MATH 189 HW2

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Last Updated: February 03, 2023

Concrete contributions

All problems were done by Xiangyi Lin, Zijian Su, Zelong Zhou. All contributing equally to this assignment. Everyone put in enough effort.

Packages

```
#install.packages("rmarkdown")
#install.packages('HSAUR3')
#install.packages('tools')
library("HSAUR3")
```

```
## Warning: package 'HSAUR3' was built under R version 4.1.3
```

```
## Loading required package: tools
```

```
data("pottery")
#tinytex::install_tinytex()
```

Probelm 1

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. The water quality dataset (water.txt) contains ten pairs of data that measure zinc concentration in bottom water and surface water. Suppose we consider the zinc concentration in bottom water and in surface water as two samples. Denote by μ_1 and μ_2 the underlying population means of the two samples. Test the null and alternative hypotheses:

$H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 != \mu_2$.

(Hint: Treat the dataset as a paired sample, or use a general univariate two-sample test)

Answer:

Using R to do the t-test(Treat the dataset as a paired sample):

```
##
##  Paired t-test
##
## data:  data$bottom and data$surface
## t = 4.8638, df = 9, p-value = 0.0008911
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.043006 0.117794
## sample estimates:
## mean of the differences
##                  0.0804
```

Since P -value < 0.05, I reject H_0 and support H_1. Therefore, there is a high probability that the zinc concentration in bottom water and surface water is different.

Probelm 2

Download the complete Romano-British Pottery Data from R:

install.packages('HSAUR3') library("HSAUR3") data("pottery")

This dataset is about chemical composition of Romano-British pottery. It contains 45 total observations on the following 9 chemicals from 5 regions (1=Gloucester, 2=Llanedeyrn, 3=Caldicot, 4=Islands Thorns, 5=Ashley Rails):

Al2O3: aluminium trioxide Fe2O3: iron trioxide MgO: magnesium oxide CaO: calcium oxide Na2O: natrium oxide K2O: kalium oxide TiO2: titanium oxide MnO: mangan oxide BaO: barium oxide

"Kiln" indicates at which kiln site the pottery was found.

Since Site 3 (Caldicot) only consists of 2 observations, we discard it and focus on the comparisons among the rest 4 sites: 1=Gloucester, 2=Llanedeyrn, 4=Islands Thorns, 5=Ashley Rails.

Analyze the dataset according to the following steps: Let $\mu_g = (\mu_{g1}, \cdots, \mu_{g9})^T$ be the population mean vector of the 9 chemical concentrations from Site g (g = 1,2,4,5). We wish to test the hypothesis

$H_0 : \mu_1 = \mu_2 = \mu_4 = \mu_5$

versus the alternative that there exist two means that are unequal.

Question 1

Decompose the problem into multiple hypotheses testing: for k = 1,...,9, consider the null $H_0^{(k)} : \mu_{1k} = \mu_{2k} = \mu_{4k} = \mu_{5k}$. Calculate F-statistics $F^{(k)}(k = 1, \cdots, 9)$ for these nine hypotheses.

