

## WEEK 11 HOMEWORK – SAMPLE SOLUTIONS

### IMPORTANT NOTE

These homework solutions show multiple approaches and some optional extensions for most of the questions in the assignment. You don't need to submit all this in your assignments; they're included here just to help you learn more – because remember, the main goal of the homework assignments, and of the entire course, is to help you learn as much as you can, and develop your analytics skills as much as possible!

#### Question 1

*Describe a situation or problem from your job, everyday life, current events, etc., for which optimization would be appropriate. What data would you need?*

Some (perhaps overzealous!) baseball fans have tried to drive around the country to see a baseball game at each of the 30 Major League stadiums, and then return home, in the shortest number of days. This can be modeled using optimization: minimize the number of days it takes, subject to the constraints that a game is seen at each stadium, and the planned sequence of games is possible given the driving times and game schedules. The necessary data would include the Major League schedule (which stadiums have games scheduled on each day, and what time they're scheduled for), and how long it takes to drive between each pair of stadiums.

#### Question 2

*In the videos, we saw the “diet problem”. (The diet problem is one of the first large-scale optimization problems to be studied in practice. Back in the 1930's and 40's, the Army wanted to meet the nutritional requirements of its soldiers while minimizing the cost.) In this homework you get to solve a diet problem with real data. The data is given in the file diet.xls.*

Here's one possible solution. Please note that a good solution doesn't have to try all of the possibilities in the code; they're shown to help you learn, but they're not necessary.

1. *Formulate an optimization model (a linear program) to find the cheapest diet that satisfies the maximum and minimum daily nutrition constraints, and solve it using PuLP. Turn in your code and the solution. (The optimal solution should be a diet of air-popped popcorn, poached eggs, oranges, raw iceberg lettuce, raw celery, and frozen broccoli. UGH!)*

Algebraically, here's what the model looks like:

### Data

$c_j$  = cost per unit of food  $j$

$a_{ij}$  = amount of nutrient  $i$  per unit of food  $j$

$m_i$  = minimum amount of nutrient  $i$  required

$M_i$  = maximum amount of nutrient  $i$  required

### Variables

$x_j$  = amount of food  $j$  eaten

### Objective

Minimize  $\sum_j c_j x_j$  (minimize total cost)

### Constraints

$\sum_j a_{ij} x_j \geq m_i$  for each nutrient  $i$  (take in at least minimum amount of each nutrient)

$\sum_j a_{ij} x_j \leq M_i$  for each nutrient  $i$  (take in no more than max amount of each nutrient)

$x_j \geq 0$  for each food  $j$  (can't eat negative amounts)

The file HW11-Q2-1-EachNutrientSeparate-fall.py shows one way to solve the problem. This file deals with each nutrient separately, to show what the problem looks like.

However, an easier way to do it is shown in file HW11-Q2-1-SimplerFormat-fall.py, where data is read and constraints are written in a loop, so individual nutrients don't need to each be written out. This will come in handy when there are more constraints (like the optional part below).

However you set up the problem, here's the optimal solution:

```
## -----The solution to the diet problem is-----
## 52.64371 units of Celery,_Raw
## 0.25960653 units of Frozen_Broccoli
## 63.988506 units of Lettuce,Iceberg,Raw
## 2.2929389 units of Oranges
## 0.14184397 units of Poached_Eggs
## 13.869322 units of Popcorn,Air_Popped
##
## Total cost of food = $4.34
```

It's a pretty cheap daily diet, but not one that looks particularly delicious!

2. Please add to your model the following constraints (which might require adding more variables) and solve the new model:
  - a. If a food is selected, then a minimum of 1/10 serving must be chosen. (Hint: now you will need two variables for each food  $i$ : whether it is chosen, and how much is part of the diet. You'll also need to write a constraint to link them.)

