CMPE 544: Pattern Recognition (Fall 2020)
Homework #1
Due November 27, 2020 by 11:59pm on Moodle

- Please type your answers and submit your homework as PDF. Any other file format will not accepted.
- Handwritten homework will not be accepted.
- Show your steps to get full points.
- Any homework sent via email after the deadline will NOT be accepted (even if it is 1 min past the deadline). Please make sure to submit your homework before the cutoff time.
- If you have difficulty in accessing Moodle email your homework to me before the deadline.
- Remember you can use one extension if you need. Please let me know beforehand if you plan to use an extension.
- Cheating by copying answers from internet, from a friend, not citing references you use in your homework is forbidden. If a cheating behavior is detected, your grade will be -100.

Questions

1. (30 points) Consider the following decision rule for a two-category univariate problem:

$$\text{Decide} \quad \omega_1 \quad \text{if} \quad x > k; \quad \text{otherwise decide} \quad \omega_2$$

   (a) (15 points) Show that the average probability of error for this rule is given by

$$P\left(\text{error}\right) = P\left(\omega_1\right) \int_{-\infty}^{k} p\left(x|\omega_1\right) dx + P\left(\omega_2\right) \int_{k}^{\infty} p\left(x|\omega_2\right) dx$$

   (b) (15 points) Derive a necessary condition to minimize $P\left(\text{error}\right)$ that k should satisfy. (Hint: Take the derivative of $P\left(\text{error}\right)$ with respect to k.)

2. (35 points) Let the two class-conditional densities be univariate Gaussian. Assume equal variances (σ^2) and different means (μ_0, μ_1). Determine the critical region and the power $(1-\beta)$ of the Neyman-Pearson rule for a given α.

3. (35 points) Consider a pattern classification problem with rejection option. Let

$$\lambda\left(\alpha_i|\omega_j\right) = \begin{cases} 0 & , \quad i = j \quad i, j = 1, \cdots, c \\ \lambda_r & , \quad i = c+1 \\ \lambda_s & , \quad \text{otherwise} \end{cases}$$

   where λ_r is the loss incurred for choosing the rejection and λ_s is the loss incurred for making a substitution error.

   (a) (15 points) Show that the following discrimination functions are optimal for such problems:

$$g_i\left(x\right) = \begin{cases} p\left(x|\omega_i\right) P\left(\omega_i\right) & , \quad i = 1, \cdots, c \\ \frac{\lambda_s - \lambda_r}{\lambda_s} \sum_{j=1}^{c} p\left(x|\omega_j\right) P\left(\omega_j\right) & , \quad i = c+1 \end{cases}$$

(b) (20 points) Plot these discriminant functions and decision regions for the two-category univariate case given below. Please include the script you used to plot in the pdf.

$$
\begin{aligned}
p\left(x|\omega_1\right) &\sim N\left(1,1\right) \\
p\left(x|\omega_2\right) &\sim N\left(-1,1\right) \\
P\left(\omega_1\right) &= P\left(\omega_2\right) \\
\frac{\lambda_r}{\lambda_s} &= \frac{1}{4}
\end{aligned}
$$