Week 3! P.1 $\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4)$

Date:

Sol-n: 1) Simplify each term.

$\log_2\left(\frac{8\sqrt{2}}{16}\right) = \frac{8\cdot 2^{\frac{1}{2}}}{16} = \frac{8}{16}\cdot 2^{\frac{1}{2}} = \frac{1}{2}\cdot 2^{\frac{1}{2}} = 2^{-1}\cdot 2^{\frac{1}{2}} = 2^{-\frac{1}{2}}$ $\quad 2^{-1+\frac{1}{2}}$

So 1st term: $\log_2(2^{-\frac{1}{2}}) = -\frac{1}{2}$

2) $\log_2(32) = \log_2(2^5) = 5$

3. $2\log_2(4) = 2\cdot\log_2(2^2) = 2\cdot 2 = 4$

Now combine all terms: $-\frac{1}{2} + 5 - 4 = 0.5$

$\log_b a = c \Leftrightarrow b^c = a$

Ex: $\log_3 27 = 3 \,|\, 3^3$

P-2. Solve for x $\log_3(x-1) + \log_3(x+1) = 2$

$\log_b(A) + \log_b(B) = \log_b(AB)$

So, $\log_3((x-1)(x+1)) = 2$

$\log_3(x^2-1) = 2$

ln(x) – what power must we raise e to get x?
A nat. log. – is a log. with base e:
$e \approx 2.71828$

Convert from logarithmic to exponential form:

$x^2 - 1 = 3^2 = 9$

Solve for x, $x^2 - 1 = 9$

$x^2 = 10$

$x = \pm\sqrt{10}$

1) $x = \sqrt{10} \approx 3{,}16$ $\quad$ • $x-1>0 \Rightarrow \sqrt{10}-1>0$ (True)

• $x+1>0 \Rightarrow \sqrt{10}+1>0$ (True)

2) $x = -\sqrt{10} \approx -3{,}16$. $\quad$ • $x-1>0 \Rightarrow -\sqrt{10}-1>0$ (False)

So $x = -\sqrt{10}$ is extraneous (посторонний) $\quad$ Answer: $x = \sqrt{10}$

P-3. Initial inv. – 10.000\$

annual interest rate – 6%

How many years – ? to grow at least 20.000\$

The Compound interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$

A = final amount (20.000\$)

P = principal amount (10.000\$)

r = annual int. rate (0.06)

n = number of time interest is compounded per year (4)

t = number of years

$20000 = 10000\left(1+\frac{0.06}{4}\right)^{4t}$

$2 = 1 + 0.015$

$2 = (1.015)^{4t}$

Take natural logarith. of both sides.

$\ln(2) = \ln(1.015)^{4t}$

$\ln(2) = 4t\ln(1.015)$

Solve for t.

$t = \frac{\ln(2)}{4\ln(1.015)} \approx \frac{0{,}6931}{4\times 0{,}014889} \approx \frac{0{,}6931}{0{,}059556}$

$\approx 11{,}64$ years.

Answer: approx. 11,64 years

P4. $N(t) = N_0 e^{-kt}$. Radioactive Decay exercise

Where N_0 is the initial amount, k is the decay constant, & t is the time in years

If a sample has a half-life of 5 years, find the decay constant k.

S-n: The half life $t_{\frac{1}{2}}$ is the time when $N(t_{\frac{1}{2}}) = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-kt_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-k\cdot 5}$$

Take the natural logarithms: $\ln\left(\frac{1}{2}\right) = -5k$

$$-0.6931 = -5k$$

$$k = \frac{0.6931}{5} \approx 0,1386$$

Answer: The decay constant k is approx 0,1386 per year.

P5. If 100g of radio subs decays to 70 grams in 3hrs, find the time it will take to decay to 20 grams. $N(t) = N_0 e^{-kt}$

$N(3) = 70gr$

$$70 = 100e^{-k\times 3}$$

$$0.7 = e^{-3k}$$

Natural log: $\ln(0.7) = -3k$

$$-0,3567 = -3k$$

$$k = \frac{0,3567}{3} \approx 0.1189$$

Now, find t when $N(t) = 20gr$.