Answer:

```
data <- pottery[pottery$kiln != 3, ]
data
```

```
##    Al2O3 Fe2O3  MgO  CaO Na2O  K2O TiO2   MnO   BaO kiln
## 1   18.8  9.52 2.00 0.79 0.40 3.20 1.01 0.077 0.015    1
## 2   16.9  7.33 1.65 0.84 0.40 3.05 0.99 0.067 0.018    1
## 3   18.2  7.64 1.82 0.77 0.40 3.07 0.98 0.087 0.014    1
## 4   16.9  7.29 1.56 0.76 0.40 3.05 1.00 0.063 0.019    1
## 5   17.8  7.24 1.83 0.92 0.43 3.12 0.93 0.061 0.019    1
## 6   18.8  7.45 2.06 0.87 0.25 3.26 0.98 0.072 0.017    1
## 7   16.5  7.05 1.81 1.73 0.33 3.20 0.95 0.066 0.019    1
## 8   18.0  7.42 2.06 1.00 0.28 3.37 0.96 0.072 0.017    1
## 9   15.8  7.15 1.62 0.71 0.38 3.25 0.93 0.062 0.017    1
## 10  14.6  6.87 1.67 0.76 0.33 3.06 0.91 0.055 0.012    1
## 11  13.7  5.83 1.50 0.66 0.13 2.25 0.75 0.034 0.012    1
## 12  14.6  6.76 1.63 1.48 0.20 3.02 0.87 0.055 0.016    1
## 13  14.8  7.07 1.62 1.44 0.24 3.03 0.86 0.080 0.016    1
## 14  17.1  7.79 1.99 0.83 0.46 3.13 0.93 0.090 0.020    1
## 15  16.8  7.86 1.86 0.84 0.46 2.93 0.94 0.094 0.020    1
## 16  15.8  7.65 1.94 0.81 0.83 3.33 0.96 0.112 0.019    1
## 17  18.6  7.85 2.33 0.87 0.38 3.17 0.98 0.081 0.018    1
## 18  16.9  7.87 1.83 1.31 0.53 3.09 0.95 0.092 0.023    1
## 19  18.9  7.58 2.05 0.83 0.13 3.29 0.98 0.072 0.015    1
```

```
## 20  18.0  7.50 1.94 0.69 0.12 3.14 0.93 0.035 0.017    1
## 21  17.8  7.28 1.92 0.81 0.18 3.15 0.90 0.067 0.017    1
## 22  14.4  7.00 4.30 0.15 0.51 4.25 0.79 0.160 0.019    2
## 23  13.8  7.08 3.43 0.12 0.17 4.14 0.77 0.144 0.020    2
## 24  14.6  7.09 3.88 0.13 0.20 4.36 0.81 0.124 0.019    2
## 25  11.5  6.37 5.64 0.16 0.14 3.89 0.69 0.087 0.009    2
## 26  13.8  7.06 5.34 0.20 0.20 4.31 0.71 0.101 0.021    2
## 27  10.9  6.26 3.47 0.17 0.22 3.40 0.66 0.109 0.010    2
## 28  10.1  4.26 4.26 0.20 0.18 3.32 0.59 0.149 0.017    2
## 29  11.6  5.78 5.91 0.18 0.16 3.70 0.65 0.082 0.015    2
## 30  11.1  5.49 4.52 0.29 0.30 4.03 0.63 0.080 0.016    2
## 31  13.4  6.92 7.23 0.28 0.20 4.54 0.69 0.163 0.017    2
## 32  12.4  6.13 5.69 0.22 0.54 4.65 0.70 0.159 0.015    2
## 33  13.1  6.64 5.51 0.31 0.24 4.89 0.72 0.094 0.017    2
## 36  18.3  1.28 0.67 0.03 0.03 1.96 0.65 0.001 0.014    4
## 37  15.8  2.39 0.63 0.01 0.04 1.94 1.29 0.001 0.014    4
## 38  18.0  1.50 0.67 0.01 0.06 2.11 0.92 0.001 0.016    4
## 39  18.0  1.88 0.68 0.01 0.04 2.00 1.11 0.006 0.022    4
## 40  20.8  1.51 0.72 0.07 0.10 2.37 1.26 0.002 0.016    4
## 41  17.7  1.12 0.56 0.06 0.06 2.06 0.79 0.001 0.013    5
## 42  18.3  1.14 0.67 0.06 0.05 2.11 0.89 0.006 0.019    5
## 43  16.7  0.92 0.53 0.01 0.05 1.76 0.91 0.004 0.013    5
## 44  14.8  2.74 0.67 0.03 0.05 2.15 1.34 0.003 0.015    5
## 45  19.1  1.64 0.60 0.10 0.03 1.75 1.04 0.007 0.018    5
```

list to save all F statistics value

```
f_value <- c()
```

Get the Critical value F m,n-m,a

```
df1 = 3
df2 = 39
a = 0.05
F_Critical <-qf(a, df1, df2, lower.tail=FALSE)
F_Critical
```

```
## [1] 2.845068
```

