hw3

Ali Kolenovic

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(iv)

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##   [1] 6.389450 6.638294 6.716207 7.550852 7.607430 6.846084 7.442304 6.831434
##   [9] 7.530910 7.144445 6.244452 7.245595 8.141389 6.868463 6.864039 7.373329
##  [17] 7.832494 7.152456 7.135815 6.726840 7.557423 7.177814 7.737924 5.390775
##  [25] 7.874377 6.929192 5.695719 6.835512 7.217394 6.088265 7.296419 6.777736
##  [33] 6.235720 7.056432 6.468100 7.236714 6.585866 8.092537 7.323175 7.459868
##  [41] 7.337576 7.921700 6.043258 5.516182 7.053226 6.186484 6.836767 8.471672
##  [49] 6.257493 6.451560 6.667605 7.490952 5.480281 6.615361 8.483408 7.547164
##  [57] 7.241227 6.324116 6.888809 6.282110 7.449274 7.684191 6.718847 6.715843
##  [65] 6.864768 6.549147 8.326986 7.162429 6.207736 5.944185 6.681432 7.836664
##  [73] 6.009621 6.634583 6.587675 5.685798 7.552373 6.680417 7.389672 6.934828
##  [81] 7.025972 7.783550 6.921705 7.377258 6.968290 8.666783 6.660207 7.740187
##  [89] 7.152286 7.109968 6.351715 6.806230 7.150575 6.853929 6.549196 6.913445
##  [97] 7.398447 7.036599 7.722430 6.581129
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##   [1] 6.811850 6.661909 5.859906 6.930748 6.584577 6.739358 6.245075 6.518588
