Part 1: Description of SGD+ and Adam.

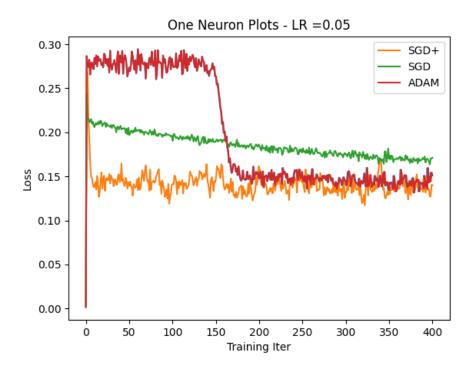
SGD+ helps solve the problem of getting oscillatory behavior near the true minimum by introducing a momentum coefficient. This momentum coefficient, μ , multiplies with the previous step size (v_t), and adds that resulting product to the current gradient (g_{t+1}). The result of that addition is the new step size (i.e, v_{t+1} = μ * v_t + g_{t+1}). This step size is then multiplied by the constant learning rate, so, the set of parameter values is actually dependent on the previous step size! Mathematically, this is represented as p_{k+1} = p_k - α *v_{t+1}, where α is a fixed learning rate. Overall, this makes the step sizes larger if the previous step size and current gradient are in the same direction (because instead of multiplying learning rate by just the current gradient, we are adding a fraction of the previous gradient as well). On the other hand, if the previous step size and current gradient point in different directions (i.e, one is positive and the other is negative), then, the next set of parameters values is incremented more slowly than without SGD+.

SGD+ assumes that we should set a fixed learning rate for each parameter. However, it does not make sense to have a fixed learning rate for each parameter when the resulting gradient is sparse. In order to adaptively set a learning rate, AdaGrad aims to divide the sum of all the parameters in that dimension by the learning rate and then multiply that quotient by the parameter (i.e., the next step is $(\alpha / \sum (g_k)^2)^* g_k$. So, if the current gradient is zero, we actually do not update that parameter. However, as the learning process goes on, the sum of that certain parameter increases monotonically and therefore the impact of the adaptive learning rate actually becomes smaller, until we increase the step size by infinitesimally small steps. This becomes counterproductive to learning when there are actually discriminatory features in the data. Therefore, a separate algorithm named RMSprop actually keeps a running average of the gradient, which doesn't increase monotonically. This effectively multiplies the k-th iteration of i-th component of the gradient $g_{k,i}$ by the learning rate α , as shown here: $[\alpha/sqrt(\sum ((g(k, i))^2 + \varepsilon)] * g(k, i)$, where ε is a very small value to prevent division by zero. The Adam optimizer put these two ideas (adaptive momentum due to better convergence to a minimum and adaptive gradient to deal with sparse gradients) into one algorithm. Adam keeps a running average of both of these factors in two "moments". The first moment, $m_{t+1} = \beta_1$ * $m_t + (1 - \beta_1) * g_{t+1}$, where $\beta_1 = 0.9$ and stays constant. The second moment, $v_{t+1} = \beta_2 * v_t + (1 - \beta_2)$ * $(q_{t+1})^2$. The moments are initialized as $m_0 = 0$ and $v_0 = 0$, which makes resulting next values of the moments quite small at the beginning. Therefore, we adjust the moments to $m_k^* = m_k / (1 - m_k)$ β_1^k) and $v_k^a = v_k / (1 - \beta_2^k)$, where k is the training iteration of the parameter in question. v_k^a and m_k^{Λ} are then both contributing to the next step size by the next parameters, $p_{t+1} = p_t - \alpha * (m^{\Lambda} / m_t^{\Lambda})$ $sqrt(v^{\Lambda} + \epsilon)$).

