1. Question ID 509: Let R(ABCDEFGH) satisfy the following functional dependencies:

   $A \rightarrow B$, $CH \rightarrow A$, $B \rightarrow E$, $BD \rightarrow C$, $EG \rightarrow H$, $DE \rightarrow F$.

   Which of the following FD's is also guaranteed to be satisfied by R?

   Question Explanation: The secret is to compute the closure of the left side of each FD that you get as a choice. Recall that the closure of a set of attributes is computed by starting with that set, and repeatedly looking for left sides of given FD's that are wholly contained within your current set of attributes. If you find such an FD, you add its right side to your set, until you can add no more. If the right side of the candidate FD is contained in your set, then the candidate does follow from the given FD's; otherwise not.

2. Question ID 510: Find all the keys of the Relation R(ABCDE) with FD's:

   $D \rightarrow C$, $CE \rightarrow A$, $D \rightarrow A$, and $AE \rightarrow D$

   Demonstrate your knowledge by indicating which of the following is a key.

   Question Explanation: In order for a set of attributes S to be a key, given a set of FD's, not only must the closure of this set of attributes be all attributes, but no proper subset of S must also have a closure that is all attributes. It is sufficient to check that no proper subset missing exactly one of the attributes of S has a closure equal to all the attributes.

   Recall that the closure of a set of attributes is computed by starting with that set, and repeatedly looking for left sides of given FD's that are wholly contained within your current set of attributes.

3. Question ID 511: Find all the keys of the relation R(ABCDEF) with FD's:

   $CDE \rightarrow B$, $ACD \rightarrow F$, $BEF \rightarrow C$, and $B \rightarrow D$

   Demonstrate your knowledge by identifying which of the following is a key.

   Question Explanation: In order for a set of attributes S to be a key, given a set of FD's, not only must the closure of this set of attributes be all attributes, but no proper subset of S must also have a closure that is all attributes. It is sufficient to check that no proper subset missing exactly one of the attributes of S has a closure equal to all the attributes.

Recall that the closure of a set of attributes is computed by starting with that set, and repeatedly looking for left sides of given FD's that are wholly contained within your current set of attributes.

4. Question ID 512: Determine the keys and superkeys of the relation R(ABCDEF) with FD's:

AEF → C, BF → C, EF → D, and ACDE → F

Then, demonstrate your knowledge by selecting the true statement from the list below.

Question Explanation: First, notice that A, B, and E don't appear in any right side, so they must be in any key. Thus, we might as well assume A, B, and E are in any closure we care about, and remove them from left sides. That leaves us with F → C, F → D, and CD → F. To get C, D, F in a closure, we can start with either F alone, or C and D; no subsets will serve, and any other subset of {C,D,F} is a superset of either {F} or {C,D}. Thus, the only keys are ABCDE and ABEF.

There are three superkeys other than the two keys above. These are the proper superses of ABCDE or ABEF, namely ABCDEF, ABCEF, and ABDEF. Thus, there are two keys, five superkeys, and three superkeys that are not keys.

5. Question ID 513: Find all keys of the relation R(ABCDEFG) with functional dependencies

AB → C, CD → E, EF → G, FG → E, DE → C, and BC → A

Demonstrate your knowledge by identifying which of the following is a key.

Question Explanation: First, notice that B, D, and F do not appear on the right side of any FD, and so must be a member of any key. A closure test tells us BDF is not a key by itself.

If we assume these FD's are already in any key, then for any relevant closure we do not need to have them on the left sides of FD's. That is, we can pretend the FD's are A → C, C → E, E → G, G → E, E → C, and C → A. Now we see that any of A, C, E, and G (along with B, D, and F) functionally determines all the others. That tells us all the keys are BDF plus exactly one of A,C,E,G.

6. Question ID 515: Consider relation R(A,B,C,D,E) with functional dependencies AB → C, BC → D, CD → E, DE → A, and AE → B. Project these FD's onto the relation S(A,B,C,D). Which of the following FD's holds in the projected relation?

Question Explanation: Think of A, B, C, D, E as arranged in a circle. If we start with any two adjacent attributes, such as BC or AE, then the closure includes all the attributes. However, if we start with any one attribute or any two nonadjacent attributes, then the closure is just what we start with. Notice also that any three or more attributes must include two adjacent attributes, so its closure is all the attributes.

To see what FD's hold in S, we must close every subset of {A,B,C,D} (but allowing E in the closure) and see which among those four attributes are in the closure. In our problem, AB, BC, CD, and any superset has ABCDE as the closure, and thus we get, for S, AB → CD, BC → AD, and CD → AB. We also get FD's that follow from these, such as ABC → D. However, S has no nontrivial FD's with singletons on the left or with other pairs on the left.