

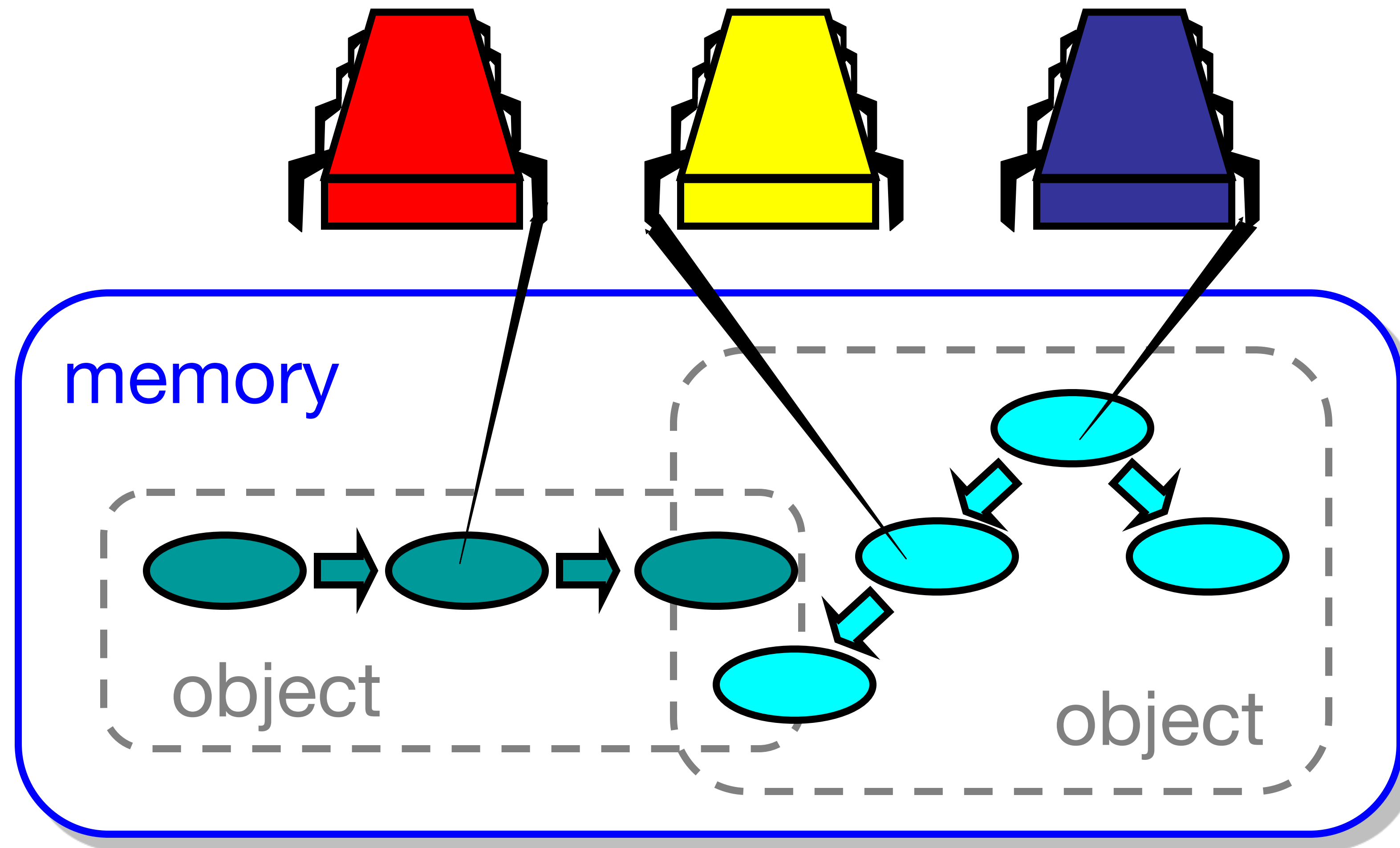
03 Concurrent Objects

CS 6868: Concurrent Programming

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Spring 2026, IIT Madras

Concurrent Computation



Objectivism

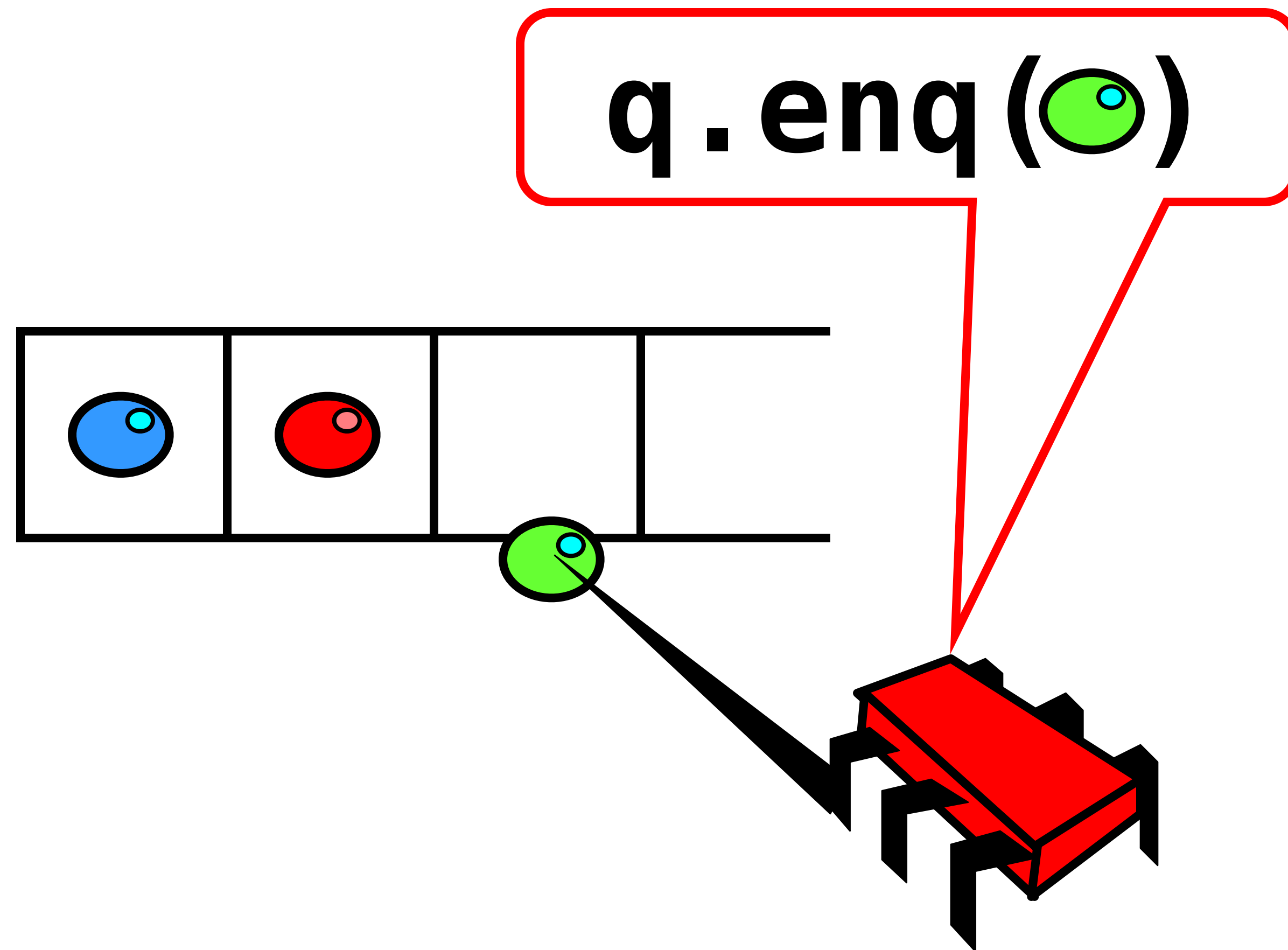
- What is a concurrent object?
 - How do we *describe* one?
 - How do we *implement* one?
 - How do we *tell if we're right*?

Objectivism

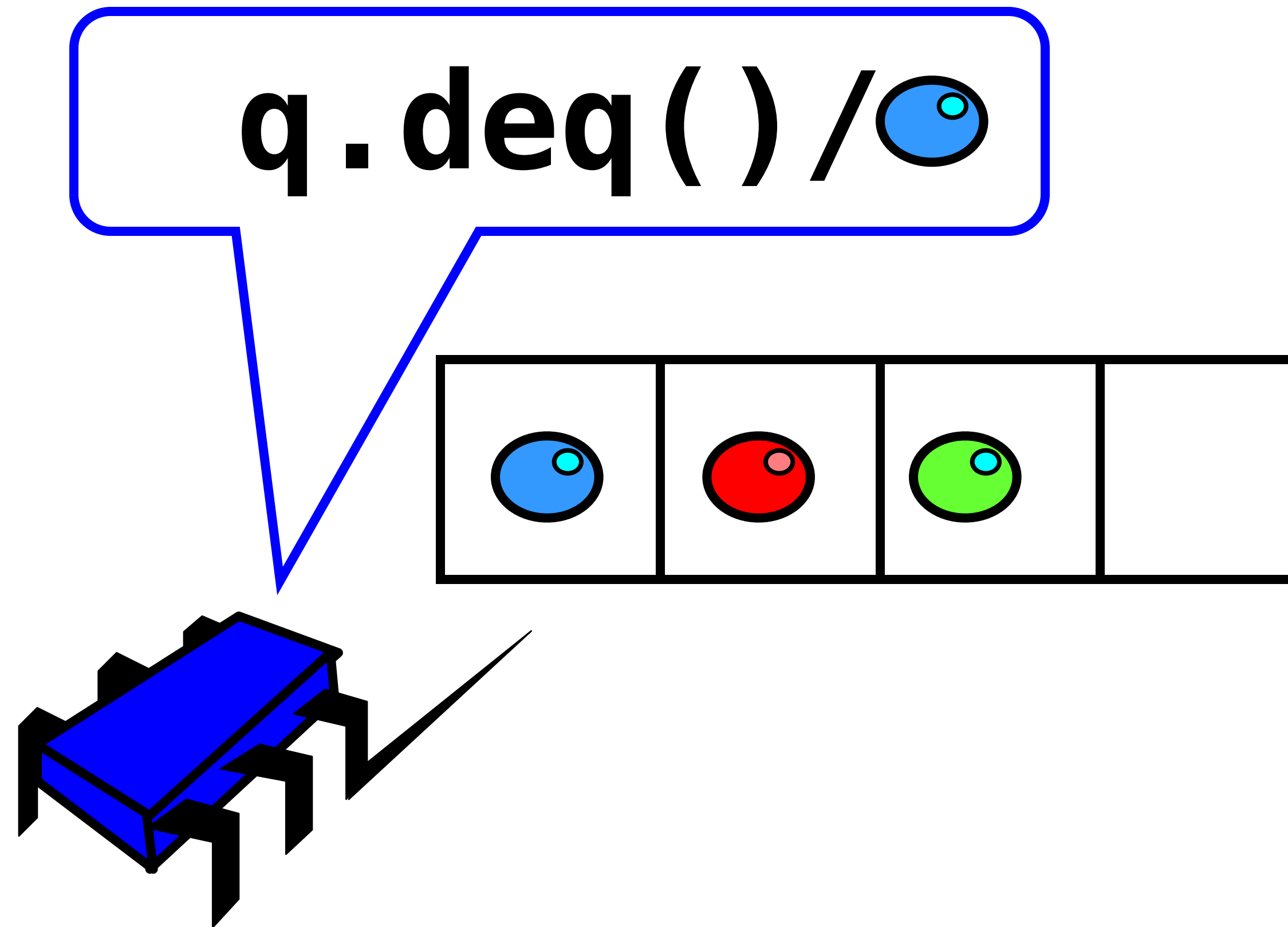
- What is a concurrent object?
 - How do we *describe* one?
 - ~~How do we *implement* one?~~
 - How do we *tell if we're right*?

Concurrent Queues

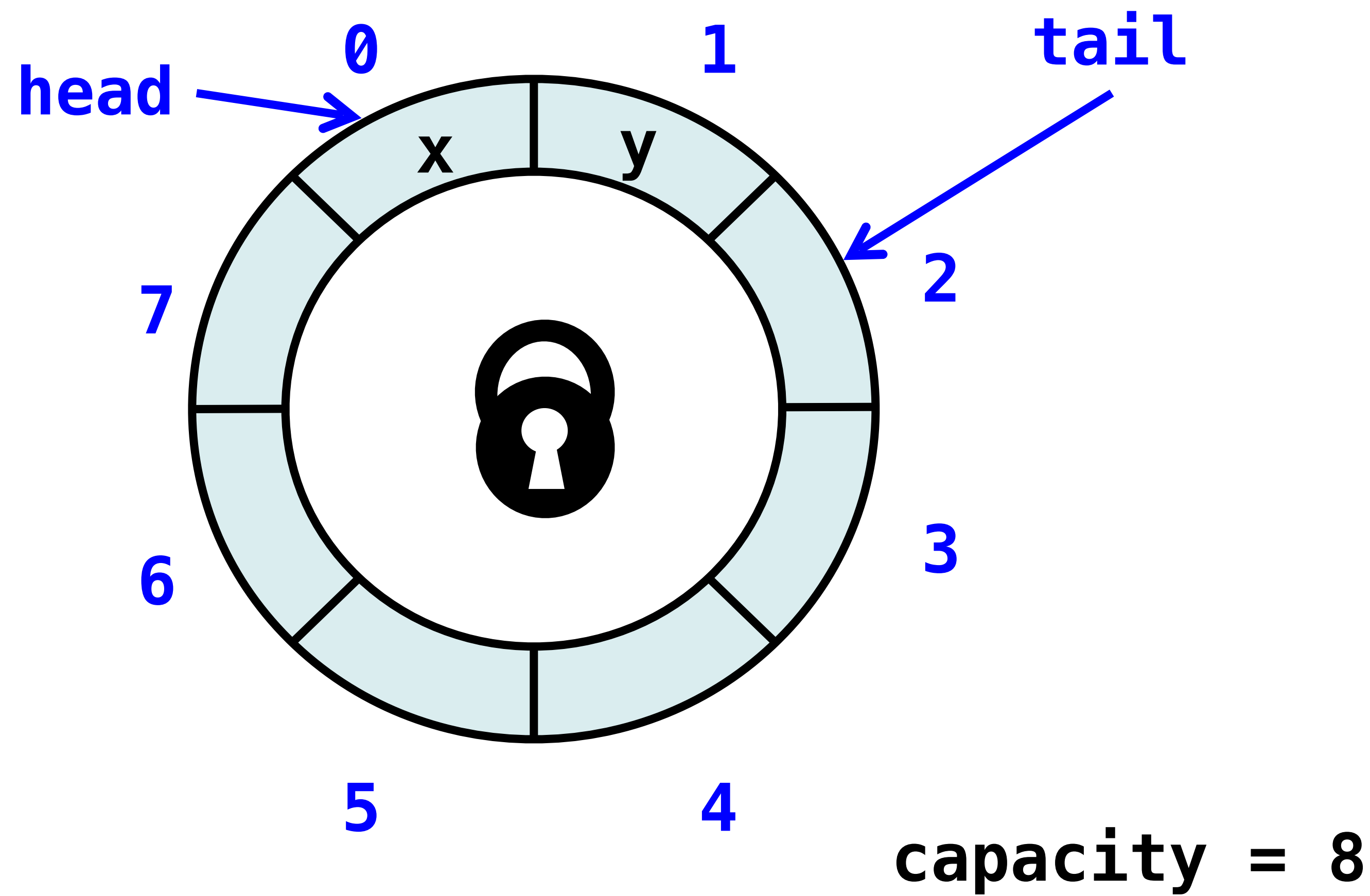
FIFO Queue – Enqueue method



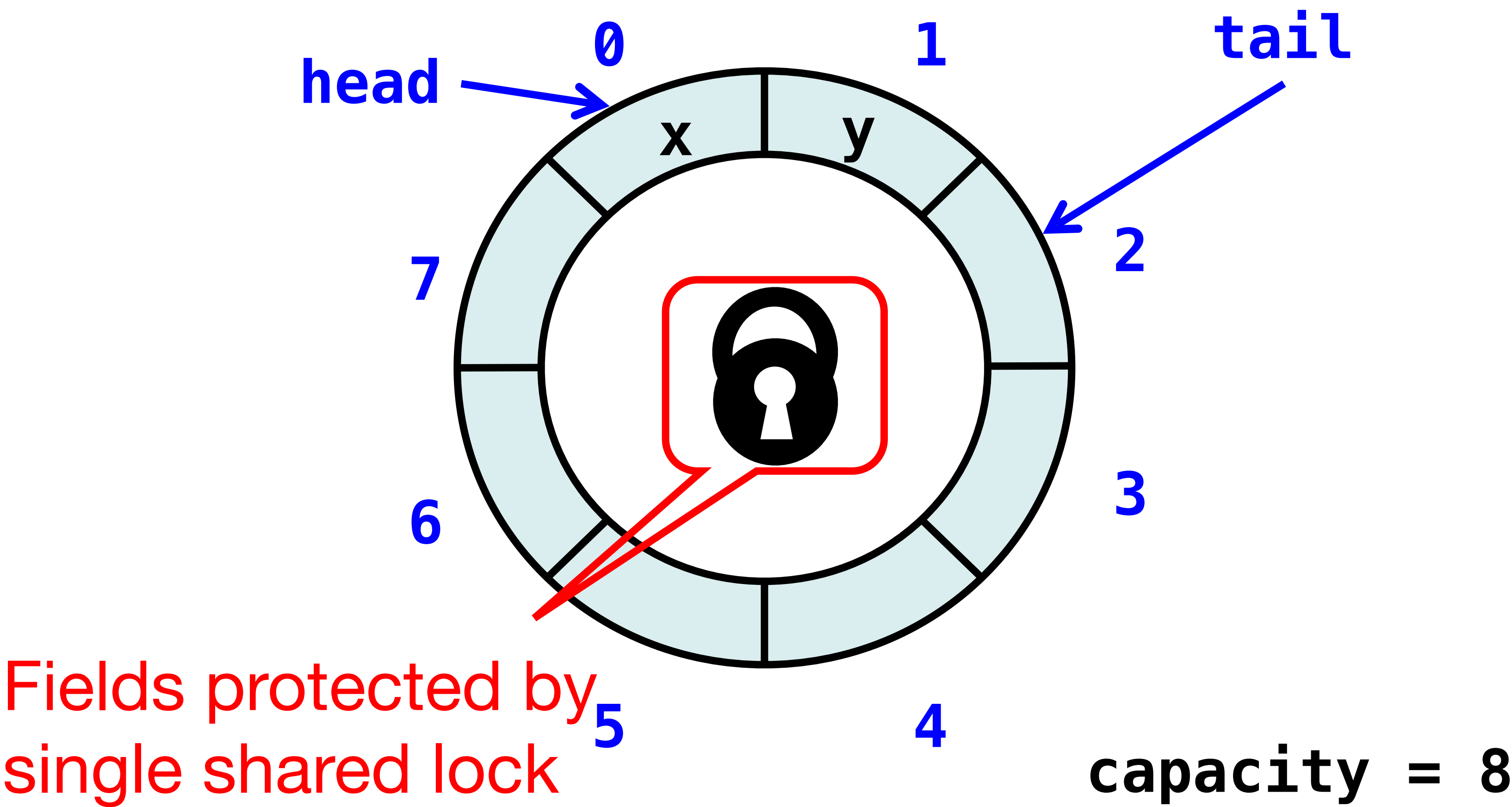
FIFO Queue – Dequeue method



Lock-based Queue



Lock-based Queue



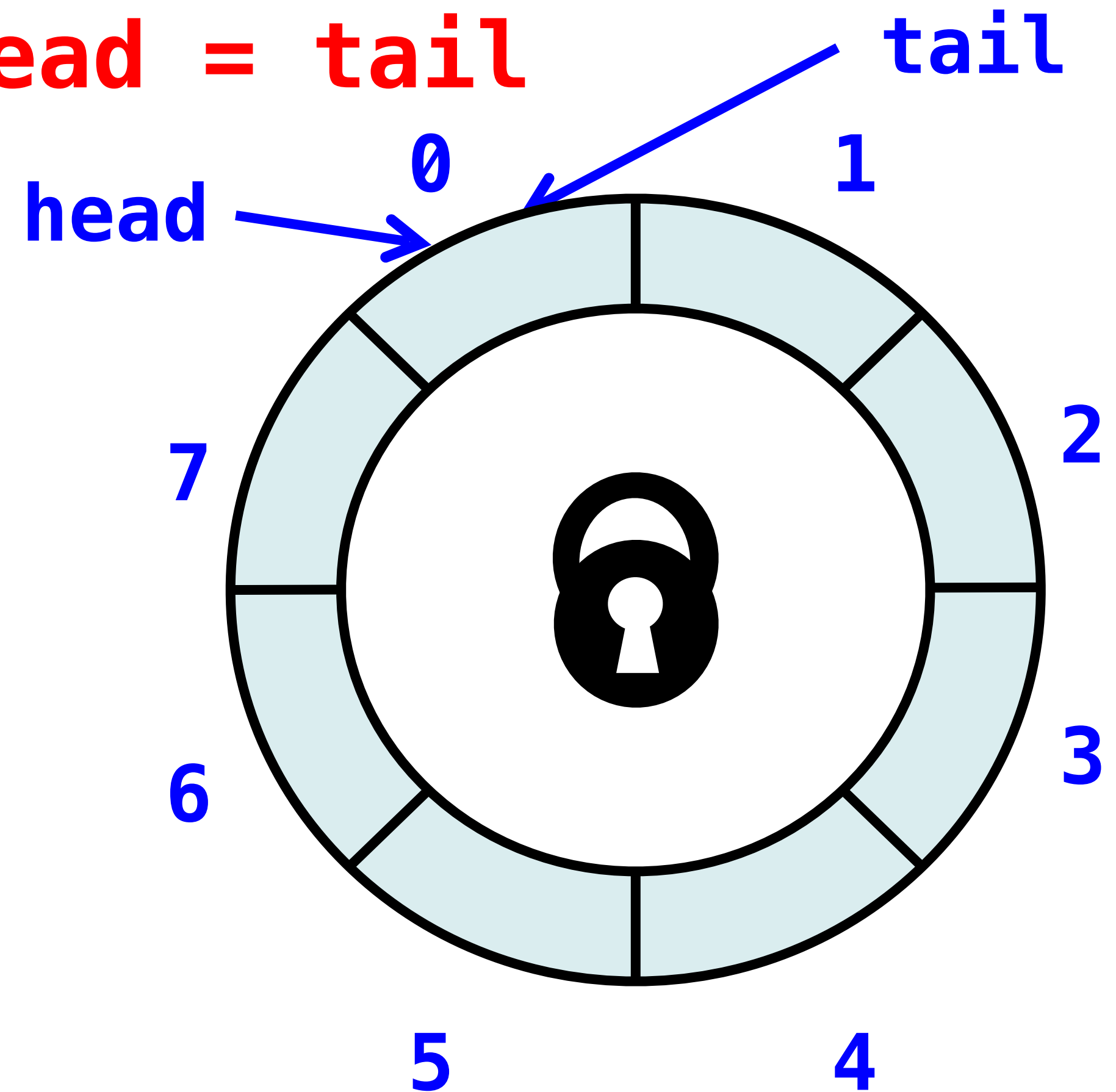
Lock-based Queue

```
exception Full  
exception Empty
```

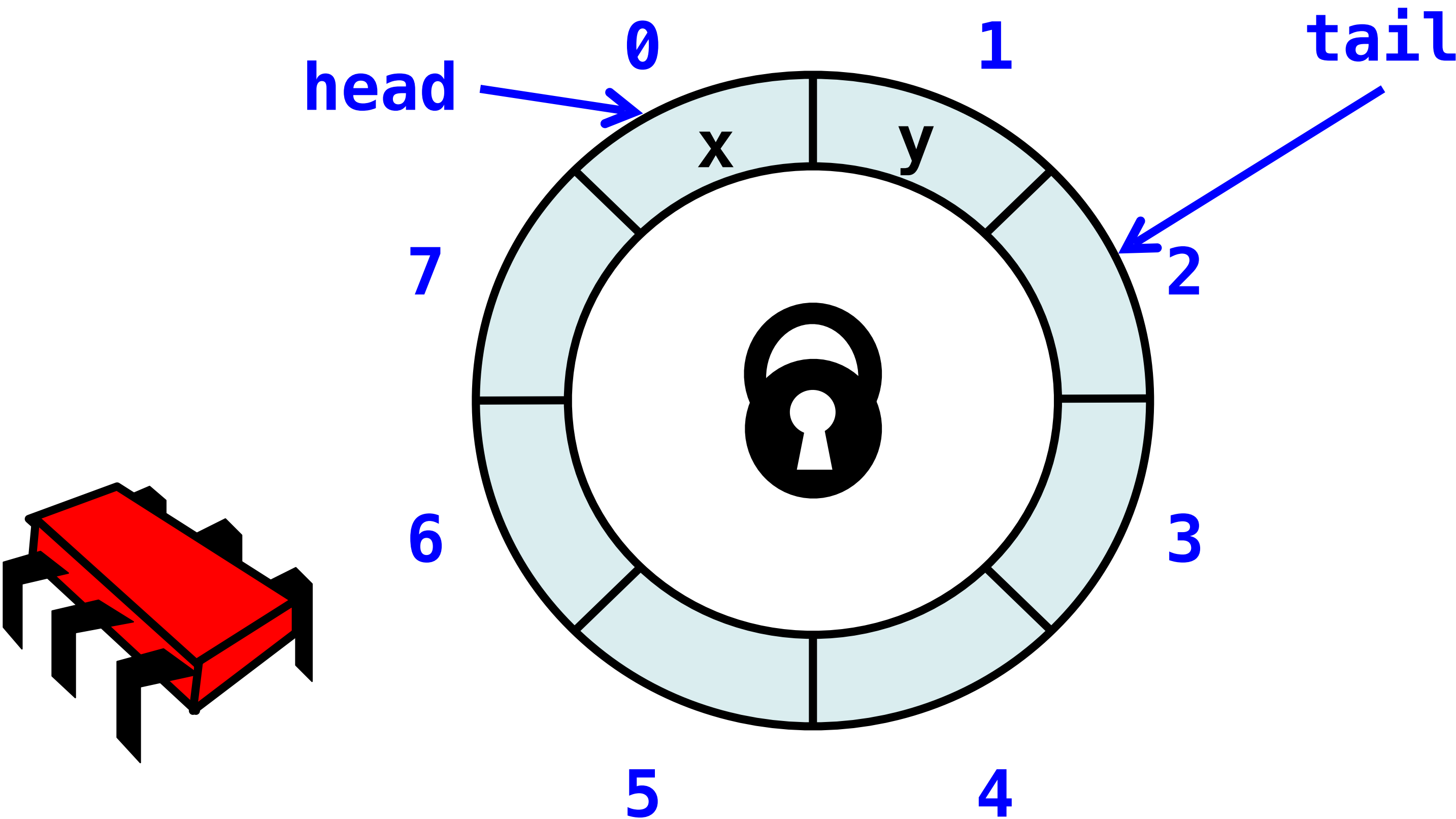
```
type 'a t = {  
  items : 'a option array;  
  capacity : int;  
  mutable head : int;  
  mutable tail : int;  
  lock : Mutex.t;  
}
```

```
let create capacity =  
  {  
    items = Array.make capacity None;  
    capacity;  
    head = 0;  
    tail = 0;  
    lock = Mutex.create ();  
  }
```