Part 2: Comparative Plots for SGD, SGD+, and Adam

Part 2a: One Neuron Classifier Plots (Figure 1 and 2)

Figure 1 below



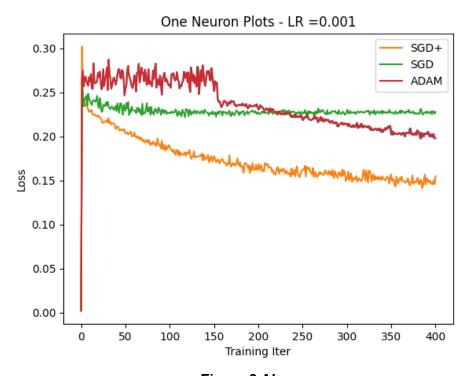
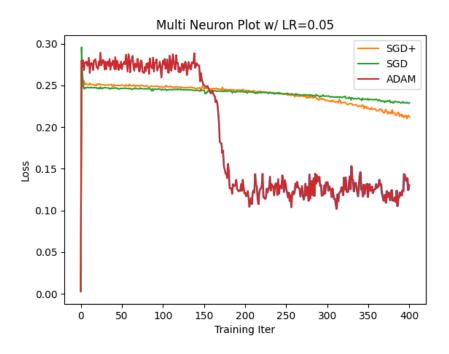


Figure 2 Above

Part 2b: Multi Neuron Classifier Plots Figure 3 Below



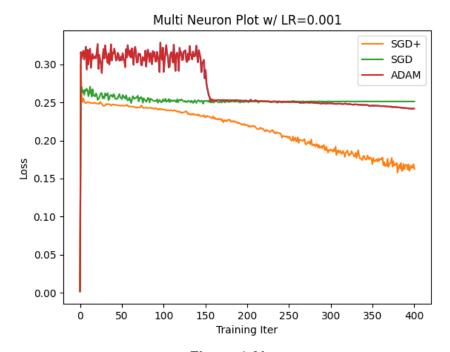


Figure 4 Above

Part 2c: Discussion of findings

Overall, SGD+ and Adam both perform better than SGD in both cases (with learning rate set to 0.001 or 0.05). It was interesting to note that Adam had a significant jump about halfway through the training process. It isn't super clear why Adam doesn't seem to learn anything and then all of a sudden takes large leaps. Adam also performed better compared to SGD+ with a higher learning rate (learning rate set to 0.05 lead to Adam having a loss between 0.10 and 0.15) in both the multi neuron case, but Adam and SGD+ had the same loss after 40,000 iterations in the single neuron case (both losses were around 0.14). SGD+ on the other hand, performed better than Adam when the learning rate was set to 0.001 (had a loss around 0.16 for both the single and multi neuron cases). Lastly, Adam seems to oscillate much more than either SGD or SGD+. SGD and SGD+ seem to decrease monotonically while Adam can have some small oscillations when it is already at a smaller loss.

Part 3: Code

Summary: I created 2 classes (one for Adam, one for SGD+) that inherit ComputationalGraphPrimer and overrode the multi neuron and one neuron backpropagation functions in each of those classes. I also created code that runs each scenario (one neuron for SGD, SGD+, and Adam and multi neuron for SGD, SGD+, Adam) and plots the results. Furthermore, I added empty lists in the training loop functions which were passed to the overridden functions. However, since those are so small, I didn't explicitly add them here. Additionally, one can see the overriding function definitions contain more input parameters than in the original CGP code.

Here is the class definition for the SGD+ class:

```
learning_rate = 1e-3,
    #learning_rate = 5 * 1e-2,
    training_iterations = 40000,
    batch_size = 8,
    display_loss_how_often = 100,
    debug = True,
) #initialize base class ComputationalGraphPrimer
```

