VARMA versus VAR for Macroeconomic Forecasting

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Outline

- VAR models dominate
- Why VARMA?
  - More parsimonious representation
  - Closed with respect to linear transformations

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Identification Problem

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} - \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix}$$

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- SCM framework: Tiao & Tsay (1989) completed by Athanasopoulos & Vahid (2006)
- Echelon form: Hannan & Kavalieris (1984); Poskitt (1992); Lütkepohl & Poskitt (1996), Athanasopoulos, Poskitt & Vahid (2007)

Outline

Definition of a SCM: For a given K-dimensional $VARMA(p,q)$

$$\mathbf{y}_t = \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{\Phi}_p \mathbf{y}_{t-p} + \eta_t - \mathbf{\Theta}_1 \eta_{t-1} - \ldots - \mathbf{\Theta}_q \eta_{t-q} \qquad (1)$$

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$$z_{r,t} = {\alpha_r}' \mathbf{y}_t \sim SCM(p_r, q_r)$$

$$\text{if } \alpha_r \text{ satisfies} \quad \begin{array}{l} {\alpha_r}' \mathbf{\Phi}_{p_r} \neq \mathbf{0}^T \text{ where } 0 \leq p_r \leq p \\ {\alpha_r}' \mathbf{\Phi}_l = \mathbf{0}^T \quad \text{for } l = p_r + 1, ..., p \\ {\alpha_r}' \mathbf{\Theta}_{q_r} \neq \mathbf{0}^T \text{ where } 0 \leq q_r \leq q \\ {\alpha_r}' \mathbf{\Theta}_l = \mathbf{0}^T \text{ for } l = q_r + 1, ..., q \end{array}$$

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SCM Methodology: Find K-linearly independent vectors $\mathbf{A} = (\alpha_1, \ldots, \alpha_K)'$ which transform (1) into

$$\mathbf{A}\mathbf{y}_t = \mathbf{\Phi}_1^*\mathbf{y}_{t-1} + \ldots + \mathbf{\Phi}_p^*\mathbf{y}_{t-p} + \varepsilon_t - \mathbf{\Theta}_1^*\varepsilon_{t-1} - \ldots - \mathbf{\Theta}_q^*\varepsilon_{t-q} \qquad (2)$$

where $\mathbf{\Phi}_i^* = \mathbf{A}\mathbf{\Phi}_i, \varepsilon_t = \mathbf{A}\eta_t$ and $\mathbf{\Theta}_i^* = \mathbf{A}\mathbf{\Theta}_i\mathbf{A}^{-1}$

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Series of C/C tests: $E\begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix}\begin{bmatrix} y_{1,t} & y_{2,t} \end{bmatrix}[\alpha_1] = \mathbf{0}$

$$\alpha_1'\mathbf{y}_t \sim SCM(0,0)$$

Example: $K = 3$

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- Reduce parameters of **A** to produce a Canonical SCM

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Empirical Results:

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Empirical Results:

1. Average performance across many trivariate systems
2. A four variable example
3. Simulation: Why do VARMA models do better than VARs

Outline

1. Introduction
2. Canonical SCM
3. **Forecast performance**
4. Example
5. Simulation
6. Summary of findings and future research

Forecasting: 40 monthly macroeconomic variables from 8 general categories of economic activity, 1959:1-1998:12 (N=480)

