



الجامعة المستنصرية

كلية العلوم

قسم: علوم الحاسوب

عنوان التقرير

Two dimension Geometric transformation (Translation)

أسم الطالب الرباعي: يوسف عبد الستار شيخان احمد

التوقيع:

المرحلة: الثالثة

القسم: علوم الحاسوب

الفرع: CS

الشعبة: A1

البريد الالكتروني: yousafabdalstar1@gmail.com

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لمادة رسم بالحاسوب

يملئ من قبل أستاذ المادة					
اسم الأستاذ:					
					الدرجة
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توقيع المدقق

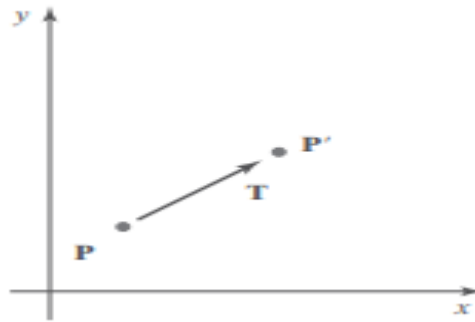
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Introduction

Two-Dimensional Translation

We perform a **translation** on a single coordinate point by adding offsets to its coordinates so as to generate a new coordinate position. In effect, we are moving the original point position along a straight-line path to its new location. Similarly, a translation is applied to an object that is defined with multiple coordinate positions, such as a quadrilateral, by relocating all the coordinate positions by the same displacement along parallel paths. Then the complete object is displayed at the new location.

To translate a two-dimensional position, we add translation distances t_x and t_y to the original coordinates (x, y) to obtain the new coordinate position (x', y') as shown in Figure 1



[Figure 1] Translating a point from position P to position P' using a translation vector T.

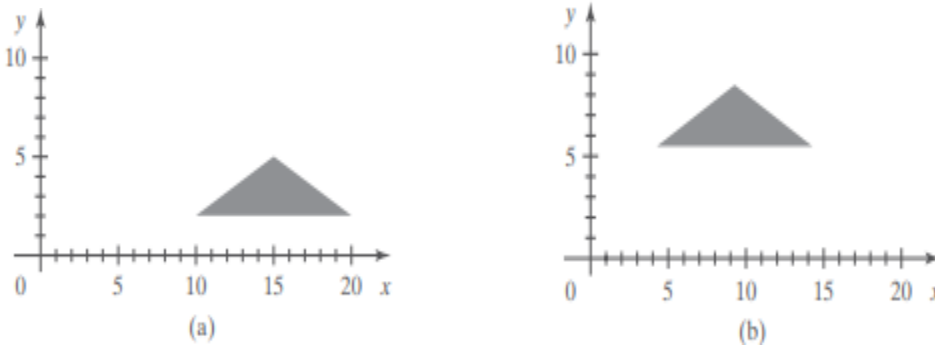
$$x' = x + t_x, y' = y + t_y \quad (1)$$

The translation distance pair (t_x, t_y) is called a translation vector or shift vector. We can express Equations 1 as a single matrix equation by using the following column vectors to represent coordinate positions and the translation vector:

$$p = \begin{pmatrix} x \\ y \end{pmatrix}, \quad p' = \begin{pmatrix} x' \\ y' \end{pmatrix}, \quad t = \begin{pmatrix} t_x \\ t_y \end{pmatrix} \quad (2)$$

This allows us to write the two-dimensional translation equations in the matrix form

$$P' = P + T \quad (3)$$



[Figure 2]

Moving a polygon from position (a) to position (b) with the translation vector $(-5, 0)$.

Translation is a rigid-body transformation that moves objects without deformation. That is, every point on the object is translated by the same amount. A straight-line segment is translated by applying Equation 3 to each of the two line endpoints and redrawing the line between the new endpoint positions. A polygon is translated similarly. We add a translation vector to the coordinate position of each vertex and then regenerate the polygon using the new set of vertex coordinates. **Figure 2** illustrates the application of a specified translation vector to move an object from one position to another. The following routine illustrates the translation operations. An input translation vector is used to move the n vertices of a polygon from one world-coordinate position to another, and OpenGL routines are used to regenerate the translated polygon.

Two-Dimensional Translation Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = T(t_x, t_y) \cdot P$$

Inverse translation matrix v

Translate in the opposite direction

$$T^{-1} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- If two successive translation vectors (t_{1x}, t_{1y}) and (t_{2x}, t_{2y}) are applied to a 2-D coordinate position P , the final transformed location P' is

$$\begin{aligned} P' &= T(t_{2x}, t_{2y}) \cdot \{T(t_{1x}, t_{1y}) \cdot P\} \\ &= \{T(t_{2x}, t_{2y}) \cdot T(t_{1x}, t_{1y})\} \cdot P \end{aligned}$$

- The composite transformation matrix for this sequence of translations is

$$\begin{bmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(t_{2x}, t_{2y}) \cdot T(t_{1x}, t_{1y}) = T(t_{1x} + t_{2x}, t_{1y} + t_{2y})$$



Example, to translate a triangle with vertices at original coordinates (10,20), (10,10), (20,10) by $t_x=5$, $t_y=10$, we compute as followings:

Translation of vertex (10,20):

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 1*10+0*20+5*1 \\ 0*10+1*20+10*1 \\ 0*10+0*20+1*1 \end{bmatrix} = \begin{bmatrix} 15 \\ 30 \\ 1 \end{bmatrix}$$

Translation of vertex (10,10):

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 1*10+0*10+5*1 \\ 0*10+1*10+10*1 \\ 0*10+0*10+1*1 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \\ 1 \end{bmatrix}$$

Translation of vertex (20,10):

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 1*20+0*10+5*1 \\ 0*20+1*10+10*1 \\ 0*20+0*10+1*1 \end{bmatrix} = \begin{bmatrix} 25 \\ 20 \\ 1 \end{bmatrix}$$

The resultant coordinates of the triangle vertices are (15,30), (15,20), and (25,20) respectively.



Reference

1. <https://www.gatevidyalay.com/2d-transformation-in-computer-graphics-translation-examples/>
2. <https://www.cs.rit.edu/~icss571/clipTrans/2DTransBack.html>
3. https://www.tutorialspoint.com/computer_graphics/2d_transformation.htm
4. <https://www.javatpoint.com/computer-graphics-introduction-of-transformations>
5. <http://www.it.hiof.no/~borres/j3d/math/twod/p-twod.html>