$$20 = 100e^{-0,1189t}$$

$$0,2 = e^{-0,1189t}$$

Take natural logarithms:

$$\ln(0,2) = -0,1189t$$

$$-1,6094 = -0,1189t$$

$$t = \frac{1,6094}{0,1189} \approx 13.54 \text{ hours.}$$

Answer: it will take approx. 13,54 hours

Geometric

P6. Find the unit vector in the direction from point $A(1,2,3)$ to point $B(4,6,9)$

Sln. 1) Find the vector $\vec{AB}$ $\quad \vec{AB} = \langle 4-1, 6-2, 9-3\rangle = \langle 3, 4, 6\rangle$

Magnitude of $\vec{AB}$: $\vec{AB} = \sqrt{3^2+4^2+6^2} = \sqrt{9+16+36} = \sqrt{61} \approx 7,81$

Unit vector $\breve{u}$: $\frac{\vec{AB}}{|\vec{AB}|} = \left\langle \frac{3}{\sqrt{61}}, \frac{4}{61}, \frac{6}{61} \right\rangle$ — просто надо так написать?

Answer:

P7 Matrix Form

Express the vector $\vec{v} = 7i - 2j + 4k$ in the matrix form & find its magnitude

$$\xrightarrow{\text{Matrix form}} \vec{v} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$$

Magnitude: $|\vec{v}| = \sqrt{7^2+(-2)^2+4^2} = \sqrt{49+4+16} = \sqrt{69} \approx 8.31.$

P9. Dot Product.

Find the angle between vectors $\vec{p} = \langle 1,2,3 \rangle$ & $q = \langle 4,-5,6 \rangle$

1) Compute the dot product

$$\vec{p}\cdot\vec{q} = (1)(4)+(2)(-5)+(3)(6) = 4-10+18 = 12$$

Compute magnitudes:

$$|\vec{p}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$|\vec{q}| = \sqrt{4^2+(-5)^2+6^2} = \sqrt{16+25+36} = \sqrt{77}$$

Compute the angle θ:

$$\cos\theta = \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|\cdot|\vec{q}|} = \frac{12}{\sqrt{14}\sqrt{77}} = \frac{12}{\sqrt{1078}} \approx 0,3647$$

$$\theta = \arccos(0.3647) \approx 68.58^\circ$$

P10. Determine if the vectors $\vec{u} = \langle 2,-1,4 \rangle$ and $\vec{v} = \langle -8,4,-16 \rangle$ are ortho

Compute: $\vec{u}\cdot\vec{v} = (2)\cdot(-8)+(-1)\cdot(4)+(4)+(-16) = -16-4+64 = -84$

Since $\vec{u}\cdot\vec{v} = \emptyset$ the vectors are not orthogonal (=perpendicular)

P11. Adding & substracting Matrices

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$$

Compute 2A-3B

1) Compute 2A $\quad 2A = 2\times\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix}$

2) Compute 3B $\quad 3B = 3\cdot\begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix}$

Substract: $2A-3B = \begin{bmatrix} 4-12 & -2-15 \\ 0-(-6) & 6-3 \end{bmatrix} = \begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$

A: $2A-3B = \begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$

P12 Multiplying Matrices

Compute the product

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \quad E = CD$$

$$E = \begin{bmatrix} (1)(5)+(2)(7) & (1)(6)+(2)(8) \\ (3)(5)+(4)(7) & (3)(6)+(4)(8) \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$E = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

P. 13. Row Operations

use Gaussian elimination to solve the system.

Date:

$$\begin{cases} x+y+z=6 \\ 2x-y+3z=14 \\ -3x+2y-2z=-10 \end{cases}$$

j - column j
i - row i

Augmented matrix:
Creat matrix using only the coefficient of the variable & constants. Include the constants in the separate column on the right. This is called augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{array}\right]$$

(1) use row R_1 to eliminate x from R_2 & R_3

$R_2 = R_2 - 2R_1$

$$\left.\begin{array}{l} 2-2(1)=0 \\ -1-2(1)=-3 \\ 3-2(1)=1 \\ 14-2(6)=2 \end{array}\right\} R_2$$

$R_3 = R_3 + 3R_1$

$$\left.\begin{array}{l} -3+3(1)=0 \\ 2+3(1)=5 \\ -2+3(1)=1 \\ -10+3(6)=8 \end{array}\right\} R3$$

Now the matrix is:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 5 & 1 & 8 \end{array}\right]$$