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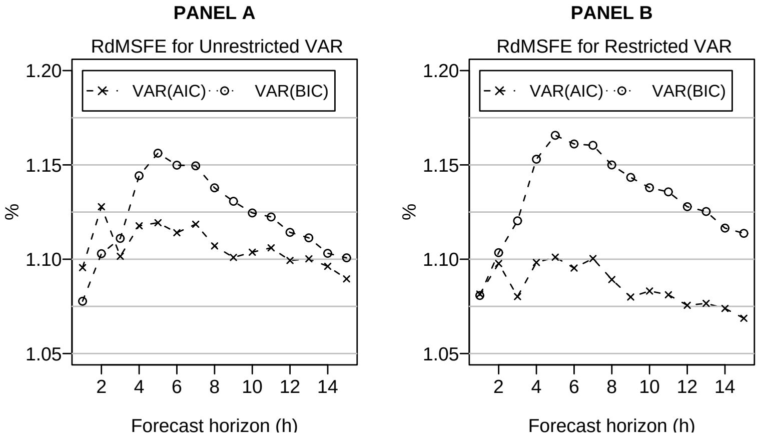
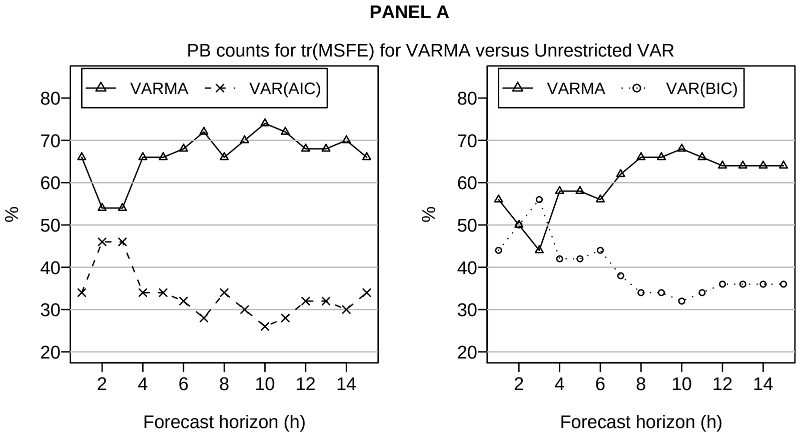
- Hold-out sample: $N_2 = 180$
  - Produced $N_2 - h + 1$ out-of-sample forecasts for each h=1 to 15
  - Forecast error measures: $|MSFE|$ and $tr(MSFE)$
  - Percentage Better: PB_h
  - Relative Ratios:
    - $\overline{RRdMSFE}_h = \frac{1}{50} \sum_{i=1}^{50} \frac{|MSFE_i^{VAR}|}{|MSFE_i^{VARMA}|}$

Relative Ratios



Percentage Better: Unrestricted VAR



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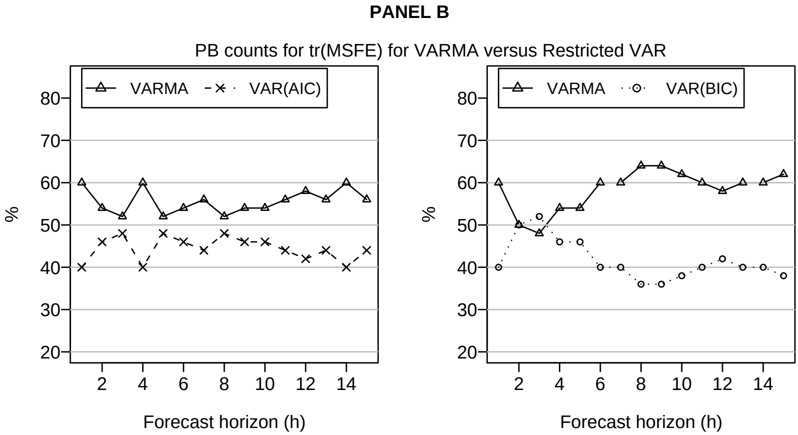

Diebold-Mariano test for tr(MSFE):

VARMA sign better (5%, 25%, 25%,5%) VARMA sign worst

	Forecast horizon (h)				
	1	4	8	12	15
$VAR(AIC)$ - Unrest	24,46,20,14	16,38,10,0	14,32,6,4	10,22,10,0	10,22,8,0
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3. As h increases the number significant differences decreases

