

hw2

Ali Kolenovic

9/16/2020

Problem 1

(i)

N = 20, 30, 50, 75, 100, respectively

```
## [1] 5.955987e-01 9.401122e-02 2.305229e-04 1.826106e-08 5.431127e-13
```

(ii)

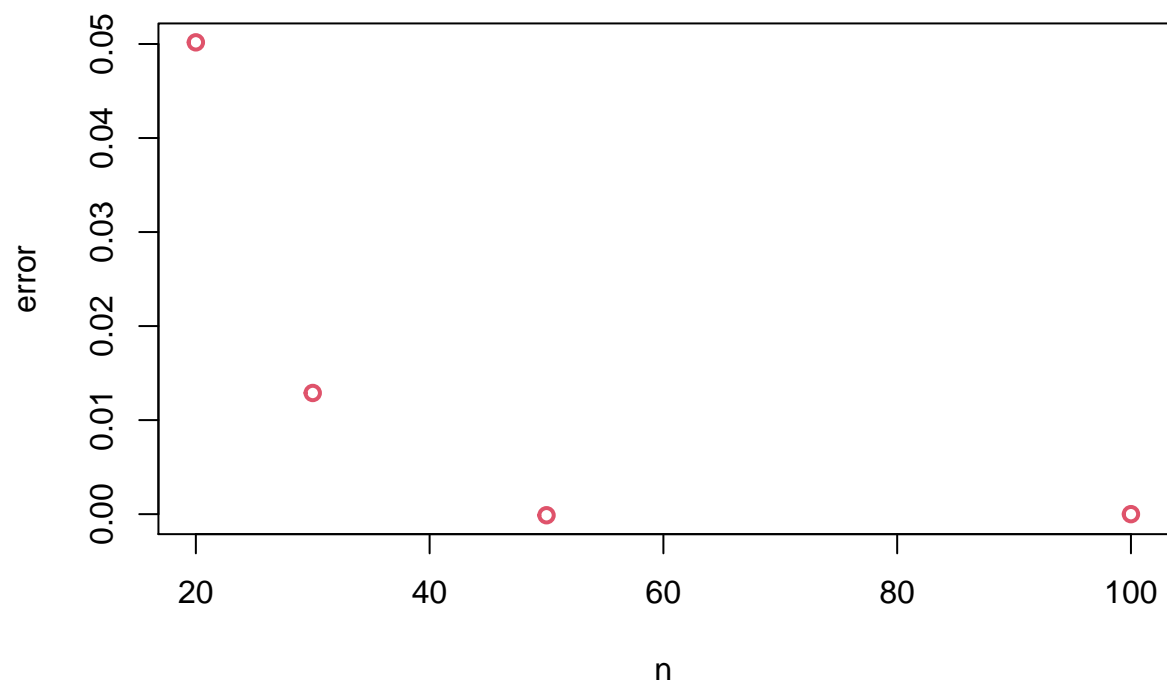
N = 20, 30, 50, 75, 100, respectively

```
## [1] 5.454243e-01 8.112525e-02 3.470073e-04 1.475701e-07 4.557597e-11
```