Al2O3

H_0^1 : $\mu_{11} = \mu_{21} = \mu_{41} = \mu_{51}$ vs H_1^1 : there has some μ_{x1} != μ_{y1}.

```
Al2O3 <- aov(Al2O3 ~ kiln, data)
f_value <- c(f_value, summary.lm(Al2O3)$f["value"])
summary.lm(Al2O3)
```

```
##
## Call:
## aov(formula = Al2O3 ~ kiln, data = data)
##
```

```
## Residuals:
##    Min     1Q Median     3Q    Max
## -3.219 -1.089 -0.019  1.161  2.620
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  16.9190     0.3423  49.434  < 2e-16 ***
## kiln2        -4.3607     0.5676  -7.683 2.52e-09 ***
## kiln4         1.2610     0.7805   1.616    0.114
## kiln5         0.4010     0.7805   0.514    0.610
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.568 on 39 degrees of freedom
## Multiple R-squared:  0.6667, Adjusted R-squared:  0.641
## F-statistic:    26 on 3 and 39 DF,  p-value: 2.083e-09
```

For Al2O3, we reject the null hypothesis since F statistics = 26 > F Critical = 2.845068. We can believe that at least one chemical concentration is different.

Fe2O3

H_0^2 : $\mu_{12} = \mu_{22} = \mu_{42} = \mu_{52}$ vs H_1^2 : there has some μ_{x2} != μ_{y2}.

```
Fe2O3 <- aov(Fe2O3 ~ kiln, data)
f_value <- c(f_value, summary.lm(Fe2O3)$f["value"])
summary.lm(Fe2O3)
```

```
##
## Call:
## aov(formula = Fe2O3 ~ kiln, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.08000 -0.36529 -0.00857  0.39143  2.09143
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.4286     0.1554  47.816  < 2e-16 ***
## kiln2        -1.0886     0.2576  -4.225 0.000139 ***
## kiln4        -5.7166     0.3543 -16.136  < 2e-16 ***
## kiln5        -5.9166     0.3543 -16.701  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7119 on 39 degrees of freedom
## Multiple R-squared:  0.9223, Adjusted R-squared:  0.9163
## F-statistic: 154.3 on 3 and 39 DF,  p-value: < 2.2e-16
```

For Fe2O3, we reject the null hypothesis since F statistics = 154.3 > F Critical = 2.845068. We can believe that at least one chemical concentration is different.

MgO

H_0^3 : $\mu_{13} = \mu_{23} = \mu_{43} = \mu_{53}$ vs H_1^3 : there has some μ_{x3} != μ_{y3}.

```
MgO <- aov(MgO ~ kiln, data)
f_value <- c(f_value, summary.lm(MgO)$f["value"])
summary.lm(MgO)
```

```
##
## Call:
## aov(formula = MgO ~ kiln, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.5017 -0.2024 -0.0040  0.1526  2.2983
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.8424     0.1363  13.518 2.66e-16 ***
## kiln2         3.0893     0.2260  13.669  < 2e-16 ***
## kiln4        -1.1684     0.3108  -3.760 0.000558 ***
## kiln5        -1.2364     0.3108  -3.978 0.000292 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6245 on 39 degrees of freedom
## Multiple R-squared:  0.8826, Adjusted R-squared:  0.8736
## F-statistic: 97.77 on 3 and 39 DF,  p-value: < 2.2e-16
```

For MgO, we reject the null hypothesis since F statistics = 97.77 > F Critical = 2.845068. We can believe that at least one chemical concentration is different.

