how to read! $x+y$ is bigger than 0.

Homework: 1.1.

13 (1.1.) Let p & q be propositions

p: You drive over 65 miles per hour

q You get a speeding ticket.

a) $\neg p$

b) $p \land \neg q$ (but means $\land$ (&))

c) $p \rightarrow q$

d) $\neg p \rightarrow \neg q$

e) $p \rightarrow q$

f) $q \land \neg p$

g) $q \rightarrow p$ whenever means "if"

33 ~~a) $p \oplus p$~~

a) $(p \lor q) \rightarrow (p \oplus q)$

exclusive OR (XOR) → It's ~~T~~ if either p is T. or q is T

p	q	$p \lor q$	$p \oplus q$	$(p \lor q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

b) $(p \oplus q) \to (p \wedge q)$

p	q	$p \oplus q$	$p \wedge q$	$(p \oplus q) \to (p \wedge q)$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

c) $(p \vee q) \oplus (p \wedge q)$

$p \vee q$	$\oplus$	$p \wedge q$
T	F	T
T	T	F
T	T	F
F	F	F

d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

$p \leftrightarrow q$	$\neg p$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	F	F	T
F	F	T	T
F	T	T	T
T	T	F	T

e). $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$.

p	q	r	$\neg p$	$\neg r$	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg r$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
T	T	T	F	F	T	T	F
T	T	F	F	T	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	F

f) $(p \oplus q) \to (p \oplus \neg q)$

$p \oplus q$	$\neg q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus \neg q)$
F	F	T	T
T	T	F	F
T	F	F	F
F	T	T	T

2.1. 35

a) $p \to \neg q$ $(p \to \neg q)$

b) $(\neg p \leftrightarrow q)$

$\neg p$ F

F

F T F T T
F F T T T F T T F T F
F F F T T

f) $(p \oplus q) \to (p \oplus \neg q)$ $(p \oplus q) \to (p \oplus \neg q)$

$p\oplus q$	$\neg q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus \neg q)$
F	F	T	T
T	T	F	F
T	F	F	F
F	T	T	T

1.1 35

a) $p \to \neg q$

p	q	$\neg q$	$p \to \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

b) $(\neg p \leftrightarrow q)$

$\neg p$	$\neg p \leftrightarrow q$
F	F
F	T
T	T
T	F

c) $(p \to q) \vee (\neg p \to q)$

d) $(p \to q) \wedge (\neg p \to q)$

p	q	$\neg p$	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$	$(p \to q) \wedge (\neg p \to q)$
T	T	F	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	F	T	T	F	T	F

e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$ $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

$\neg q$	$\neg p \leftrightarrow \neg q$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
F	T	T
T	F	T
F	F	T
T	T	T

1.1 41 The first clause $(p \vee q \vee r)$ is true if & only if at least one of p, q, & r is true. The 2nd clause $(\neg p \vee \neg q \vee \neg r)$ is true if & only if at least one of the 3 variables false. Therefore both clauses are true, & therefore the entire statement is true, if & only if there at least one T & one F amont the truth values of the variable, in other words, that they don't all have the same truth value.

1.3. 15 $(\neg q \wedge (p \to q)) \to \neg p$ is tautology?

p	q	¬p	¬q	p→q	¬q∧(p→q)
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	T	T	T

$(\neg q \wedge (p \to q)) \to \neg p$
T
T
T
T

so it's tautology, since is alwy "T".

42 45, 51.

— DNF — Disjunctive Normal form.
берем все строки где выражени истинно (T).

- Для каждой такой строки записваем коньюкцию (And, "и") перемен-х и их отрицаний.
- Затем объединяем все такие коньюкции чере дизьюнкцию (or, "или")

Ex:

p	q	F(p,q)
T	T	T
T	F	F
F	T	T
F	F	F

тут "T"

тепер строим выражени для каждой из этих строк:

$p \wedge q$ (коньюкция)

чтобы учесть что p=F записваем

¬p · q · $\neg p \wedge q$ (коньюкция)

тепер объединяем эти 2 части ч/з or (дизюнкция "или")

$(p \wedge q) \vee (\neg p \wedge q)$

Это и есть выражени в дизюнктивной норм. форме (DNF) полностью соотв. таблице)

Общая формула DNF.