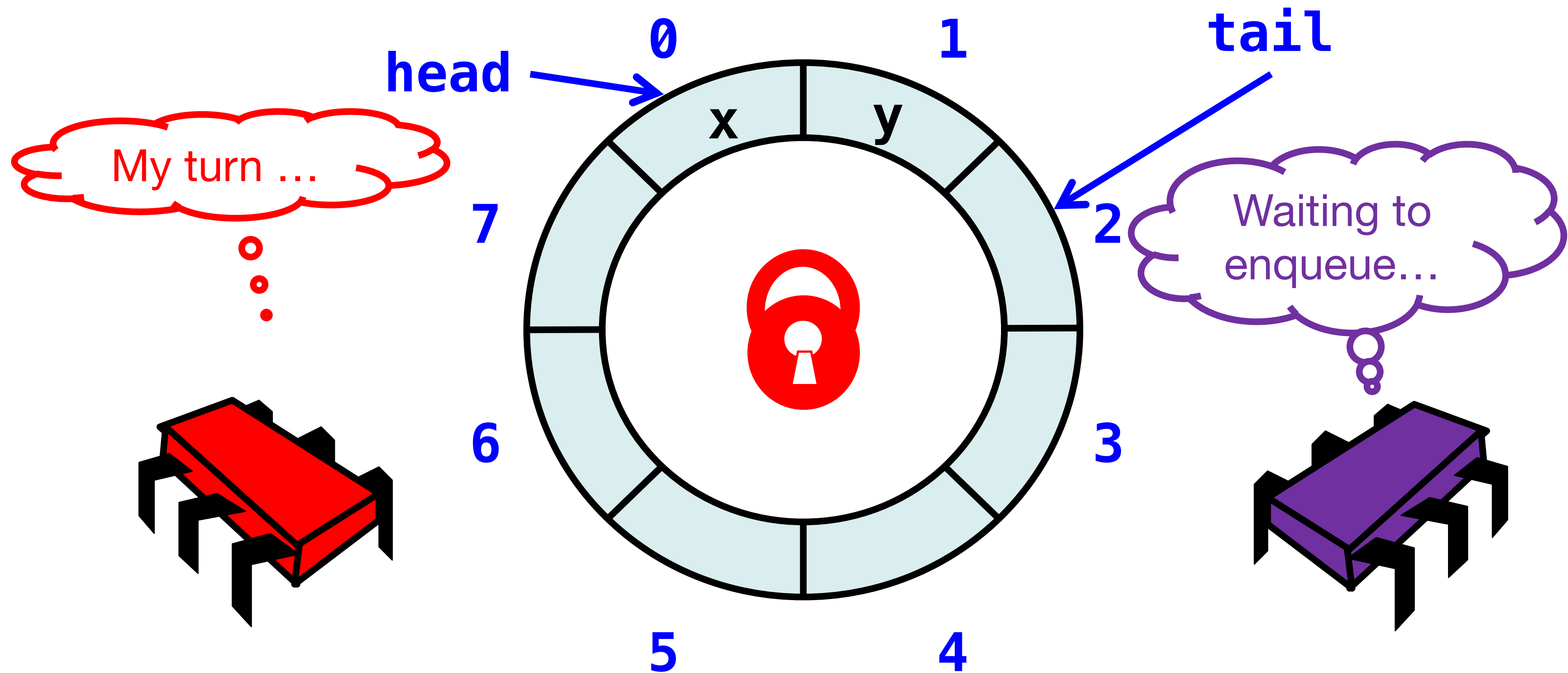
Initially: **head = tail**



Lock-based Queue — `deq()` operation



Acquire Lock

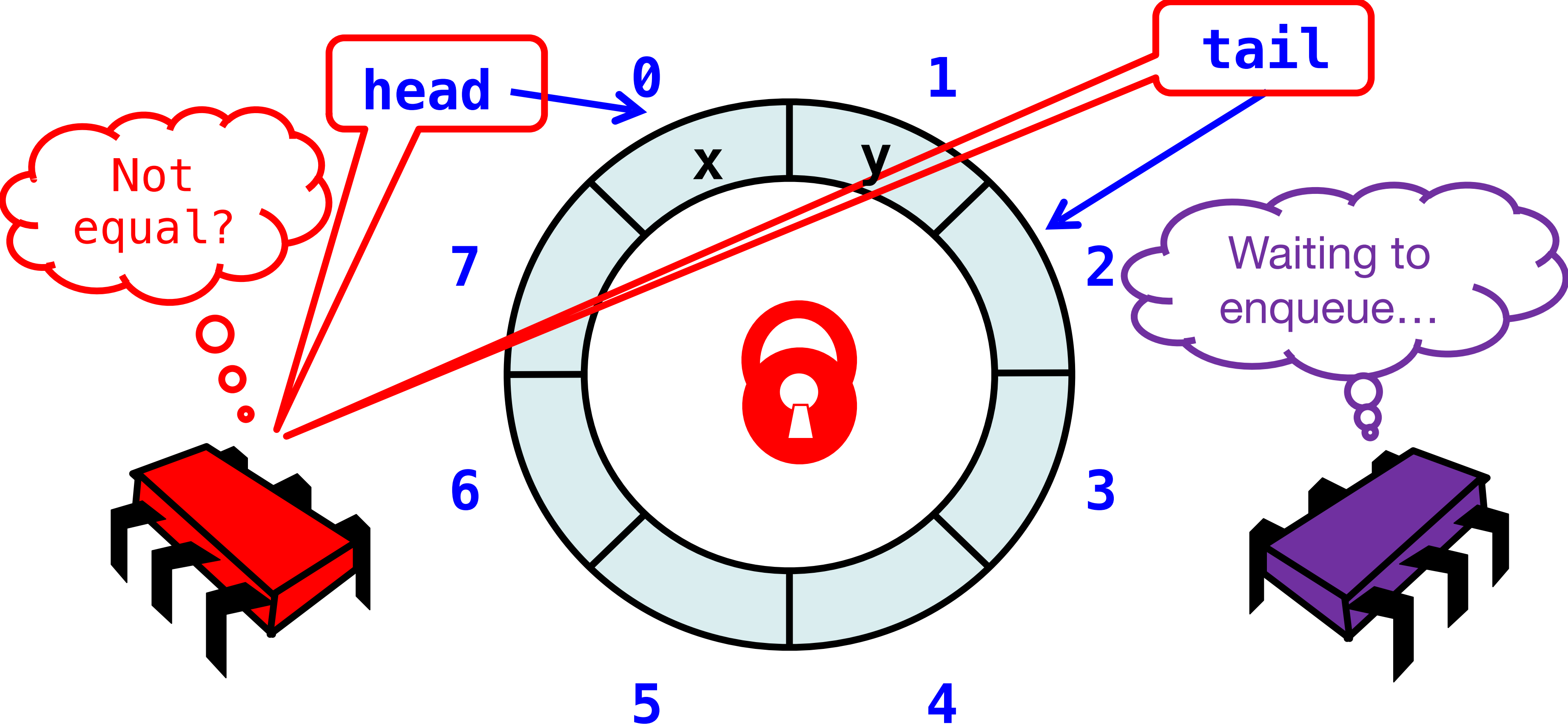


Acquire Lock

```
let def q =  
  Mutex.lock q.lock;  
  try  
    if q.tail = q.head then  
      raise Empty;  
    match q.items.(q.head mod q.capacity) with  
    | None -> assert false  
    | Some x ->  
      q.head <- q.head + 1;  
      Mutex.unlock q.lock;  
      x  
  with e ->  
    Mutex.unlock q.lock;  
    raise e
```

Acquire lock at
method start

Check if non empty



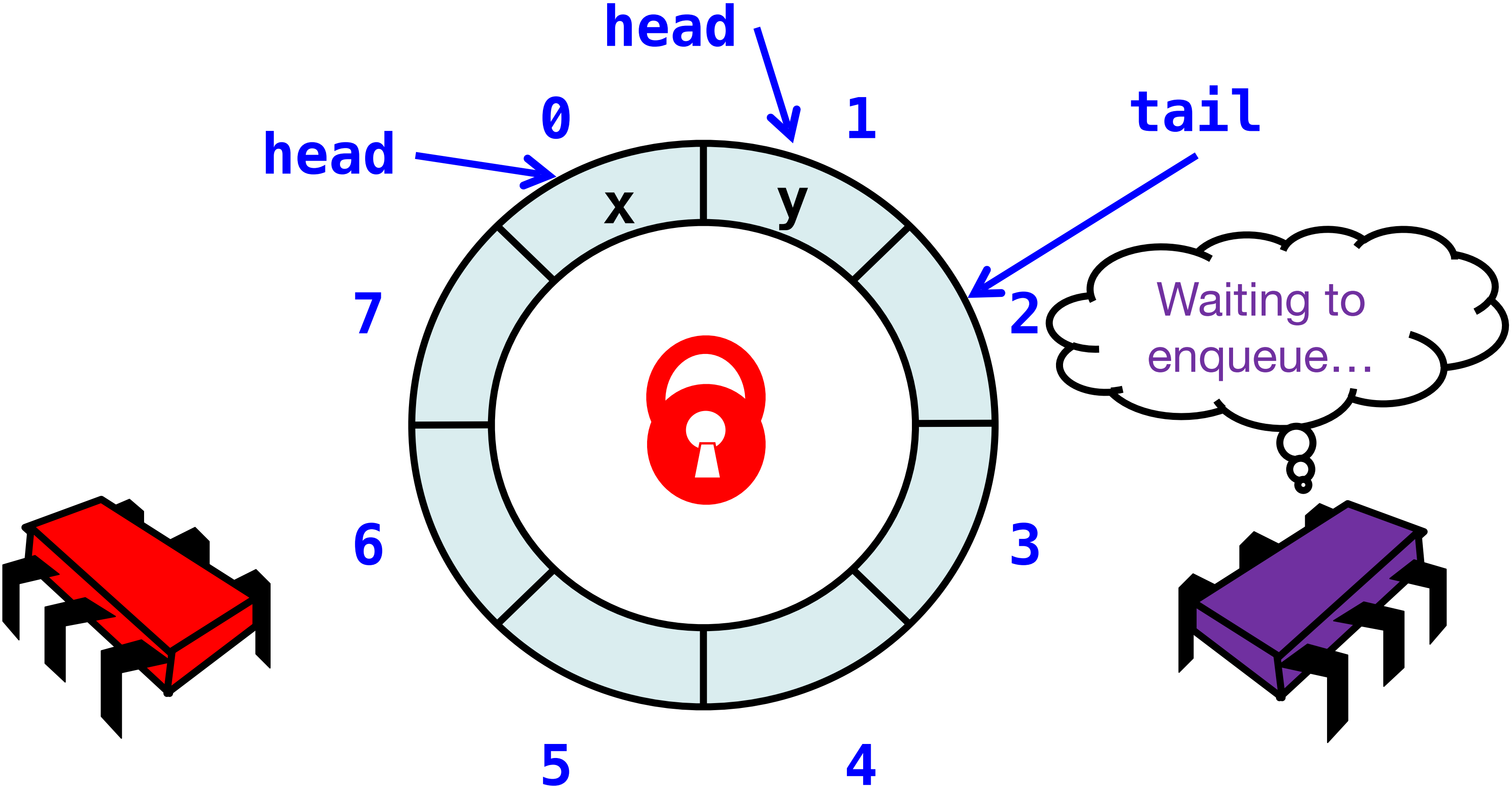
Check if non empty

```
let def q =  
  Mutex.lock q.lock;  
  try  
    if q.tail = q.head then  
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      q.head <- q.head + 1;  
      Mutex.unlock q.lock;  
      x  
  with e ->  
    Mutex.unlock q.lock;  
    raise e
```

If queue empty
throw exception

In case of
exceptions,
lock released
here

Modify the queue



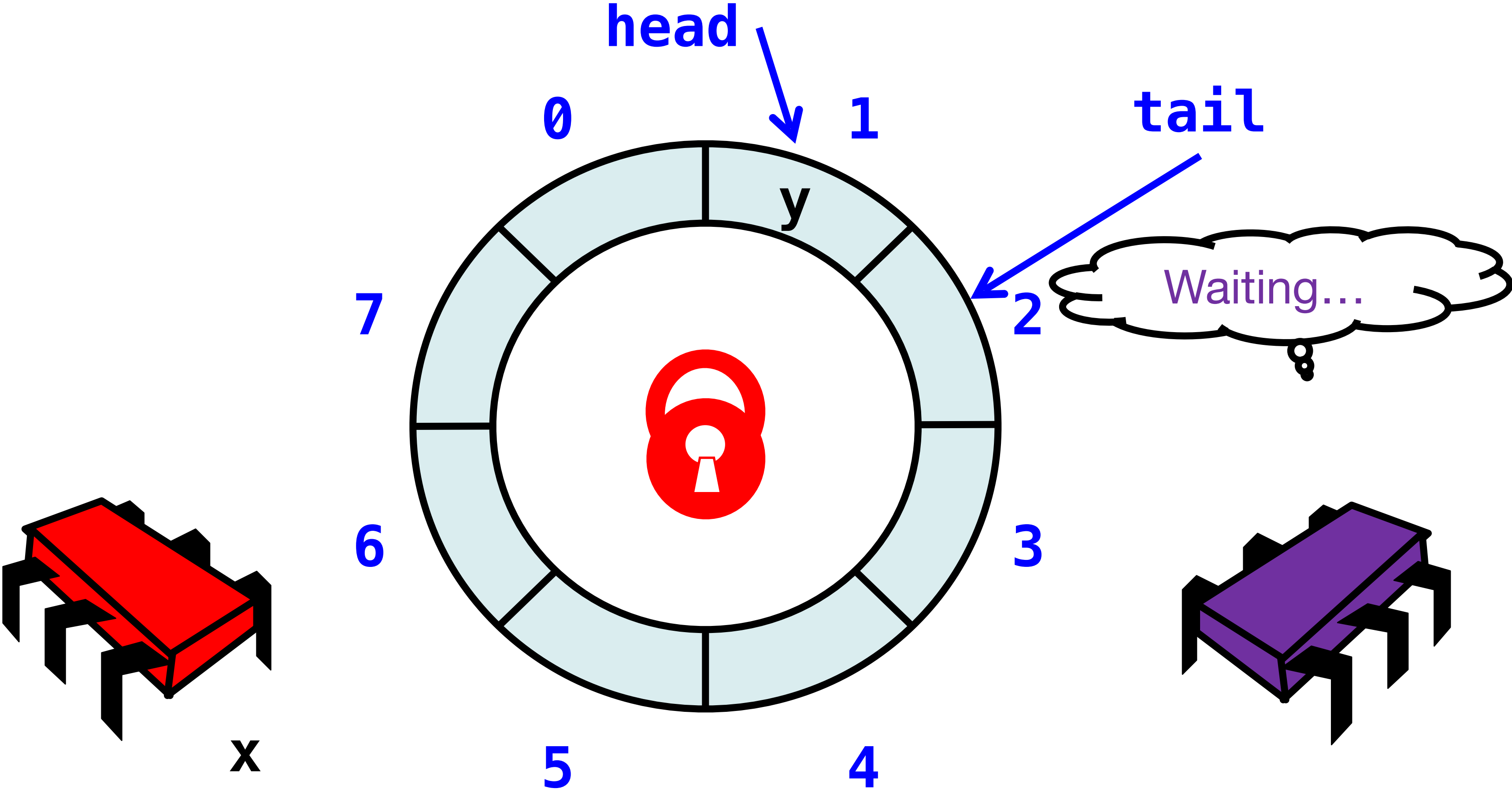
Modify the queue

```
let def q =  
  Mutex.lock q.lock;  
  try  
    if q.tail = q.head then  
      raise Empty;  
    match q.items.(q.head mod q.capacity) with  
    | None -> assert false  
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      q.head <- q.head + 1;  
      Mutex.unlock q.lock;  
      x  
  with e ->  
    Mutex.unlock q.lock;  
    raise e
```

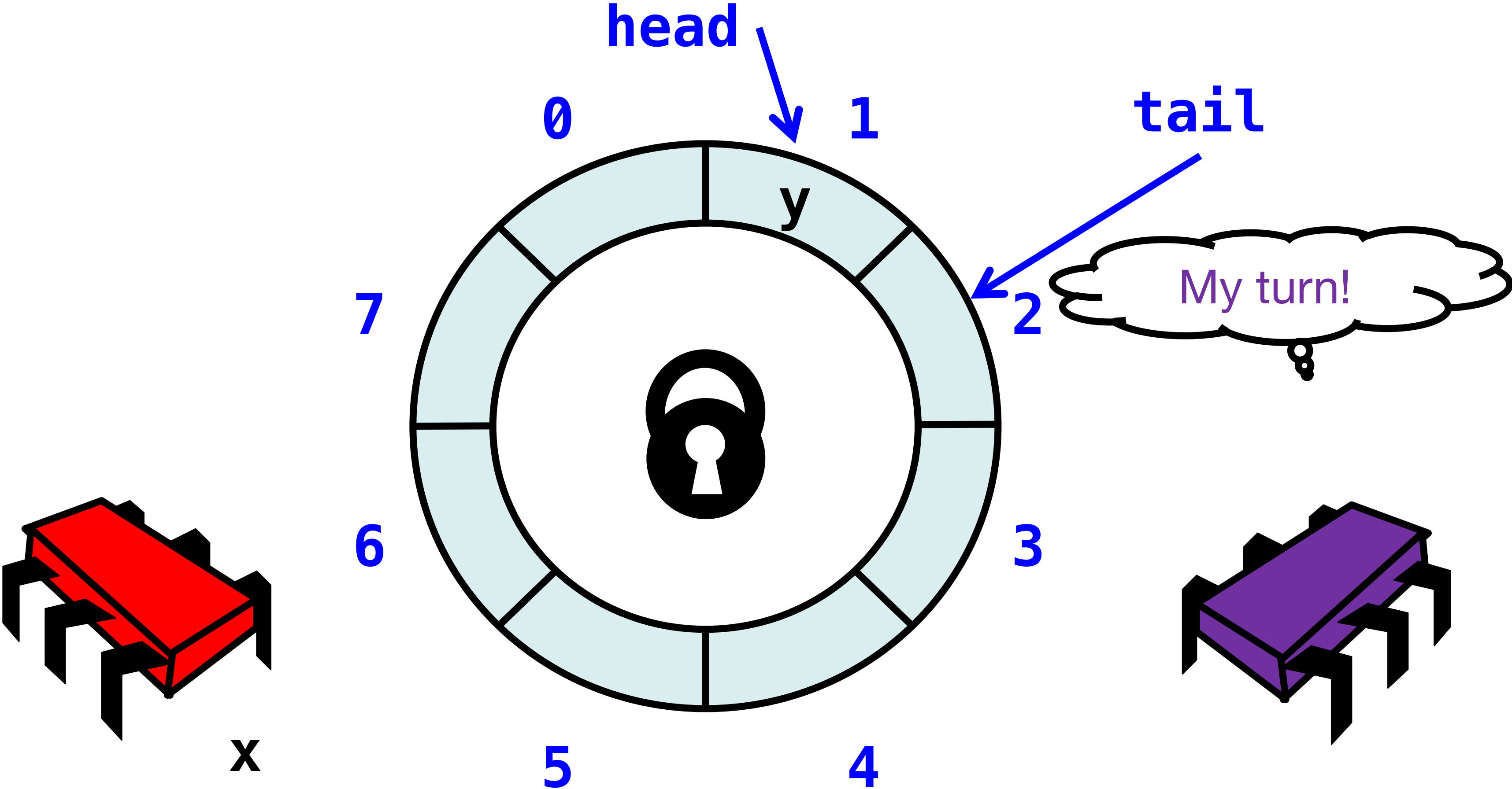
Queue not empty?

Remove item “x” and update head

Release the lock and return item



Release the lock and return item



Release the lock and return item

```
let def q =  
  Mutex.lock q.lock;  
  try  
    if q.tail = q.head then  
      raise Empty;  
    match q.items.(q.head mod q.capacity) with  
    | None -> assert false  
    | Some x ->  
      q.head <- q.head + 1;  
      Mutex.unlock q.lock;  
      x  
  with e ->  
    Mutex.unlock q.lock;  
    raise e
```

Unlock and return item “x”

Implementation — `deq()`

```
let deq q =  
  Mutex.lock q.lock;  
  try  
    if q.tail = q.head then  
      raise Empty;  
    match q.items.(q.head mod q.capacity) with  
    | None -> assert false  
    | Some x ->  
      q.head <- q.head + 1;  
      Mutex.unlock q.lock;  
      x  
  with e ->  
    Mutex.unlock q.lock;  
    raise e
```

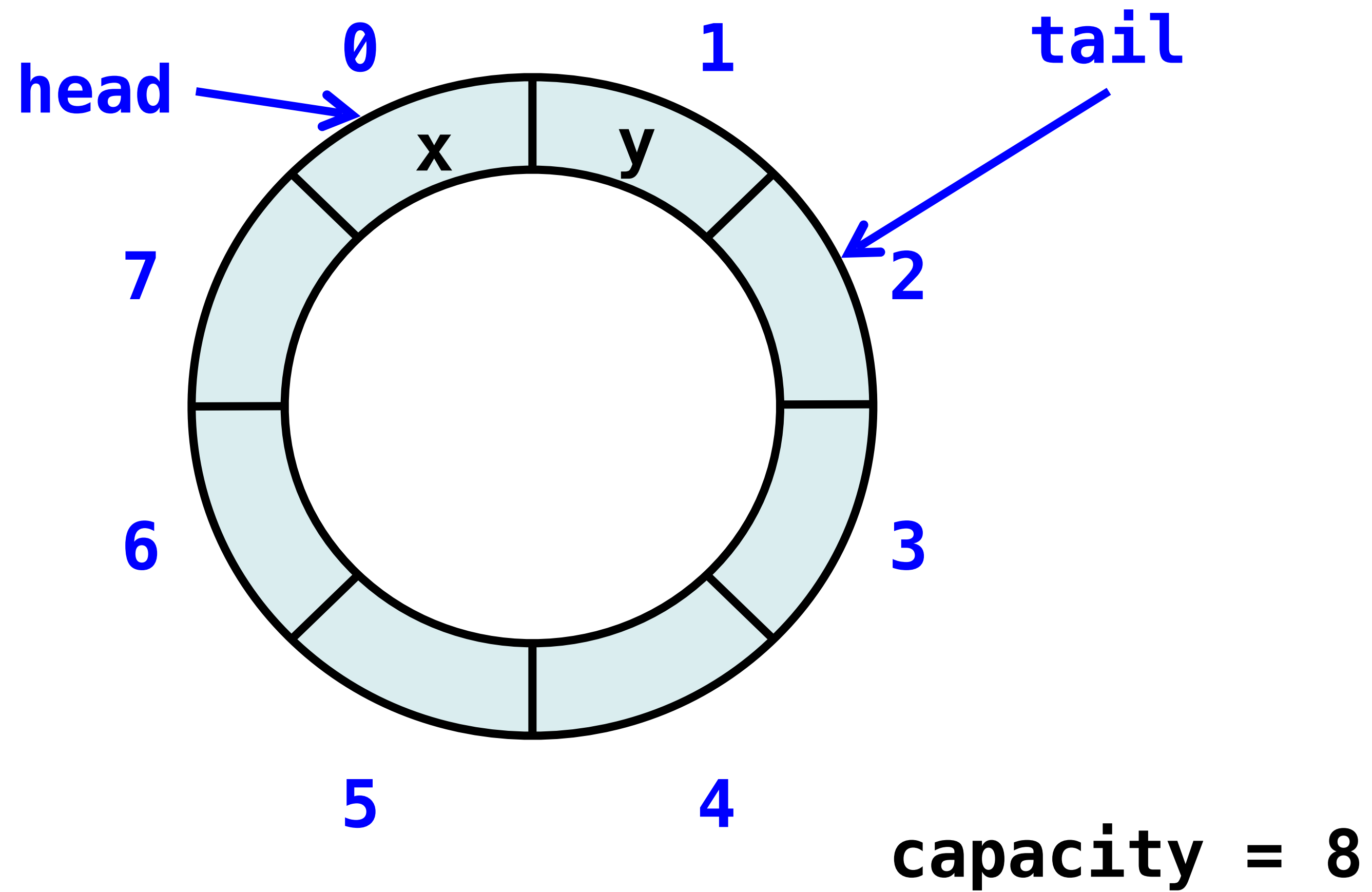
Should be correct because
modifications are mutually exclusive...