CaO

H_0^4 : $\mu_{14} = \mu_{24} = \mu_{44} = \mu_{54}$ vs H_1^4 : there has some μ_{x4} != μ_{y4}.

```
CaO <- aov(CaO ~ kiln, data)
f_value <- c(f_value, summary.lm(CaO)$f["value"])
summary.lm(CaO)
```

```
##
## Call:
## aov(formula = CaO ~ kiln, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.27905 -0.10405 -0.02200  0.01358  0.79095
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.93905    0.04630  20.283  < 2e-16 ***
## kiln2       -0.73821    0.07678  -9.615 7.68e-12 ***
```

```
## kiln4       -0.91305    0.10557  -8.648 1.32e-10 ***
## kiln5       -0.88705    0.10557  -8.402 2.77e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2122 on 39 degrees of freedom
## Multiple R-squared:  0.8045, Adjusted R-squared:  0.7895
## F-statistic:  53.5 on 3 and 39 DF,  p-value: 6.88e-14
```

For CaO, we reject the null hypothesis since F statistics = 53.5 > F Critical = 2.845068. We can believe that at least one chemical concentration is different.

Na2O

H_0^5 : $\mu_{15} = \mu_{25} = \mu_{45} = \mu_{55}$ vs H_1^5 : there has some μ_{x5} != μ_{y5}.

```
Na2O <- aov(Na2O ~ kiln, data)
f_value <- c(f_value, summary.lm(Na2O)$f["value"])
summary.lm(Na2O)
```

```
##
## Call:
## aov(formula = Na2O ~ kiln, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.22571 -0.07036 -0.01400  0.05014  0.48429
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.34571    0.02988  11.570 3.52e-14 ***
## kiln2       -0.09071    0.04955  -1.831 0.074787 .
## kiln4       -0.29171    0.06814  -4.281 0.000117 ***
## kiln5       -0.29771    0.06814  -4.369 8.94e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1369 on 39 degrees of freedom
## Multiple R-squared:  0.446,  Adjusted R-squared:  0.4034
## F-statistic: 10.47 on 3 and 39 DF,  p-value: 3.48e-05
```

For Na2O, we reject the null hypothesis since F statistics = 10.47 > F Critical = 2.845068. We can believe that at least one chemical concentration is different.

K2O

H_0^6 : $\mu_{16} = \mu_{26} = \mu_{46} = \mu_{56}$ vs H_1^6 : there has some μ_{x6} != μ_{y6}.

```
K2O <- aov(K2O ~ kiln, data)
f_value <- c(f_value, summary.lm(K2O)$f["value"])
summary.lm(K2O)
```

```
##
## Call:
## aov(formula = K2O ~ kiln, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.85286 -0.08810  0.02714  0.15214  0.76667
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.10286    0.06852  45.284  < 2e-16 ***
## kiln2        1.02048    0.11363   8.981 4.90e-11 ***
## kiln4       -1.02686    0.15625  -6.572 8.34e-08 ***
## kiln5       -1.13686    0.15625  -7.276 8.98e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.314 on 39 degrees of freedom
## Multiple R-squared:  0.8628, Adjusted R-squared:  0.8523
## F-statistic: 81.76 on 3 and 39 DF,  p-value: < 2.2e-16
```

For K2O, we reject the null hypothesis since F statistics = 10.47 > F Critical = 2.845068. We can believe that at least one chemical concentration is different.

TiO2

H_0^7 : $\mu_{17} = \mu_{27} = \mu_{47} = \mu_{57}$ vs H_1^7 : there has some μ_{x7} != μ_{y7}.

```
TiO2 <- aov(TiO2 ~ kiln, data)
f_value <- c(f_value, summary.lm(TiO2)$f["value"])
summary.lm(TiO2)
```

```
##
## Call:
## aov(formula = TiO2 ~ kiln, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.39600 -0.04583  0.00238  0.04419  0.34600
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.93762    0.02659  35.266  < 2e-16 ***
## kiln2       -0.23679    0.04409  -5.371 3.87e-06 ***
## kiln4        0.10838    0.06063   1.788   0.0816 .
## kiln5        0.05638    0.06063   0.930   0.3581
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1218 on 39 degrees of freedom
## Multiple R-squared:   0.53,  Adjusted R-squared:  0.4938
## F-statistic: 14.66 on 3 and 39 DF,  p-value: 1.525e-06
```

For TiO2, we reject the null hypothesis since F statistics = 14.66 > F Critical = 2.845068. We can believe that at least one chemical concentration is different.