1) Ищем все строки таблицы, где значение выражение = T
2) Для каждой строки запис-м коньюкцию (And) всех перменнных, причем:
   если перемен-е истинна (T) → берем без изменения
   если ложная (F) берем её отрицание (¬)
3) Объед-ем все такие коньюнкции ч/з дизюнкцию (or, "или")

Функциональная полнота: And, OR, NOT достаточно, чтобы построить любое выражение

45. Functionally complete mean — a set of logical operators is called functionally complete if you can

③ Объед-ение все ...

Функциональная полнота: And, OR, NOT достаточно, чтобы построить любое выражение

45. Functionally complete mean – a set of logical operators is called functionally complete if you can use them to express any possible logical function. For ex, you can use them to build things like AND, OR, NOT or even more compleated logical expressions.

① Show that ¬ & ∨ are functionally complete: to prove we need to show that we can use them to create any other logical operator like And, Implies & even NAND and NOR.

Step1: We already have Not (¬) & OR (∨)

$\neg p$ means "not p" If p is true then $\neg p$ is F., & vice versa.

$p \vee q$ means "p or q" It's true if at least one of p or q T.

Step2: Can we create AND (∧) using ¬ & ∨?

Yes we can use De Morgan's Laws to do this. De Morgan's Laws:

$\neg(p \vee q) = \neg p \wedge \neg q$

$\neg(p \wedge q) = \neg p \vee \neg q$ we can rewrite AND in terms of Not and OR:

using the 1st Law

$p \wedge q = \neg(\neg p \vee \neg q)$ – This means we can create AND using NOT & OR.

Step3: Can we create Implies (→) using ¬ & ∨?

Yes, The implication $p \to q$ is logically equiv-t to $\neg p \vee q$.

$p \to q = \neg p \vee q$

Step4: Can we create NAND & NOR using ¬ & ∨?

Yes: p NAND $q = \neg(p \wedge q)$. But we already know how to create AND using ¬ & ∨, so:

p Nand $q = \neg(\neg(\neg p \vee \neg q))$ – This is already in terms of Not & OR

p NOR $q = \neg(p \vee q)$

Conclusion: Since we can create AND, →, NAND & NOR using just NOT & OR this means ¬ & ∨ are functionally complete. They can express any logical f. – they are also functionally complete on their own.

Understanding NAND & NOR.

NAND → p NAND q means "Not (p AND q)"

It's true if at least one of p or q is false.

It's False only if both p & q are true

symbol $p \uparrow q$ (Sheffer stroke)

NOR. – p NOR q means "NOT (p or q)
It's T only if both p & q are false
It's F if at least one of p or q is true.
Symbol $p \downarrow q$

Why are NAND & NOR special?
- Both of them are functionally complete on their own. This means you can use just NAND or just NOR to creat any other logical operator., like NOT, AND, OR etc.

Example creating not using NAND.
$\neg p = p$ NAND p
- if p is true then p NAND p is F.
if p is F, the p NAND p is T.

Exam. creating AND using NAND.
$p \wedge q = \neg(p$ NAND $q)$
we alread know how to create NOT using NAND so;
$p \wedge q = (p$ NAND $q)$ NAND $(p$ NAND $q)$

Ex: creating OR using NAND
$p \vee q = \neg(\neg p \wedge \neg q)$
using NAND we can write: $p \vee q$ $(p$ NAND $p)$ NAND $(q$ NAND $q)$

conc-n. $\neg$ & $\vee$ are functionally complete because you can use them to create any othe logical operator.
NAND & NOR are also func-ly complete on their own, which makes them super powerful.

51 we need to find compound proposition that is logically = to $p \rightarrow q$ using only the NOR operator ($\downarrow$).
$p \downarrow q$ means "NOT (p OR q)
It's true only when both p & q are F. otherwise F.

Step 1.- Recall what $p \rightarrow q$ means
$p \rightarrow q$ is logically equiv-t to $\neg p \vee q$ means; q is.
if p is T, then q must be true
if p is false, then $p \rightarrow q$ is always true, no matter what

Step 2. Express $\neg p \vee q$ using NOR
Rewrite $\neg p \vee q$ using only the NOR ($\downarrow$)

Step 3; • Express $\neg p$ using NOR:

$p \downarrow q$ means "NOT $(p$ OR $q)$
It's true only when both p & q are F. Otherwise F.

Step 1: Recall what $p \to q$ means
$p \to q$ is logically equiv to $\neg p \vee q$ means; q is.
If p is T, then q must be true
If p is false, then $p \to q$ is always true, no matter what.

