1

Homework week 16

Ex 3. Show that the set of all integers is countable.

S: We can list all integers in a sequence by starting with 0 & alternating between positive & negative integers: 0, 1, −1, 2, −2...

Alternatively, we could find a one-to-one correspondence between the set of positive integers & the set of all integers. We leave it to the reader to show that the function $f(n) = n/2$ when n is even & $f(n) = -(n-1)/2$ when n is odd is such a function. Consequently, the set of all integers is countable.

It is not surprising that the set of odd integers & the set of all integers are both countable sets (as shown in Ex. 1 & 3). Many people are amazed to learn that the set of rational numbers is countable, as ex. 4 demonstrates.

1	2	3	4	5	6	7	8	9
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
0	1	−1	2	−2	3	−3	4	−4

(2) Prove that the set of all rational numbers ($\mathbb{Q}$) is countable.

$$\frac{1}{1} \quad -\frac{1}{1} \quad \frac{2}{1} \quad -\frac{2}{1} \quad \frac{3}{1} \quad -\frac{3}{1} \quad \frac{4}{1} \quad -\frac{4}{1}$$

$$\frac{1}{2} \quad -\frac{1}{2} \quad \frac{2}{2} \quad -\frac{2}{2} \quad \frac{3}{2} \quad -\frac{3}{2}$$

(35) Prove that the power set of a set of all natural numbers (Z^+) is uncountable.

We can follow the hint or argue as follows, which really amounts to the same thing. (See the answer key for a proof using bit strings.) Suppose there were such a one-to-one correspondence f from Z^+ to the power set of Z^+ (the set of all subsets of Z^+). Thus, for each $x \in Z^+$, $f(x)$ is a subset of Z^+. We will derive a contradiction by showing that f is not onto, we do this by finding an element not in its range. To this end, let $A = \{x \mid x \notin f(x)\}$. We claim that A is not in the range of f. If it were, then $A = f(x_0)$ for some $x_0 \in Z^+$. Let us look at weather $x_0 \in A$ or not. On the one hand, if $x_0 \in A$, then by the definition of A, it must be true that $x_0 \notin f(x_0)$, which means that $x_0 \notin A$; that is a contradiction. On the other hand, if $x_0 \notin A$, then by the definition of A, it must be true that $x_0 \in f(x_0)$, which means that $x_0 \in A$, again a contradiction. Therefore no such one-to-one correspondence exists.

How to count Past infinity

$$\aleph_0 < P(\aleph_0)$$

! Solve a version of Hilbert's Hotel problem where an infinite buses with an infinite number of people show up

	HR1	HR2	HR3	HR4
B1	B1S1	B1S2	B1S3	B1S3
B2	B2S1	B2S2	B2S3	B2S4
B3	B3S1	B3S1	B3S1	B3S1
B4				
B5				
B6				
B7				
	1	2	3	4

Just lineup each person with unique code.

! Solve a version of H. Hotel problem where a single bus with an infinite number of people with infinitely long names consisting of A & B shows up.

Ex: the 1st person's name is AAAAA, ... second person's name is ABABAB 3rd person's name is ABBAA & so on, each infinitely long

! How An Infinite Hotel Ran out of Room.

1 ⟷ AAAA...
2 ⟷ ABABAB...
3 ⟷ ABBAAAA...

R1 ABBA.....
R2 ABAB
R3 BBAA
R4 BBABA
R5
R6

BAA
1st letter of 1st person then flip A to B, 2nd letter of 2nd p. & flip etc.

Some infinities are bigger than the other.

S.4.2. Week 17. Integer Representations & Algorithms

• In everyday life, we use decimal notation (base 10) to