Now we can count the number of ways to climb larger staircases:

$$f(3) = f(2) + f(1) = 2 + 1 = 3,$$

 $f(4) = f(3) + f(2) = 3 + 2 = 5,$
 $f(5) = f(4) + f(3) = 5 + 3 = 8,$
 $f(6) = f(5) + f(4) = 8 + 5 = 13,$
 $f(7) = f(6) + f(5) = 13 + 8 = 21,$
 $f(8) = f(7) + f(6) = 21 + 13 = 34,$
 $f(9) = f(8) + f(7) = 34 + 21 = 55,$
 $f(10) = f(9) + f(8) = 55 + 34 = 89.$

Once again, we see that there are 89 ways to climb a 10-stair staircase.

If we list f(1), f(2), f(3),..., then we (almost) get the sequence that opened this chapter:

Each number in the sequence (after the first two) is the sum of the two numbers immediately before 2 = 1 + 1, 3 = 2 + 1, 5 = 3 + 2, and so on.

The only thing that's different from the sequence on page 172 is that the sequence on page 172 has an extra 1 at the start. We can add this 1 to our example, though, without any difficulty. If there are 0 stairs, then there's only 1 way to climb this staircase: do nothing! So f(0) = 1 makes sense. Also note that f(2) = f(1) + f(0) = 1 + 1 = 2, so that our equation f(n) = f(n-1) + f(n-2) holds for all positive integers $n \ge 2$.

When we add this first 1 to our sequence, we get the sequence that we first saw at the start of the chapter:

These numbers are called the **Fibonacci numbers**, in honor of the Italian mathematician Leonardo of Pisa, whose nickname was "Fibonacci," and who first published the sequence of numbers in his *Liber abaci* in 1202.

We've seen that each Fibonacci number is the sum of the previous two Fibonacci numbers. We can write this using a more formal definition.

We typically denote sequences using a variable with a subscript, such as

$$a_1, a_2, a_3, \ldots$$

Thus, a_1 is the first number in the sequence, a_2 is the second number in the sequence, and so on. Sometimes we'll start our lists with a_0 instead of a_1 , so that our sequence would be

$$a_0, a_1, a_2, \ldots$$

The **Fibonacci numbers** are defined by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all positive integers n > 2. So, for example,