Step 2. Express $\neg p \vee q$ using NOR
Rewrite $\neg p \vee q$ using only the NOR $(\downarrow)$

Step 3: • Express $\neg p$ using NOR:
$\neg p$ is $= p \downarrow p$. "(Not(p or p)" = "Not p"
• Express $\vee$ (OR) using NOR
$p \vee q = \neg(p \downarrow q)$, Because $p \downarrow q$ means "NOT (p OR q)" so
$\neg(p \downarrow q)$ means "NOT(NOT (p or q)) ⇒ which simplifies to $p \vee q$
• Combine the results.
we need $\neg p \vee q$. using the results above:
$\neg p = p \downarrow p$
$\neg p \vee q = \neg(p \downarrow p) \downarrow q$

Step 4: Simplify the expressing.
1) $\neg p = p \downarrow p$
2) $\neg p \vee q = \neg(p \downarrow p) \downarrow q$
3) But $\neg(p \downarrow p)$ is the same as $(p \downarrow p) \downarrow (p \downarrow p)$
(Because $\neg x = x \downarrow x$)
So the final expression is: $(p \downarrow p) \downarrow q$

Answer: The compound prop. logically equivalent to $p \to q$ using only the NOR $(\downarrow)$ is $(p \downarrow p) \downarrow q$

verification:

p	q	$p \downarrow q$	$(p \downarrow p) \downarrow q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

1.4 37

a) $\forall x(F(x,25000) \vee S(x,25) \to E(x))$

$E(x)$ - is "Person x qualifies as an elite flyer in a given year."
$F(x,y)$ - is "Person x flies more than y miles in a given year.
$S(x,y)$ is "Person x takes more than y flights in a given year."

b) $\forall x(((M(x) \wedge T(x,3)) \vee (\neg M(x) \wedge T(x,3.5))) \to Q(x))$

$Q(x) \to$ is "Person x qualifies for the marathon"
$M(x)$ is "Person x is a man", & $T(x,y)$ is "Person x has run the marathon in less than y hours".

c) $M \to ((H(60) \vee (H(45) \wedge T)) \wedge \forall y\, G(B,y))$

M - is the proposition. "The student received a masters degree",
$H(x)$ - is "The student took at least x course hours"
T is the proposition "The student wrote a thesis".
$G(x,y)$ is "The person got grade x or higher in his course y"

d) $\exists x((T(x,21) \wedge G(x,4.0))$, where $T(x,y)$ is "Person x took more than y credit hours",
$G(x,p)$ is "person x earned grade point average p".

53 a) $\exists!\, P(x) \to \exists x\, P(x)$

This is certainly true: if there is a unique x satisfying P(x), then there certainly is an x satisfying P(x).

b) $\forall x(P(x)) \to \exists! x\, P(x)$

unless the domain (universe of discourse) has fewer than 2 items in it, the truth of the hypothesis implies that there is more than one x such that P(x) holds. Therefore this proposition need ~~to~~ not to be true. (For ex., let P(x) be the proposition $x^2 \geq 0$ in the context of real numbers. The hypothesis is true but there is not a unique x for which $x^2 \geq 0$)

c) $\exists!\, P(x) \to xP(x)$

This is true: if there is an x (unique or not) such that P(x) is false, then we can conclude that it is not the case that

... proposition need ~~to~~ not to be true.
(For ex:, let $P(x)$ be the proposition $x^2 \geq 0$ in the context of real numbers. The hypothesis is true but there is not a unique x for which $x^2 \geq 0$)

ⓒ $\exists! P(x) \rightarrow \forall x P(x)$
This is true: if there is an x (unique or not) such that $P(x)$ is false, then we can conclude that it is not the case that $P(x)$ holds for all x.

(59) Let $P(x)$, $Q(x)$, $R(x)$ be the statement "x is a professor", "x is ignorant" & "x is vain" respectively.

a) No professors are ignorant.
This is the statement that every person who is a professor is not ignorant. In other words, for every person, if that person is a professor, then that person is not ignorant. In symbols: $\forall x(P(x) \rightarrow \neg Q(x))$
This is not the only possible answer. We could equivalently think of the statement as asserting that there does not exist an ignorant professor:
$\neg \exists x(P(x) \land Q(x))$

b) all ignorant people are vain. $\forall x(Q(x) \rightarrow R(x))$

c) No ~~$\exists$~~ professors are vain. (similar to part a) $\forall x(P(x) \rightarrow \neg R(x))$.

d) Does (c) follow from (a) & (b)?
The conclusion part c does not follow. There may well be vain professor, since the premises do not rule out the possibility that there are vain people besides the ignorant ones.