(2) use R_2 to eliminate y from R_3. Multiply R_2 by $\frac{5}{-3}$ & add to R_3

$$R_3 = R_3 + \left(\frac{5}{-3} R_2\right)$$

Compute:

New $R_3[2]$ $\quad 5+\left(\frac{5}{-3}\cdot -3\right) = 5+5=10$

New $R_3[3]$ $\quad 1+\left(\frac{5}{-3}\cdot 1\right) = 1-\frac{5}{3} = \frac{3}{3}-\frac{5}{3} = -\frac{2}{3}$

New $R_3[4]$ $\quad 8+\left(\frac{5}{-3}\cdot 2\right) = 8-\frac{10}{3} = \frac{24}{3}-\frac{10}{3} = \frac{14}{3}$

After simplification R_3 becomes $\quad R_3 = \left[0, 0, -\frac{2}{3}, \middle| \frac{14}{3}\right]$

(3) Solve for z.

$$-\frac{2}{3}z = \frac{14}{3}$$

$$z = \frac{14}{3} : \left(-\frac{2}{3}\right) = \frac{14}{3}\cdot\left(-\frac{3}{2}\right) = -7$$

(4) Back-substitude z into R_2:

$-3y + 1(-7) = 2$

$-3y - 7 = 2$

$-3y = 9$

$y = -3$

(5) Back-substitude y & z into R_1

$x + (-3) + (-7) = 6$

$x - 10 = 6$

$x = 16$

Answer is: $x=16$, $y=-3$, $z=-7$.

P14. Reduced Row Echelon Form
Find the REF of the matrix.
Date:

$$B = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

① Eliminate the -1 in R_1 using R_3

$R_1 = R_1 + R_3 \cdot 1$

? $-1 + 1(1) = -1 + 1 = 0$

$0 + 1(-1) = 0 - 1 = -1$ — от куда это?

Compute : $R_1[4] = R_1[4] + 1 \times R_3[4]$ $0 + 1(-1) = -1$

Updated $R_1 = [1, 2, 0, -1]$

Step 2 Eliminate the 3 in $R_2[3]$

$R_2 = R_2 - 3R_3$

$3 - 3(1) = 3 - 3 = 0$

тут так же не понимаю → $5 - 3(-1) = 5 + 3 = 8$

Updated $R_2 = [0, 1, 0, 8]$

Step 3. Eliminate the 2 in $R_1[2]$

$R_1 = R_1 - 2R_2$

$2 - 2(1) = 2 - 2 = 0$

$-1 - 2(8) = -1 - 16 = 17$

Updated R_1 $R_1 = [1, 0, 0, -17]$

Answer: $\begin{bmatrix} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

P-15. Matrix Inverse & RREF Relationship.

$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ find A^{-1} using row operations.

Set up augmented matrix $[A|I]$ $\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$

Goal: Transform A into I while performing the same operation on I to obtain A^{-1}

1) Make $a_{11} = 1$

$R_1 = R_1 : 2$ $R_1 = [1, \frac{1}{2} | \frac{1}{2}, 0]$

Step 2. Eliminate a_{21}:

$R_2 = R_2 - 5R_1$

$5 - 5(1) = 0$

$3 - 5\left(\frac{1}{2}\right) = 3 - \frac{5}{2} = \frac{1}{2}$

$0 - 5\left(\frac{1}{2}\right) = 0 - \frac{5}{2} = -\frac{5}{2}$

$1 - 5(0) = 1$

R_2 becomes $[0, \frac{1}{2} | -\frac{5}{2}, 1]$

R_2 becomes $[0, \frac{1}{2} | -\frac{5}{2}, 1]$

Step 3. Make $a_{22} = 1$

$R_2 = R_2 \times 2$

$R_2 = [0, 1 | -5, 2]$

Step 4. Eliminate a_{12}

$R_1 = R_1 - (\frac{1}{2} R_2)$

$1 - \frac{1}{2}(0) = 1$

$\frac{1}{2} - \frac{1}{2}(1) = \frac{1}{2} - \frac{1}{2} = 0$

$\frac{1}{2} - \frac{1}{2}(-5) = \frac{1}{2} + \frac{5}{2} = 3$

$0 - \frac{1}{2}(2) = 0 - 1 = -1$

R_1 becomes $[1, 0 | 3, -1]$

Now, the augmented matrix is: $\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right]$

So,

$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

Answer: $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

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