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VAR(*BIC*) - Rest	32,54,22,14	10,38,18,8	12,26,22,6	10,20,20,6	10,16,14,2

Messages:

1. There are cases where VARMA significantly outperform VAR and vice versa
2. VARMA models significantly outperform VAR more than the reverse
3. As h increases the number significant differences decreases
4. Restrictions do not improve VAR performance when significant differences

Outline

MONASH University

Example:

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- Four variables (also six variables):
    - GDP growth rate
    - inflation rate
    - spread (10 yr gvt bill yield) − (3-month treasury bill rate)
    - 3-month treasury bill rate

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Message: We should start considering VARMA

Outline

Why do VARMA forecast better?

Estimated a VARMA(2,1): - SCM(2,0) - SCM(1,1)
- SCM(1,0) - SCM(0,0)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ * & * & * & 1 \end{pmatrix} \mathbf{y}_t = \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{y}_{t-1} + \begin{pmatrix} * & * & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{y}_{t-2}$$

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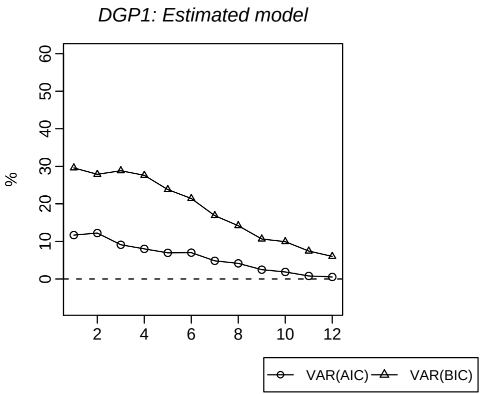
- Simulate from the benchmark estimated model assuming $\mathbf{e} \sim N(\mathbf{0}, \mathbf{\Sigma})$
- $n = 164 \longrightarrow$ estimate VARMA(2,1), VAR(AIC), VAR(BIC)
- $nout = 42 \longrightarrow$ compute 1 to 12-step ahead forecasts
- $iterations = 100 \longrightarrow$ calculate $\overline{|MSFE|}$ for all models
- Compare the percentage difference using $\overline{|MSFE_{VARMA}|}$ as a base
- Repeat by changing specific features and compare with the benchmark

Why do VARMA forecast better:



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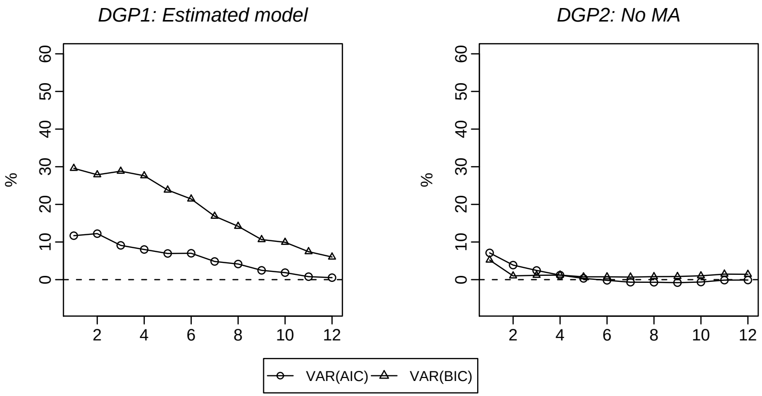

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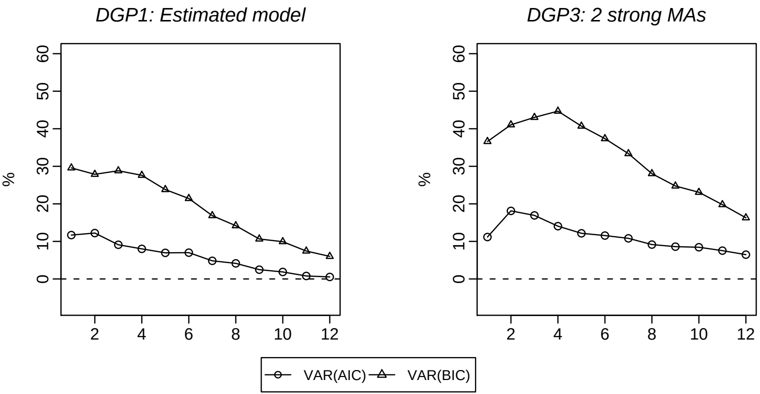