We need to add two new things to the model above:

#### New variables

$y_j = 1$  if food  $j$  is eaten, 0 if not (so it's a binary variable)

#### New constraints

$x_j \geq 0.1 y_j$  for each food  $j$  (if food  $j$  is eaten, must eat at least 0.1 units)  
 $0.0000001 x_j \leq y_j$  for each food  $j$  (if any of food  $j$  is eaten, must ensure  $y_j = 1$ )

*b. Many people dislike celery and frozen broccoli. So at most one, but not both, can be selected.*

This requires just one constraint:

#### New constraints

$y_{\text{frozen broccoli}} + y_{\text{raw celery}} \leq 1$  (can't eat both frozen broccoli and raw celery)

*c. To get day-to-day variety in protein, at least 3 kinds of meat/poultry/fish/eggs must be selected.*

This requires just one constraint, but it's a longer one:

#### New constraints

$y_{\text{Roasted Chicken}} + y_{\text{Poached Eggs}} + y_{\text{Scrambled Eggs}} + y_{\text{Bologna,Turkey}} + y_{\text{Frankfurter,Beef}}$   
 $+ y_{\text{Ham,Sliced,Extralean}} + y_{\text{Kielbasa,Prk}} + y_{\text{Pizza W/Pepperoni}} + y_{\text{Hamburger W/Toppings}}$   
 $+ y_{\text{Hotdog,Plain}} + y_{\text{Pork}} + y_{\text{Sardines in Oil}} + y_{\text{White Tuna in Water}} + y_{\text{Chicknoodl Soup}}$   
 $+ y_{\text{Splt Pea\&Hamsoup}} + y_{\text{Vegetbeef Soup}} + y_{\text{Neweng Clamchwd}} + y_{\text{New E Clamchwd,W/Mlk}}$   
 $+ y_{\text{Beanbacn Soup,W/Watr}} \geq 3$

[Note: if you identified a different set of things that counted as meat/poultry/fish/eggs, that's fine. I'm not worried about your ability to distinguish foods; I just want you to be able to write this constraint.]

The file HW11-Q2-2-SimplerFormat-fall.py shows one way to solve the problem. Here's the solution:

```
## -----The solution to the diet problem is-----
## 0.1 units of Bologna,Turkey
## 42.423026 units of Celery,_Raw
## 82.673927 units of Lettuce,Iceberg,Raw
## 3.0856009 units of Oranges
## 1.9590978 units of Peanut_Butter
## 0.1 units of Poached_Eggs
## 13.214473 units of Popcorn,Air_Popped
## 0.1 units of Scrambled_Eggs
##
## Total cost of food = $4.51
```

It's pretty close to the previous solution, but the constraints we added have had their effect. Broccoli has disappeared (celery stayed), we have three proteins (turkey bologna, poached eggs, and scrambled

eggs), and because the proteins are more-expensive, the solution uses the minimum amount of them that we allow (0.1 units).

*If you want to see what a more full-sized problem would look like, try solving the model of Question 2 for the file diet\_large.xls, which is a low-cholesterol diet model (rather than minimizing cost, the goal is to minimize cholesterol intake). I don't know anyone who'd want to eat this diet – the optimal solution includes dried chrysanthemum garland, raw beluga whale flipper, freeze-dried parsley, etc. – which shows why it's necessary to add additional constraints beyond the basic ones we saw in the video!*

Algebraically, this model looks exactly like the one in Part 1 – there's no difference.

#### Data

$c_j$  = cost per unit of food  $j$

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#### Variables

$x_j$  = amount of food  $j$  eaten

#### Objective

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$\sum_j a_{ij} x_j \geq m_i$  for each nutrient  $i$  (take in at least minimum amount of each nutrient)

$\sum_j a_{ij} x_j \leq M_i$  for each nutrient  $i$  (take in no more than max amount of each nutrient)

$x_j \geq 0$  for each food  $j$  (can't eat negative amounts)

The file HW11-Q2-LargeProb-SimplerFormat-fall.py shows one way to solve this problem. Note that the data file is less clean, so the code includes skipping the first empty row, replacing blank values with zero, etc. But other than that, we don't need to change much in the code from Part 1 to solve this much-larger problem; that's the benefit of having a good, general algebraic model in the code.

