Week 2

P1. $S = 5 + 9 + 13 + \ldots + 89$ express the arithm. series in sigma notation.

Date:

$a_1 = 5$ $\quad a_n = a_1 + (n-1)d$ — formula for finding general form

common differ. $d = 4$

$89 = 5(n-1) \cdot 4$

$84 = 4(n-1)$

$n - 1 = 21$

$n = 22$

Sigma notation: $S = \sum_{k=1}^{22} [5 + (k-1) + 4]$ — 22: ending value; $k=1 \rightarrow$ starting value

P2. Rewrite the arithmetic series $\sum_{k=3}^{15} (2k+1)$ to start at $k=1$

Since this sum starts at 3, we introduce a new variable j so that our new summation is $j=1$

$j = k - 2$

So, when $k = 3$, then $j = 3 - 2 = 1$

when $k = 15$, then $j = 15 - 2 = 13$

Summary of the method:
1) Define a new index $m = k - C$, C is number needed to shift the start to 1.
2) Express m in terms of j as $k = j + C$
3) Replace k is the summation formula
4) Adjust the limits of summ-n accordingly

Thus the limits of summation change: $\sum_{j=1}^{13}$

Now express k in terms of j $\quad k = j + 2$

Substitute $k = j + 2$ in the formula. The formula is $2k+1$

Substituting $k = j + 2$ $\quad 2(j+2) + 1 = 2j + 4 + 1 = 2j + 5$

Now rewrite the summation in terms of j $\quad \sum_{j=1}^{13} (2j + 5)$

Recursive notation.

P3. An arithm. seq. is defined recursively by $a_1 = 12$, & $a_n = a_{n-1} + d$, if $a_{10} = 57$ find the value of d & then find a_{25}.

1) find d $\quad a_n = a_1 + (n-1)d$

$a_{10} = 12 + (10-1)d = 12 + 9d = 57$

$9d = 45$

$d = 5.$

2) find a_{25}.

$a_{25} = 12 + (25-1) \cdot 5 = 12 + 120 = 132$ $\quad$ A: $d = 5$, $a_{25} = 132$

P4. Find the sum of all multiples of 7 between 100 & 1000.

$a_1 = 7 \times 15 = 105$ $\rightarrow$ First multiple above 100

$a_n = 7 \times 142 = 994$ $\rightarrow$ Last multiple below 1000

Number of terms $\boxed{n = \frac{a_n - a_1}{d} + 1} = \frac{994 - 105}{7} + 1 = 128$

Sum $\boxed{S = \frac{n}{2}(a_1 + a_n)} = \frac{128}{2}(105 + 994) = 64 \times 1099 = 70336$

formula

P5. $S = \sum_{k=1}^{n} (3k+2)$ find the value of n such that $S=2650$

(?) Date: 1) ~~find the sum of~~ arithmetic series. $S = \frac{n}{2}[a_1 + a_n]$

where

$a_1 = 3(1)+2 = 5$

$a_n = 3n+2$

So, $S = \frac{n}{2}[5 + (3n+2)] = \frac{n}{2}(3n+7)$

Set the sum equal to $S = 2650$.

$\frac{n}{2}(3n+7) = 2650$

$n(3n+7) = 5300$

$3n^2 + 7n - 5300 = 0$

Solve the quadratic equation.

$n = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 3 \cdot (-5300)}}{2 \cdot 3}$

$n = \frac{-7 \pm \sqrt{49 + 63600}}{6}$

$n = \frac{-7 \pm \sqrt{63649}}{6}$

Since n must be +, take the positive root.

$\sqrt{63649} \approx 252.29$

$n = \frac{-7 + 252.29}{6} \approx \frac{245.29}{6} \approx 40.88$.

Since n must be an integer $n = 41$

P6. Arith. seq. the 5th term is 20, the 15th term is 60. Show that the 10th term is the arith. mean of the 5th & 15th terms.

Date:

1) use the formula for the n^{th} term. $a_n = a + (n-1) \cdot d$ (arith. seq.)

for the 5^{th} term $a + (5-1) \cdot d = 20$

$a + 4d = 20$

for the 15^{th} term $a + (15-1) \cdot d = 60$

$a + 14d = 60$

2) Now sunstract the 2 equations:

$(a+14d) - (a+4d) = 60-20$

$\cancel{a} + 14d - \cancel{a} - 4d = 40$

$10d = 40$

$d = 4$

3) Find a, Substitute $d=4$ into equation $a+4d=20$

$a + 4(4) = 20$

$a + 16 = 20$

$a = 4$

4) Find the 10^{th} term. Using the formula: $a_{10} = a + (10-1) \cdot d$

$a_{10} = 4 + 9 \cdot 4$

$a_{10} = 4 + 36 = 40$

5. Show that a_{10} = is the Arithmetic Mean

The Arithmetic mean of the 5th & 15th terms:

$$\frac{a_5 + a_{15}}{2} = \frac{20+60}{2} = \frac{80}{2} = 40$$

Final answer.

$$a_{10} = \frac{a_5 + a_{15}}{2}$$

P7. A staircase has 20 steps. The first step is 5cm high, & each subsequent step is 0,5cm higher than the previous one. What is the total height of the staircase?