This is the overriding function of the backpropagation for SGD+ in the one neuron case. Note that the parts that were modified are denoted (about halfway through the function).

```
def backprop and update params one neuron model (self, y error, vals for input vars,
deriv sigmoid, vt1 list, bias vt1 list, k):
      input vars = self.independent vars
      input vars to param map = self.var to var param[self.output vars[0]]
input vars to param map.items() }
       for i,param in enumerate(self.vals for learnable params):
          gradient = y error * vals for input vars dict[param to vars map[param]] *
deriv sigmoid #orig step computation.
              vt1 list.append(mu vt + gradient) #append Vt+1 to step list. We will
need to access this as Vt for future computations.
```

```
vtl_list.append(vtl)
    #step = (momentum * prev_step) + step #here, step is Vt+1, momentum is mu,
step_list[i-1] is Vt, and step is Gt+1
    ## Update the learnable parameters
    self.vals_for_learnable_params[param] =
self.vals_for_learnable_params[param] + (self.learning_rate * vtl_list[-1]) #implement
-momentum * dpk-1
    bias_gradient = y_error * deriv_sigmoid #COMPUTE GRADIENT OF BIAS TERM.
    if (len(bias_vtl_list) == 0): #SAME LOGIC AS VT1 LIST, BUT KEEPING TRACK OF
BIAS GRADIENTS
    mu_vt = 0 #INITIALIZE MU TO 0
    bias_vtl_list.append(mu_vt + bias_gradient)
    else:
        bias_vtl = (momentum * bias_vtl_list[-1]) + bias_gradient
        bias_vtl_list.append(bias_vtl)
        self.bias = self.bias + self.learning_rate * bias_vtl_list[-1] #UPDATE
PARAMETER FOR BIAS
    ###END MODIFICATION FOR SGD+
```

This is the overriding function of the backpropagation for SGD+ in the multi neuron case. Note that the parts that were modified are denoted.

```
layer params = self.layer params[back layer index]
          transposed layer params = list(zip(*layer params))  ## creating a
          backproped_error = [None] * len(vars_in_next_layer_back)
          for k,varr in enumerate(vars in next layer back):
              for j,var2 in enumerate(vars in layer):
                  backproped_error[k] =
sum([self.vals for learnable params[transposed layer params[k][i]] *
pred err backproped at layers[back layer index][i]
                                             for i in range(len(vars in layer))])
          pred err backproped at layers[back layer index - 1] = backproped error
          input vars to layer = self.layer vars[back layer index-1]
          for j,var in enumerate(vars in layer):
              layer params = self.layer params[back layer index][j]
derivatives stored away
calculations is presented
              for i,param in enumerate(layer params):
                  gradient of loss for param = input vals avg[i] *
pred err backproped at layers[back layer index][j] * deriv sigmoid avg[j]
```

```
if len(Vt1_list) < 10: #for the first 10 parameters, Vt = 0, so,</pre>
                       Vt1 list.append(mu vt + gradient of loss for param)
look 10 indices back on the list to get
                       Vt1 list.append(Vt1)
                   self.vals for learnable params[param] += self.learning rate *
Vt1 list[-1] #update current parameters based on
          bias gradient = sum(pred err backproped at layers[back layer index]) \
sum(deriv sigmoid avg)/len(deriv sigmoid avg)
there are only 2 bias parameters
parameters for the first backprop)
              bias_vt1_list.append(mu_vt + bias_gradient)
               bias vt1 list.append(momentum * bias vt1 list[-2] + bias gradient)
           self.bias[back layer index-1] += self.learning rate * bias vtl list[-1]
```

This is the new class for Adam which inherits from CGP:

```
dataset_size = 5000,
    #learning_rate = 1e-6,
    #learning_rate = 1e-3,
    learning_rate = 5 * 1e-2,
    #learning_rate = 1e-2,
    training_iterations = 40000,
    batch_size = 8,
    display_loss_how_often = 100,
    debug = True,
) #initialize base class ComputationalGraphPrimer
    self.loss_oneN
    self.loss_multiN
```