Demo

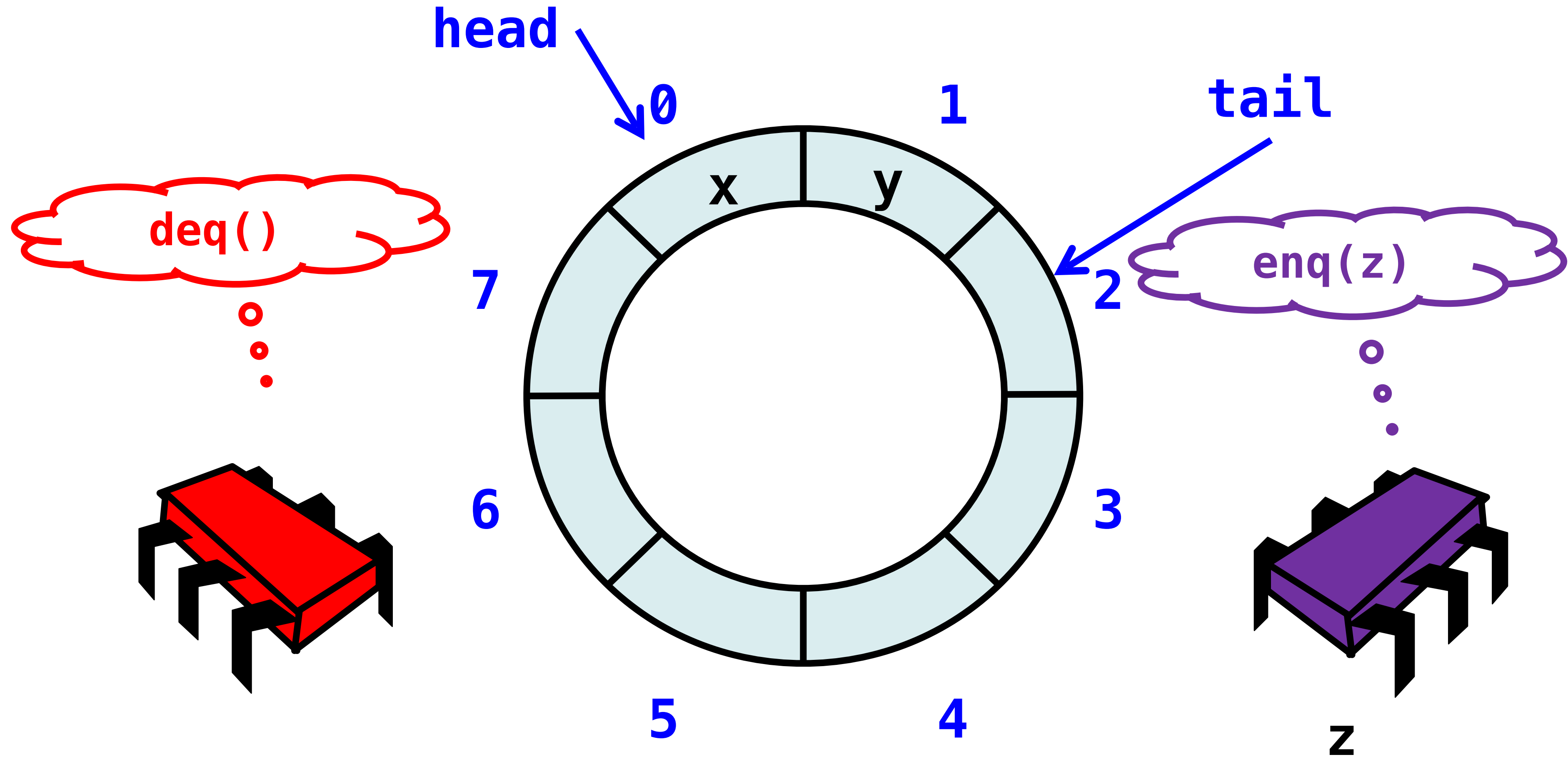
Consider the following implementation

- The same thing without mutual exclusion
- For simplicity, only **two** threads
 - One thread **enq only**
 - The other **deq only**

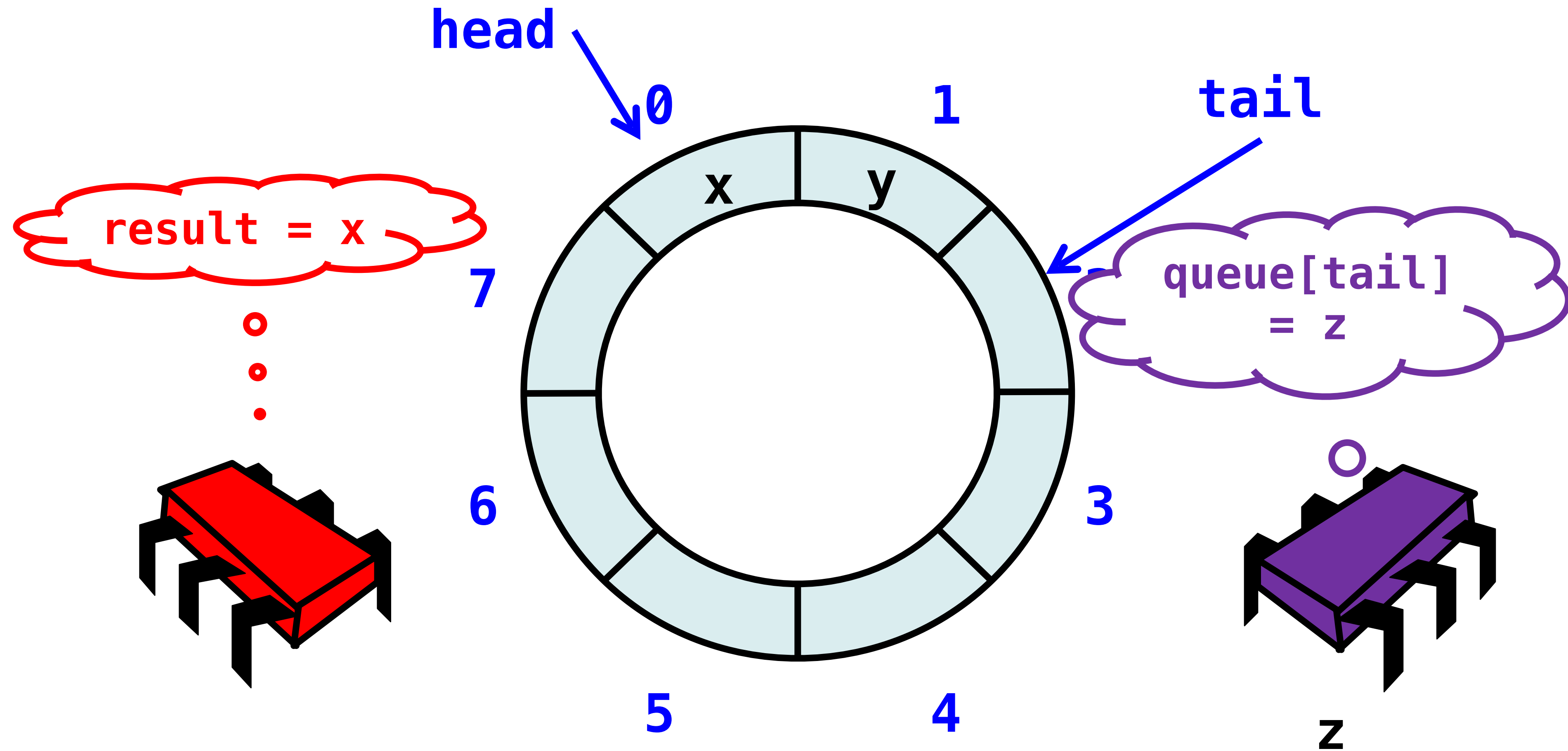
Wait-free 2-thread queue



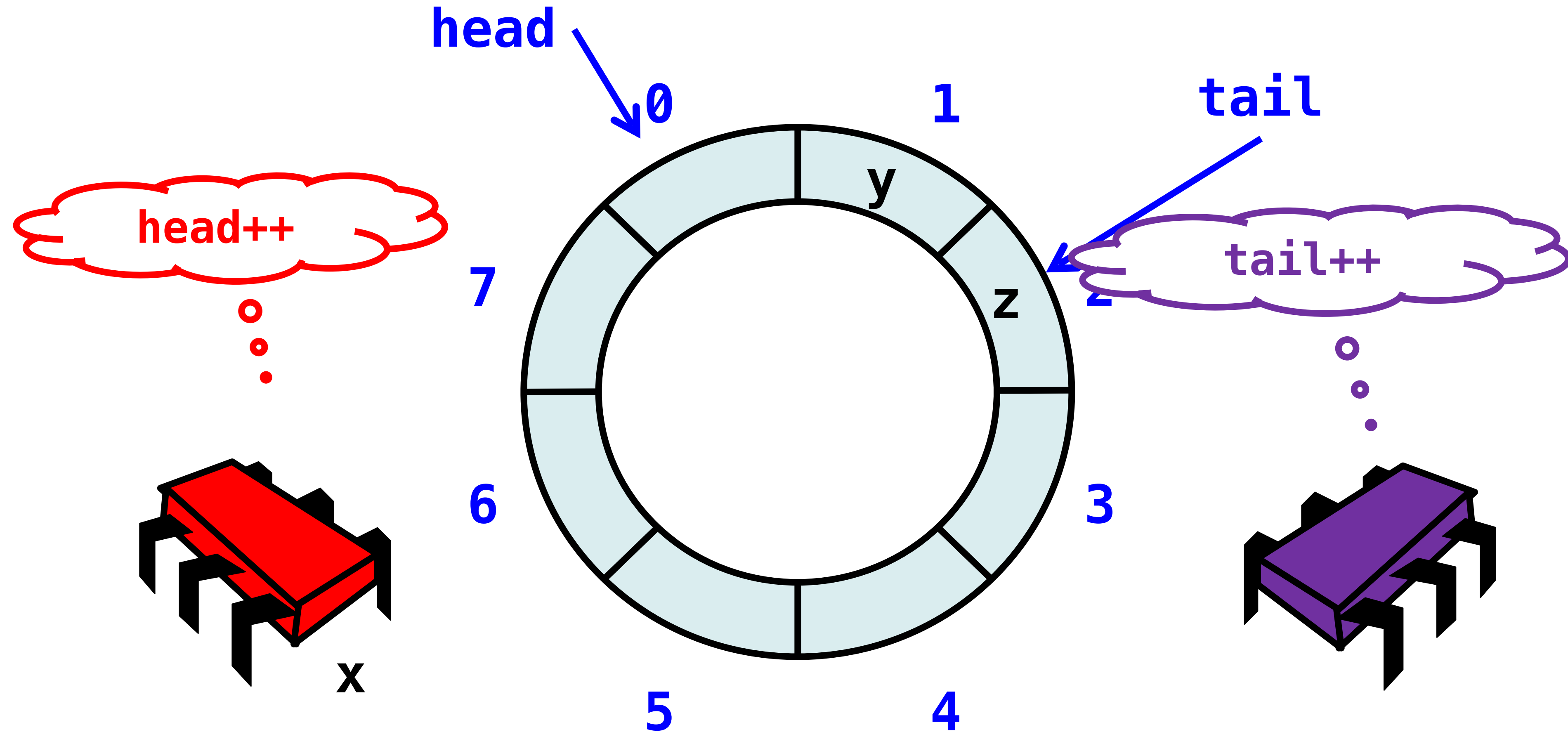
Wait-free 2-thread queue



Wait-free 2-thread queue



Wait-free 2-thread queue



Wait-free 2-thread queue

No locks needed!

```
(** Enqueue – should be called by only ONE thread *)
let enq q x =
  (* Check if queue is full *)
  if q.tail - q.head = q.capacity then
    raise Full;
  (* Write to the array *)
  q.items.(q.tail mod q.capacity) <- Some x;
  (* Advance tail *)
  q.tail <- q.tail + 1;

(** Dequeue – should be called by only ONE thread *)
let deq q =
  (* Check if queue is empty *)
  if q.tail = q.head then
    raise Empty;
  (* Read from the array *)
  match q.items.(q.head mod q.capacity) with
  | None -> raise Empty;
  | Some x -> q.head <- q.head + 1;
  | _ -> (* Should never happen *)
```

How do we define “correct” when
modifications are not mutually
exclusive?

Demo

Concurrency Specification

What *is* a concurrent queue?

- Need a way to *specify* a concurrent queue object
- Need a way to *prove* that an algorithm implements the object's specification
- Lets talk about object specifications ...

Correctness and Progress

- In a concurrent setting, we need to specify both the ***safety*** and the ***liveness*** properties of an object
- Need a way to define
 - when an implementation is ***correct***
 - the conditions under which it guarantees ***progress***

Lets begin with correctness

Sequential Objects

- Each object has a ***state***
 - Usually given by a set of ***fields***
 - Queue example: sequence of items
- Each object has a set of ***methods***
 - Only way to manipulate state
 - Queue example: **enq** and **deq** methods

Sequential Specifications

- If (*precondition*)
 - the object is in such-and-such a state
 - before you call the method,
- Then (*postcondition*)
 - the method will return a particular value
 - or throw a particular exception.
- and (*postcondition, cont*)
 - the object will be in some other state
 - when the method returns,

Pre and Post Conditions for Dequeue

- Precondition:
 - Queue is *non-empty*
 - Postcondition:
 - Returns first item in queue
 - Postcondition:
 - Removes first item in queue
- Precondition:
 - Queue is *empty*
 - Postcondition:
 - Raises Empty exception
 - Postcondition:
 - Queue state is unchanged

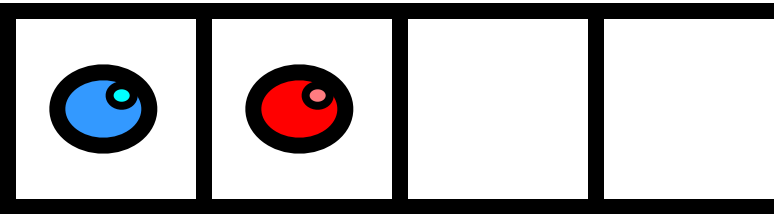
Why Sequential Specifications Totally Rock

- Interactions among *methods* captured by side-effects on object state
 - State meaningful between method calls
- *Documentation* size is linear in the number of methods
 - Each method described in isolation
- Can add *new methods*
 - Without changing descriptions of old methods

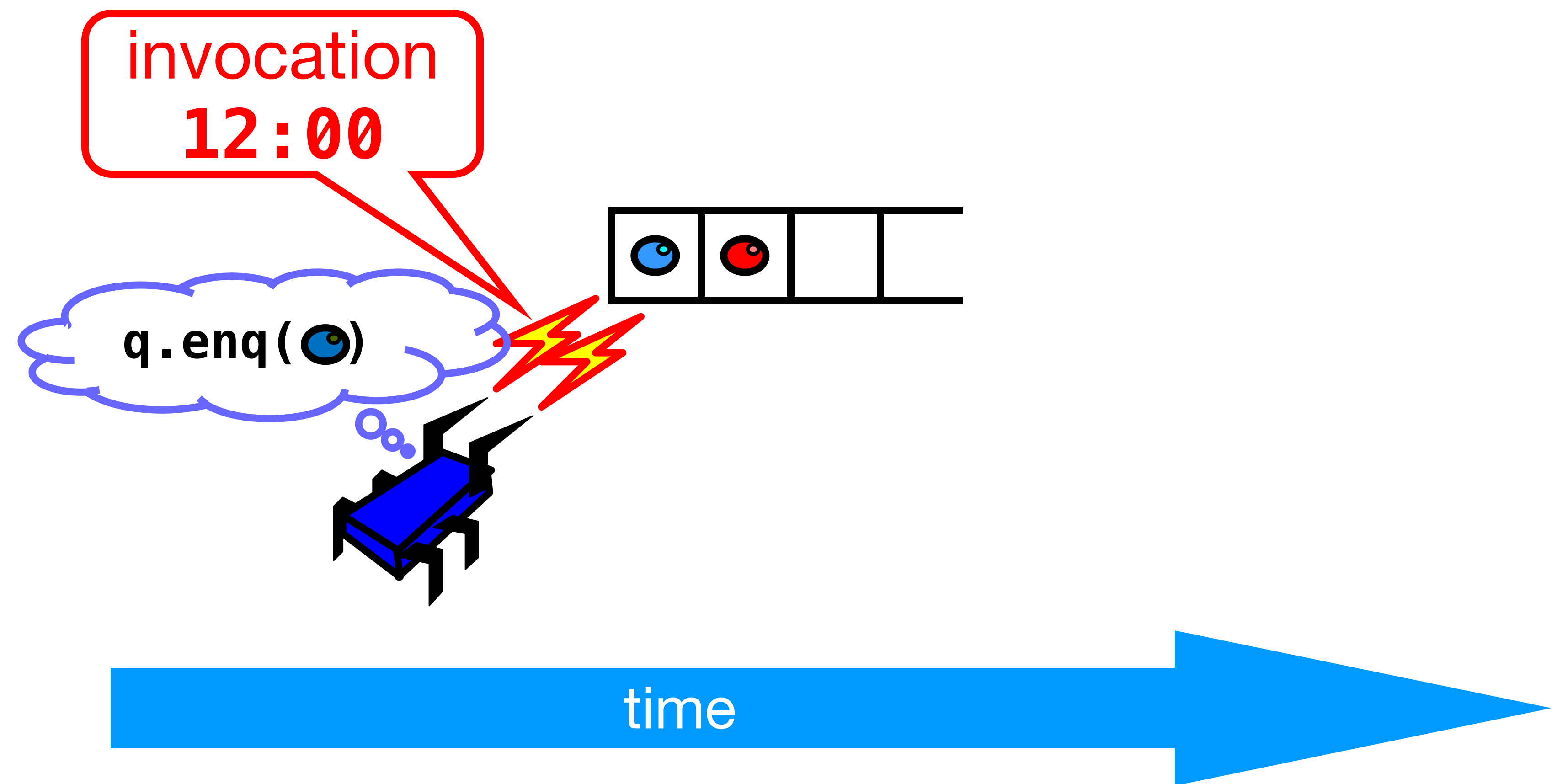
What about concurrent Specifications?

- Methods?
- Documentation?
- Adding new methods?

