5. Exercises 175

have detected |k| as the order of  $SH_1(L_k)$ , whereas all  $M_{k,-k}$  have the same homology and we have used a more subtle invariant to distinguish them.

Finally, we construct an (orientation-reversing) diffeomorphism from  $M_{k,\ell}$  to  $M_{-k,-\ell}$  by mapping  $D^4 \times S^3$  to  $D^4 \times S^3$  via  $(x,y) \mapsto (\bar{x},y)$  and  $-D^4 \times S^3$  to  $-D^4 \times S^3$  via  $(x,y) \mapsto (\bar{x},y)$ . Thus we conclude:

**Theorem 18.6.** Two Milnor manifolds  $M_{k,-k}$  and  $M_{r,-r}$  are diffeomorphic if and only if |k| = |r|.

## 5. Exercises

- (1) Let E be a complex vector bundle over  $S^{4k}$ . Give a formula for the Pontrjagin class  $p_k(E)$  in terms of  $c_{2k}(E)$ .
- (2) Let E be a 2k-dimensional oriented vector bundle. Prove that  $p_k(E) = e(E) \smile e(E)$ .
- (3) Let E be a not necessarily oriented 2k-dimensional vector bundle. Prove that the class represented by  $p_k(E)$  in  $\mathbb{Z}/2$ -cohomology is equal to  $w_{2k}(E) \smile w_{2k}(E)$ .
- (4) Prove that  $\langle p_k(E), [S^{4k}] \rangle$  is even for all vector bundles E over  $S^{4k}$ . You can use (or better prove it as an application of Sard's theorem) that an r-dimensional vector bundle over  $S^n$  with r > n is isomorphic to  $F \oplus (S^n \times \mathbb{R}^{r-n})$  for some n-dimensional vector bundle F.