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**Instructions:**

Solutions to this assignment must be submitted through Gradescope by **Sunday, August 21st 2022 at 10 PM**— no late submissions will be accepted.

Please use the submission template provided for this PA to get started.

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1. **(Counting Inversions)**

We are given an array  $L$  of  $n > 0$  distinct integers. We say that two indices  $i < j$  form an inversion if  $L[i] > L[j]$ ; that is, if the two elements  $L[i]$  and  $L[j]$  are “out of order.”

In this problem, we seek to count the number of inversions in our array.

Using *divide and conquer*, write a procedure called `myCount(L)` that takes as input

- $L$ : an array of  $n > 0$  distinct integers.

and returns a pair  $(count, L_{sorted})$  containing

- *count*: the total number of inversions in  $L$ .
- $L_{sorted}$ : an array containing every element of  $L$ , but sorted.

For example, consider the array

$$L = [6, 1, -4, 10, 2, 7].$$

This array has six inversions: the index pairs  $(0, 1), (0, 2), (0, 4), (1, 2), (3, 4), (3, 5)$ . Therefore, our algorithm would return the tuple  $(6, [-4, 1, 2, 6, 7, 10])$ .

Your algorithm should run in  $O(n \log n)$  time.

**Important Note:** You will receive **no points** for this question if found using a brute-force method with a time complexity of  $O(n^2)$  or worse. Your program may be manually verified to ensure this.

*Hint.* Modify the MergeSort algorithm accordingly.

## 2. (Closest pair of points)

Given an array of  $n$  points on a plane  $P = [(x_1, y_1), \dots, (x_n, y_n)]$ , find the *square* of the distance of the closest pair of points in the array.

Using *divide and conquer*, write a procedure called `myMinDistance(P)`. The input format for `myMinDistance` will differ depending on whether C++ or Python is used.

If you choose to write your algorithm in C++, then the input will be an array  $P$ , where

- $P$  is an array of Points.
- Points are a class with two defined fields:  $x$  and  $y$ . The tuple  $(x, y)$  represents a point in the Cartesian plane.

If you choose to write your algorithm in Python, then the input will be an array  $P$ , where

- $P$  is an array of  $n > 0$  tuples  $(x_j, y_j)$  where  $x_j, y_j$  are integers. The tuple  $(x_j, y_j)$  represents a point in the Cartesian plane.

For both C++ and Python, we return

- *min*: the square of the minimum distance of points in  $P$ .

For example, consider the array

$$P = [(0, 1), (-5, 3), (4, 2), (2, 0)].$$

The two closest points in this array are  $(0, 1)$  and  $(2, 0)$ . The square of their distance is 5, so our algorithm would return 5.

Your algorithm should have  $O(n(\log n)^2)$  runtime or better.

**Important Note:** You will receive **no points** for this question if found using a brute-force method with a time complexity of  $O(n^2)$  or worse. Your program may be manually verified to ensure this.