Discrete Math

2023eBCS072

Staff Graded Assignment-2

1. The seven letters provided are a, b, c, e, f, g, h

ⓐ 5 from ~~$~~ 7 letters need to be permuted

$$P(7,5) = \frac{7!}{(7-5)!}$$

$$= \frac{7!}{2!}$$

$$= 2{,}520$$

ⓑ There are 2 ways to interpret "these strings" in this question.
If it refers to all strings of length 5 → ⓘ, strings from ⓐ → ⓘⓘ

ⓘⓘ $ba\cancel{h}\underline{x}\,\underline{y}$

Since 3 letters are used in "bah", x can be one of the remaining 4 letters.
After x is chosen, y can be 1 of the 3 remaining letters
So total possibilities are
$4 \times 3 = 12$ strings

ⓘ $bah\,\underline{x}\,\underline{y}$

Since there are no constraints ~~r~~egarding repetition,
x and y can be any of the 7 letters, so
$7 \times 7 = 49$ possible strings

ⓒ Since d is not in the list of elements, "bad" cannot be a Substring, so the possible number of strings with no repeats is
2,520 (as discussed in ⓐ)

② Let X be a given string.

X_i is the ith letter of the string, so X_1 is the first letter

$X - X_i$ is the string X without the i^{th} letter.

XA is the string A appended to X

So, if $X =$ 'abcd', $X_2 =$ 'b', $X - X_2 =$ 'acd', $XX_2 =$ 'abcdb'

R_x is the reverse of a string.

Basis step:

If length of $X = 1$,

$R_x = X$

Recursive step

$R_x = R_{X-X_1} X_1$

Using this with the given example of abcd

$R_{abcd} = R_{bcd}\, a = dcba$

$R_{bcd} = R_{cd}\, b = dcb$

$R_{cd} = R_d\, c = dc$

$R_d = d$

③ Let the 2 partitions formed by $K_{m,n}$ be M (m vertices) and N (n vertices respectively)

- ~~Eleme~~ Vertices in M are all ~~be~~connected by an edge to each vertex in N, so their degree is n for each Vertex in M
- Likewise all Vertices in N have degree m as they are each connected to all ~~n~~ m vertices in M with an edge.

~~Case~~ i) $m > n$

In this case, all vertices in N will be listed first so the degree sequence is

$$\underbrace{m, m, \ldots, m}_{n \text{ times}}, \underbrace{n, n, \ldots, n}_{m \text{ times}}$$

ii) $n > m$

Here, the vertices in M are listed first

So, the degree sequence is

$$\underbrace{n, n, \ldots, n}_{m \text{ times}}, \underbrace{m, m, \ldots, m}_{n \text{ times}}$$

iii) $n = m$

In this case, the order doesn't matter

The degree sequence is

$$\underbrace{m, m, \ldots, m}_{m+n \text{ times}} \quad \overbrace{n, n, \ldots, n}$$

| both are the same as $m = n$

(4) (a) An Euler circuit can be represented by the cycle C_i, C_i is a subgraph of G, $i = |V_G|$

For C_i to be a bipartite graph, i should be even and the two partitions are each of size $i/2$

So, if $C_i \subseteq K_{m,n}$, $i = |V_{m,n}| = m+n$, $m = n = i/2$

So $K_{m,n}$ has an Euler circuit iff $\underline{\underline{m=n}}$

(b) If any one edge is removed from an Euler circuit, it becomes an Euler path. ~~So $K_{m,n}$ ... Euler path~~

So if $m=n$, $K_{m,n}$ has an Euler path.

Similarly, if a terminal vertex and the edge incident to it are removed, it is an Euler path for G, $V = V - v$, $E = E - e$ | V is vertex removed

Say one vertex (terminal) with the corresponding edge also,

~~...~~ The vertex removed was removed from one of the partition.

So the new graph is an Euler path for either $K_{m-1,n}$ or $K_{m,n-1}$

~~...~~ Now, both the terminal vertices lie in ~~...~~ same partitions since edges in the Euler path are odd. It starts and ends in the partition with more vertices, so if another ~~...~~ terminal vertex and edge were to be removed it'd be from that partition, forming an Euler Path for $K_{m-1,n-1}$.

~~So~~ For every edge removed, it is an Euler path for $K_{m,n}$, $|m-n| \leq 1$

So there is an Euler path for every $K_{m,n}$ if $\underline{\underline{|m-n| \leq 1}}$