COMP 335: Assignment 3
Fall 2021
Submission through Moodle is due by Sunday November 14th at 23:55

Part A (Graded): Submit your solution to each of the following questions.

1. (15 pts) Apply the Pumping Lemma to prove that the following languages are not regular.
   (a) $L_1 = \{a^k b^n : n = 2^k\}$
   (b) $L_2 = \{ww : w \in \{a^i b^j : i, j \geq 0\}\}$
   (c) $L_3 = \{vw : v \in \{a, b\}^*, w \in \{c, d\}^*, |v| = |w|\}$

2. (10 pts) Give context-free grammars for each of the following languages.
   (a) $\{a^h b^k a^m b^n : h + k = m + n\}$
   (b) $\{a^i b^j a^k : (i = j \text{ and } k \geq 0) \text{ or } (i \geq 0 \text{ and } j > k)\}$

3. Let the CFG G be defined by productions $S \to aS \mid Sb \mid a \mid b$.
   (a) (10 pts) Prove by an induction of number of derivations steps that no string $w \in L(G)$ has ba as substring.
   (b) (5 pts) Describe $L(G)$ formally.

4. (10 pts) Design a PDA to accept each of the following languages. You may design your PDA to accept either by final state or empty stack, whichever is more convenient.
   (a) $\{a^h b^k a^m b^n : h + k = m + n\}$
   (b) $\{a^n b : n \geq 0\} \cup \{ab^n : n \geq 0\} \cup \{a^n b^n : n \geq 0\}$

5. (10 pts) Convert the following grammars into Chomsky Normal Form
   (a) $S \to ASB \mid \epsilon,\ A \to aAS \mid a,\ B \to SbS \mid A \mid bb.$
   (b) $S \to 0A0 \mid 1B1 \mid BB,\ A \to C,\ B \to S \mid A,\ C \to S \mid \epsilon.$

Total grade: 60 pts.

Part B (Not graded): Questions in this part are for your extra practice. Do NOT submit your solutions to questions in this part.

6. In each case, what language is generated by CFG's below. Justify your claim (prove it!)

   (a) G with productions $S \to aSa|bSb|aAb|bAa,\ A \to aAa|bAb|a|b|\epsilon$

   (b) G with productions $S \to aS|bS|a$

   (c) $S \to SS|bS|a$

   (d) G with productions $S \to SaS|b,\ \ S \to aT|bT|\epsilon,\ T \to aS|bS.$

7. Find a CFG for each of the languages below.

   (a) $L = \{a^n b^m : n \neq m - 1\}$

   (b) $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$

   (c) $L = \{w \in \{a,b\}^* : n_a(w) \neq n_b(w)\}$

   (d) $\overline{L}$, where $L = \{w \in \{a,b\}^* : w = a^n b^n,\ n \geq 0\}$

8. (10 pts) In each case below, show that the grammar is ambiguous, and find an equivalent unambiguous grammar.

   (a) $S \to SS\,|\,ab\,|\,a$

   (b) $S \to ABA,\ \ A \to aA\,|\,\epsilon,\ \ B \to bB\,|\,\epsilon$

9. Design a PDA to accept each of the following languages. You may design your PDA to accept either by final state or empty stack, whichever is more convenient.

   (a) The set of strings over $\{0,1\}$ such that no prefix has more 1's than 0's.

   (b) The set of strings with twice as many 0's as 1's.

   (c) The set of strings over $\{a,b\}$ that are *not* of the form ww, that is, not equal to any string repeated.