$a = 5$ $\quad S_n = \frac{n}{2}[2a_1 + (n-1)] d$

$d = 0,5$ $\quad S_{20} = \frac{20}{2}[2 \cdot 5 + (20-1) \cdot 0,5]$

$n = 20$

$S_{20} = 10[10 + \underbrace{19 \cdot 0,5}_{9,5}] = 10 \cdot 19,5 = 195$ cm.

Answer the total height is 195cm

P.8. Sum of the first N terms

$a_1 = 11$ What is the smallest ~~of~~ value of n such that the sum S_n exceeds 1000.

$d = 3$

Sum formula: $S_n = \frac{n}{2}[2a_1 + (n-1)\cdot d]$

$S_n = \frac{n}{2}[22 + (n-1)3$

Simplify $S_n = \frac{n}{2}[22 + 3n - 3] = \frac{n}{2}[3n+19]$

$S_n = \frac{n}{2}[3n+19]$

Set $S_n > 1000$ $\quad \frac{n(3n+19)}{2} > 1000$

$n(3n+19) > 2000$

$3n^2 + 19n - 2000 > 0$ $\quad \boxed{ax^2 + bx + c = 0}$

Solve the quadratic inequality using quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — discriminant

$n = \frac{-19 + \sqrt{19^2 - 4\cdot 3(-2000)}}{6}$ $\quad \approx 156,08$

$n = -19 \pm \sqrt{361 + 24000} = -19 \pm \sqrt{24361}$

$n = \frac{-19 + 156,08}{6} \approx \frac{137.03}{6} \approx 22,84$

Since n must be an integer, the smallest int $n = 23$

Check $n=23$ $\quad S_{23} = \frac{23(3\cdot 23 + 19)}{2} = \frac{23(69+19)}{2} = \frac{23 \times 88}{2} = 23 \cdot 44 = 1012$

So, the smallest integer satisfying the cond is $n = 23$.

P9 ? Shifting Index in Sigma notation

Rewrite $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k$ as a sum starting from $k=0$

вообще не поняла почему так

— let $j = k - 3$ when $k = 3$, $j = 0$, when $k = 12$, $j = 9$

Rewriting: $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k = \sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3} = \sum_{j=0}^{9} 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^j = \sum_{j=0}^{9}\left(\frac{1}{2}\right)^j \frac{1}{2}$

Simplify: $\sum_{j=0}^{9}\left(\frac{1}{2}\right)^j \times \frac{1}{2} = \sum_{j=0}^{9}\left(\frac{1}{2}\right)^{j+1}$

P10. N^{th} Term Formula $\boxed{a_n = a_1 (r)^{n-1}}$

Find 10^{th} term of geometric sequence if $a_2 = -6$ & $a_5 = 48$

common ratio
$2, 6, 18, 54$
$a_1 = 2$
$r = 3$

from a_2 $\quad -6 = a_1 r^1$

from a_5 $\quad +48 = a_1 r^4$

Divide equation(2) by equation 1

$$\frac{+48}{-6} = \frac{a_1 r^1}{a_1 r^4} = -8 = r^3$$

$$r^3 = -8$$

$$r = -2$$

Now find a_1 $\quad -6 = a_1(-2)$

$$a_1 = 3$$

find $a_{10} = 3(-2)^9 = 3 \cdot (-512) = -1536$ $\quad$ Answer $a_{10} = -1536$.

P11. Finding Common Ratio

In geomet. seq. $a_4 = 54$ $a_7 = 1458$ $\quad r - ?$

Divide a_7 b a_4 using $a_n = a_1 r^{n-1}$

$$\frac{a_7}{a_4} = \frac{a_1 r^6}{a_1 r^3} = r^3$$

$$\frac{1458}{54} = r^3$$

$$r^3 = 27$$

$$r = 3$$

P12. Sum of the 1st N Terms,

Calculate the sum of the 1st 15 terms of the geom. seq. where $a_1 = 8$ & $r = \frac{3}{4}$

Use the sum formula: $S_n = a_1 \frac{1 - r^n}{1 - r}$

Compute r^{15} $\quad r^{15} = \left(\frac{3}{4}\right)^{15}$

Compute S^{15} $\quad S_{15} = 8 \frac{1 - \left(\frac{3}{4}\right)^{15}}{1 - \frac{3}{4}} = 8 \frac{1 - r^{15}}{\frac{1}{4}} = 32(1 - r^{15})$

$$\approx 32 \times 1 \approx 32$$

answer is $S_{15} \approx 32$

P13. Definitions. classify it by degree & number
Date: $P(x) = x^5 - 4x^3 + x^2 - 7$
terms.