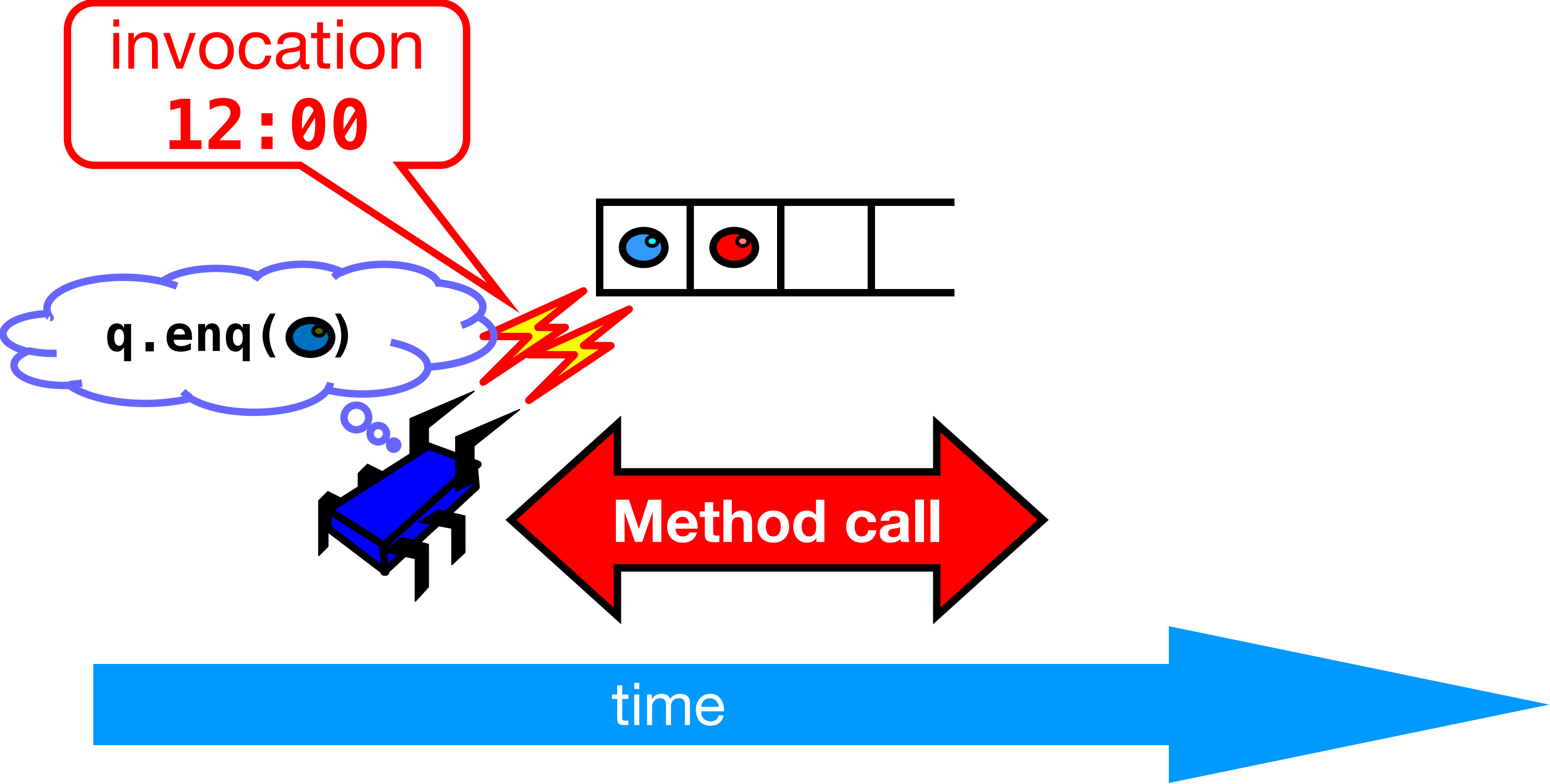
Methods take time



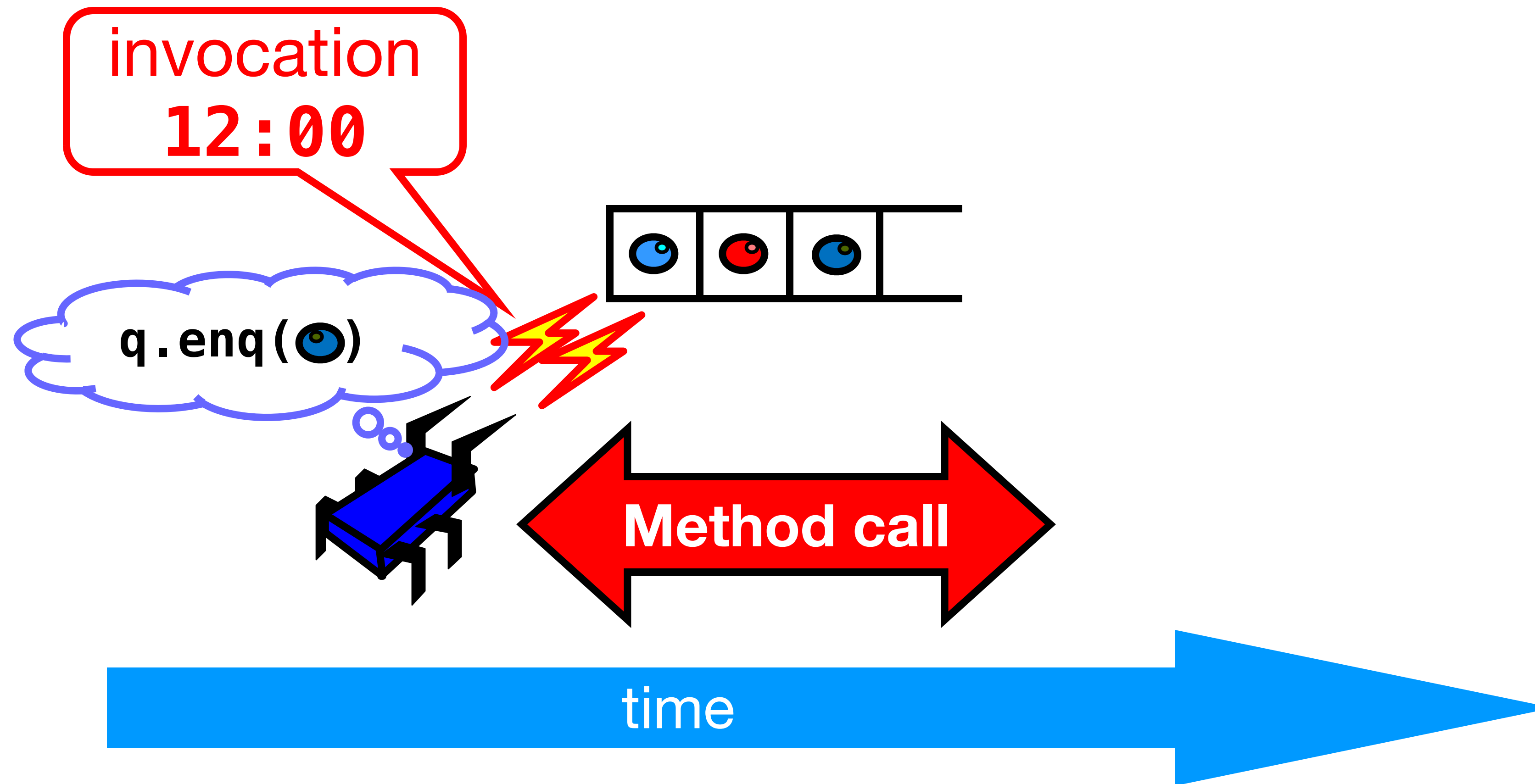
Methods take time



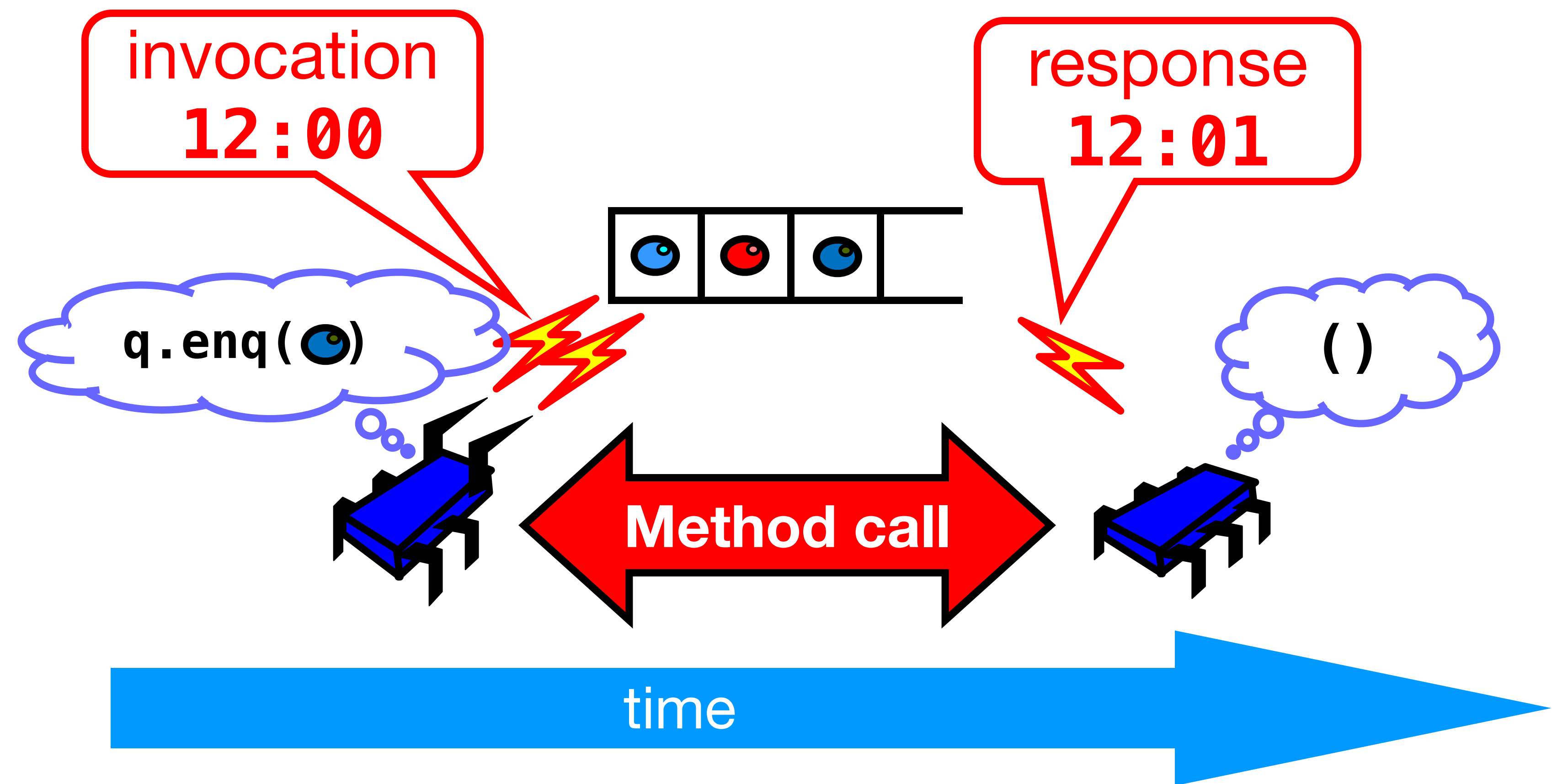
Methods take time



Methods take time



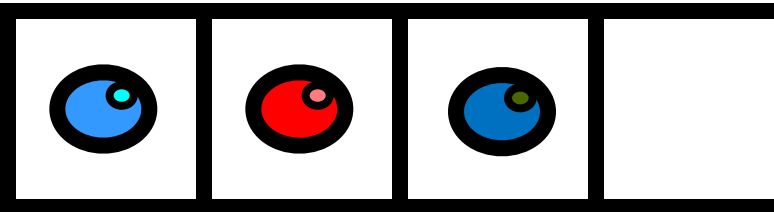
Methods take time



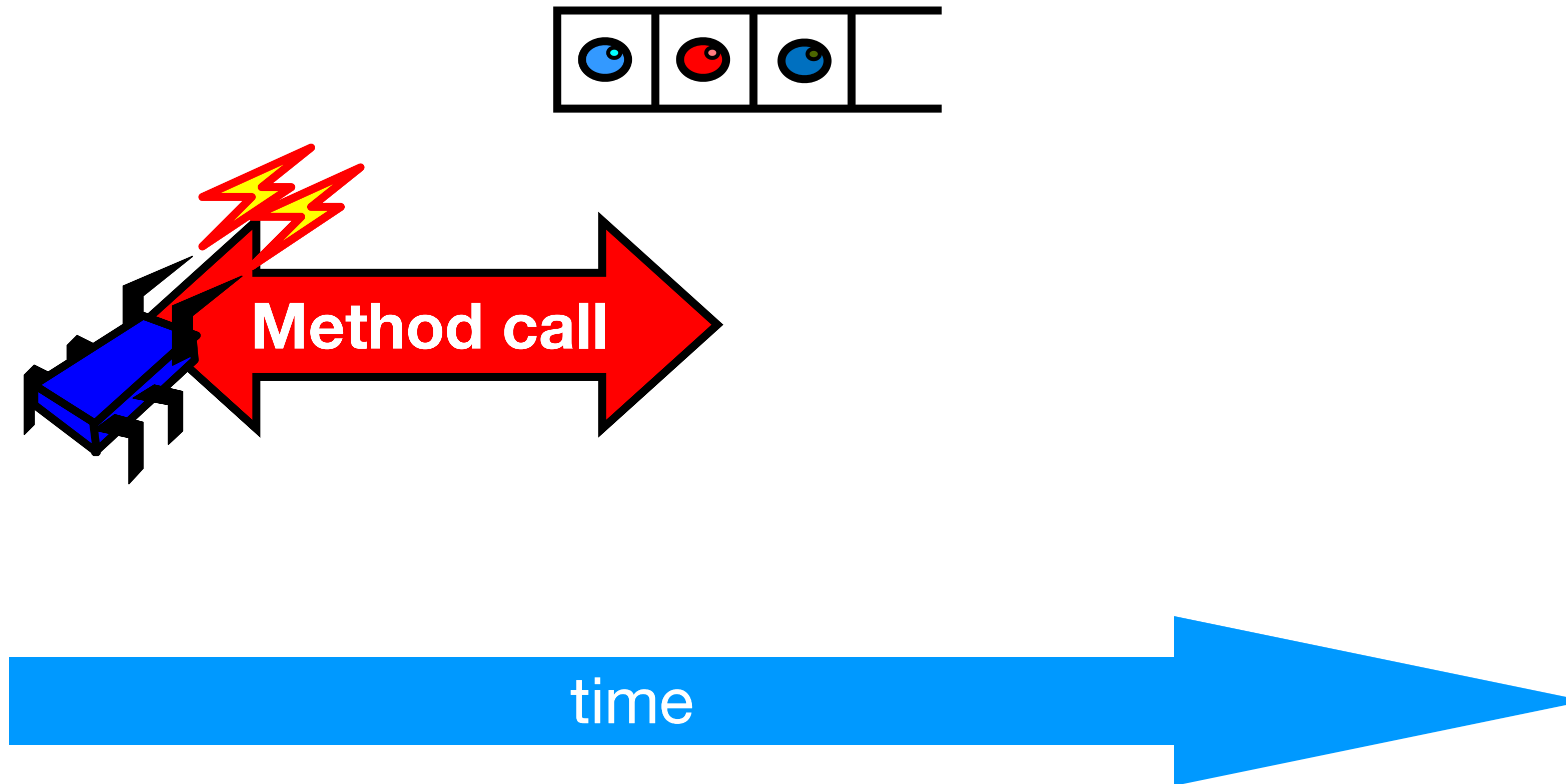
Sequential vs Concurrent

- Sequential
 - Methods take time? Who knew?
- Concurrent
 - Method call is not an *event*
 - Method call is an *interval*

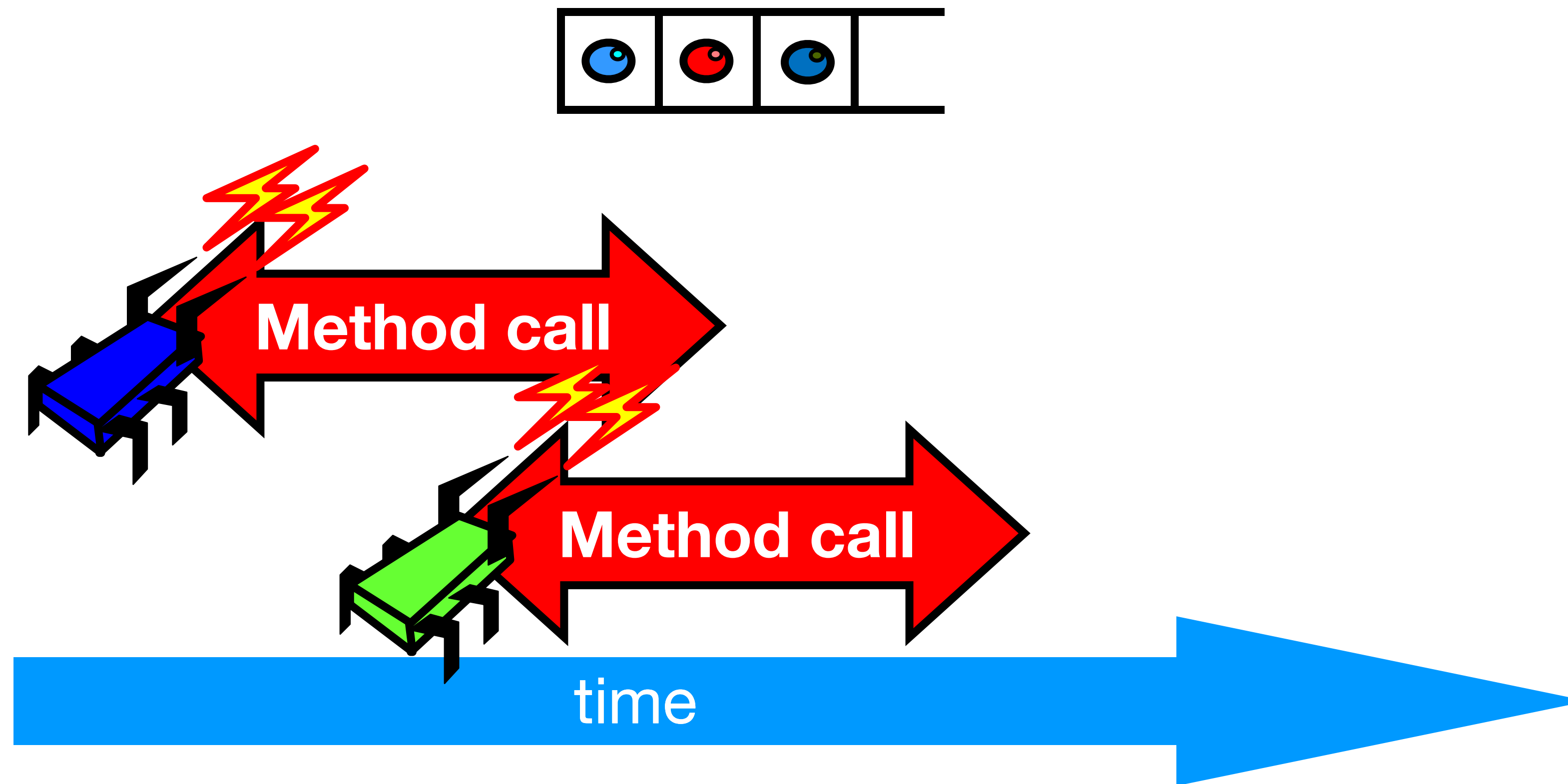
Concurrent Methods Take Overlapping Time



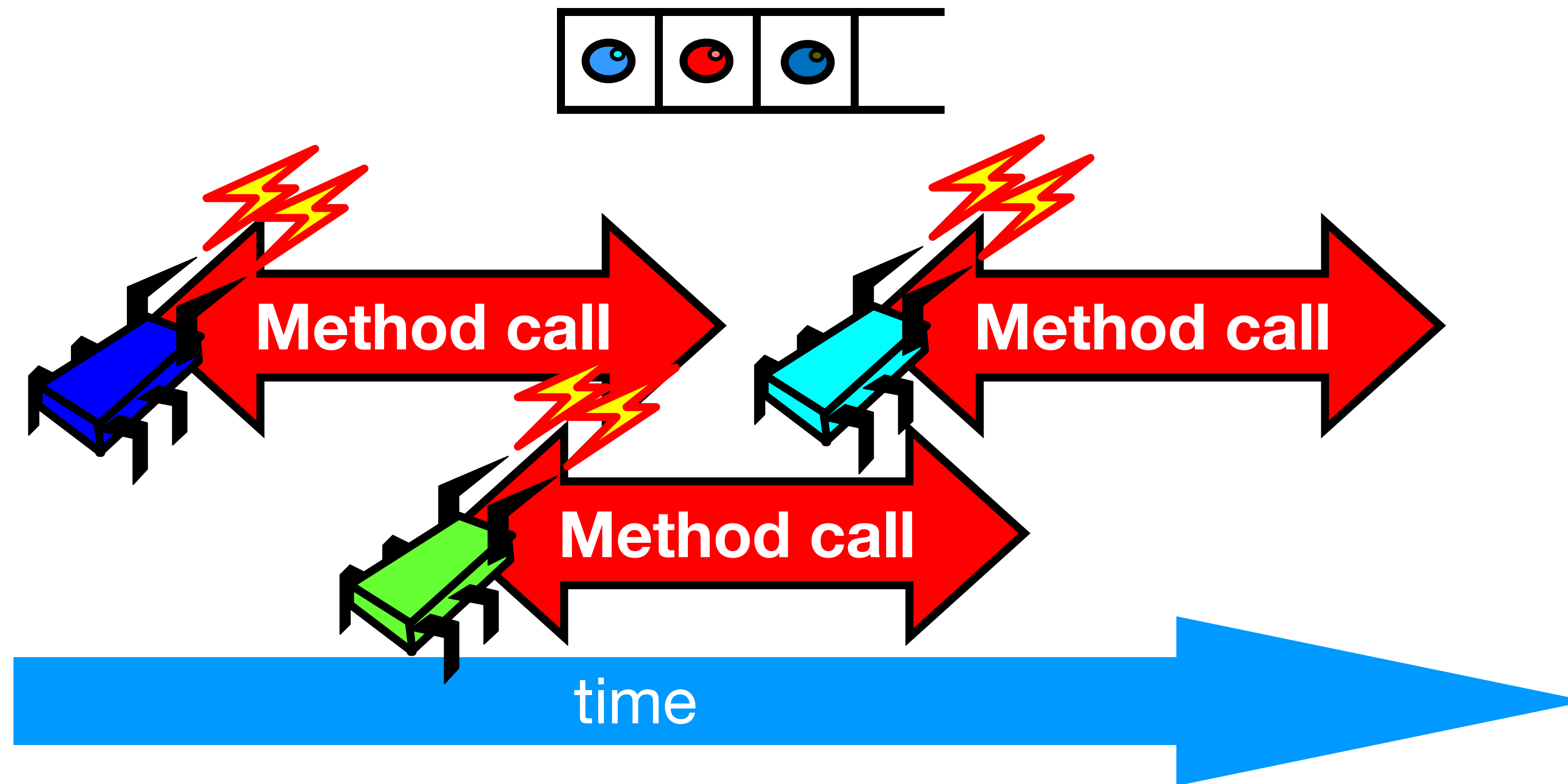
Concurrent Methods Take Overlapping Time



Concurrent Methods Take Overlapping Time



Concurrent Methods Take Overlapping Time



Sequential vs Concurrent

- Sequential
 - Object needs a meaningful state only ***between*** method calls
- Concurrent
 - Because method calls overlap, the object might ***never*** be between method calls

Sequential vs Concurrent

- Sequential:
 - Each method described in isolation
- Concurrent
 - Must characterize ***all*** possible interactions with concurrent calls
 - What if two **enq ()** calls overlap?
 - Two **deq ()** calls? **enq ()** and **deq ()**? ...

Sequential vs Concurrent

- Sequential:
 - Can add new methods without affecting older methods
- Concurrent:
 - Everything can potentially interact with everything else

Sequential vs Concurrent

- Sequential:
 - Can add new methods without affecting older methods
- Concurrent:
 - Everything can potentially interact with everything else



Panic!

The Big Question

- What does it *mean* for a *concurrent* object to be correct?
 - What *is* a concurrent FIFO queue?
 - FIFO means *strict temporal order*
 - Concurrent means *ambiguous temporal order*

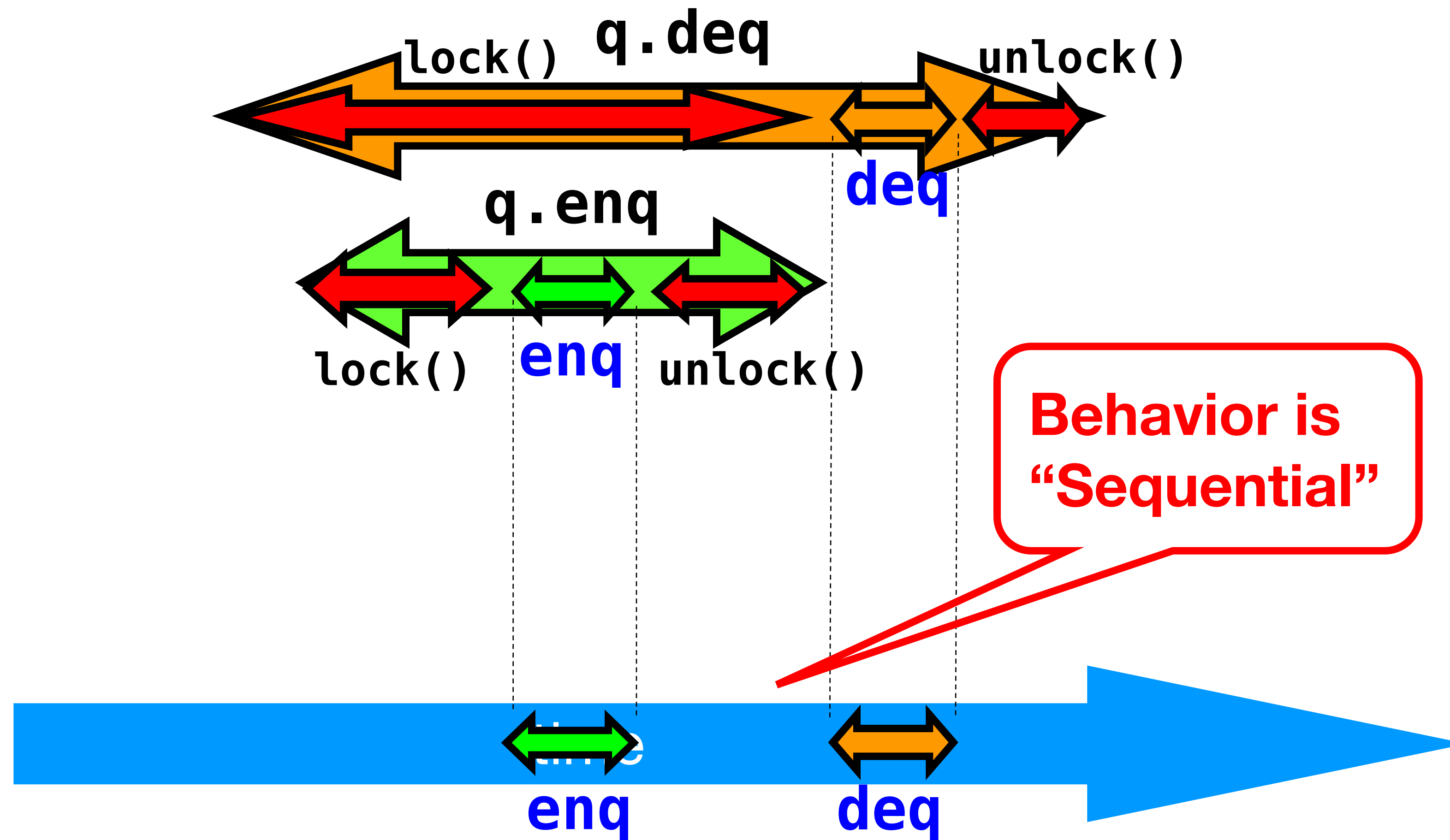
Intuitively

```
let def q =  
  Mutex.lock q.lock;  
  try  
    if q.tail = q.head then  
      raise Empty;  
    match q.items.(q.head mod q.capacity) with  
    | None -> assert false  
    | Some x ->  
      q.head <- q.head + 1;  
      Mutex.unlock q.lock;  
      x  
  with e ->  
    Mutex.unlock q.lock;  
    raise e
```

All queue modifications are mutually exclusive

Intuitively

Lets capture the idea of describing
the concurrent via the sequential



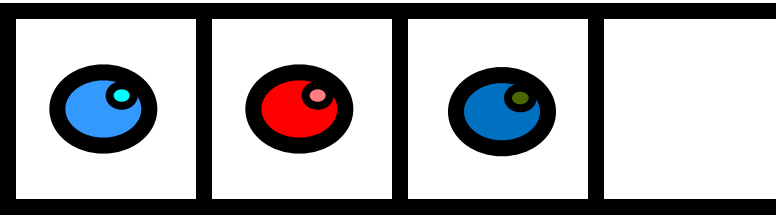
Linearizability

- Each method should
 - “take effect”
 - Instantaneously
 - Between *invocation* and *response* events
- Object is correct if this “sequentialised” behaviour is correct
- Any such concurrent object is
 - **Linearizable™**

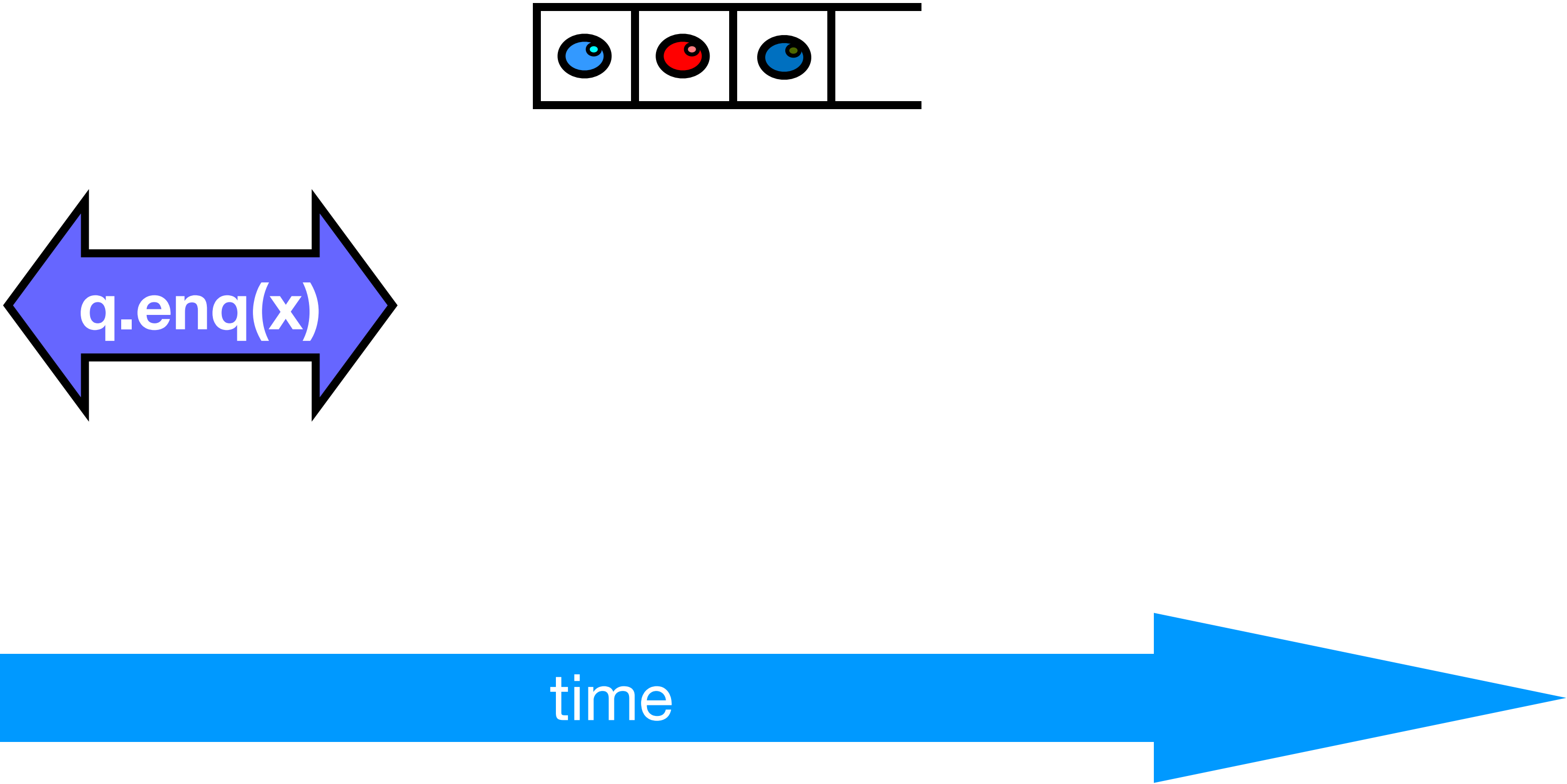
Is it really about the object?

- Each method should
 - “take effect”
 - Instantaneously
 - Between *invocation* and *response* events
- Sounds like a property of *an execution...*
- A linearizable *object*
 - One of whose all possible executions are linearizable