Below is the overriding class for the backpropagation to do the one neuron Adam optimization.

```
def backprop and update params one neuron model(self, y error, vals for input vars,
deriv sigmoid, mt list, vt list, mt bias list, vt bias list, iteration):
       input vars = self.independent vars
       input vars to param map = self.var to var param[self.output vars[0]]
       param to vars map = {param : var for var, param in
input vars to param map.items() }
          grad = y error * vals for input vars dict[param to vars map[param]] *
deriv_sigmoid
FIRST ITERATION ARE HAVE vt and mt = 0
              mt list.append((1 - B1) * grad)
              vt list.append((1 - B2) * grad**2)
power of the iteration, but in this case iter = 1
power of the iteration, but in this case iter = 1
```

```
mt list.append(B1 * mt list[-4] + ((1 - B1) * grad))
              vt list.append(B2 * vt list[-4] + ((1 - B2) * grad**2))
          self.vals for learnable params[param] =
self.vals for learnable params[param] + ((self.learning rate * (mt list[-1]) /
(np.sqrt(vt list[-1] + epsilon))))
          mt bias list.append((1 - B1) * bias gradient)
          vt_bias_list.append((1 - B2) * bias_gradient**2)
          mt bias list.append(B1 * mt bias list[-1] + (1 - B1) * bias gradient)
          vt bias list.append(B2 * vt bias_list[-1] + (1 - B2) * bias_gradient**2)
          mt bias list[-1] = mt bias list[-1] / (1 - B1**power)
       self.bias = self.bias + (self.learning rate * (mt_bias_list[-1] /
np.sqrt(vt bias list[-1] + epsilon)))
```

Below is the overriding class for the backpropagation function to do the multi neuron Adam optimization.

```
def backprop_and_update_params_multi_neuron_model(self, y_error, class_labels,
mt_list, vt_list, mt_bias_list, vt_bias_list, iteration):

    pred_err_backproped_at_layers = {i : [] for i in range(1,self.num_layers-1)}
    pred_err_backproped_at_layers[self.num_layers-1] = [y_error]
    for back_layer_index in reversed(range(1,self.num_layers)):
        input_vals = self.forw_prop_vals_at_layers[back_layer_index -1]
        input_vals_avg = [sum(x) for x in zip(*input_vals)]
```

```
input_vals_avg = list(map(operator.truediv, input_vals_avg,
[float(len(class labels))] * len(class labels)))
          deriv sigmoid = self.gradient vals for layers[back layer index]
          deriv sigmoid avg = [sum(x) for x in zip(*deriv sigmoid)]
          deriv sigmoid avg = list(map(operator.truediv, deriv sigmoid avg,
 len(class labels)))
          vars in layer = self.layer vars[back layer index]
          vars in next layer back = self.layer vars[back layer index - 1] ## a
          layer params = self.layer params[back layer index]
          transposed layer params = list(zip(*layer params)) ## creating a
          backproped error = [None] * len(vars in next layer back)
          for k, varr in enumerate(vars in next layer back):
              for j, var2 in enumerate(vars in layer):
                  backproped error[k] =
sum([self.vals for learnable params[transposed layer params[k][i]] *
pred err backproped at layers[back layer index][i]
                                             for i in range(len(vars_in_layer))])
          pred err backproped at layers[back layer index - 1] = backproped error
          input vars to layer = self.layer vars[back layer index-1]
          for j,var in enumerate(vars in layer):
              layer params = self.layer params[back layer index][j]
derivatives stored away
```

```
for i,param in enumerate(layer params):
                   epsilon = 1e-08
                   gradient of loss for param = input vals avg[i] *
pred err backproped at layers[back layer index][j] * deriv sigmoid avg[j]
                      mt list.append((1 - B1) * gradient of loss for param)
                       vt list.append((1 - B2) * gradient of loss for param**2)
PARAMETERS NETWORKS IN THIS LIST
                       mt_list.append(B1 * mt_list[-10] + (1 - B1) *
gradient of loss for param)
                       vt list.append(B2 * vt list[-10] + (1 - B2) *
gradient of loss for param**2)
                      mt list[-1] = mt list[-1] / (1 - B1**power)
                       vt list[-1] = vt list[-1] / (1 - B2**power)
                   self.vals for learnable params[param] += self.learning rate *
mt list[-1] / np.sqrt(vt list[-1] + epsilon)
          bias gradient = sum(pred err backproped at layers[back layer index]) \
sum(deriv sigmoid avg)/len(deriv sigmoid avg)
WEIGHTS
              mt bias list.append((1 - B1) * bias gradient)
              vt bias list.append((1 - B2) * bias gradient**2)
```

