CHINESE REMAINDER THEOREM (P4)

d) $x \equiv 5 \pmod 9$ & $x \equiv 6 \pmod{10}$ & $x \equiv 7 \pmod{11}$

By the CRT, there exists a unique solution modulo 990,
since $990 = 9 \times 10 \times 11$.

We can start with the solution $x = 5$ to substitute in the second congruence, $x \equiv 6 \pmod{10}$, as $x = 5 + 9y$:

$$5 + 9y \equiv 6 \pmod{10}$$

$$9y \equiv 1 \pmod{10}$$

$$y \equiv 9 \pmod{10}$$

find the inverse of 9 modulo 10:

guess & check

$9 \cdot \boxed{9} = 1 \bmod 10$

$9^{-1} = 9$

$$x = 5 + 9y = 5 + 9(9) = 86$$

Now, the solution to the first two congruences, $x = 86 + 90z$, can be substituted in $x \equiv 7 \pmod{11}$:

$$86 + 90z \equiv 7 \pmod{11}$$

$$90z \equiv 7 - 86 \pmod{11}$$

$$90z \equiv -79 \pmod{11}$$

$$90z \equiv 9 \pmod{11}$$

$$z \equiv 9 \cdot 6 \equiv 54 \equiv 10 \pmod{11}$$

find the inverse of 90 modulo 11:

guess & check

$90 \cdot \boxed{6} = 1 \bmod 11$

$90^{-1} = 6$

Now, we can substitute $z = 10$ into $x = 86 + 90z$ to get:

$$x = 86 + 90(10) = 986$$