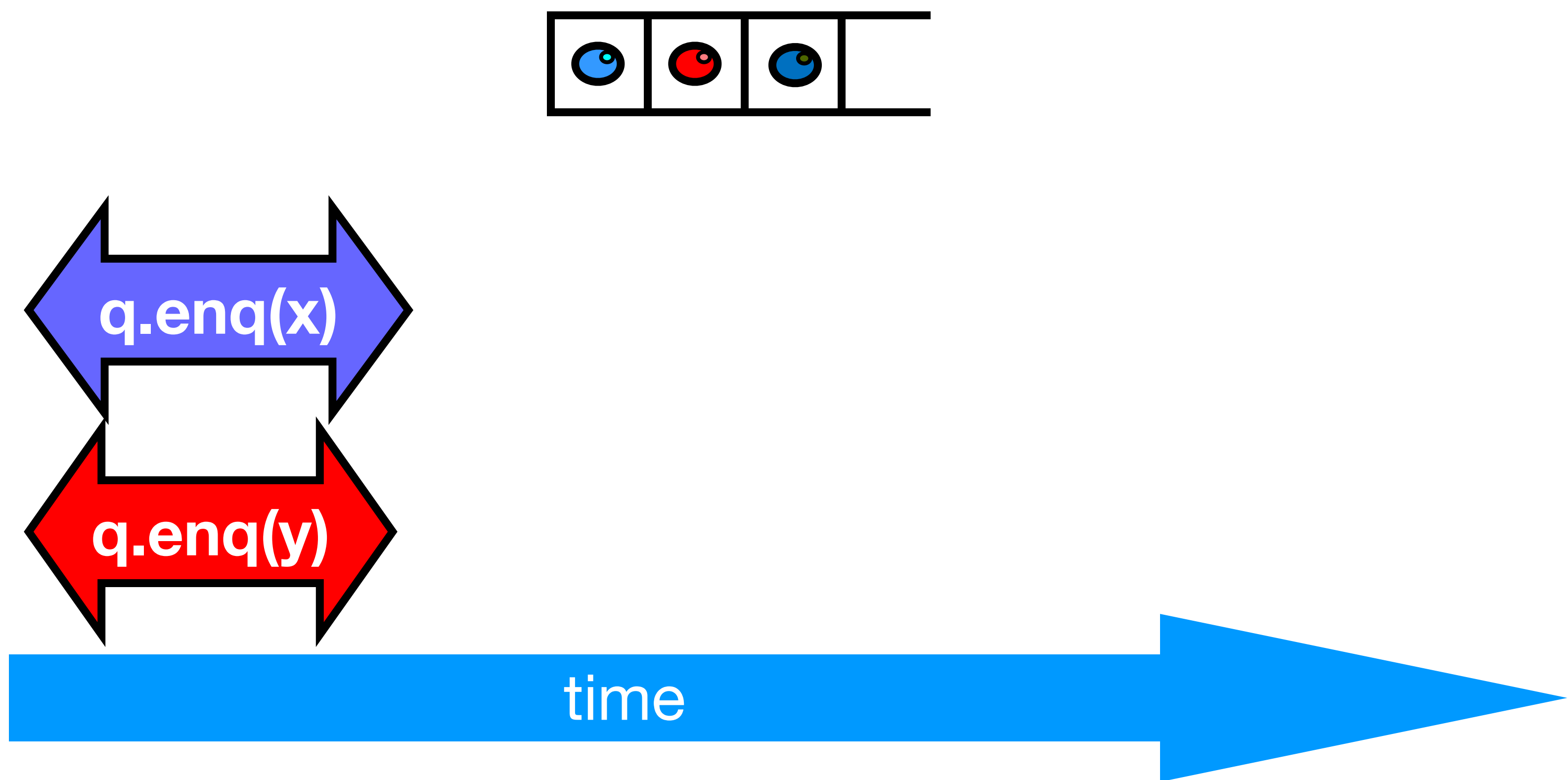
Example



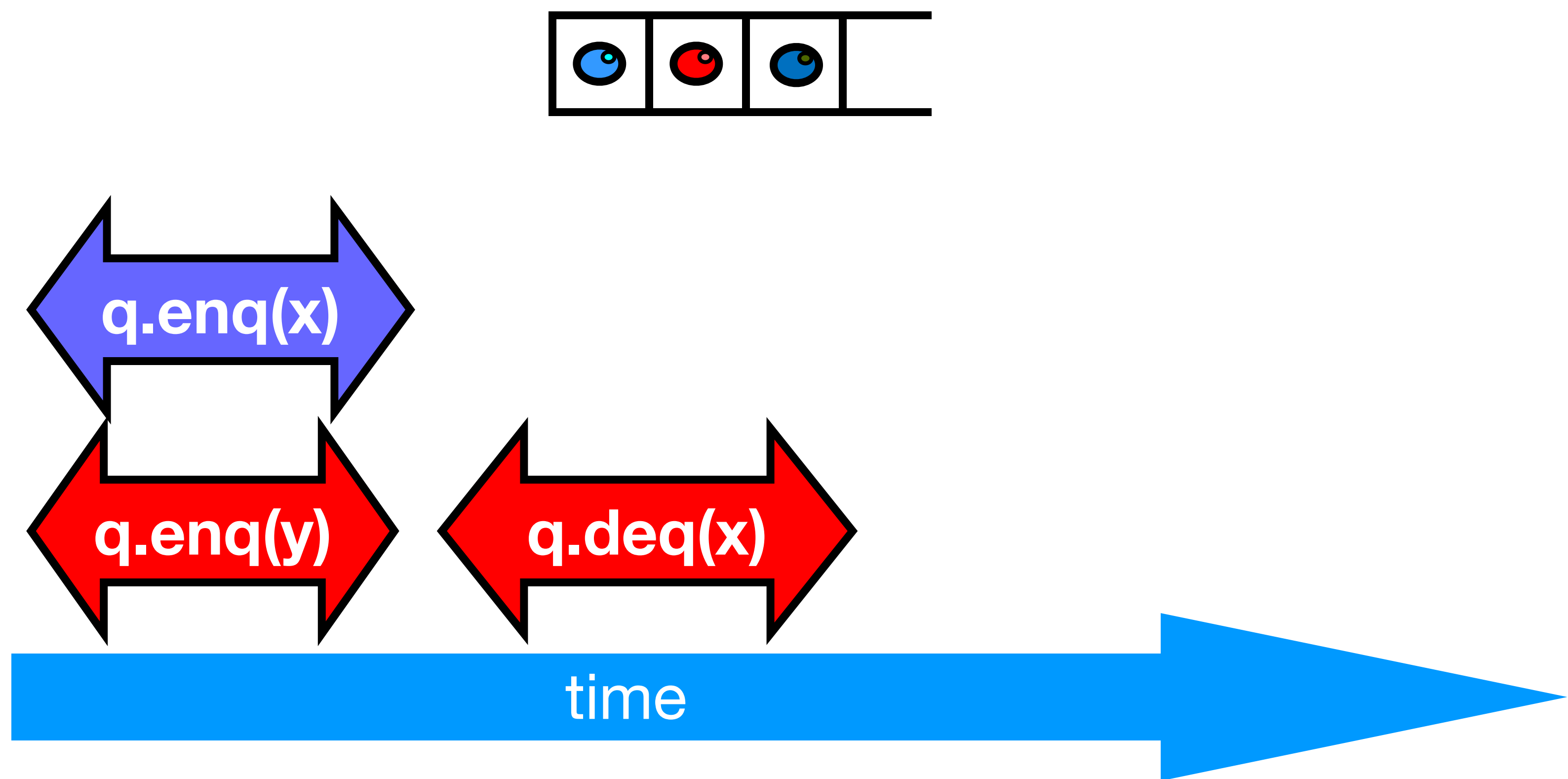
Example



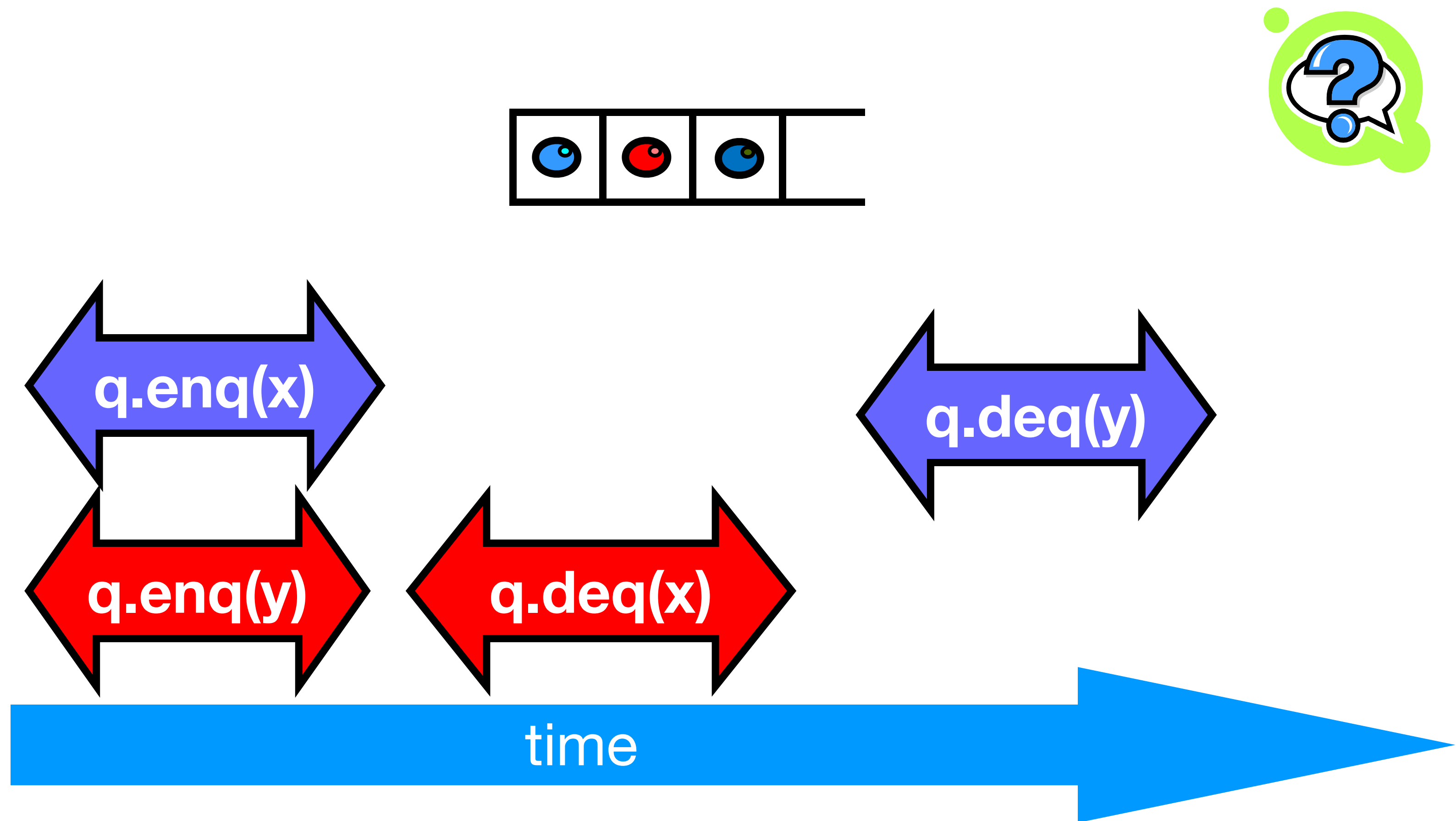
Example



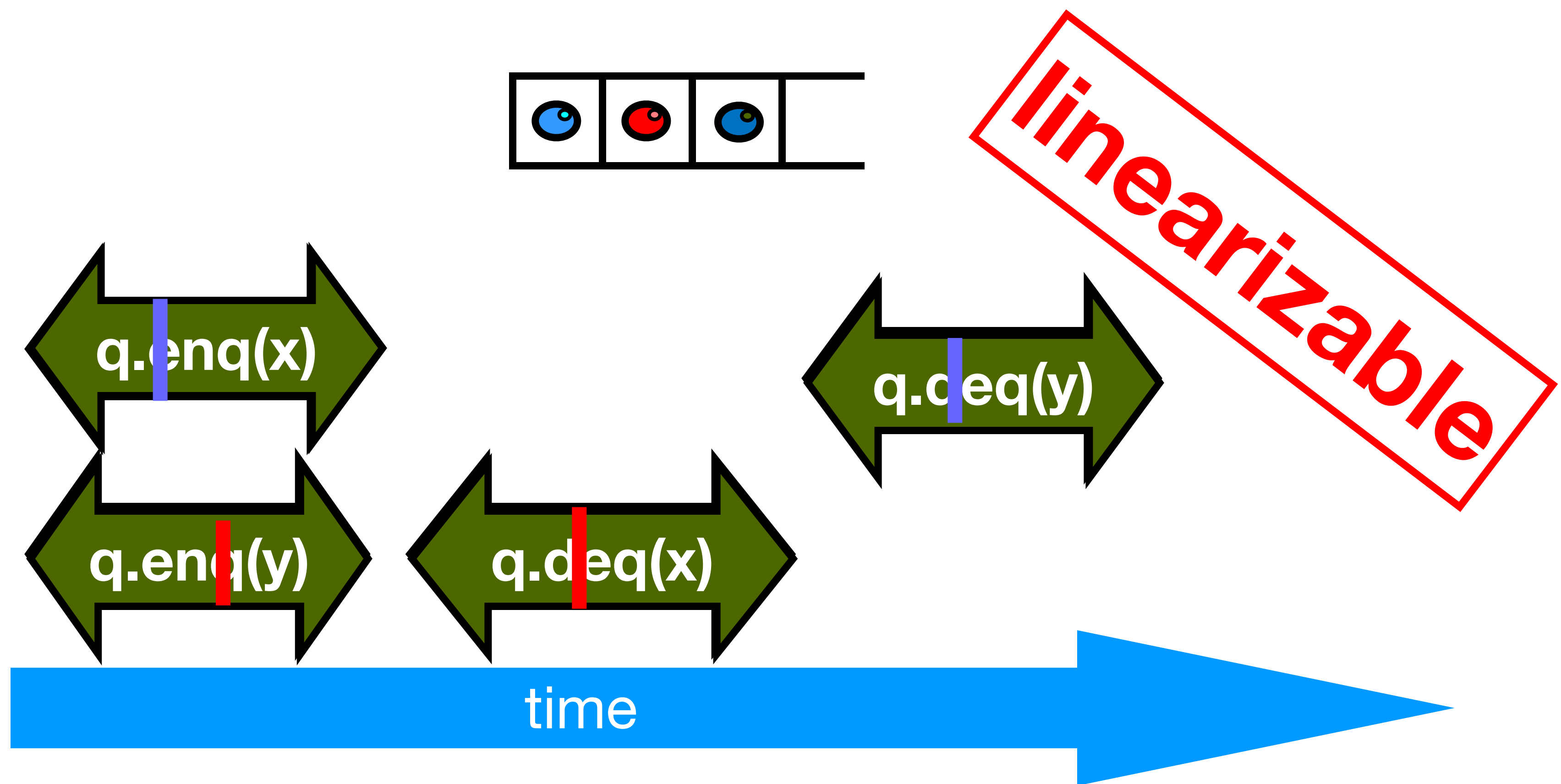
Example



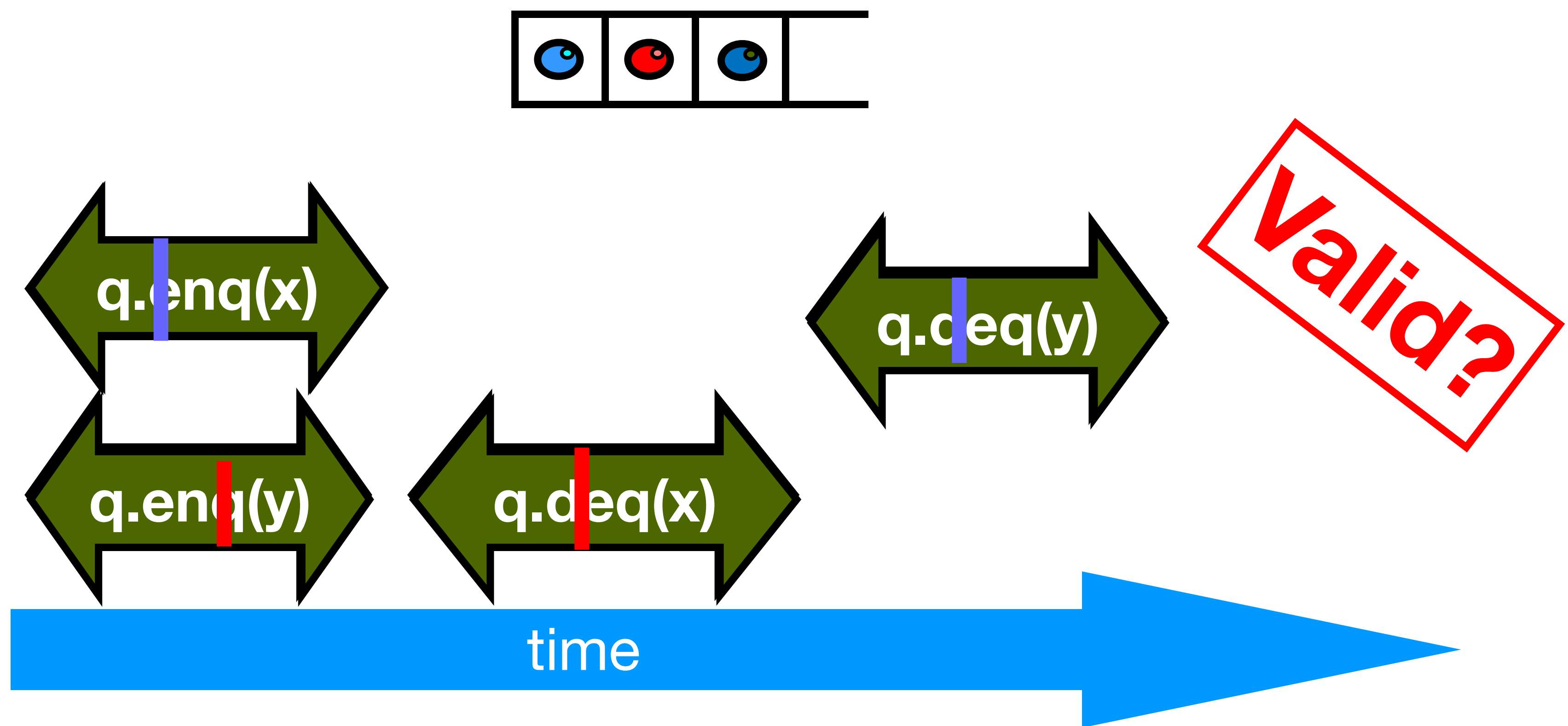
Example



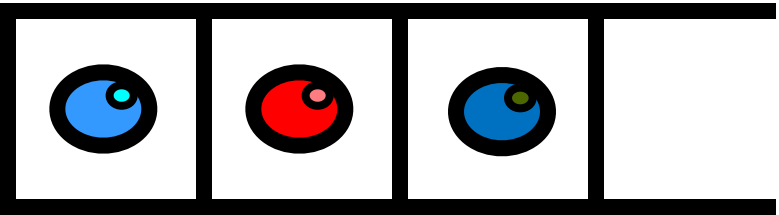
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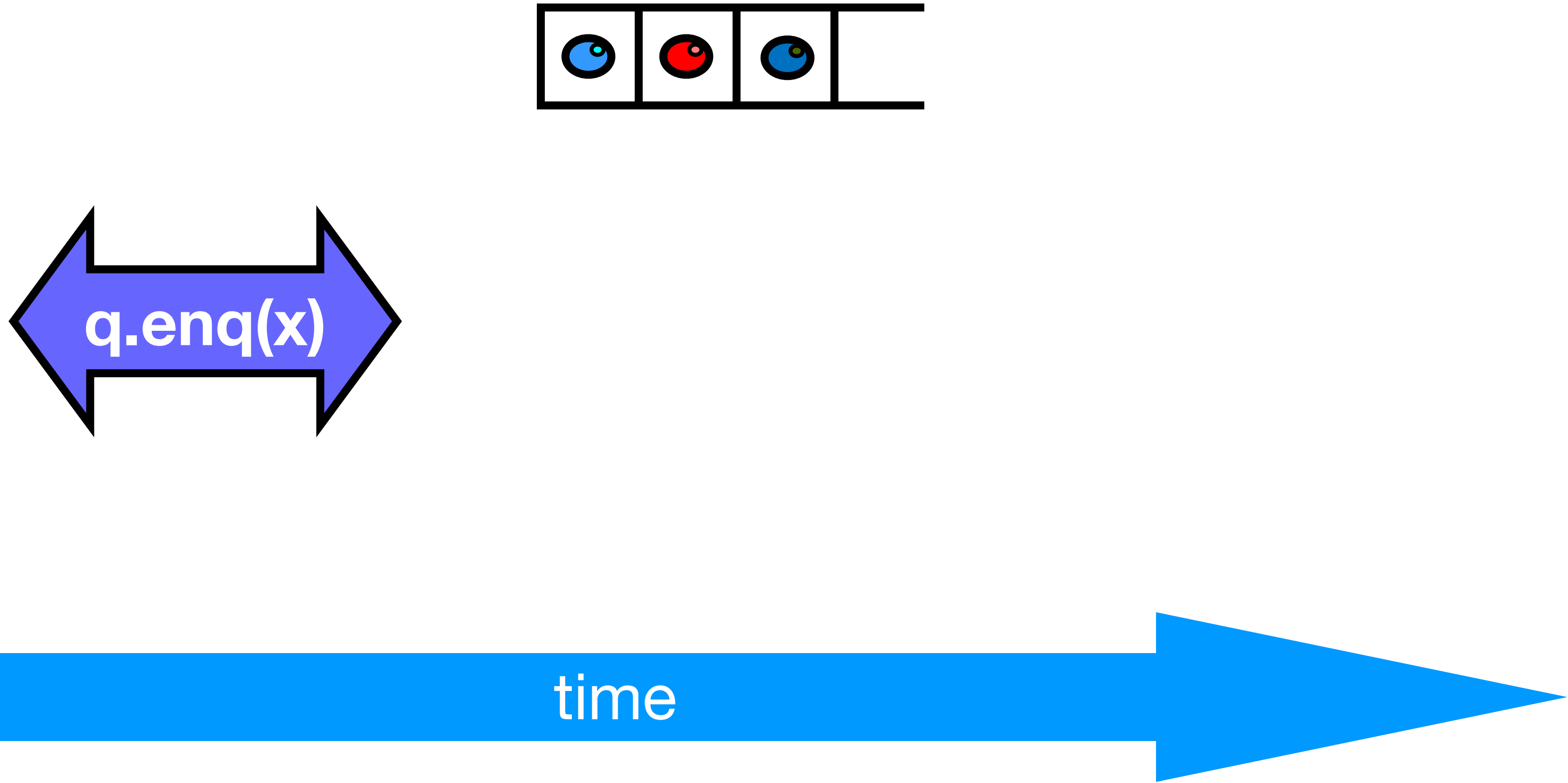
Example



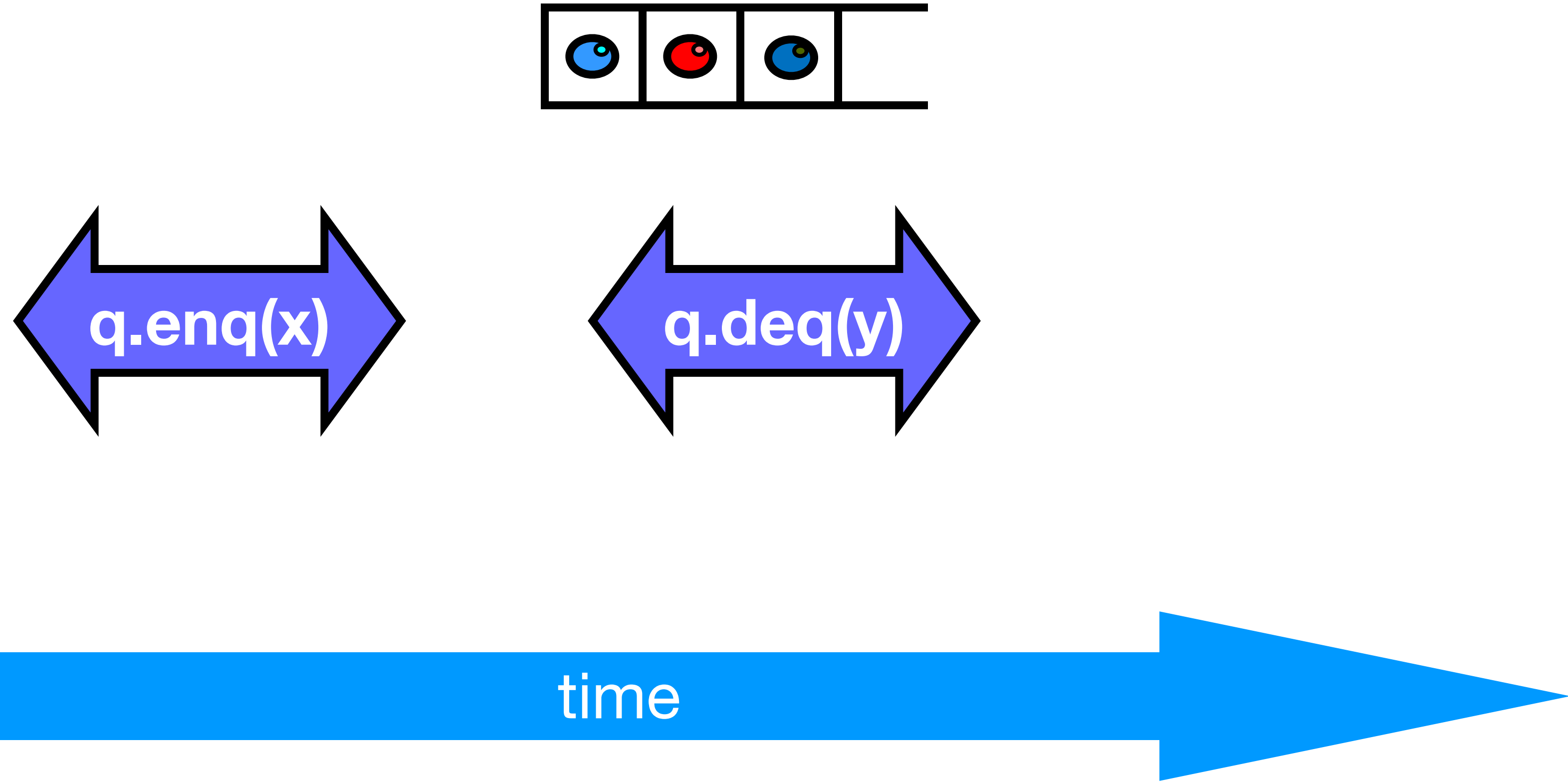
Example



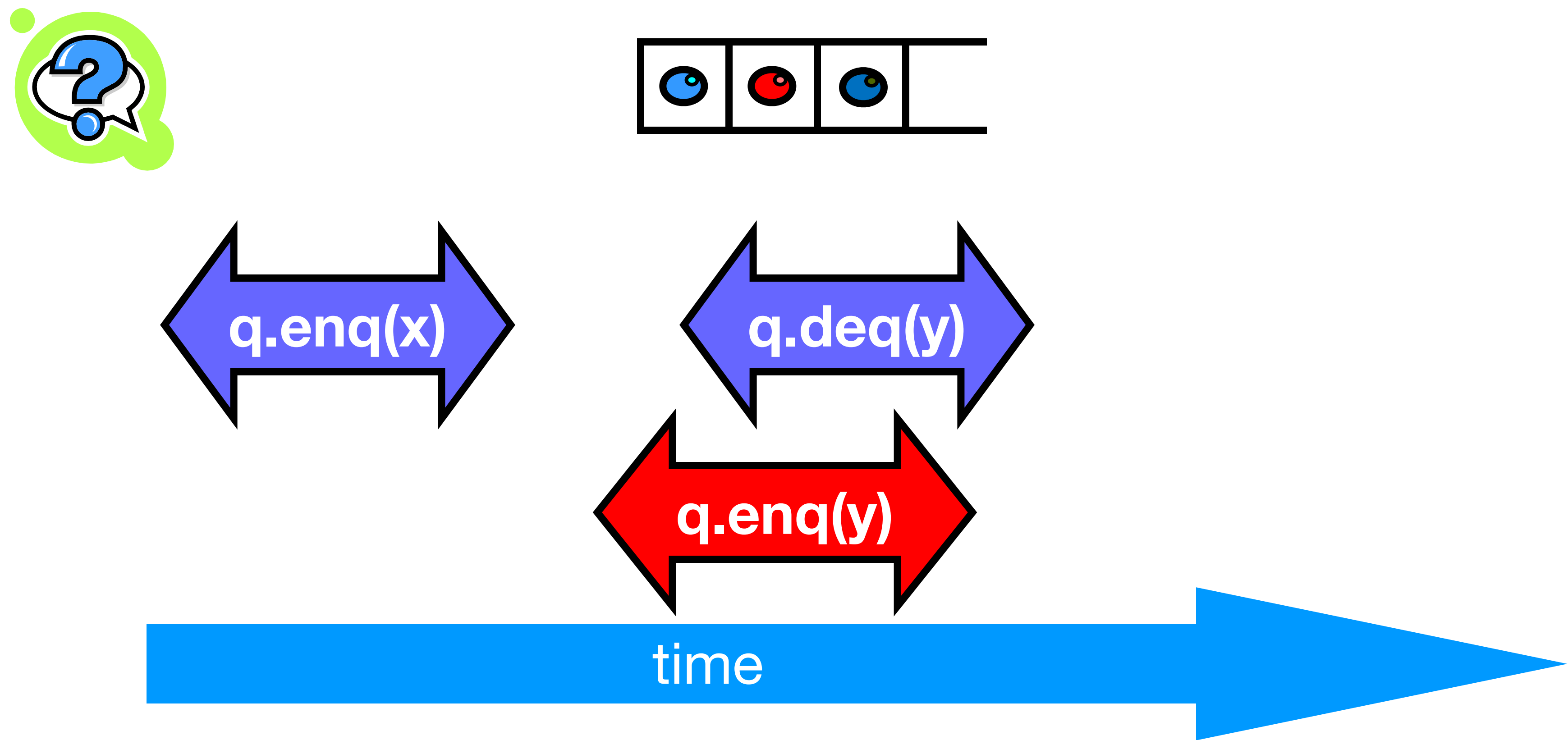
Example



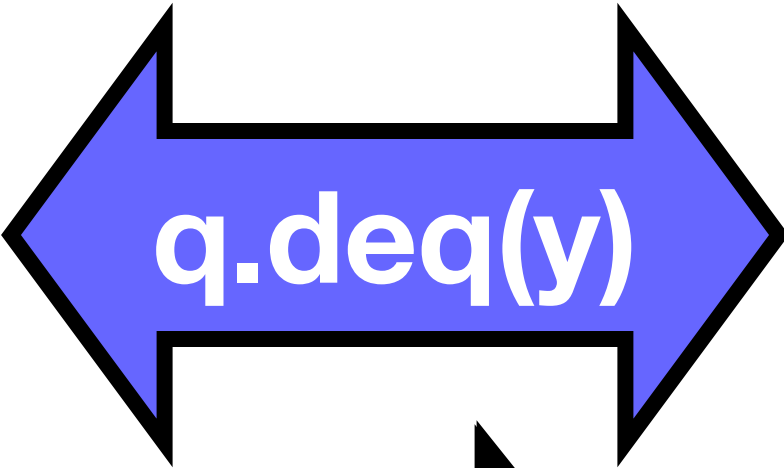
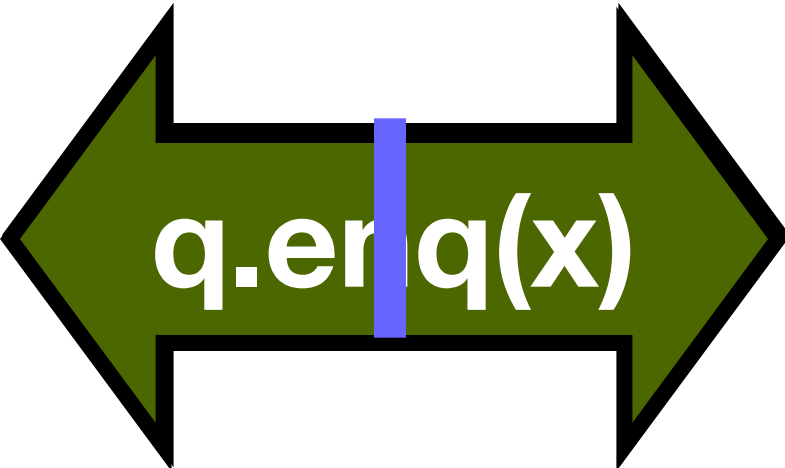
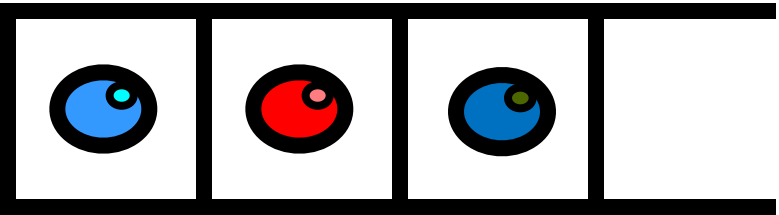
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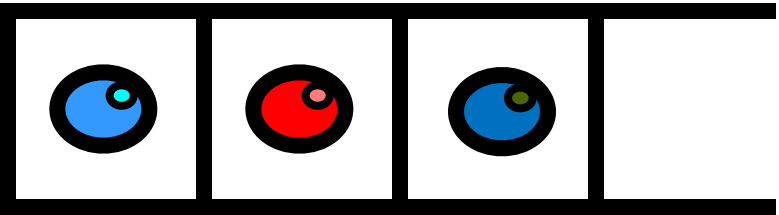
Example



