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CS 180 – HW7

11.

Consider a network that consists of  $N * N + 2$  vertices -  $s$  and  $t$  are source and sink vertices and  $N * N$  vertices that are connected only with the direct vertical and horizontal neighbors. Let all edges have capacities 1 and have only left-to-right and down-to-up directions. The maximum flow value in this network is  $N$  (simply use only left-to-right edges when looking for an  $s$ -to- $t$  path). Now consider a path that starts from  $s$  and goes right and eventually up to get to  $t$ , found by the forward-only algorithm. If this path is considered first by the algorithm, it blocks all other paths that are considered by the algorithm, as  $s$  and  $t$  are then disconnected in the “forward-only” residual graph and the returned maximum flow is 1. Since  $N$  is arbitrary, the claim that the forward-only algorithm always finds a constant approximation solution is wrong.

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To solve this, I create a network flow problem. Assign unit capacity to all existing edges. Introduce source node  $s$  and connect it to each node in  $X$  with a unit capacity directed edge. Introduce sink node  $t$  and connect each node in  $S$  with a directed edge to  $t$  with capacity  $|X|$ . Then calculate max-flow. We argue that  $\text{max-flow} = |X|$  if and only if such routes exist.

First: If required routes exist,  $\text{max-flow} = |X|$ . Use  $|X|$  unit capacity edges from source to each of the nodes in  $X$ . From there, we have a path from each node in  $X$  to some node in  $S$  using unique edges. Thus the  $|X|$  units of flow can reach nodes in  $S$  from where they have  $|X|$  capacity paths to the sink.

Second: If  $\text{max-flow} = |X|$ , required routes exist. If  $\text{max-flow} = |X|$ , each node in  $X$  is receiving unit capacity flow from  $s$ , and since all this flow is reaching  $t$  and each edge between  $X$  and  $S$  is unit capacity, all of this flow must be using unique edges. Thus we have a unique path from every node in  $X$  to some node in  $S$ . Note that you may have edges between  $X$  and they might even be used. It is also not required to have  $|X| = |S|$ . Consider a small example of a graph with nodes  $X_1, X_2, X_3, S_1, S_2$  and edges  $(X_1, X_2), (X_2, S_1), (X_2, S_2), (X_3, S_2)$ . The required routes exist in this graph. Also, there can be any number of interconnecting nodes between  $X$  and  $S$ .

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An algorithm that accomplishes this task using only  $O(K \log n)$  pings. Here is restricted to  $O(K \log n)$  pings. The run time should not be restricted, instead it should be polynomial time. The maximum flow  $R^*$  -- value of arc capacity is 1. We have  $K$ -disjoint paths. The  $K$  - disjoint paths are:  $p_1, p_2, \dots, p_n$ . Using Ford - Fulkerson algorithm, these paths can be formed is  $O(mk)$ . The intruder has  $L$  arc. One are destroyed by each path. We cannot space two - paths for a single arc. The path has  $O(n)$  length each. Bisection - search ping algorithm is used to remove  $p_i$ . If the vertex is formed ping it otherwise vertex-3-quietus along  $p_i$ . Therefore,  $O(\log n)$  remove edges, last vertex on the path is pinged and the first vertex which cannot be pinged is the removed edge.  $e^*$  is the removed edge. then perform BFS search, the  $k$ -disjoint paths are return vertices that don't exist

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I use Min-cut to solve this problem.

Let's create an undirected graph as graph has vertex for every SW applications, I call them  $S_1, S_2, \dots, S_n$ . There is a special vertex  $t$ . If application  $i$  and  $j$  have an associated value  $x_{ij}$ , then we have an edge between  $S_i$  and  $S_j$  of capacity  $x_{ij}$ . For every vertex  $S_i$ , I not equal to, we have an edge  $(S_i, t)$  of capacity  $b_{S_i}$ . We can say that  $S_1$ - $t$  min-cut would give the desired answer.

Let  $X$  be such a cut so that  $S_1$  is in  $X$ . and  $t$  is not. Capacity of  $X$  is Total of  $x_{ij}$  – Total of  $b_{S_i}$  + total  $b_s$ .

This is same as expense – benefit of moving application which are not in set  $X$

Hence, finding a min-cut is the same as finding the set of applications for which expense – benefit is minimized.