MnO

H_0^8 : $\mu_{18} = \mu_{28} = \mu_{48} = \mu_{58}$ vs H_1^8 : there has some μ_{x8} != μ_{y8}.

```
MnO <- aov(MnO ~ kiln, data)
f_value <- c(f_value, summary.lm(MnO)$f["value"])
summary.lm(MnO)
```

```
##
## Call:
## aov(formula = MnO ~ kiln, data = data)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.041000 -0.009643 -0.000200  0.009357  0.042000
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.071143   0.004777  14.892  < 2e-16 ***
## kiln2        0.049857   0.007922   6.294 2.03e-07 ***
## kiln4       -0.068943   0.010893  -6.329 1.81e-07 ***
## kiln5       -0.066943   0.010893  -6.145 3.25e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02189 on 39 degrees of freedom
## Multiple R-squared:  0.8023, Adjusted R-squared:  0.7871
## F-statistic: 52.76 on 3 and 39 DF,  p-value: 8.562e-14
```

For MnO, we reject the null hypothesis since F statistics = 52.76> F Critical = 2.845068. We can believe that at least one chemical concentration is different.

BaO

H_0^9 : $\mu_{19} = \mu_{29} = \mu_{49} = \mu_{59}$ vs H_1^9 : there has some μ_{x9} != μ_{y9}.

```
BaO <- aov(BaO ~ kiln, data)
f_value <- c(f_value, summary.lm(BaO)$f["value"])
summary.lm(BaO)
```

```
##
## Call:
## aov(formula = BaO ~ kiln, data = data)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0072500 -0.0016964 -0.0001429  0.0018571  0.0058571
##
```

```
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0171429  0.0006660  25.742   <2e-16 ***
## kiln2       -0.0008929  0.0011044  -0.808    0.424
## kiln4       -0.0007429  0.0015186  -0.489    0.627
## kiln5       -0.0015429  0.0015186  -1.016    0.316
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003052 on 39 degrees of freedom
## Multiple R-squared:  0.03411,    Adjusted R-squared:  -0.04019
## F-statistic: 0.459 on 3 and 39 DF,  p-value: 0.7125
```

For BaO, we support the null hypothesis since F statistics = 0.459 < F Critical = 2.845068. We can believe that all chemical concentration are the same.

```
for (i in f_value){
  print(i)
}
```

```
## [1] 26.00087
## [1] 154.3197
## [1] 97.7673
## [1] 53.50213
## [1] 10.46517
## [1] 81.76186
## [1] 14.65845
## [1] 52.75629
## [1] 0.4590205
```

The above is the result of my calculation of 9 F-statistics, sorted from 1-9 in order.

Question 2

At significance level a = 5%, apply the Bonferroni correction method to test $H_0 : \mu_1 = \mu_2 = \mu_4 = \mu_5$. Summarize your conclusions.

Answer:

```
new_a = 1/9 * 0.05  #m = 9 since we have 9 h0
critical_value = qf(new_a, 3, 39, lower.tail=FALSE)
print(sprintf("new a = %f",new_a))
```

```
## [1] "new a = 0.005556"
```

```
print(sprintf("Critical value = %f",critical_value))
```

```
## [1] "Critical value = 4.892638"
```

```
Al2O3 <- aov(Al2O3 ~ kiln, data)
Fe2O3 <- aov(Fe2O3 ~ kiln, data)
MgO <- aov(MgO ~ kiln, data)
CaO <- aov(CaO ~ kiln, data)
Na2O <- aov(Na2O ~ kiln, data)
K2O <- aov(K2O ~ kiln, data)
TiO2 <- aov(TiO2 ~ kiln, data)
MnO <- aov(MnO ~ kiln, data)
BaO <- aov(BaO ~ kiln, data)
```