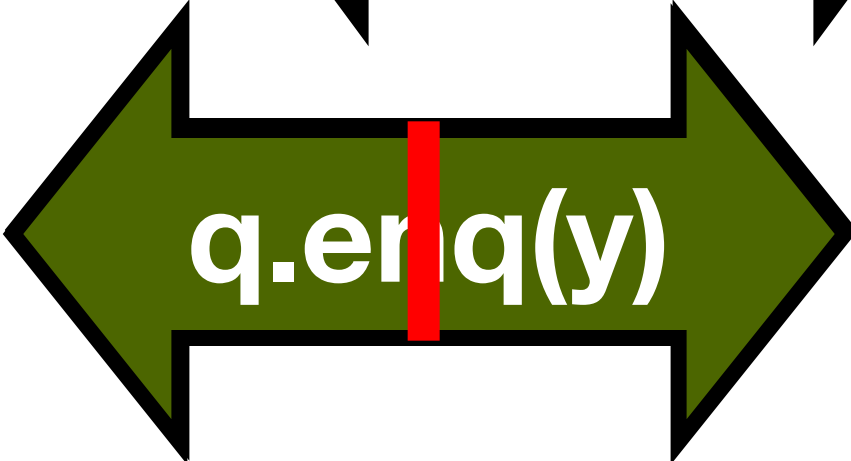
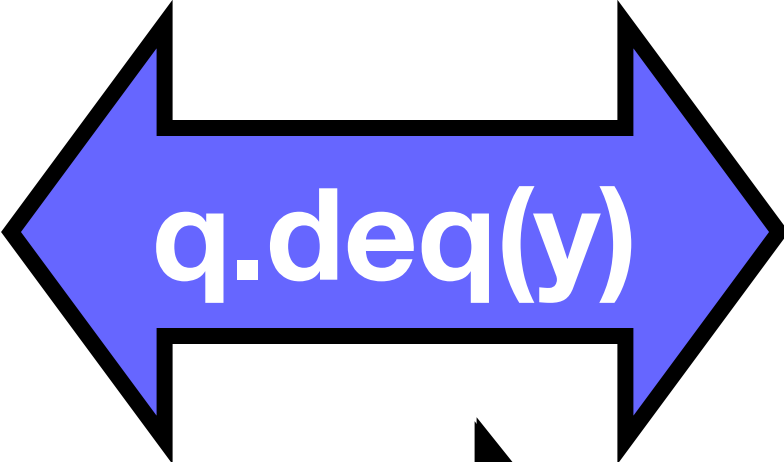
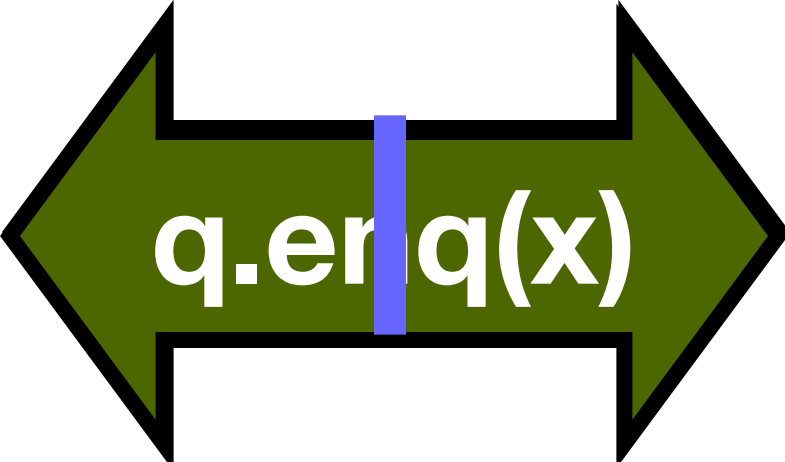
Example



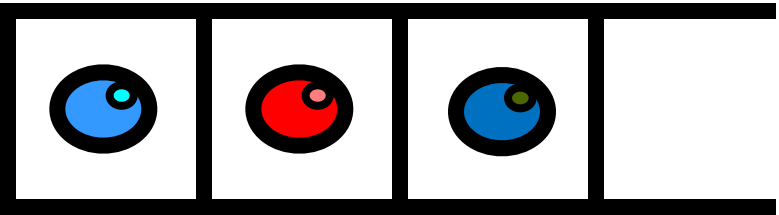
Example



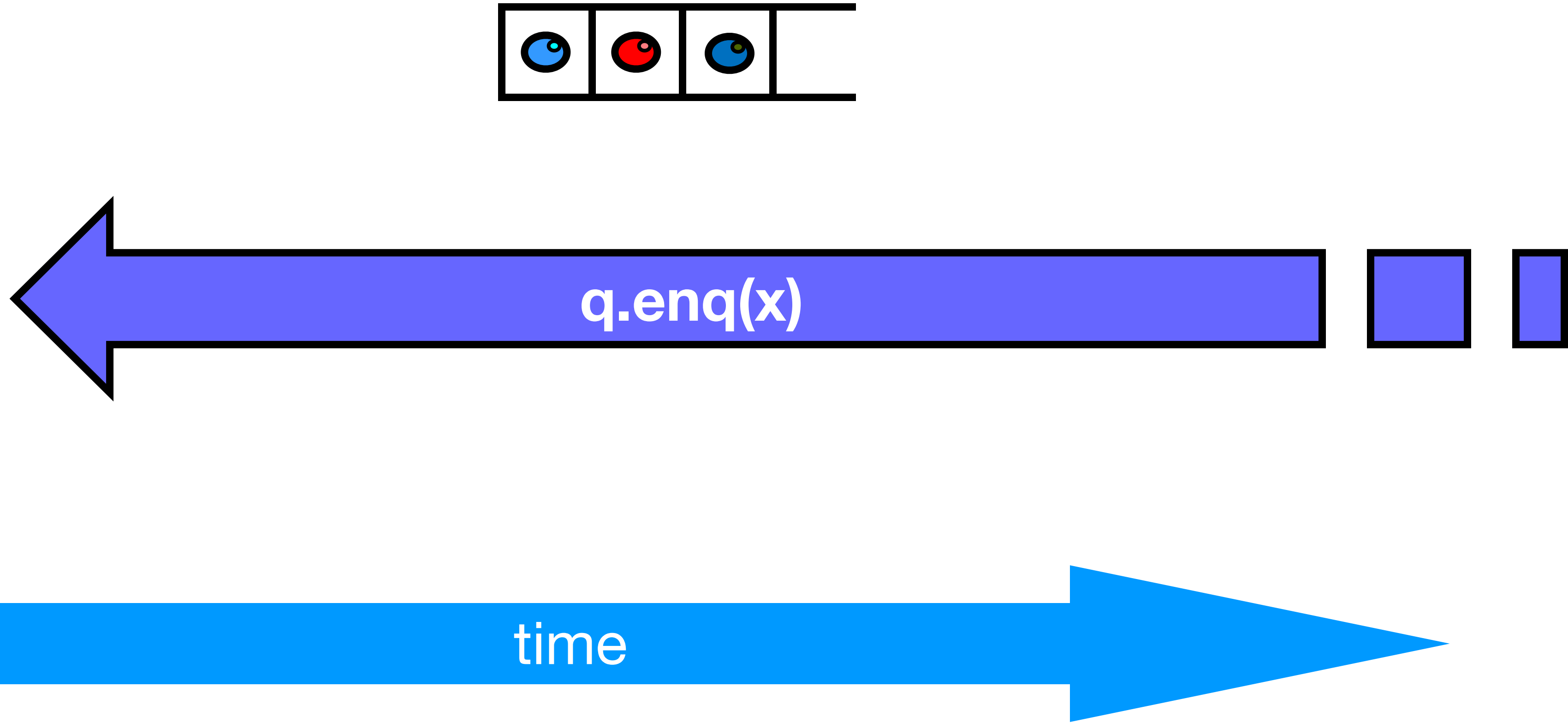
not linearizable



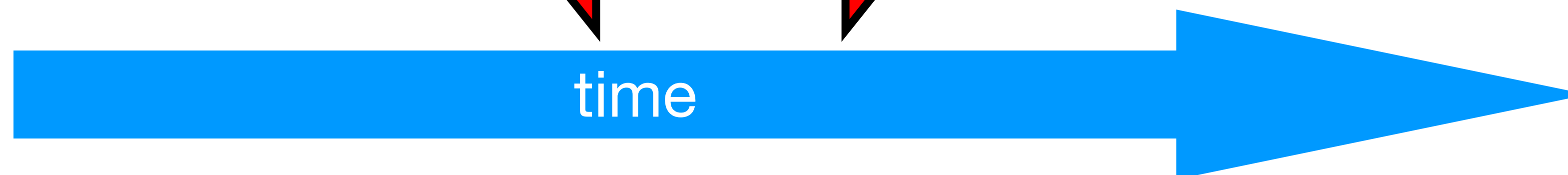
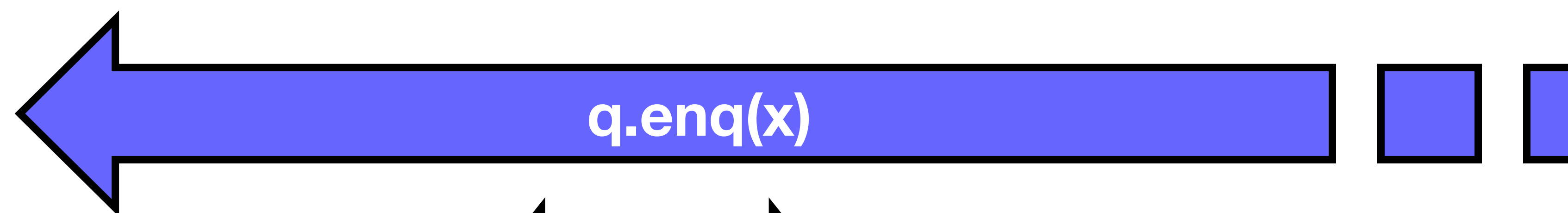
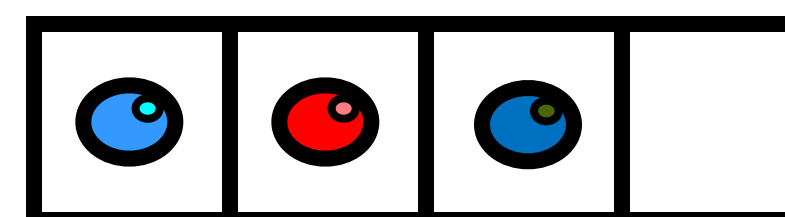
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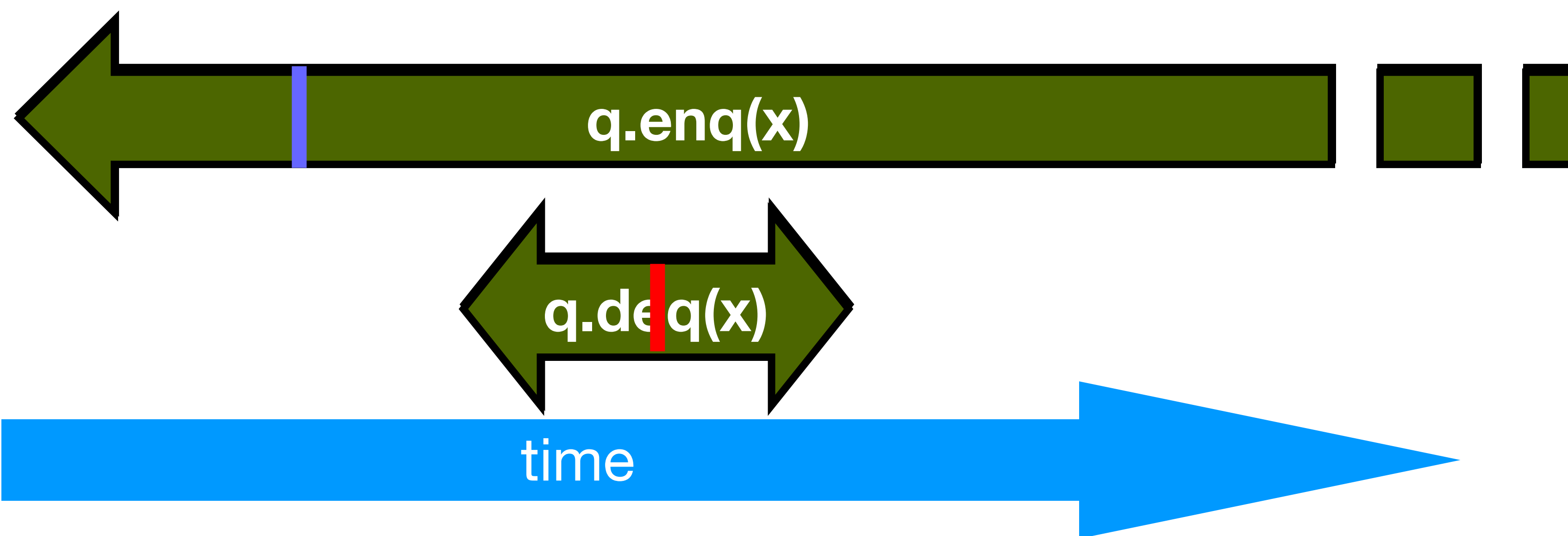
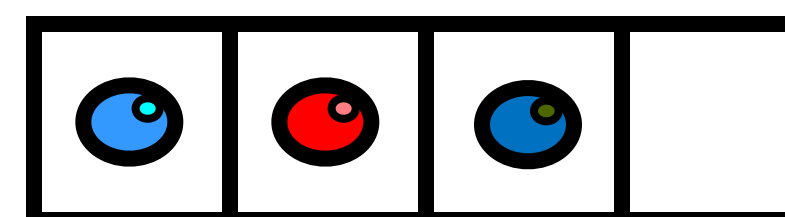
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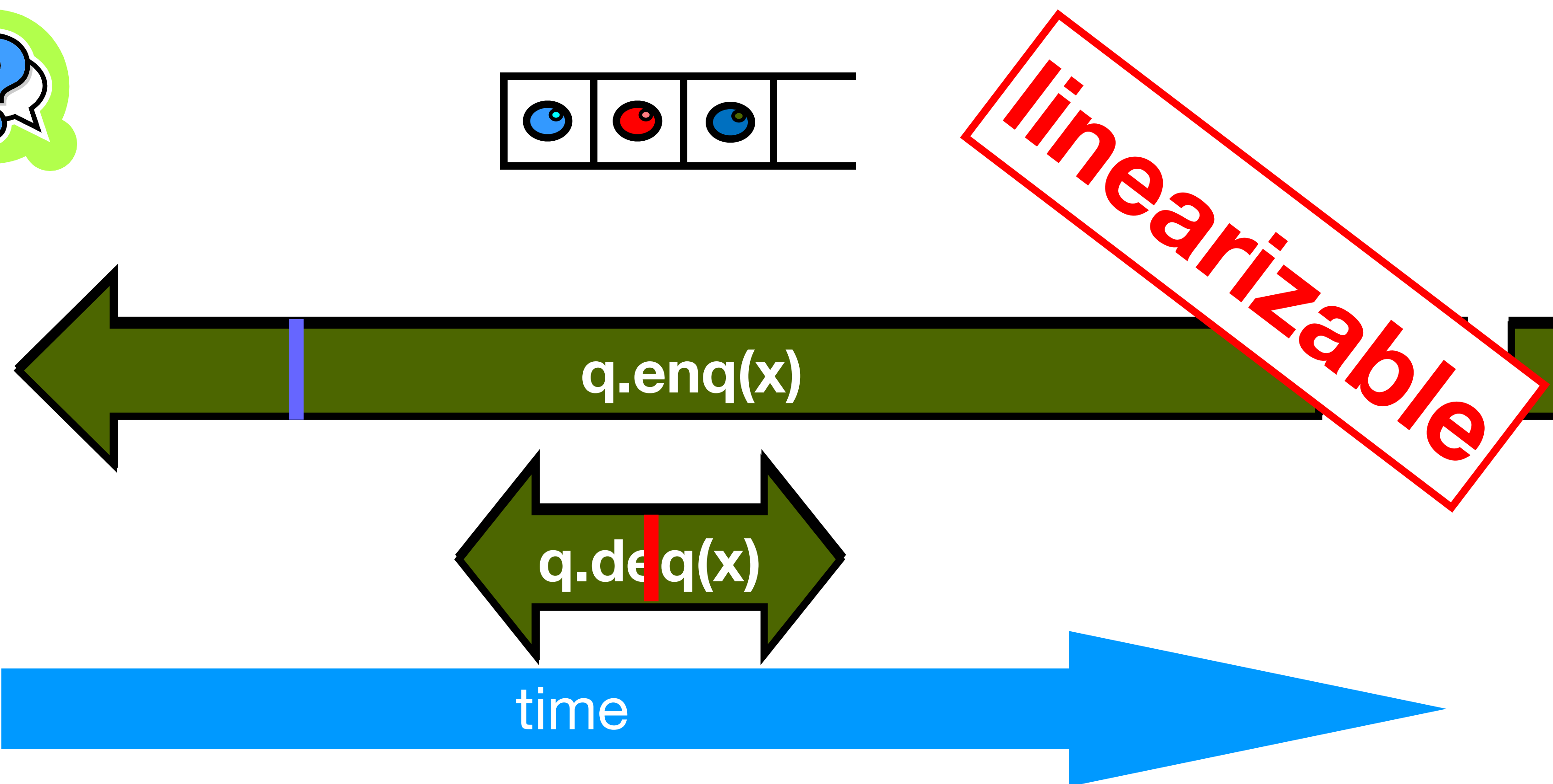
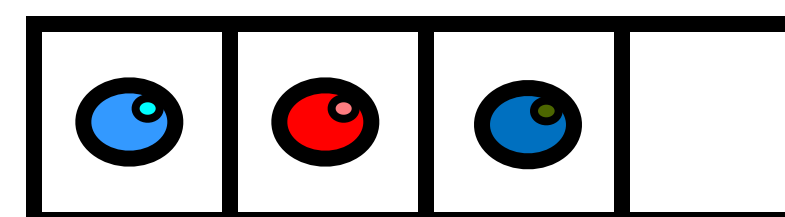
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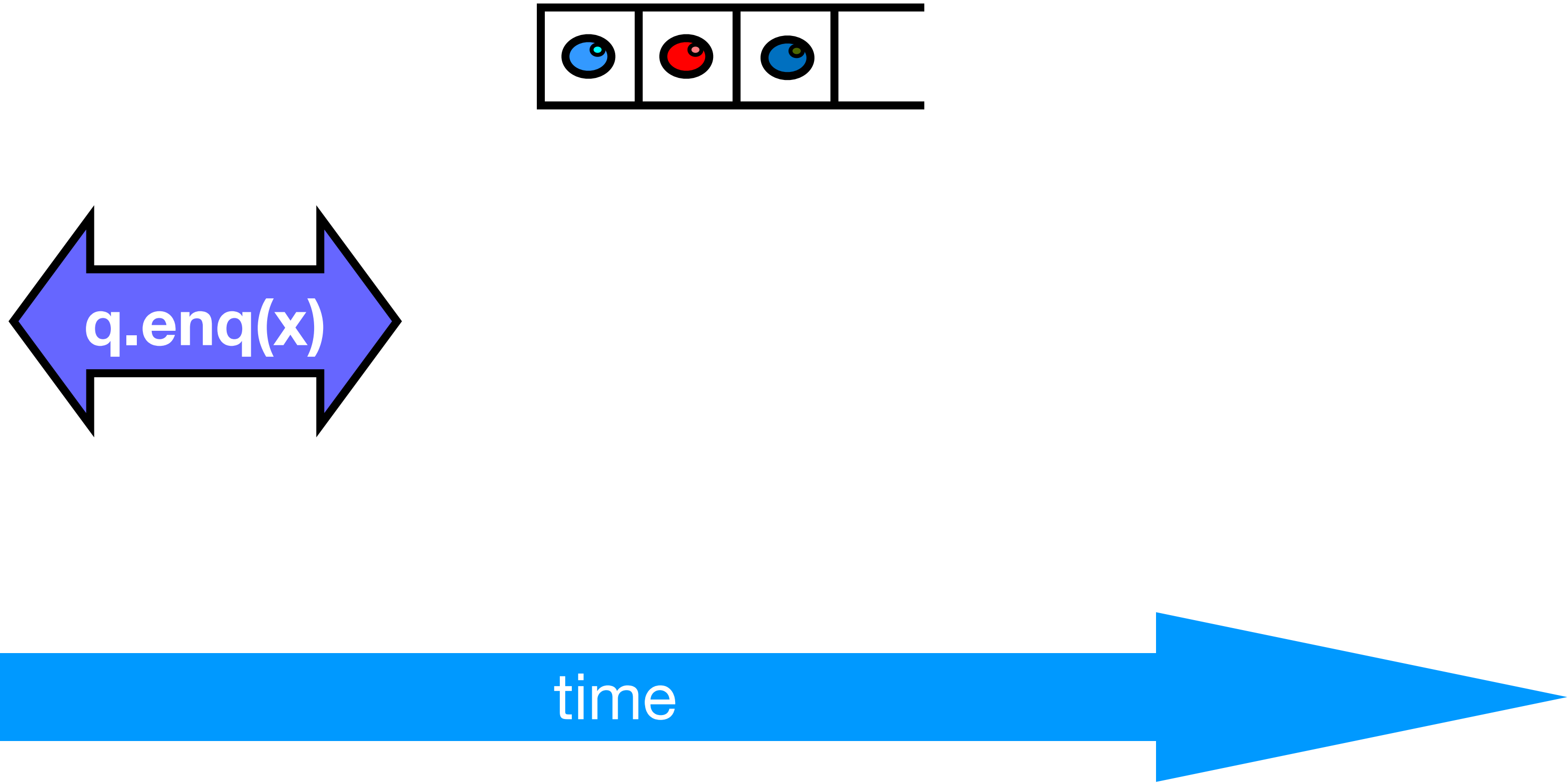
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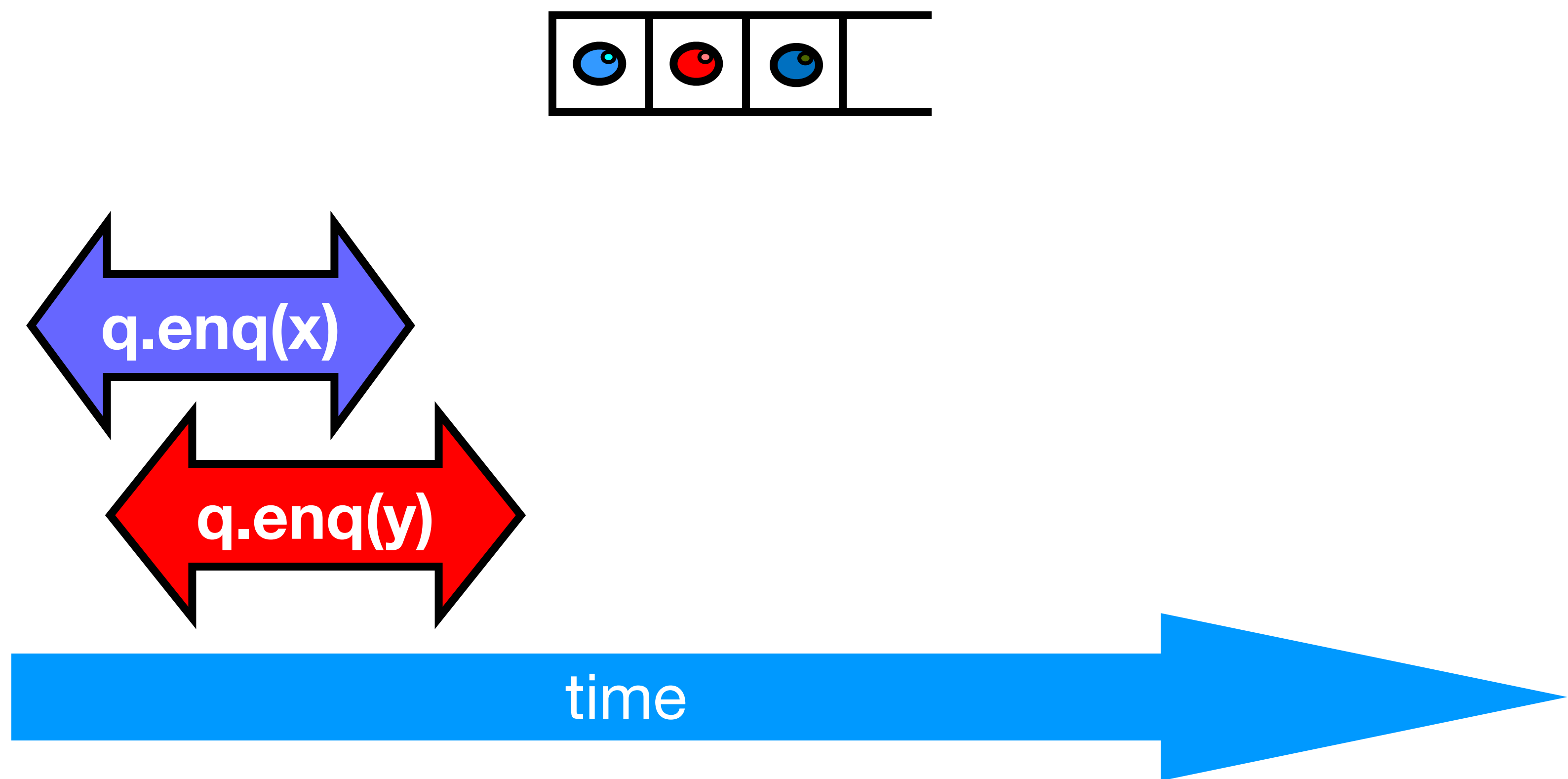
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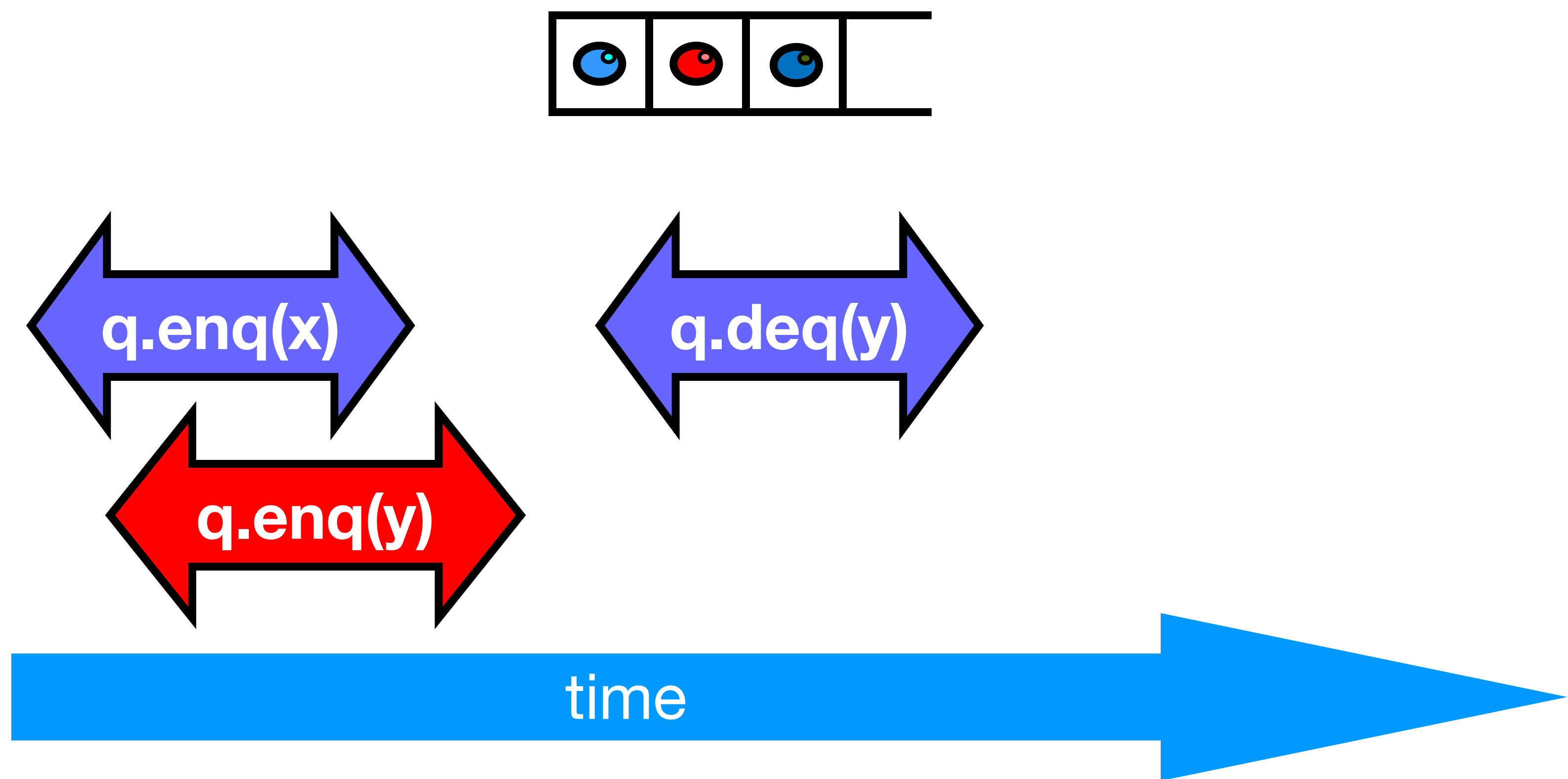
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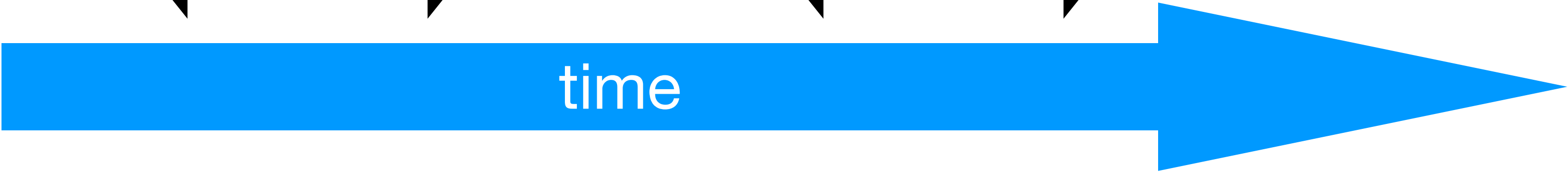
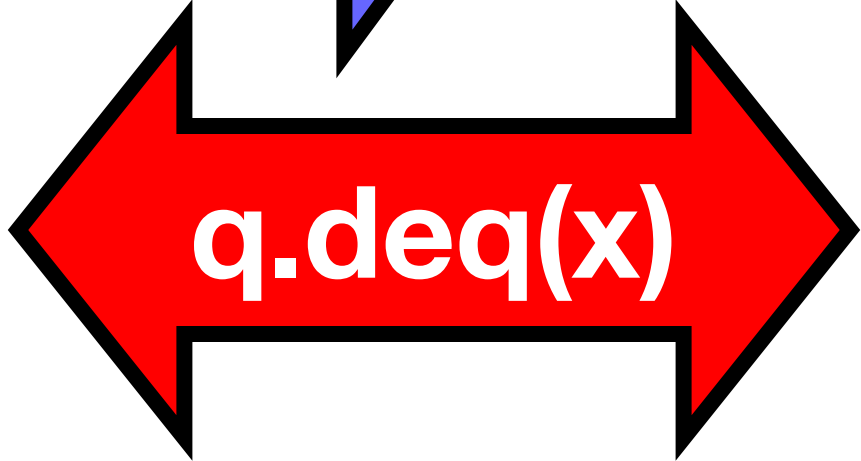
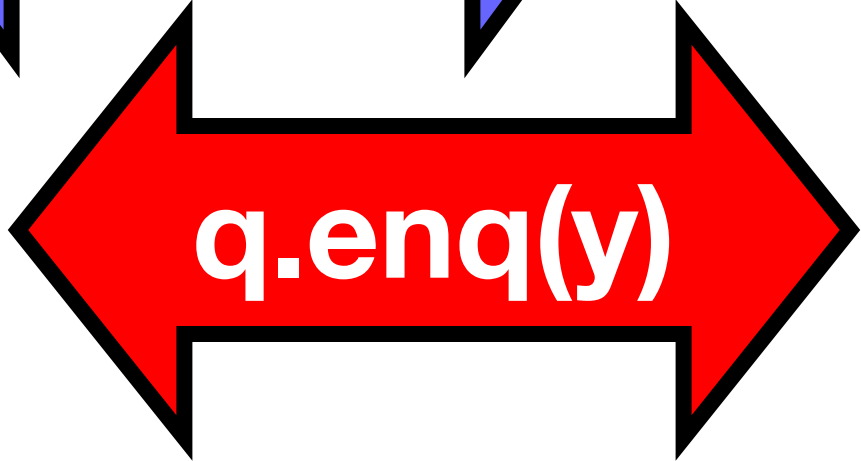
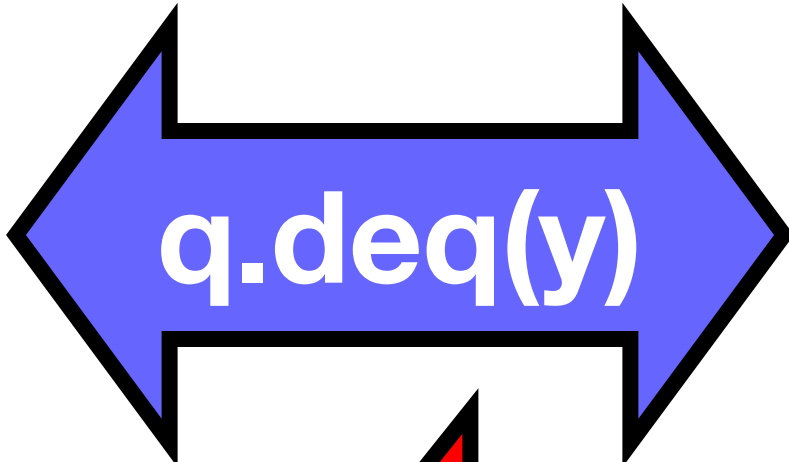
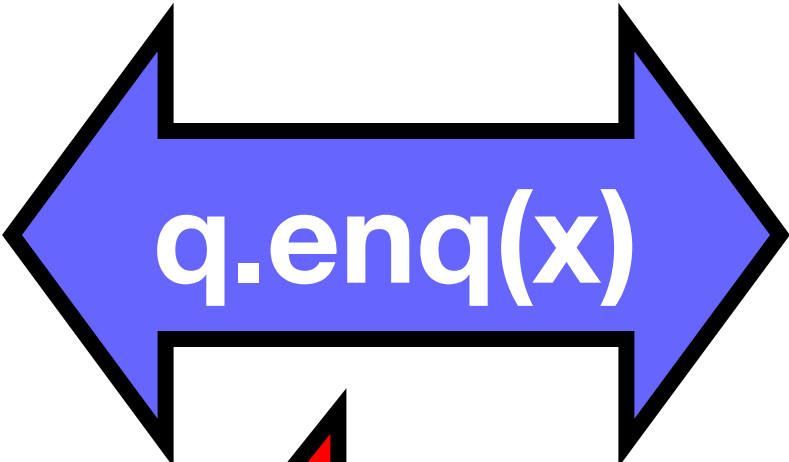
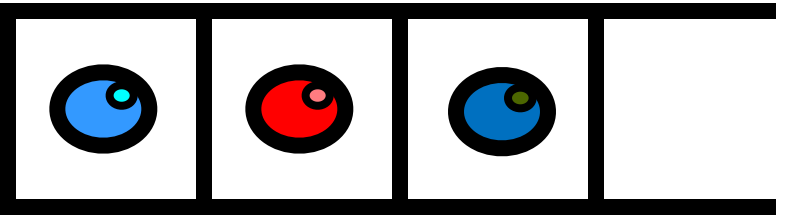
Example