##   [9] 6.974886 6.949042 6.778201 6.765212 6.960547 6.769239 6.758773 6.933042
##  [17] 6.956813 6.893764 6.991706 6.180677 6.833446 6.800008 6.897626 6.998692
##  [25] 6.746833 6.506971 6.978769 6.761985 6.978444 6.769636 6.906862 6.812876
##  [33] 6.827041 6.950809 6.970810 6.894773 6.834928 6.914177 6.961456 6.441282
##  [41] 6.949485 6.996125 6.605893 6.822061 6.746772 6.895132 6.718180 6.959419
##  [49] 6.973722 6.750137 6.946235 6.850047 6.689338 6.956941 6.962293 6.905555
##  [57] 6.670935 6.614630 6.839309 6.292031 6.951447 6.693605 6.794666 6.832477
##  [65] 6.901981 6.722332 6.912441 6.667148 6.473349 6.827591 6.975584 6.930961
##  [73] 6.885510 6.764180 6.967877 6.141826 6.902514 6.603506 6.770773 6.516729
##  [81] 6.850791 6.866318 6.974882 6.708939 6.747970 6.976699 6.860943 6.849410
##  [89] 6.971567 6.840966 6.972148 6.877760 6.810608 6.998299 6.850547 6.698261
##  [97] 6.466779 6.419180 6.818790 6.960532
```

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## [1] 6.988026
```

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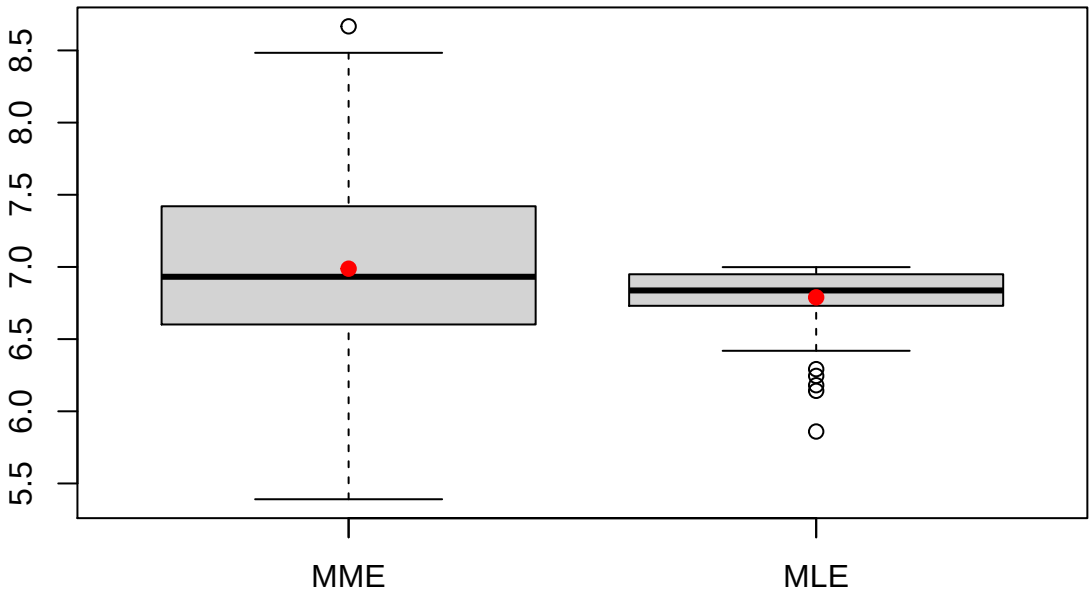
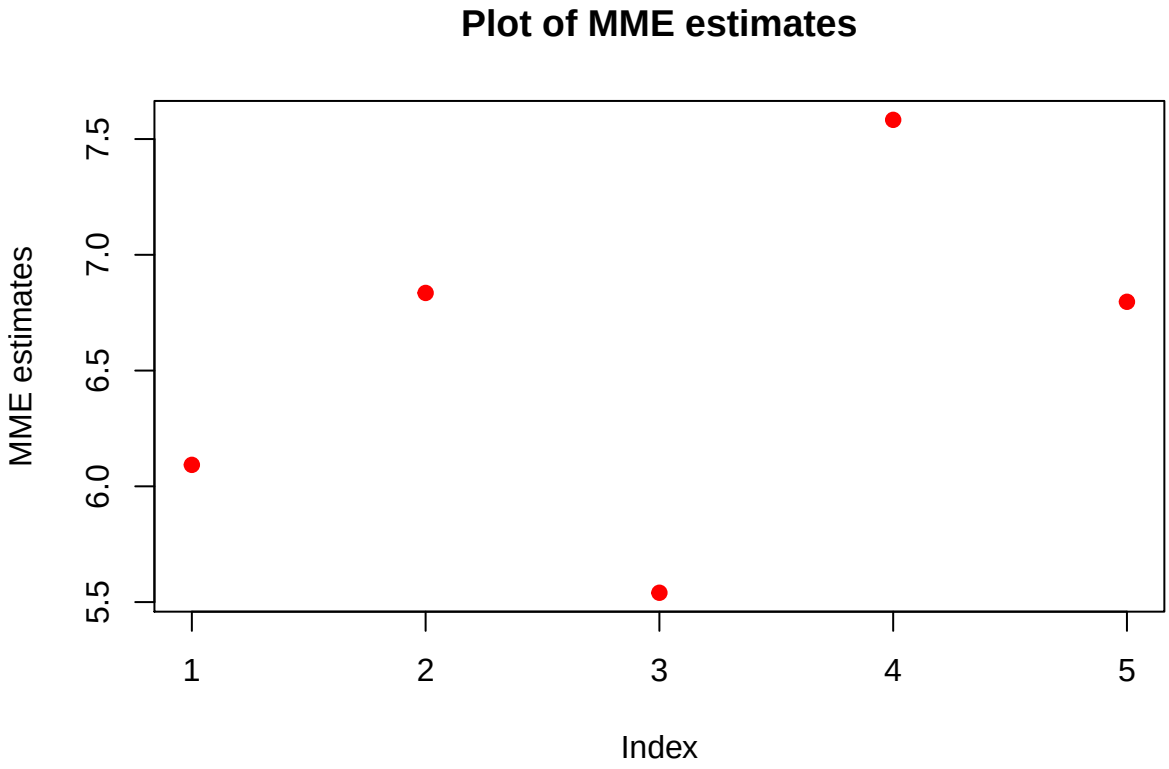
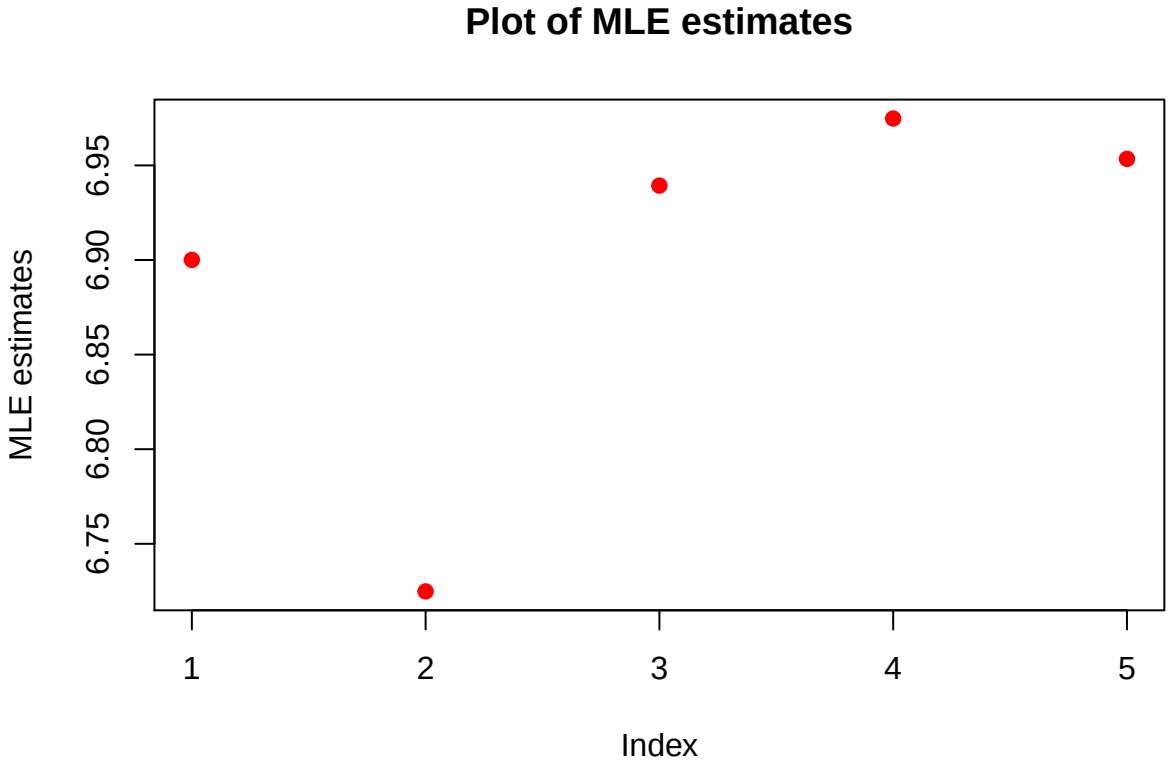