Since the F-statistics have not changed, we can reuse the F-statistics obtained in Q1. We do not output the result of the above code here

```
name <- c("Al2O3","Fe2O3","MgO","CaO","Na2O","K2O","TiO2","MnO","BaO")

for (i in 1:9){
  print(paste(name[i],"'s F-statistics:",signif(f_value[i],3), if (f_value[i] > critical_value)", F-sta
}
```

```
## [1] "Al2O3 's F-statistics: 26 , F-statistics > Critical value, rejected H0"
## [1] "Fe2O3 's F-statistics: 154 , F-statistics > Critical value, rejected H0"
## [1] "MgO 's F-statistics: 97.8 , F-statistics > Critical value, rejected H0"
## [1] "CaO 's F-statistics: 53.5 , F-statistics > Critical value, rejected H0"
## [1] "Na2O 's F-statistics: 10.5 , F-statistics > Critical value, rejected H0"
## [1] "K2O 's F-statistics: 81.8 , F-statistics > Critical value, rejected H0"
## [1] "TiO2 's F-statistics: 14.7 , F-statistics > Critical value, rejected H0"
## [1] "MnO 's F-statistics: 52.8 , F-statistics > Critical value, rejected H0"
## [1] "BaO 's F-statistics: 0.459 , F-statistics < Critical value, accepted H0"
```

After using Bonferroni correction method,the end result is that BaO support H0. All others still reject H0. Therefore, the chemical concentration of BaO at each site can be considered as not significantly different. The remaining compound concentrations at each site can be considered to have significant differences

Question 3

Apply the Benjamini-Hochberg method to test the nine hypotheses $H_0^{(k)}$: $\mu_{1k} = \mu_{2k} = \mu_{4k} = \mu_{5k}$, k=1,. . . ,9 simultaneously with FDR controlled at $\alpha = 5\%$. Summarize your conclusions.

Answer:

```
result <- manova(as.matrix(data[, 1:9]) ~ kiln, data = data)
result <- summary.aov(result)
p_value <- c()
for (i in 1:9){
  p_value <- c(p_value, (result[[i]]$`Pr(>F)`)[1])
}

p_sorted <- sort(p_value)
for (i in 1:length(p_sorted)){
  if (p_sorted[i] <= i / length(p_sorted) * 0.05){
    k <- i
  }
}
new_a2 <- p_sorted[k]
new_critical_value = qf(new_a2, 3, 39, lower.tail=FALSE)

for (i in 1:9){
  print(paste(name[i],"'s p_value:",signif(p_value[i],3), if (p_value[i] <= new_a2)", p_value <= a, rej
}
```

```
## [1] "Al2O3 's p_value: 2.08e-09 , p_value <= a, rejected H0"
## [1] "Fe2O3 's p_value: 1.13e-21 , p_value <= a, rejected H0"
## [1] "MgO 's p_value: 3.43e-18 , p_value <= a, rejected H0"
## [1] "CaO 's p_value: 6.88e-14 , p_value <= a, rejected H0"
## [1] "Na2O 's p_value: 3.48e-05 , p_value <= a, rejected H0"
## [1] "K2O 's p_value: 7.13e-17 , p_value <= a, rejected H0"
## [1] "TiO2 's p_value: 1.52e-06 , p_value <= a, rejected H0"
## [1] "MnO 's p_value: 8.56e-14 , p_value <= a, rejected H0"
## [1] "BaO 's p_value: 0.712 , p_value > a, accepted H0"
```

After using Benjamini-Hochberg method, the new a is 3.480171e-05.
The end result is that BaO support H0. All others still reject H0.
Therefore, the chemical concentration of BaO at each site can be considered as not significantly different.
The remaining compound concentrations at each site can be considered to have significant differences.
The results obtained through this methodare not different from the previous ones.