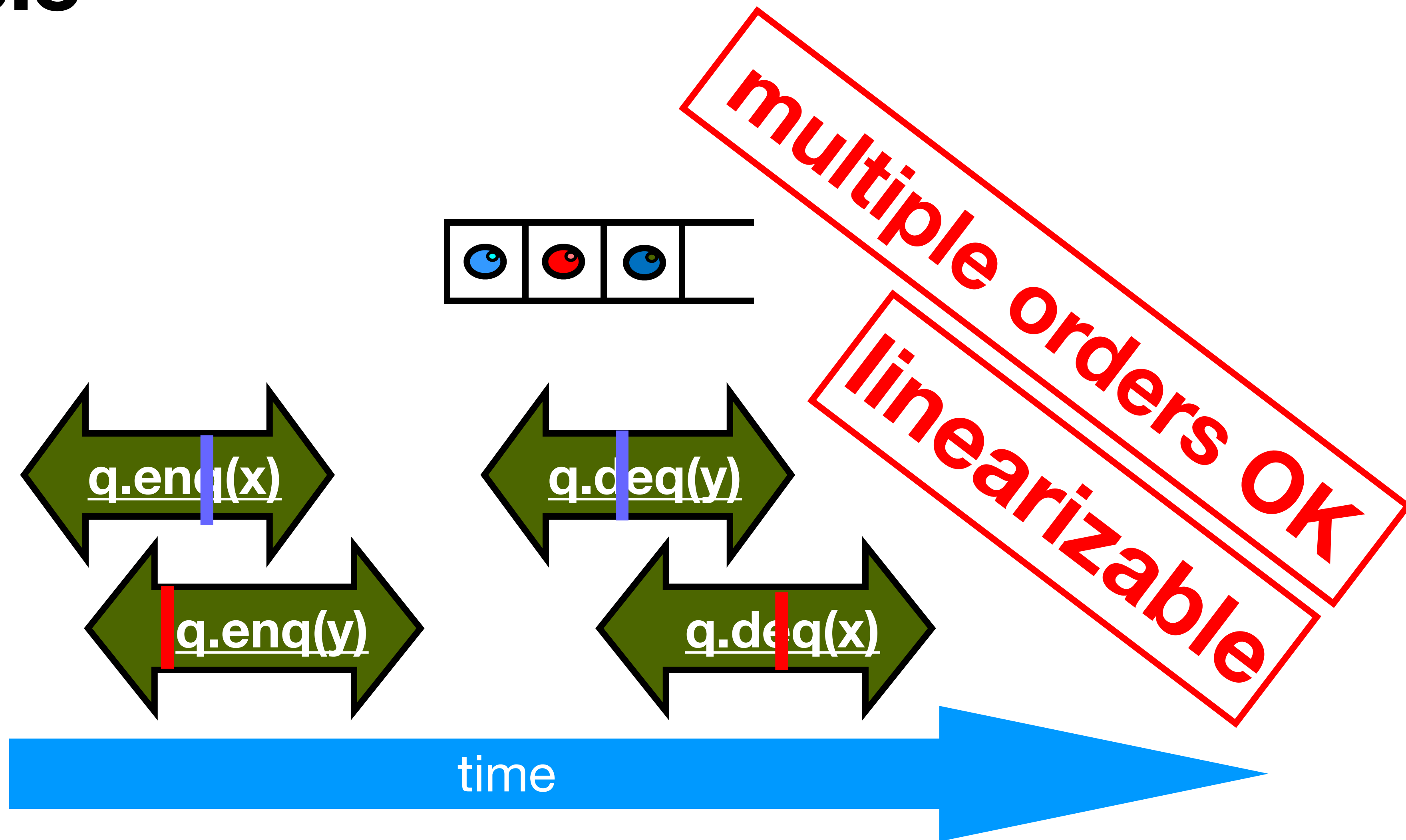
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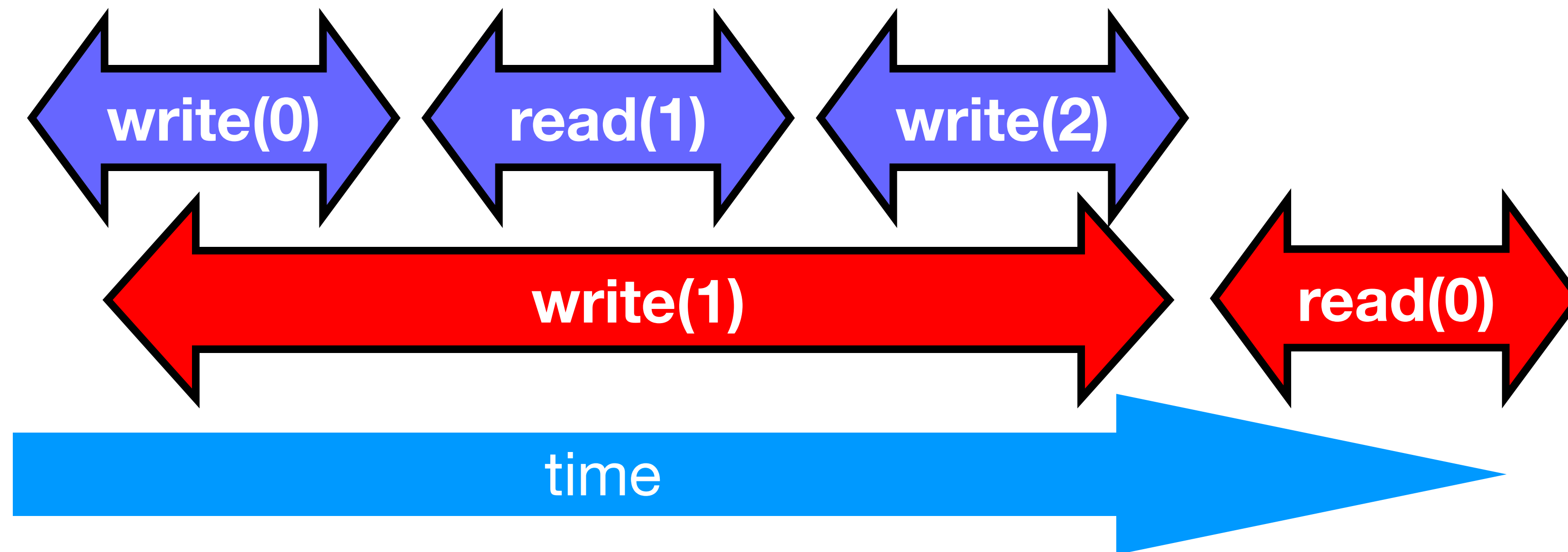
Example



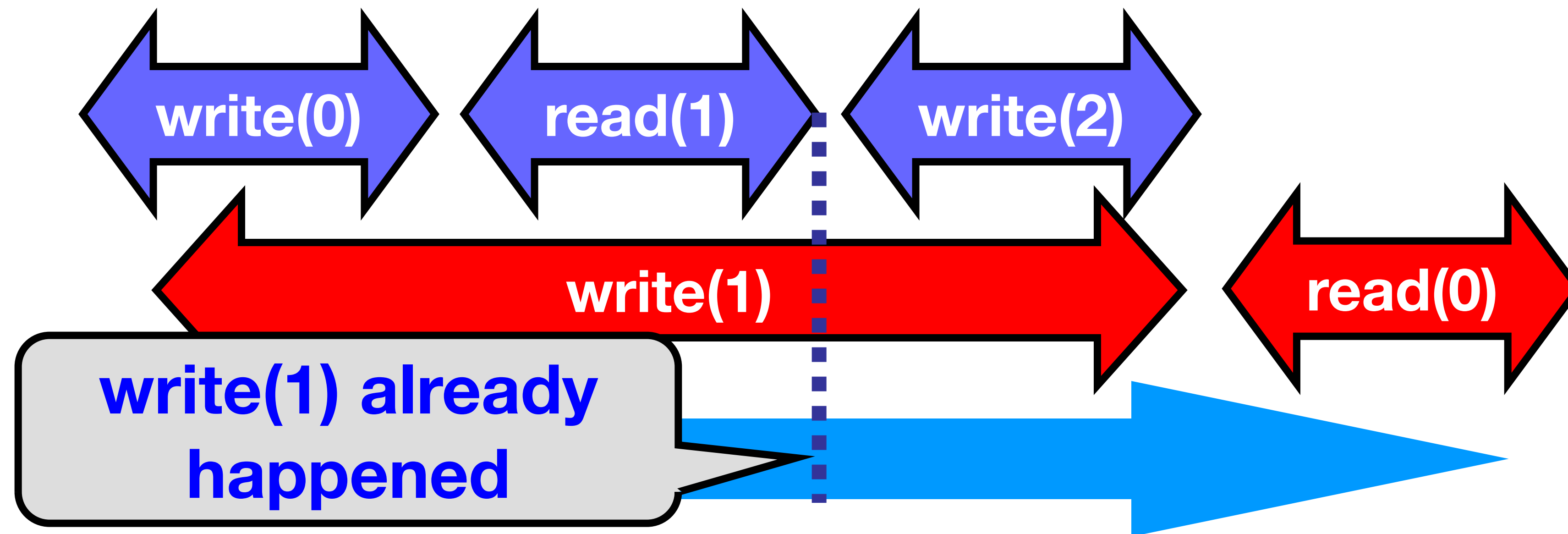
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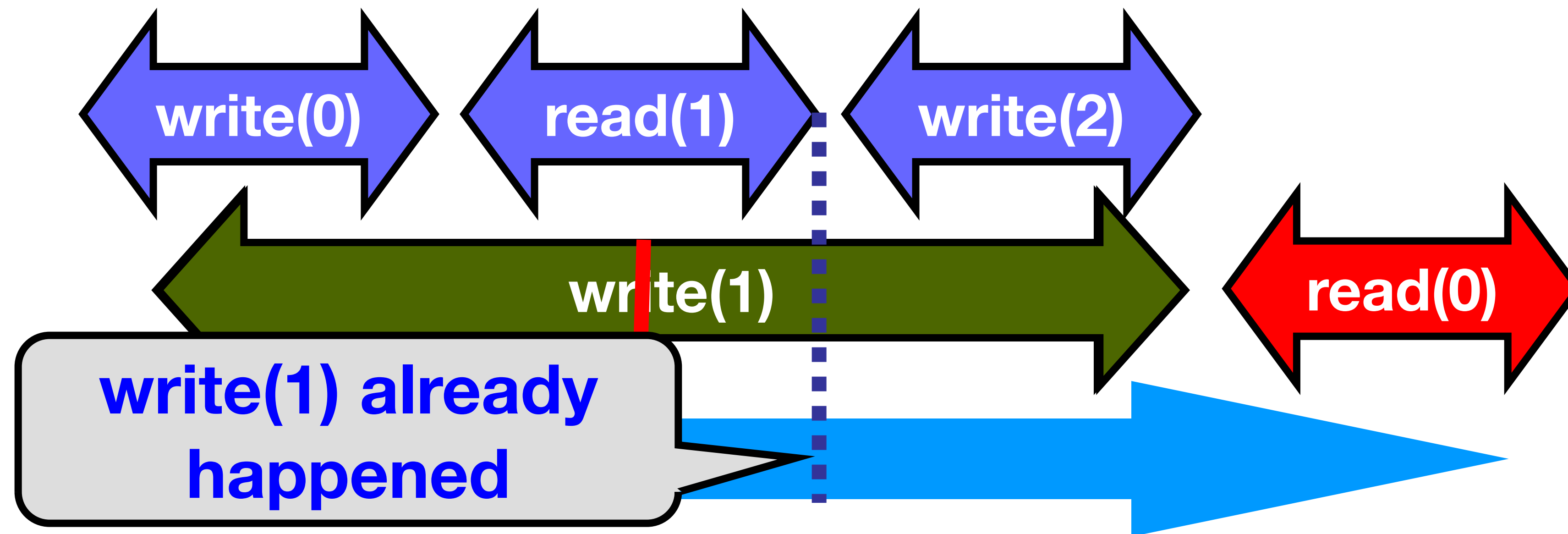
Read/Write Register Example