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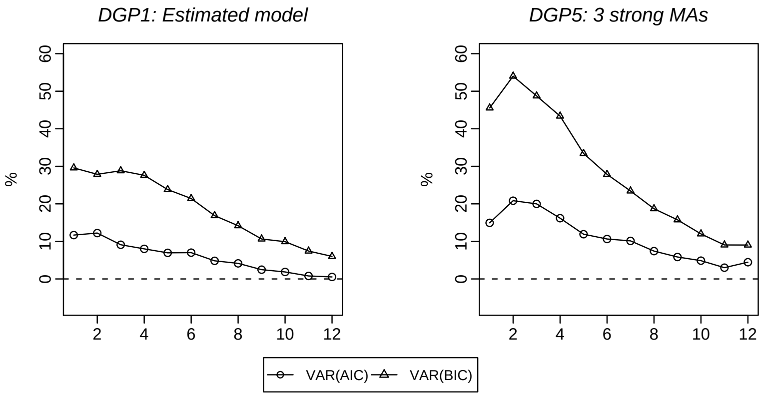

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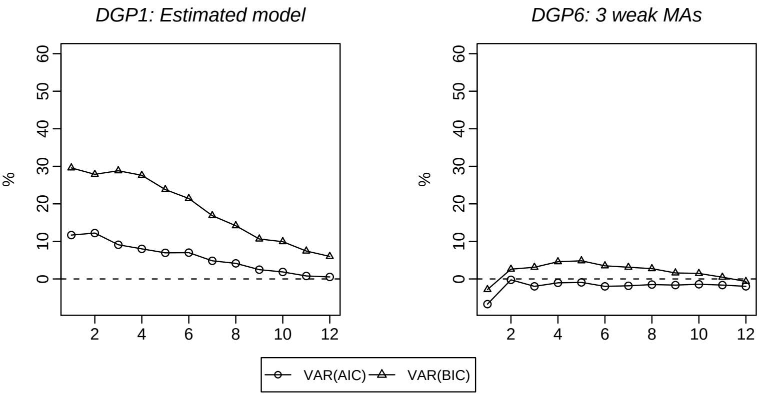

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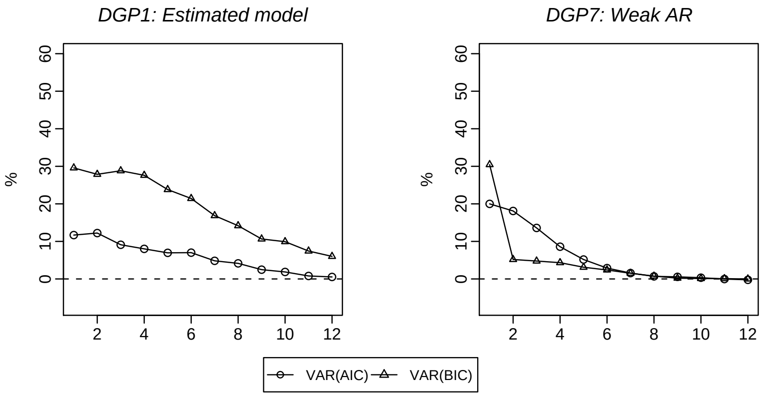

Outline

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Future Research:

1. Developing a fully automated identification process
2. Developing an alternative estimation approach which avoids fitting a long VAR to estimate the lagged innovations
3. Move into the non-stationary world

Thank you!!!!!