Ali Mirabzadeh
305179067
HW2 – CH 8
15.
a.
Nearby Electromagnetic Observation belongs to NP-complete class because given a set of k locations, there is no polynomial time algorithm to determine the locations where frequency is unblocked. However, this solution can be verified in polynomial time.
b.
I chose vertex cover problem to be X; it is a NP-Complete. I prove that Nearby Electromagnetic Observation, call it Y, is NP-Complete by reduction; or in other words,
A problem $Y \in NP$ with the property that for every $X \in NP$, X ≤ pY.
$Vertex\ Cover$ ≤ p $Nearby\ Electromagnetic\ Observation.$
I define a Graph G = (V, E) and an integer k. We want to find a subset of vertices $S \subseteq V$ such that $|S| \geq k$, and for each edge, at most one of its endpoints is in S. Note that the subsets of S are vertex covers.
Let each location li correspond to each node and fh correspond to each edge.
For an edge at (lx , ly), there is an interference source that blocks fh at all but locations lx and ly.
If there is a vertex cover of at least size k, then each frequency does not have any interference in at least one of the locations. This is equivalent to the vertex cover that for the set S of nodes, each edge has at most one of its endpoints in S. Therefore, a vertex cover S exists in graph G. Therefore, we can conclude that
$Vertex\ Cover$ ≤ p$Nearby\ Electromagnetic\ Observation.$
This proves that the Nearby Electromagnetic Observation is NP-complete.

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22.
Based on the given information:
If $V = \emptyset$, then G contains an independent set because it has no edges.
If $V \neq \emptyset$ and $k > 1$, I Add an extra node v' to G and add one edge between v' and each node in G. Call the new graph G'.
Use the given black-box algorithm A to see if G' has an independent set of at least size k.
If G has an independent set of size at least k, then G' also has an independent set of at least k.
G and G' have the same independent set of size at least k because v' can be considered another independent set in G' since v' is connected to each node only once.
G only has an independent set of size at least k if G' does.
Therefore, the Independent Set Problem is solved in polynomial time since A is called once and runs in polynomial time

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36.
We can show that a Hamilton Cycle reduces to the Daily Special Scheduling.
In other words, $Hamilton\ Cycle$ ≤ p$Daily\ Special\ Scheduling$.
Consider the graph G = (V, E). Let each node correspond to a special; let each edge correspond to an ingredient In. Edges In are incident on a node that requires those ingredients.
The goal is to find a Hamilton Cycle in G that uses up the total money at most x.
Consider i edges and j nodes for the graph G. The cost is twice the ingredients minus the nodes because each ingredient can be reused for another node. Thus, the cost of a Hamilton Cycle is $2i - j + 1$. For every pair of special nodes, there is an edge between them. As a result, there is a simple cycle that can transverse once through without repeating any nodes. Thus, the Hamilton Cycle reduces to the Daily Special Scheduling.
This makes the Daily Special Scheduling NP-complete