20 $\quad 20 : 13 = 1$ remainder 12

# Homework Week 17 4.2 7, 19, 27, 37, 43

(7) a) $(80E)_{16} = (1000\,0000\,1110)$

b) $(135AB)_{16} = (0001\ 0011\ 0101\ 1010\ 1011)_2$

c) $(ABBA)_{16} = (1010\ 1011\ 1011\ 1010)_2$

d) $(DEFACED)_{16} = (1101\ 1110\ 1111\ 1010\ 1100\ 1110\ 1101)_2$

Hexadecim. to Binary

| | | | |
|---|---|---|---|
| 0 | 0000 | A | 1010 |
| 1 | 0001 | B | 1011 |
| 2 | 0010 | C | 1100 |
| 3 | 0011 | D | 1101 |
| 4 | 0100 | E | 1110 |
| 5 | 0101 | F | 1111 |
| 6 | 0110 | | |
| 7 | 0111 | | |
| 8 | 1000 | | |
| 9 | 1001 | | |

(19) Give a procedure for converting from the octal expan-n of an integer to its hexadec-l expan-n using binary notation as an intermediate step.

S: Since we have procedures for converting both octal & hexadecimal to & from binary (Ex.7) to convert from octal to hexadec. we first convert from octal to binary & then convert from binary to hexadec.

(27) Use Algorithm 5 to find $3^{2003}$ mod 99

$3^{2003}$ mod 99.

In effect of this algorithm 3 mod 99, $3^2$ mod 99, $3^4$ mod 99, $3^8$ mod 99, $3^{16}$ mod 99 ... & then multiplies (modulo 99) the required values.

Since $2003 = (11111010011)_2$ we need to multiply together 3 mod 99, $3^2$ mod 99, $3^{16}$ mod 99, $3^{64}$ mod 99, $3^{128}$ mod 99, $3^{256}$ mod 99, $3^{512}$ mod 99, & $3^{1024}$ mod 99 reducing mod 99 at each step. We compute by repeatedly squaring: $3^2$ mod 99 = 9, $3^4$ mod 99 = 81, $3^8$ mod 9 = $81^2$ mod 99 = 6561 mod 99 = 27, $3^{16}$ mod 99 = $27^2$ mod 99 = 729 mod 99 = 36, $3^{32}$ mod 99 = $36^2$ mod 99 = 1296 mod 99 = 9 & then the pattern repeats, so $3^{64}$ mod 99 = 81, $3^{128}$ mod 99 = 27, $3^{256}$ mod 99 = 36, $3^{512}$ mod 9 = 9 & $3^{1024}$ mod 99 = 81. Thus, our final answer will be the product of 3, 9, 36, 81, 27, 36, 9 & 81. We compute these one at a time modulo 99: $3 \cdot 9 = 27 \cdot 36 = 81 \cdot 81$ is 27. $27 \cdot 27$ is 36, $36 \cdot 36$ is 9, $9 \cdot 9$ is 81 & finally $81 \cdot 81$ is 27 so $3^{2003}$ mod 99 = 27

(37) We must assume that the sum actually represents a number in the appropriate range. Assume that n bits are being used, so that numbers strictly between $-2^{n-1}$ & $2^{n-1}$ can be represented. The answer is almost, but not quite, that to obtain the one's complement representation of the sum of 2 numbers, we simply add the 2 strings representing these numbers using Algorithm (3): instead after performing this operation there may be a carry out of the left-most column, in such a case, we then add 1 more to the answer.

Ex: suppose that $n=4$, then numbers from -7 to 7 can be represented.

To add $-5$ & $3$ we add 1010 & 0011 obtaining 1101 there was no carry out the left-most column. Since 1101 is the one's complement representation of $-2$, we have the correct answer. On the other hand to add $-4$ & $-3$ we add 1011 & 1100, obtaining 1 0111. The 1 that was carried out of the left-most column is instead added to 0111, yeilding 1000, which is the one's complement representation of $-7$. A proof that this method works entails considering the various cases determined by the signs & magnitudes of the addends.

43. Answer ex 37. for 2's complement expansions.

Sln: The nice thing about two's complement arithmetic is that we can just work as if it were all in base 2, since $-x$ (where $x$ is positive) is represented by $2^n - x$; in other words modulo $2^n$ negative numbers represent themselves. However, if overflow occurs, then we must recognize an error. Let us look at some examples where $n=5$ (i.e. we use 5 bits to represent numbers between $-15$ & $15$). To add $5+7$, we write $00101 + 00111 = 01100$ in base 2, which gives us the correct answer, 12. However if we try to add $13+7$ we obtain $01101 + 00111 = 101000$ which represents $-12$, rather than 20, so we report an overflow error. (of course these 2 numbers are congruent modulo 32). Similarly for $5+(-7)$ we write $00101 + 11001 = 11110$ in base 2, if we ignore the extra 1 in the left-most column (which doesn't exist) then this is the 2's complement representations of $-12$ again the right answer. To summarize, to obtain the two's complement representation of the sum of 2 integers given in 2's complement represen-n, add them as if they were binary integers, & ignore any carry out of the ~~most~~ left-most column. However, if the left-most digits of the 2 addends agree & the left most digit of the answer is different from their common value then an overflow has occurred & the answer is not valid.

Week 18 41:41 till 1:13:41 to watch!

## Primes & Greatest Common Divisors

Primes - Let $p$ be an integer such that $p>1$. The integer $p$ is called prime if the only positive factors of $p$ are