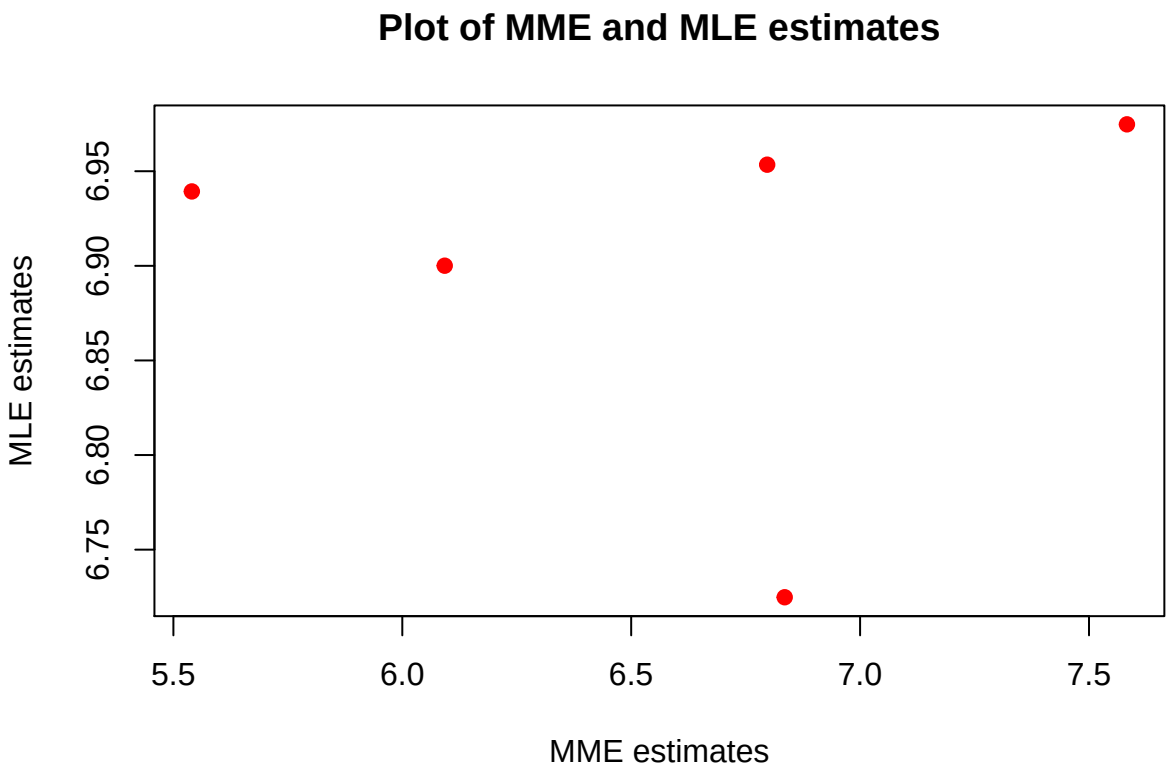
## [1] 6.789535
```


Boxplot of estimates
5.5
6.0
6.5
7.0
7.5
8.0
8.5
MME
MLE

(v)


Plot of MME estimates
MME estimates
5.5
6.0
6.5
7.0
7.5
1
2
3
4
5
Index


Plot of MLE estimates
MLE estimates
6.75
6.80
6.85
6.90
6.95
1
2
3
4
5
Index


Plot of MME and MLE estimates
MLE estimates
6.75
6.80
6.85
6.90
6.95
5.5
6.0
6.5
7.0
7.5
MME estimates

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Homework 3

Problem 1

(i) $X_1, \ldots, X_n$ be the random sample

$\therefore f(x) = \frac{1}{\Theta}$

pop first moment $= \frac{\Theta}{2}$

sample first moment $= \frac{X_1 + \ldots + X_n}{n}$

$$\frac{\Theta}{2} = \frac{X_1 + \ldots + X_n}{n}$$

$$\rightarrow \hat{\Theta}_m = \frac{2}{n} \sum_{i=1}^{n} X_i$$

(ii) Likelihood function is

$$L(\Theta | x) = \begin{cases} \frac{1}{\Theta^n} \\ 0 \end{cases}$$

$$\hat{\Theta}_L = \max(X_1, X_2 \ldots, X_n)$$

(iii) $\Theta = 7$

$$\hat{\Theta}_m = \frac{2}{7}(18.2) = 5.2$$

$$\hat{\Theta}_L = \max(1.0, \ldots, 6.8) = 6.8$$

$\hat{\Theta}_m$ is better because it more accurately fits the data.

Problem 2

(i) moment estimator $(\hat{\mu}, \hat{\sigma}^2)$ –

Since, $x \sim N(\mu, \sigma^2)$

$$E(X) = \mu, \quad V(X) = \sigma^2$$

