P2 - FAST POWERS

1) $17^{183} \bmod 256$

Let's write 183 in binary sum:

$$183 = \underset{128}{2^7} + \underset{32}{2^5} + \underset{16}{2^4} + \underset{4}{2^2} + \underset{2}{2^1} + \underset{1}{2^0} \rightarrow 10110111$$

Then, we can rewrite 17^{183}:

$$17^{183} = 17^{(2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0)} = 17^{2^7} \cdot 17^{2^5} \cdot 17^{2^4} \cdot 17^{2^3} \cdot 17^{2^2} \cdot 17^{2^1} \cdot 17^{2^0}$$

We can table the multiplications to speed calculation:

i	0	1	2	3	4	5	6	7
$17^{2^i} \bmod 256$	17	33	65	129	1	1	1	1

Using the table, we can construct a smaller computation:

$$\begin{aligned} 17^{183} &\equiv 17^{2^7} \cdot 17^{2^5} \cdot 17^{2^4} \cdot 17^{2^3} \cdot 17^{2^2} \cdot 17^{2^1} \cdot 17^{2^0} \\ &\equiv 1 \cdot 1 \cdot 1 \cdot 129 \cdot 65 \cdot 33 \cdot 17 \pmod{256} \\ &\equiv 241 \pmod{256} \end{aligned}$$

Therefore, $17^{183} \bmod 256 \equiv 241$.

2) $2^{477} \bmod 1000$

Let's write 477 in binary sum:

$$477 = \underset{256}{2^8} + \underset{128}{2^7} + \underset{64}{2^6} + \underset{16}{2^4} + \underset{8}{2^3} + \underset{4}{2^2} + \underset{1}{2^0} \rightarrow 111011101$$

Then, we can rewrite 2^{477}:

$$2^{477} = 2^{(2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2 + 2^0)} = 2^{2^8} \cdot 2^{2^7} \cdot 2^{2^6} \cdot 2^{2^4} \cdot 2^{2^3} \cdot 2^{2^2} \cdot 2^{2^0}$$

We can table the multiplications to speed calculation:

i	0	1	2	3	4	5	6	7	8
$2^{2^i} \bmod 1000$	2	4	16	256	536	296	616	456	936

Using the table, we can construct a smaller computation:

$$\begin{aligned} 2^{477} &\equiv 2^{2^8} \cdot 2^{2^7} \cdot 2^{2^6} \cdot 2^{2^4} \cdot 2^{2^3} \cdot 2^{2^2} \cdot 2^{2^0} \\ &\equiv 936 \cdot 456 \cdot 616 \cdot 536 \cdot 256 \cdot 16 \cdot 2 \pmod{1000} \\ &\equiv 272 \pmod{1000} \end{aligned}$$

Therefore, $2^{477} \bmod 1000 \equiv 272$.