S: Degree: 5 (the highest exponent)
Number of term: 4 (quartic)

P14. Adding Polynomyals.
$(2x^4 - 3x^3 + x - 5) + (x^3 - 2x^2 + 4x + 7)$
$2x^4 + (-3x^3 + x^3) + (-2x^2) + (x + 4x) + (-5 + 7) = 2x^4 - 2x^3 - 2x^2 + 5x + 2$

Answer: $2x^4 + 2x^3 + 2x^2 + 5x + 2$

P15. Multiplying Polynomyals.
Find the product. $(x^2 - x + 2)(x^2 + x + 1)$
-? кажется ошибка в ответе. у меня по-другому
$x^2(x^2 + x + 1) + (-x)(x^2 + x + 1) + 2(x^2 + x + 1) = x^4 + \cancel{x^3} + \cancel{x^2} - \cancel{x^3} - \cancel{x^2}$
$+ 2x^2 + 2x + 2 = x^4 + 2x^2 + x + 2$

P-16. GCD & LCM of Monomials.
Find GCD & LCM of $24x^3y^2z^5$ & $36x^5y^3z^2$
Надо повторить

GCD: Coefficient GCD: $\gcd(24, 36) = 12$
Variables: x^3, y^2, z^2 (min. exponents)

LCM: Coefficient LCM: $\operatorname{lcm}(24, 36) = 72$
Variables: x^5, y^3, z^5 (max. exponents)

Answer: GCD is $12x^3y^2z^2$, LCM is $72x^5y^3z^5$

P-17. Factoring quadratics.
Factor $x^4 - 13x^2 + 36$
Let $u = x^2$

$u^2 - 13u + 36 = 0$

factor: $(u - 9)(u - 4) = 0$

Back to x: $(x^2 - 9)(x^2 - 4) = (x - 3)(x + 3) \cdot (x - 2)(x + 2)$

Answer: $(x - 3)(x + 3)(x - 2)(x + 2)$

P-18. Special binomial products.
Expand $(2x + 3y)^5$ using the Binomial Theorem.

Using $(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

Compute terms for $n = 5$

1) $k = 0$: $\binom{5}{0} (2x)^5 (3y)^0 = 1 \times 32x^2 \times 1 = 32x^5$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ where $n!$ (n factorial) means $n \cdot (n-1) \times (n-2) \times .. \times 1$

$k!$ is the factorial of k

Date:

$(n-k)!$ is the factorial of $(n-k)$

So, we have $n=5$ & we calculate $\binom{5}{k}$ for diff. values of k.

$\binom{5}{0} = \frac{5!}{0!(5-0)!} = \frac{5!}{0!5!}$ since $0! = 1$ & $5!$ cancels out: $\binom{5}{0} = \frac{1}{1} = 1$.

② $k=1$ $\binom{5}{1}$ $(2x)^4(3y)^1 = 5 \cdot 16x^4 \cdot 3y = 240x^4y$

3) $k=2$ $\binom{5}{2}$ $(2x)^3(3y)^2 = 10 \cdot 8x^3 \cdot 9y^2 = 720x^3y^2$

4) $k=3$ $\binom{5}{3}$ $(2x)^2(3y)^3 = 10 \cdot 4x^2 \cdot 27y^3 = 1080x^2y^3$

5) $k=4$ $\binom{5}{3}$ $(2x)^1(3y)^4 = 5 \cdot 2x \cdot 81y^4 = 810xy^4$

6) $k=5$ $\binom{5}{5}$ $(2x)^0(3y)^5 = 1 \cdot 1 \times 243y^5 = 243y^5$

Combine: Answer: $(2x+3y)^5 = 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

P-19. Divide $6x^3 + 11x^2 - 31x + 15$ by $3x - 2$

$$\begin{array}{l|l} 6x^3 + 11x^2 - 31x + 15 & 3x-2 \\ \hline -(6x^3 - 4x^2) & 2x^2 + 5x + (-7) \\ \quad 15x^2 - 31x & \\ \quad -(15x^2 - 10x) & \\ \quad\quad -21x + 15 & \\ \quad\quad -(-21x + 14) & \\ \quad\quad\quad 1 \text{ remainder.} & \end{array}$$

Тут между минус ставится + ?

P-20. The remainder theorem

If a polynomial $f(x)$ is divided by $x-k$, then the remainder is the value $f(k)$

List all possible rational zeros of $f(x) = 2x^4 - 5x^3 - x^2 - 4$

states, that any rational root $\frac{p}{q}$ must be factor of the constant term divided by a factor of the leading coefficient.

1) Identify factors of Constant & Leading Coefficient
 - Constant term $= -4$ Factors of -4: $\pm 1, \pm 2, \pm 4$
 - Leading coefficient $= 2$. Factors of 2: $\pm 1, \pm 2$.

2. List Possible Rational Roots

$\frac{\text{Factors of constant term}}{\text{F. of leading coeff.}} = \frac{\pm 1, \pm 2 \pm 4}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{4}{2}$

Since $\frac{2}{2} = 1$ $\frac{4}{2} = 2$ are already listed, the final list of p.r. zeros:

$\pm 1, \pm 2, \pm 4; \pm \frac{1}{2}$.