Plotting code:

```
sub = Subclasses SGD plus()
sub.parse_expressions()
#sub.display_one_neuron_network()
training data = sub.gen training data()
sub.run training loop one neuron model( training data )
loss sgd plus = loss in run #copy current loss into another obj so that we can store
losses for normal SGD run in loss
cgp = ComputationalGraphPrimer(
              output vars = ['xw'],
cgp.parse expressions()
training data = cgp.gen training data()
cgp.run training loop one neuron model( training data )
```

```
loss_sgd_orig = cgp.loss_oneN
print(cgp.loss_oneN)
###CODE FOR ADAM ONE NEURON ####
adam = Subclasses ADAM()
adam.somefunc(5)
adam.parse expressions()
#adam.display one neuron network()
training data = adam.gen training data()
adam.run_training_loop_one neuron_model( training data )
loss adam = adam.loss oneN
### END CODE FOR ADAM ONE NEURON ###
##PLOTTING CODE FOR ONE NEURON ###
x axis len = len(loss sgd plus)
#print(loss_sgd_plus)
#print('sgd orig:', loss sgd orig)
plt.plot(np.linspace(0, x axis len, x axis len), loss sgd plus, label = "SGD+")
plt.plot(np.linspace(0, x axis len, x axis len), loss sgd orig, label = "SGD")
plt.plot(np.linspace(0, x axis len, x axis len), loss adam, label = "ADAM")
plt.xlabel("Training Iter")
plt.ylabel("Loss")
plt.legend()
plt.title("One Neuron Plots - LR =" + str(adam.learning rate))
plt.show()
#CODE FOR SGD+ MULTI NEURON NETWORK
sub = Subclasses SGD plus()
sub.parse multi layer expressions()
#sub.display multi neuron network()
training_data = sub.gen_training_data()
sub.run training loop multi neuron model( training data )
loss sgd plus MN = loss in run
##END CODE FOR SGD+ MULTI NEURON NETWORK
##CODE FOR SGD MULTI NEURON NETWORK #####
cgp = ComputationalGraphPrimer(
              num_layers = 3,
```

```
layers_config = [4,2,1],
              training iterations = 40000,
              display_loss_how_often = 100,
cgp.parse multi layer expressions()
training_data = cgp.gen_training_data()
cgp.run training loop multi neuron model( training data )
loss sqd orig = cqp.loss multiN
## END CODE FOR SGD MULTI NEURON NETWORK #####
##CODE FOR ADAM MULTI NEURON ####
adam = Subclasses ADAM()
adam.parse multi layer expressions()
#adam.display_multi_neuron_network
training data = adam.gen training data()
adam.run training loop multi neuron model(training data)
loss adam multiN = adam.loss multiN
##END CODE FOR ADAM MULTI NEURON
###PLOTTING CODE FOR MULTI NEURON ####
x axis len = len(loss adam multiN)
plt.plot(np.linspace(0, x axis len, x axis len), loss sgd plus MN, label = "SGD+")
plt.plot(np.linspace(0, x axis len, x axis len), loss sgd orig, label = "SGD")
plt.plot(np.linspace(0, x_axis len, x axis len), loss adam multiN, label = "ADAM")
plt.xlabel("Training Iter")
plt.ylabel("Loss")
```

```
plt.legend()
plt.title("Multi Neuron Plot w/ LR=" + str(adam.learning_rate))
plt.show()
```