$$\hat{\mu} = \bar{x}$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Therefore $(\hat{\mu}^2, \hat{\sigma}^2)$ moment estimator $= \left(\bar{x}, \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2\right)$

(ii) maximum likelihood estimator

$$\mu = \bar{x}$$

$$\hat{\sigma}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2)$$

Therefore $(\hat{\mu}, \hat{\sigma}^2)$ mar Lt $= \left(\bar{x}, \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2\right)$

Problem 3

6.17 (a) $z = 1.96$ for $C = 95\%$

$$5.4 \pm 1.96(2.3/\sqrt{340})$$

$$= (5.16, 5.64)$$

(b) 99% is (5.077, 5.723) the results are wider because we have to be more confident on the interval.

6.27

a) $\bar{x} = 11.5 \quad \sigma = 8.3 \quad n = 1200$

$11.5 \pm (1.96 (8.3/\sqrt{1200}))$

$= (11.03, 11.97)$

b) No it is not true because we are talking about the individual students in the sample.

c) Many students did not listen to the radio, so this arises error. Because of this error normal distribution should nevertheless be a good approximation.

6.28

$\bar{x} = (11.5 \text{ hrs})(60) = 690 \text{ min}$

$\sigma = (8.3)(60) = 498 \text{ min}$

b) $690 \pm (1.96 (498/\sqrt{1200}))$

$= (661.82, 718.18)$

c) By changing the hours to minutes we could have directly calculated it