(61) a) babies are illogical.
This is assirting that every person who is a baby is necessarily not logical
$\forall x(B(x) \rightarrow \neg Q(x))$

b) Nobody is despised who can manage a crocodile.
~~Every person who is not logical is necessarily~~
If a person can manage a crocodile, then the person is not despised: ~~$\forall x(\neg Q(x) \rightarrow S(x))$~~ $\forall x(R(x) \rightarrow \neg S(x))$.

c) Illogical persons are despised. $\forall x(\neg Q(x) \rightarrow S(x))$.

d) babies cannot manage crocodiles. $\forall x(P(x) \rightarrow \neg R(x))$

e) Does (d) follow from (a), (b) & (c). If not, is there a correct conclusion?
The conclusion follows: Suppose that x is a baby. Then by the 1st premise, x is illogical, & hence by the 3rd premise, x is despised. But the second premise says that if x could manage a crocodile, then x would not be despised. Therefore x cannot manage a crocodile. Thus we have proved that babies cannot manage crocodiles.

(1.5) 19, 21, 31, 49.

(19) a) The sum of 2 negative integers is negative.

$\forall x \forall y ((x<0) \wedge (y<0) \rightarrow (x+y)<0))$

b) The difference of 2 + integers is not necessarily positive.

$\exists x \exists y ((x>0) \wedge (y>0) \wedge (x-y) \leq 0))$

c) The sum of the squares of 2 integers is greater than or equal to the square of their sum.

$\forall x \forall y (x^2 + y^2 \geq (x+y)^2)$.

d) The absolute value of the product of their absolute values

$\forall x \forall y (|xy| = |x||y|)$

(21) every positive integer is the sum of the squares of 4 integers.

$\forall x \exists a \exists b \exists c \exists d ((x>0) \rightarrow x = a^2+b^2+c^2+d^2$ - Here the domain (universe of discourse) consists of all integers.

(31) As we push the negation symbol inside, each quantifier it passes must changes its type. For logical connectives we either use De Morgan's laws or recall that $\neg(p \rightarrow q) \equiv p \wedge \neg q$

a) $\forall x \exists y \forall z T(x,y,z)$.

$\neg \forall x \exists y \forall z T(x,y,z) \equiv \exists x \neg \exists y \forall z T(x,y,z)$.

$\equiv \exists x \forall y \neg \forall z T(x,y,z)$.

$\equiv \exists x \forall y \exists z \neg T(x,y,z)$.

b) $\forall x \exists y P(x,y) \vee \forall x \exists y Q(x,y)$

$\neg (\forall x \exists y P(x,y) \vee \forall x \exists y Q(x,y)) \equiv \neg \forall x \exists y P(x,y) \wedge \neg \forall x \exists y Q(x,y)$

$\equiv \exists x \neg \exists y P(x,y) \wedge \exists x \neg \exists y Q(x,y)$

$\equiv \exists x \forall y \neg P(x,y) \wedge \exists x \forall y \neg Q(x,y)$

c) $\forall x \exists y (P(x,y) \wedge \exists z R(x,y,z))$

$\neg \forall x \exists y (P(x,y) \wedge \exists z R(x,y,z)) \equiv \exists x \neg \exists y (P(x,y) \wedge \exists z R(x,y,z))$

$\equiv \exists x \forall y \neg (P(x,y) \wedge \exists z R(x,y,z))$

$\equiv \exists x \forall y (\neg P(x,y) \vee \neg \exists z R(x,y,z))$

d) $\forall x \exists y (P(x,y) \rightarrow Q(x,y))$

$\neg \forall x \exists y (P(x,y) \rightarrow Q(x,y)) \equiv \exists x \neg \exists y (P(x,y) \rightarrow Q(x,y))$

$\equiv \exists x \forall y \neg (P(x,y) \rightarrow Q(x,y))$

$\equiv \exists x \forall y (P(x,y) \wedge \neg Q(x,y))$

(49) $\forall x P(x) \wedge \exists x Q(x) \equiv \forall x \exists y (P(x) \wedge Q(y))$ → show

$\forall x P(x)$ means $P(x)$ is true for all x

$\exists x Q(x)$ means there is at least 1 x, for which $Q(x)$ is true.

$\forall x \exists y (P(x) \wedge Q(y))$ means for each x there exists y, which $P(x$ & $Q(y)$ are true.

Week 15. Intro-n to Proofs (Textbook p76) F80 (1.8.

Proofs Theorems