(iii)

```
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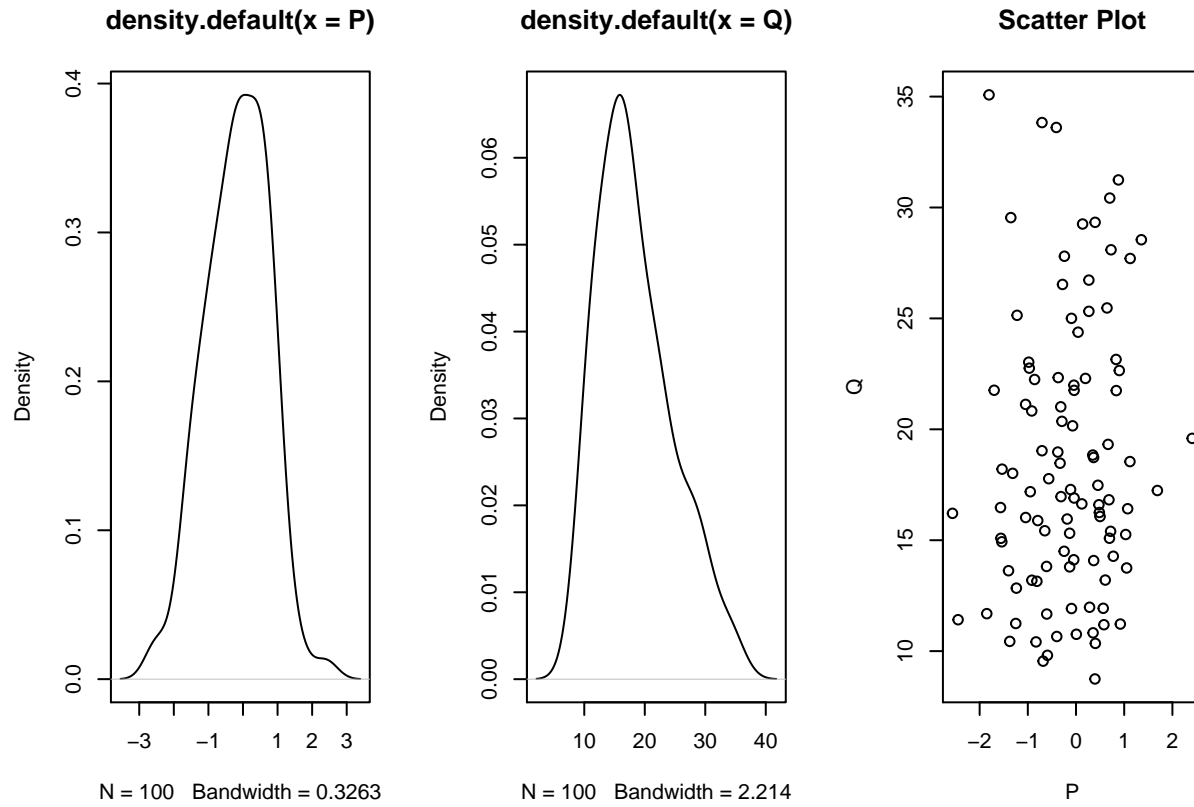


(iv)

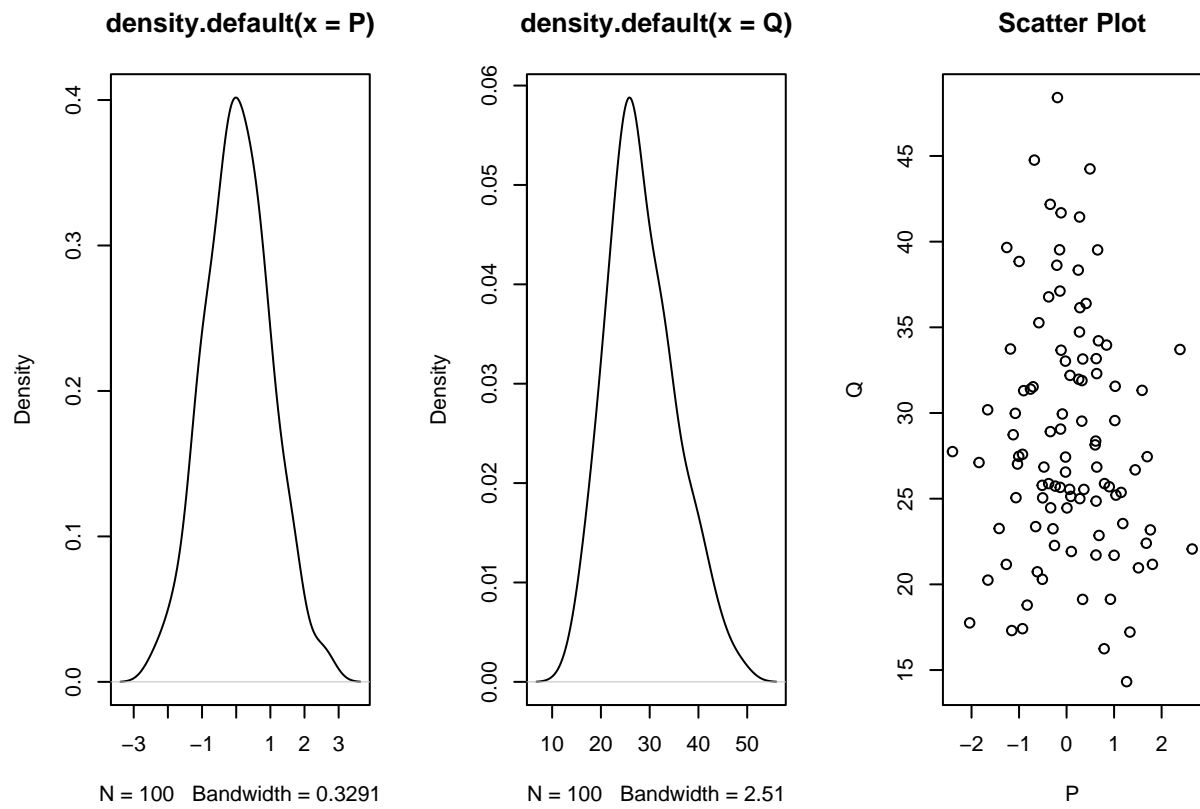
The error plots shows that as N gets greater the error decreases, and when $N=50$ the approximation holds up well because the error is low.

Problem 2

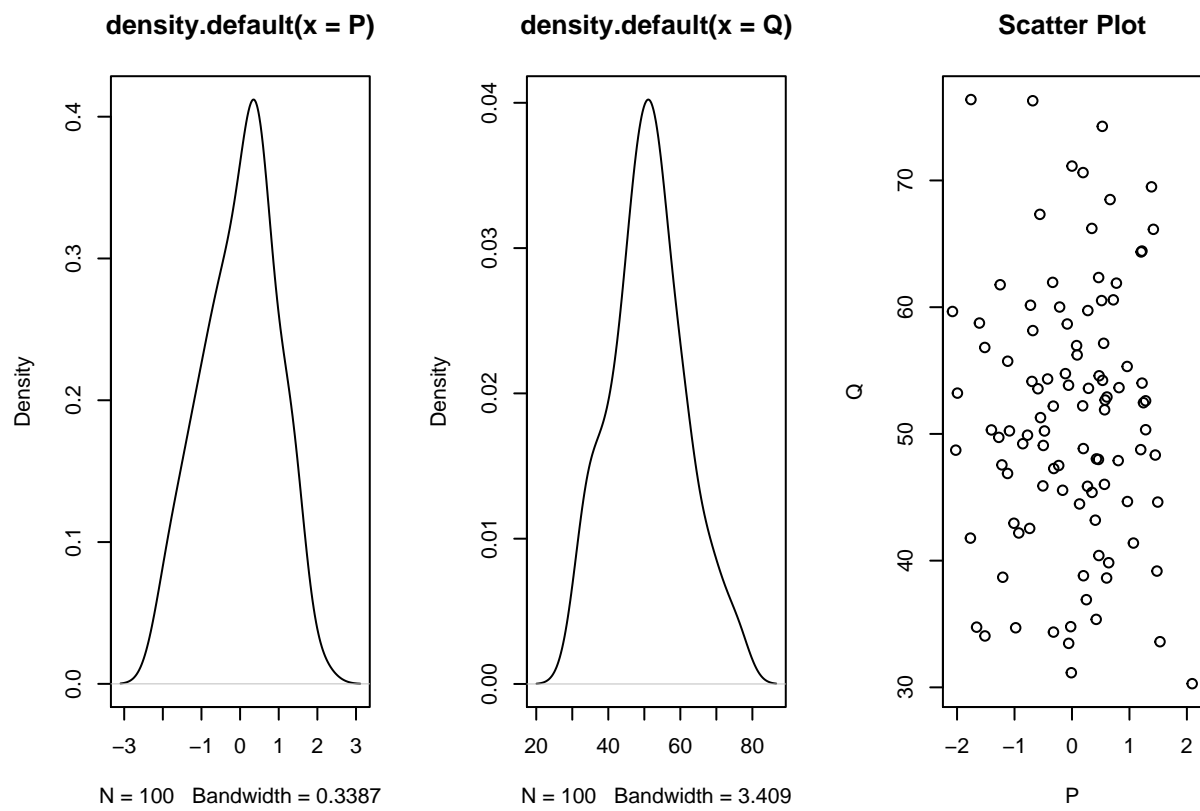
(i)



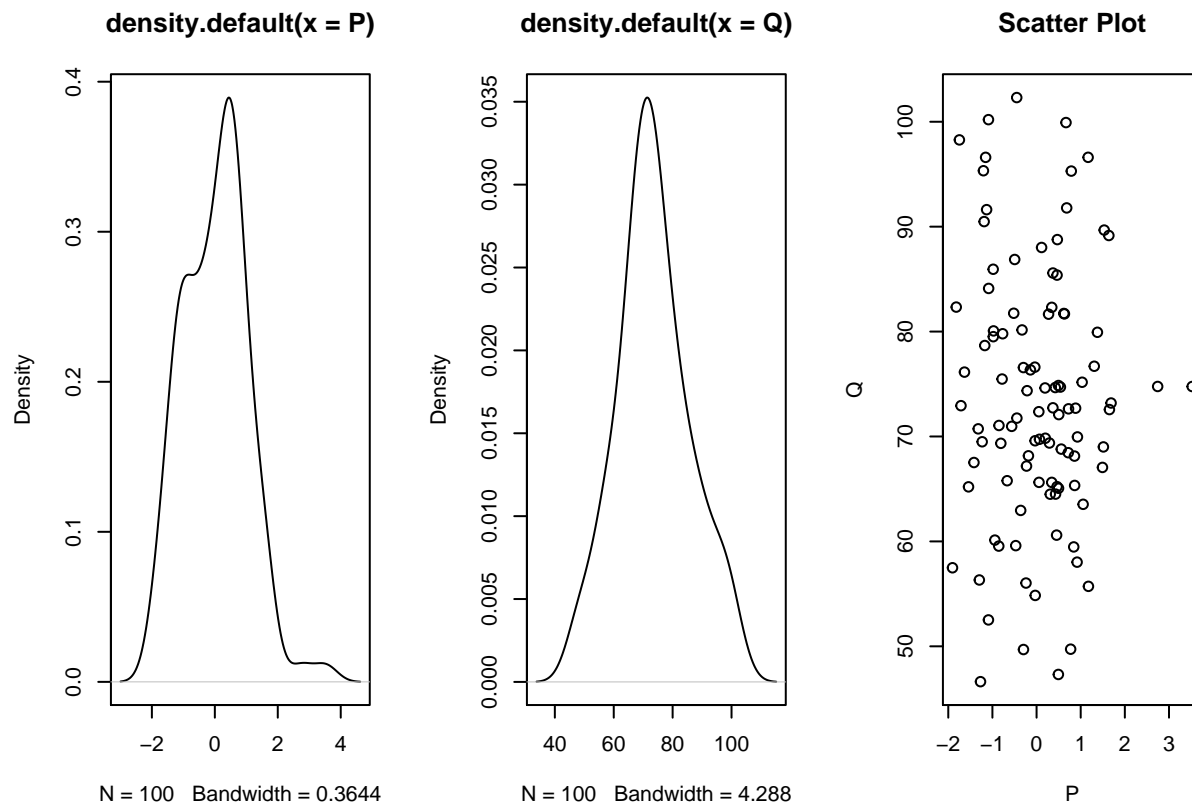
(ii)



(iii)



(iv)



(v)

The plots of x average - 2 are symmetrical and therefore follows normal distribution. The plots of $(n - 1)$ is right skewed and therefore follows chi square distribution.

(v1)

The scatter plot shows that there is no association between them, therefore they are independently distributed.

$$E(x) = np$$

Problem 3

$$E(x+y) = E(x) + E(y)$$

$$y \begin{cases} P(y=1) = p \\ P(y=0) = 1-p \end{cases}$$

$$E(x) = n E(y) = \boxed{np}$$

$$E(y) = p + (1-p) \cdot 0 = p$$

Problem 4

$$E[x] = \frac{1}{\sqrt{n}} \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{n}{2})} \int_{-\infty}^{+\infty} x \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}} dx$$

given that n is greater than 1 so it is defined,
the integral on the right evaluates to 0.

$$\text{Therefore, } E[x] = 0$$