Eq. (12.39) made use of a quantity $\delta\lambda$, defined to be the shift of the renormalized coupling λ needed to preserve the values of the bare Green's functions when the renormalization point is shifted infinitesimally. Since the bare Green's functions depend on the bare coupling λ_0 and the cutoff, this definition can be rewritten as

$$\beta(\lambda) = M \frac{\partial}{\partial M} \lambda \Big|_{\lambda_0, \Lambda}. \tag{12.64}$$

Thus the β function is the rate of change of the renormalized coupling at the scale M corresponding to a fixed bare coupling. Recalling our analysis in Section 12.1, it is tempting to associate $\lambda(M)$ with the coupling constant λ' obtained by integrating out degrees of freedom down to the scale M. With this correspondence, the β function is just the rate of the renormalization group flow of the coupling constant λ . A positive sign for the β function indicates a renormalized coupling that increases at large momenta and decreases at small momenta. We can see explicitly that this relation works for ϕ^4 theory, to leading order in λ , by comparing Eqs. (12.28) and (12.46). We will justify this correspondence further in the following section.

The equality of the exact formula (12.64) with the first-order formula (12.53) again follows from the counterterm definitions (10.17). As with (12.63), it is not obvious that this formula for $\beta(\lambda)$ is independent of Λ , but that fact again follows from renormalizability. Conversely, it is possible to prove the