Proposition 7.2. Let $X_1, ..., X_n$ be a basis of g and $\omega_1, ..., \omega_n$ the 1-forms on G determined by $\omega_i(\tilde{X}_j) = \delta_{ij}$. Then

$$d\omega_i = -\frac{1}{2} \sum_{j,k=1}^n c^i{}_{jk} \omega_j \wedge \omega_k \tag{3}$$

if $c^i_{\ jk}$ are the structural constants given by

$$[X_j, X_k] = \sum_{i=1}^n c^i_{jk} X_i.$$

Equations (3) are known as the Maurer-Cartan equations. They follow immediately from (1). They also follow from Theorem 8.1, Chapter I if we give G the left invariant affine connection for which α in Prop. 1.4 is identically 0. Note that the Jacobi identity for g is reflected in the relation $d^2 = 0$.

Example. Consider as in §1 the general linear group GL(n, R) with the usual coordinates $\sigma \to (x_{ij}(\sigma))$. Writing $X = (x_{ij})$, $dX = (dx_{ij})$, the matrix

$$\Omega = X^{-1} dX$$

whose entries are 1-forms on G, is invariant under left translations $X \to \sigma X$ on G. Writing

$$dX = X\Omega$$
,

we can derive

$$0=(dX)\wedge\Omega+X\wedge d\Omega,$$

where \wedge denote the obvious wedge product of matrices. Multiplying by X^{-1} , we obtain

$$d\Omega + \Omega \wedge \Omega = 0,$$
 (4)

which is an equivalent form of (3).

More generally, consider for each x in the Lie group G the mapping

$$dL(x^{-1})_x:G_x\to \mathfrak{g}$$

and let Ω denote the family of these maps. In other words,

$$\Omega_x(v) = dL(x^{-1})(v)$$
 if $v \in G_x$.

Then Ω is a 1-form on G with values in g. Moreover, if $x, y \in G$, then

$$\Omega_{xy} \circ dL(x)_y = \Omega_y,$$