I wrote above that the optimal solution included some odd stuff, like dried chrysanthemum garland, raw beluga whale flipper, etc. It turns out that there's more than one optimal solution – both solutions (and others) are able to find a completely cholesterol-free diet. I got the solution above on a different machine using commercial-grade optimization software (CPLEX); using my laptop and PuLP, I got this solution:

```
## -----The solution to the diet problem is-----
## 0.059863415 units of Beans,_adzuki,_mature_seeds,_raw
## 0.069514608 units of Broccoli_raab,_raw
## 0.42866218 units of Cocoa_mix,_no_sugar_added,_powder
## 0.14694398 units of Egg,_white,_dried,_flakes,_glucose_reduced
## 0.73805891 units of Infant_formula,_MEAD_JOHNSON,_ENFAMIL,_NUTRAMIGEN,_with_iron,_p
## 0.4258564 units of Infant_formula,_NESTLE,_GOOD_START_ESSENTIALS__SOY,__with_iron,
## 0.050114149 units of Infant_formula,_ROSS,_ISOMIL,_with_iron,_powder,_not_reconstitu
## 0.15033656 units of Margarine_like_spread,_approximately_60%_fat,_tub,_soybean_(hyd
```

```
## 0.25918767 units of Mung_beans,_mature_seeds,_raw
## 0.18052856 units of Nuts,_mixed_nuts,_dry_roasted,_with_peanuts,_with_salt_added
## 1.184482 units of Oil,_vegetable,_sunflower,_linoleic,_(hydrogenated)
## 0.10375187 units of Seeds,_sunflower_seed_kernels,_dry_roasted,_with_salt_added
## 0.031866196 units of Snacks,_potato_chips,_fat_free,_made_with_olestra
## 0.070710308 units of Spices,_paprika
## 0.55106575 units of Tomatoes,_sun_dried
## 9999.6864 units of Water,_bottled,_non_carbonated,_CALISTOGA
##
## Total cholesterol = 0.000000
```

It's also a pretty odd diet; in addition to eating three kinds of infant formula, you'd have to drink more water than a human body can hold!

I also attached the file HW11-Q2-LargeProb-MaxProtein-fall.py, to see what would happen if, using the large data set, we wanted to maximize protein intake rather than minimize cholesterol. Here too, the diet looks pretty weird: lots of water, three types of beluga whale meat, and a frozen dinner!

```
## -----The solution to the diet problem is-----
## 7.0117007 units of BANQUET_Salisbury_Steak_Meal,_Gravy_and_Salisbury_Steak_with_Ma
## 0.20365743 units of Cereals_ready_to_eat,_KASHI_Heart_to_Heart_by_KELLOGG
## 0.23412086 units of Collards,_raw
## 25.855235 units of Fish,_devilfish,_meat_(Alaska_Native)
## 31.46708 units of Fish,_lingcod,_meat,_raw_(Alaska_Native)
## 0.02 units of Fish_oil,_cod_liver
## 2.2140307 units of Gelatins,_dry_powder,_unsweetened
## 0.037489833 units of Mollusks,_oyster,_eastern,_canned
## 57.437865 units of Rhubarb,_wild,_leaves_(Alaska_Native)
## 621.79859 units of Sweeteners,_tabletop,_aspartame,_EQUAL,_packets
## 9.5089609 units of Tea,_brewed,_prepared_with_distilled_water
## 9552.2849 units of Water,_bottled,_non_carbonated,_CALISTOGA
## 276.5536 units of Water,_bottled,_non_carbonated,_DANNON
## 0.076732592 units of Whale,_beluga_flipper,_raw_(Alaska_Native)
## 9.6405544 units of Whale,_beluga_liver,_raw_(Alaska_Native)
## 1.7353546 units of Whale,_beluga_meat,_air_dried,_raw_(Alaska_Native)
##
## Total protein = 2994.899576
```

**IMPORTANT:** These odd solutions might make you wonder whether optimization solutions are really useful/realistic. But that actually is looking in the wrong direction. The problem with the solutions in this homework is that the model is too simplistic. Obviously, none of us eats this way; if we wanted to find a realistic high-protein diet, for example, we'd have to write more-complicated constraints – we wouldn't allow more water than we can drink, we'd require that the food be easily obtainable (so, probably no beluga whale meat), we'd add constraints for our personal taste, for price, for variety of diet, etc. As the model would get more and more realistic, the solution would too. We didn't see much detail about that in this course, because we didn't have time; modeling is probably at least 1/3 of the Georgia Tech's elective course on optimization.