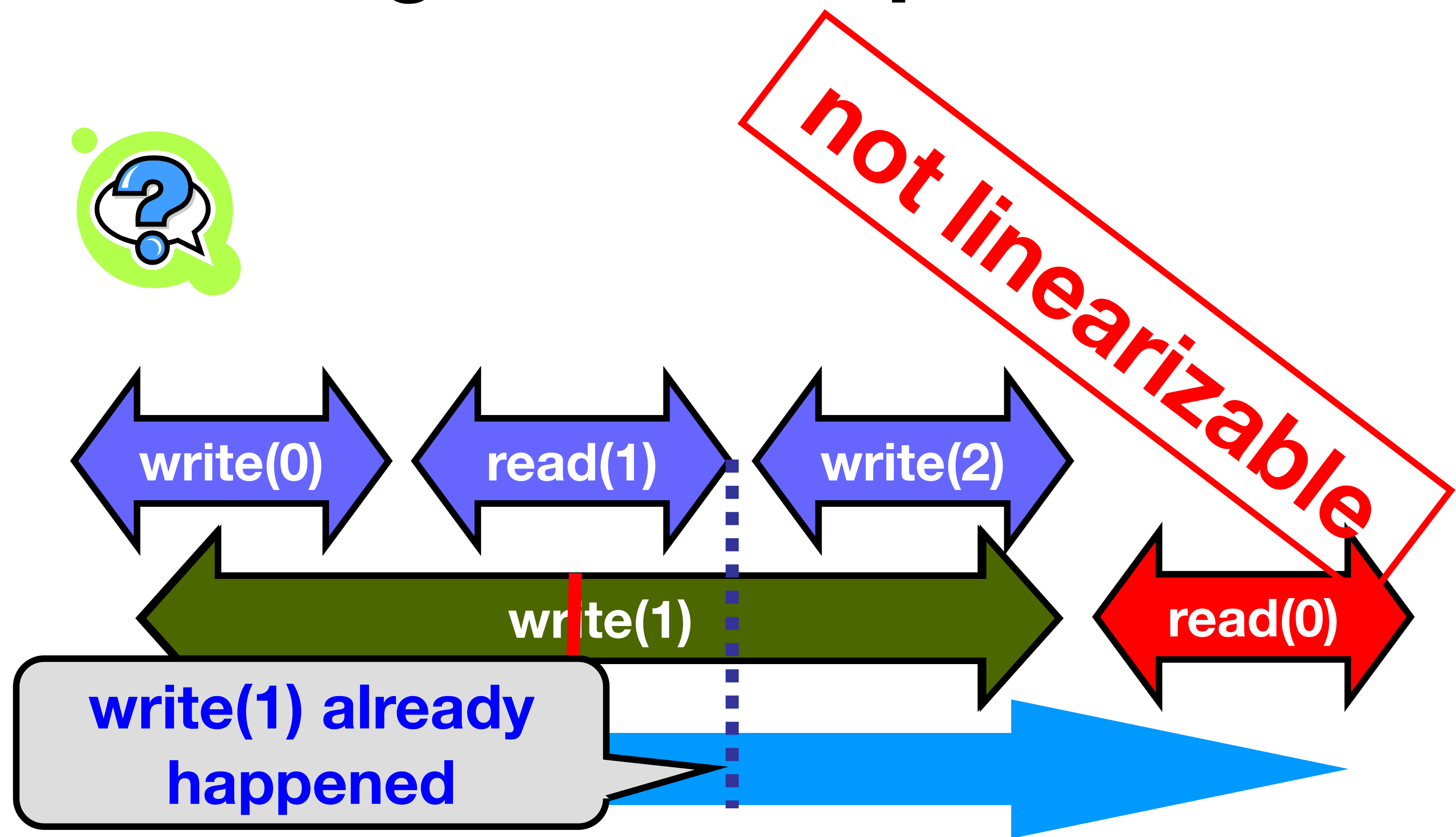
Read/Write Register Example



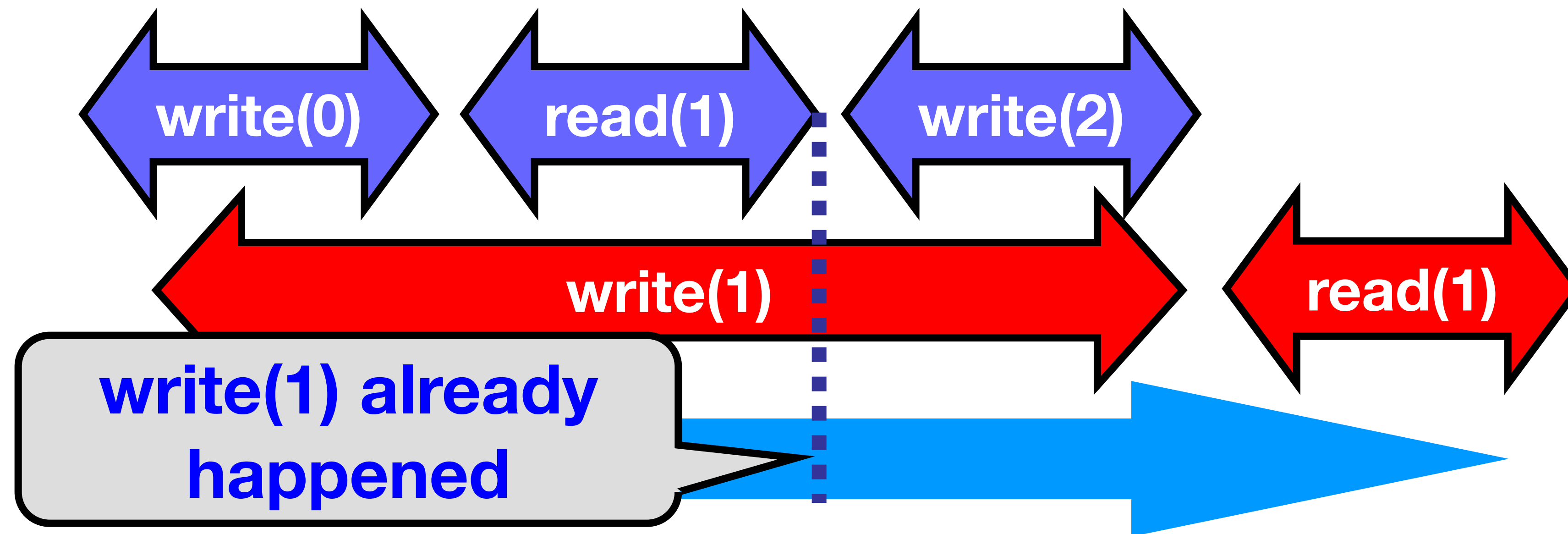
Read/Write Register Example



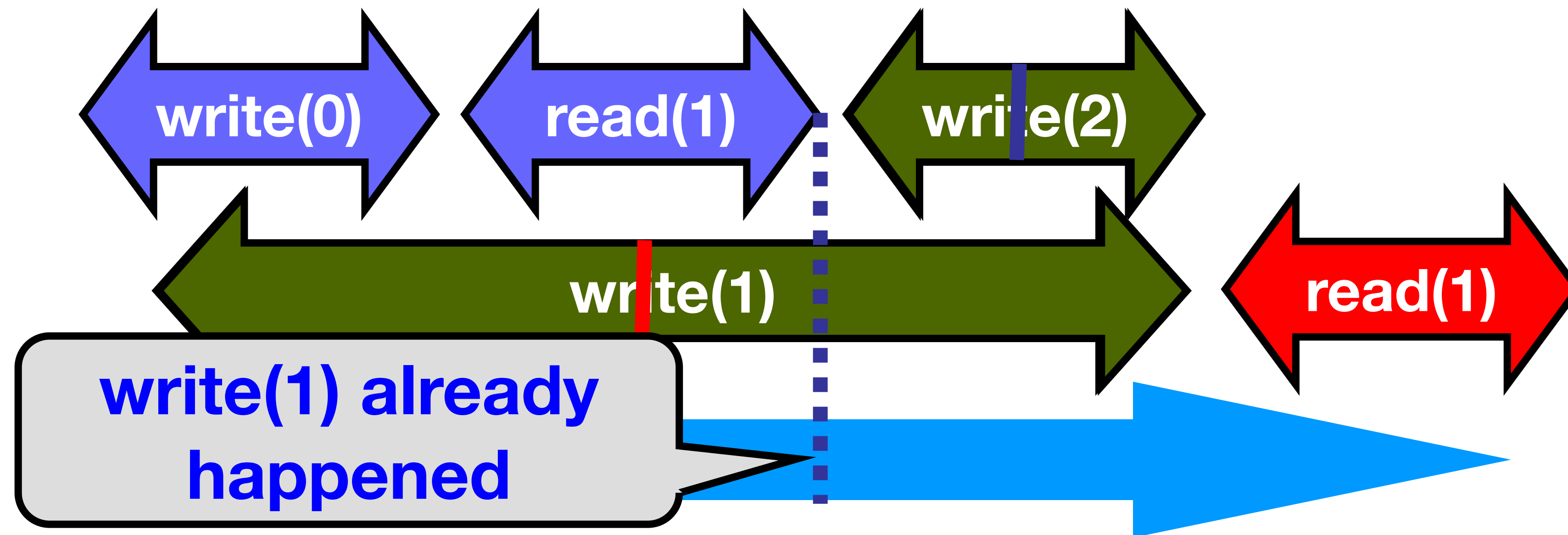
Read/Write Register Example



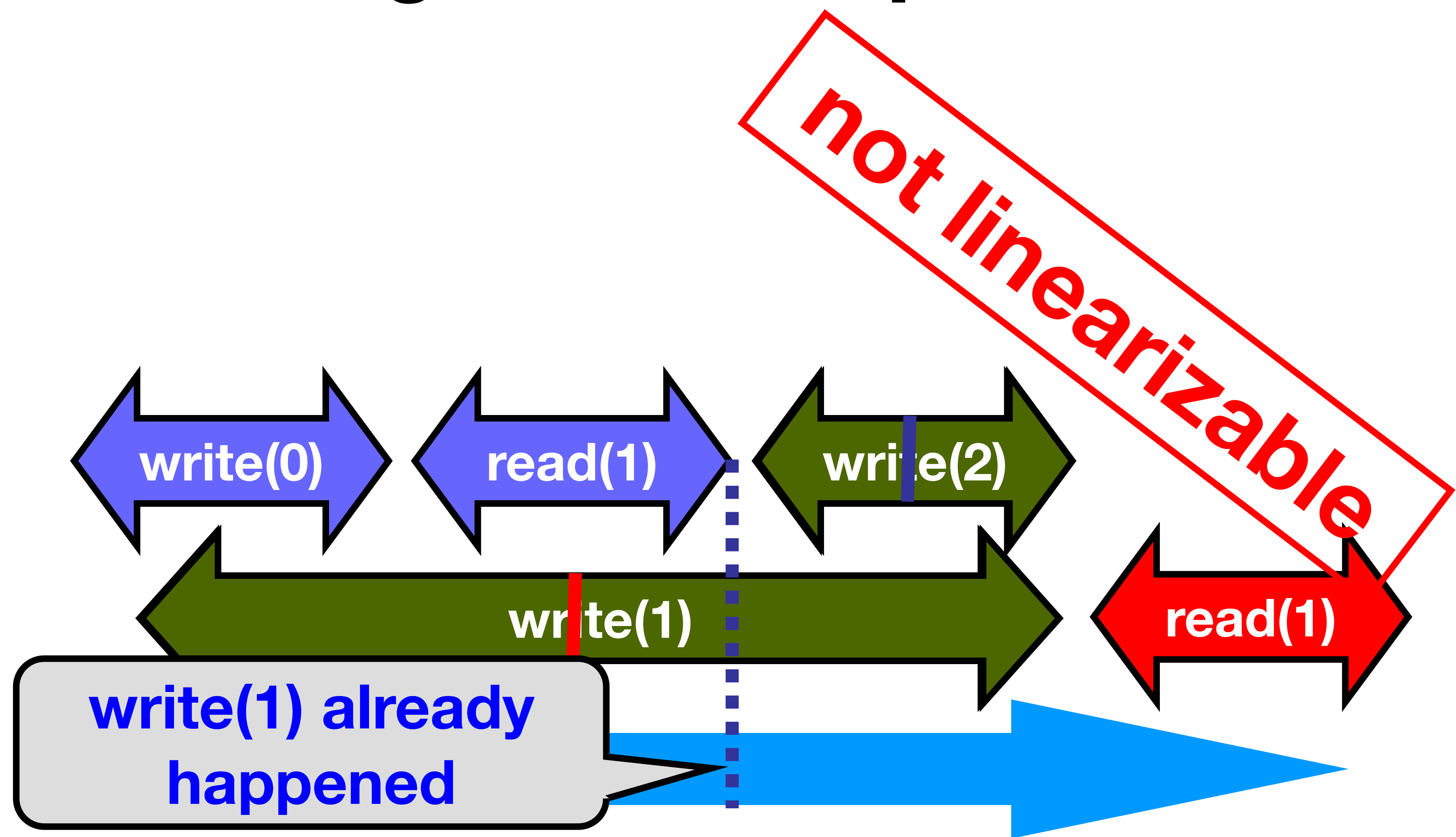
Read/Write Register Example



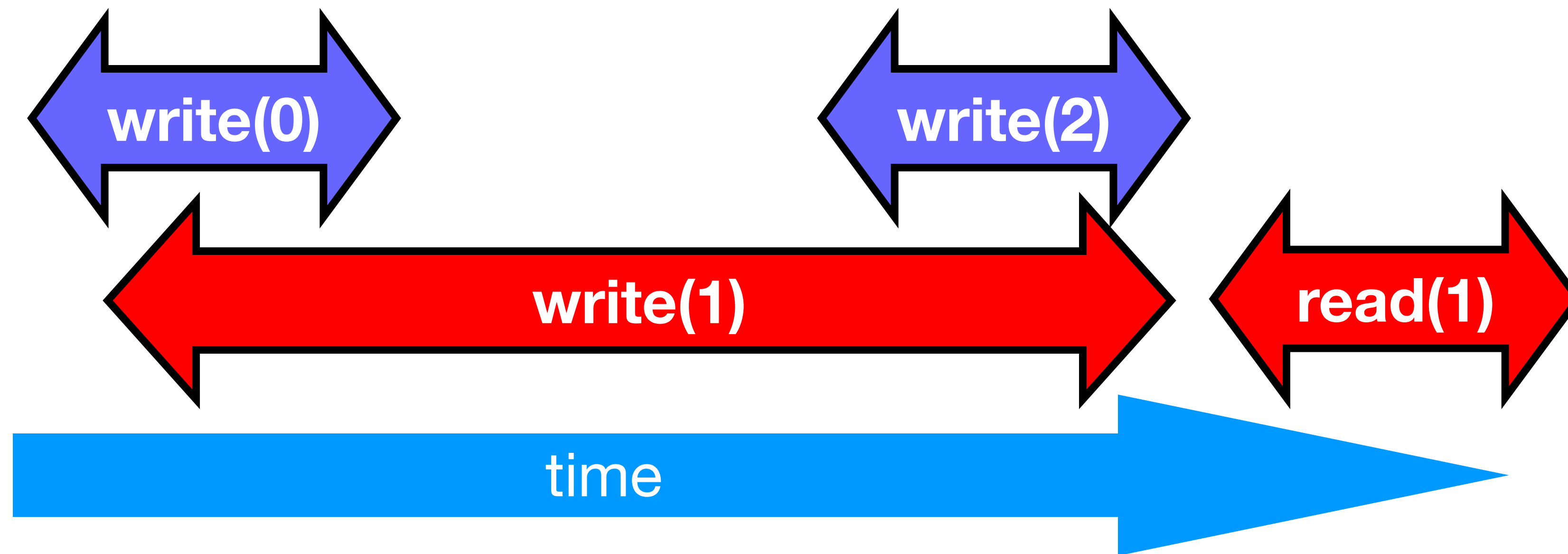
Read/Write Register Example



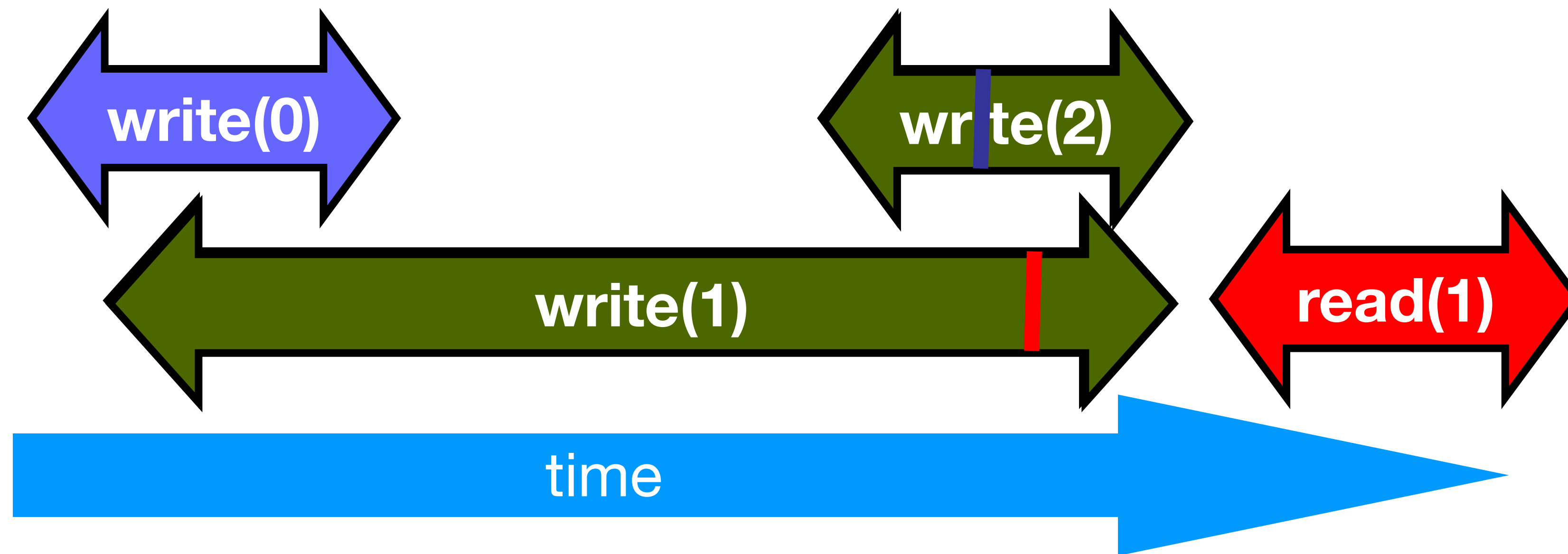
Read/Write Register Example



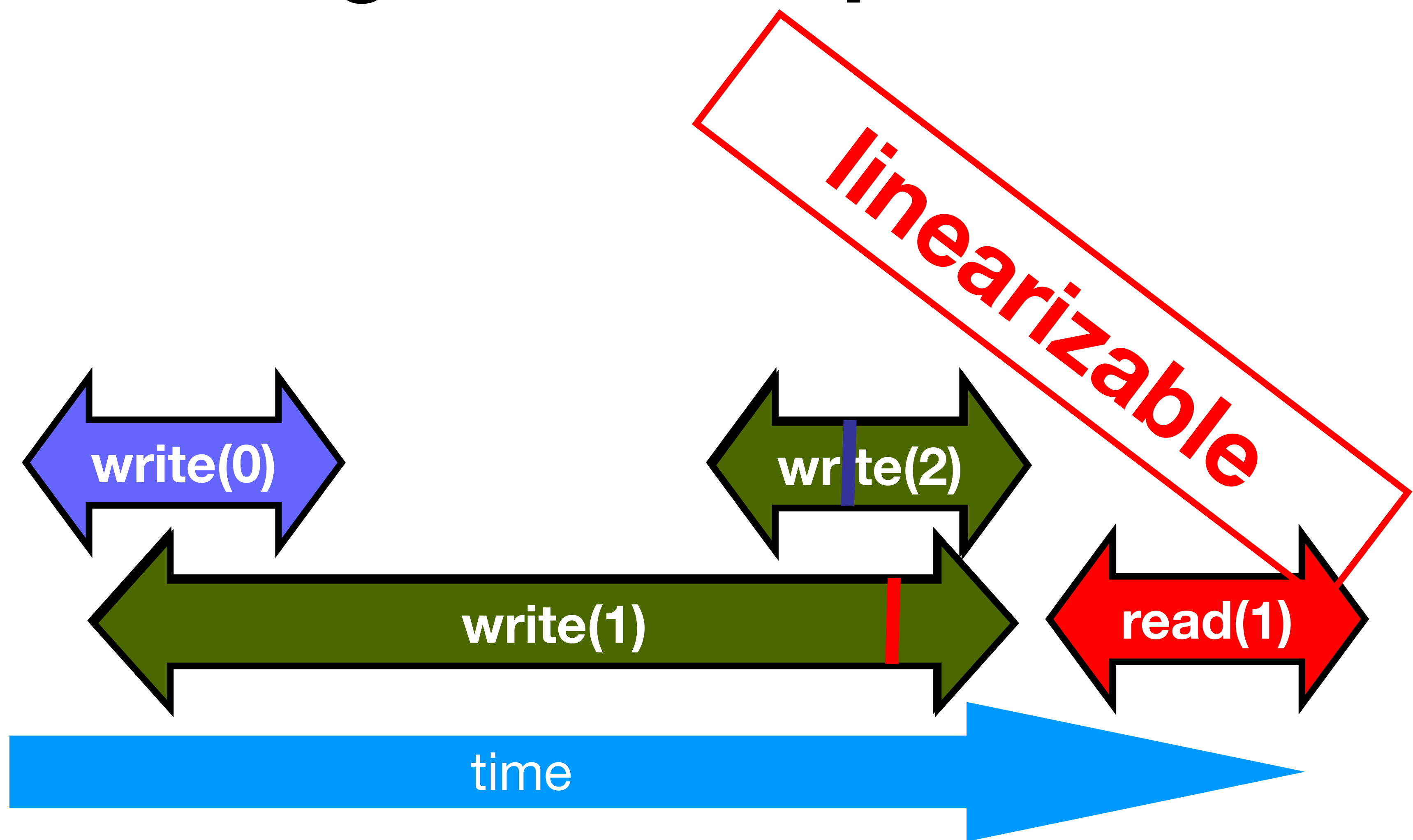
Read/Write Register Example



Read/Write Register Example



Read/Write Register Example



Talking About Executions

- Why?
 - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
 - In some cases, linearization point ***depends on the execution***

Formal Model of Executions

- Define precisely what we mean
 - Ambiguity is bad when intuition is weak
- Allow reasoning
 - Formal
 - But mostly informal
 - In the long run, actually, more important

Split Method Calls into Two Events

- Invocation
 - method name & args
 - **q.enq(x)**
- Response
 - result or exception
 - **q.enq(x)** returns **void**
 - **q.deq()** returns **x**
 - **q.deq()** throws **empty**
- Note that I'm following the convention of the book
 - Book uses OO
 - Code in this course uses FP
- Note that we're still reasoning using *objectivism*
- For the current discussion, distinction doesn't matter
 - **q.enq(x)** is read as **enq q x** in code
 - Returns **void** is read as returns **()**
 - Throws **empty** is read as raises **Empty**

Invocation Notation

A q.enq(x)

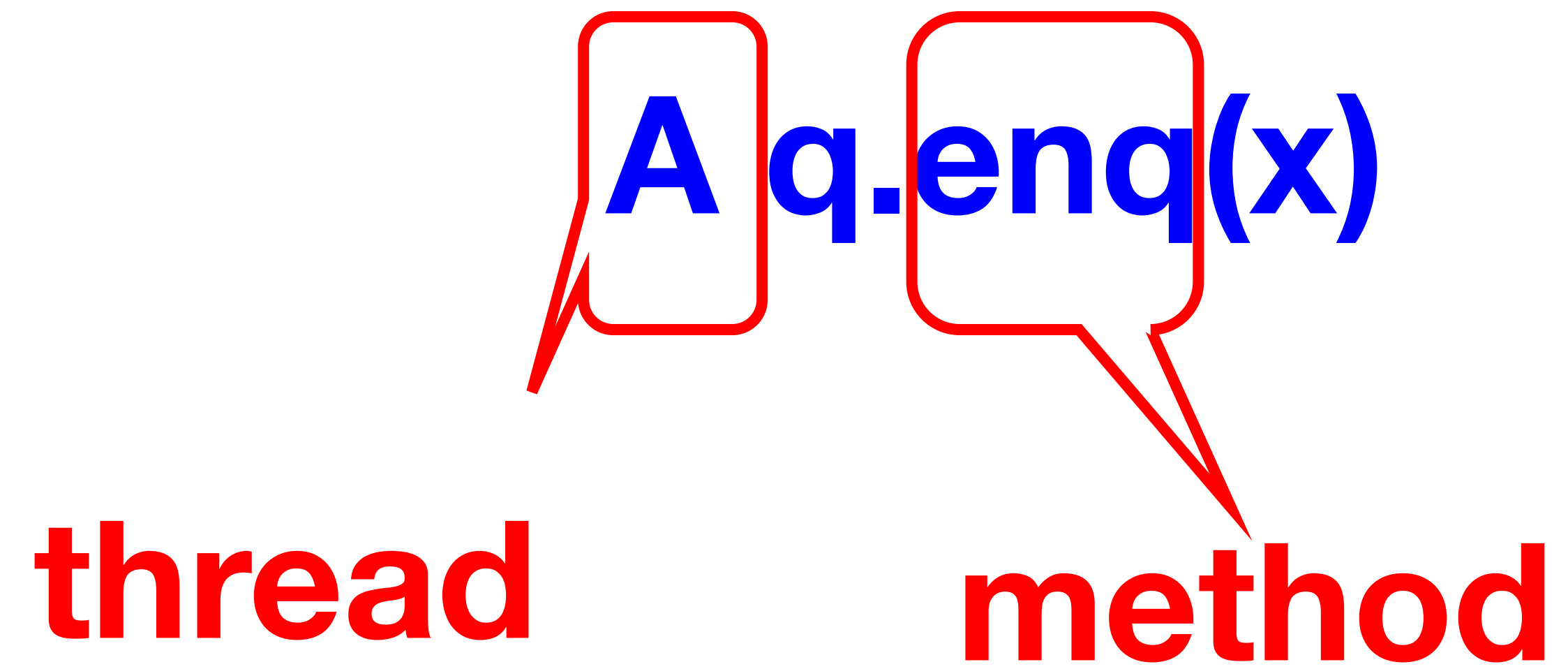
Invocation Notation

Aq.enq(x)

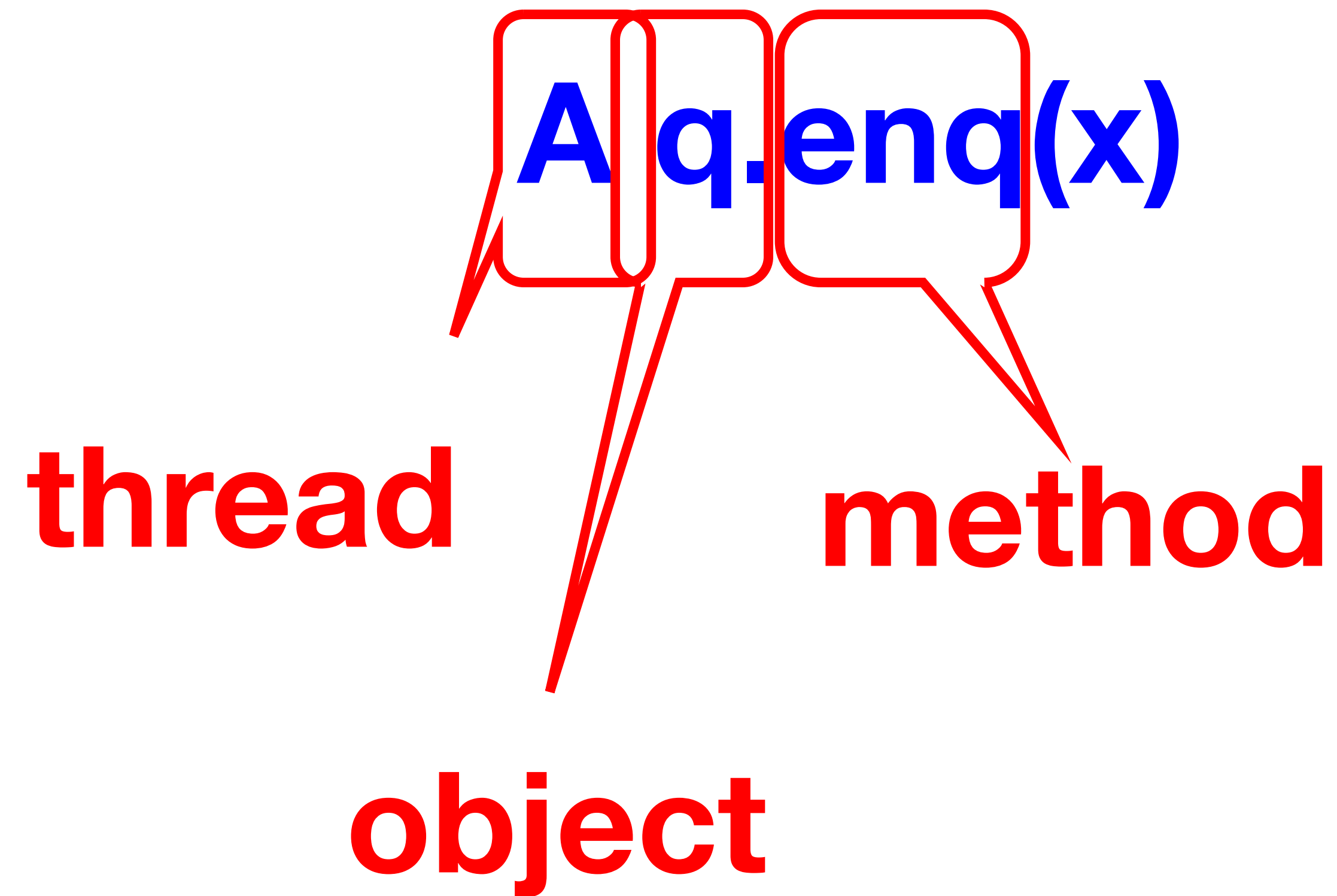


thread

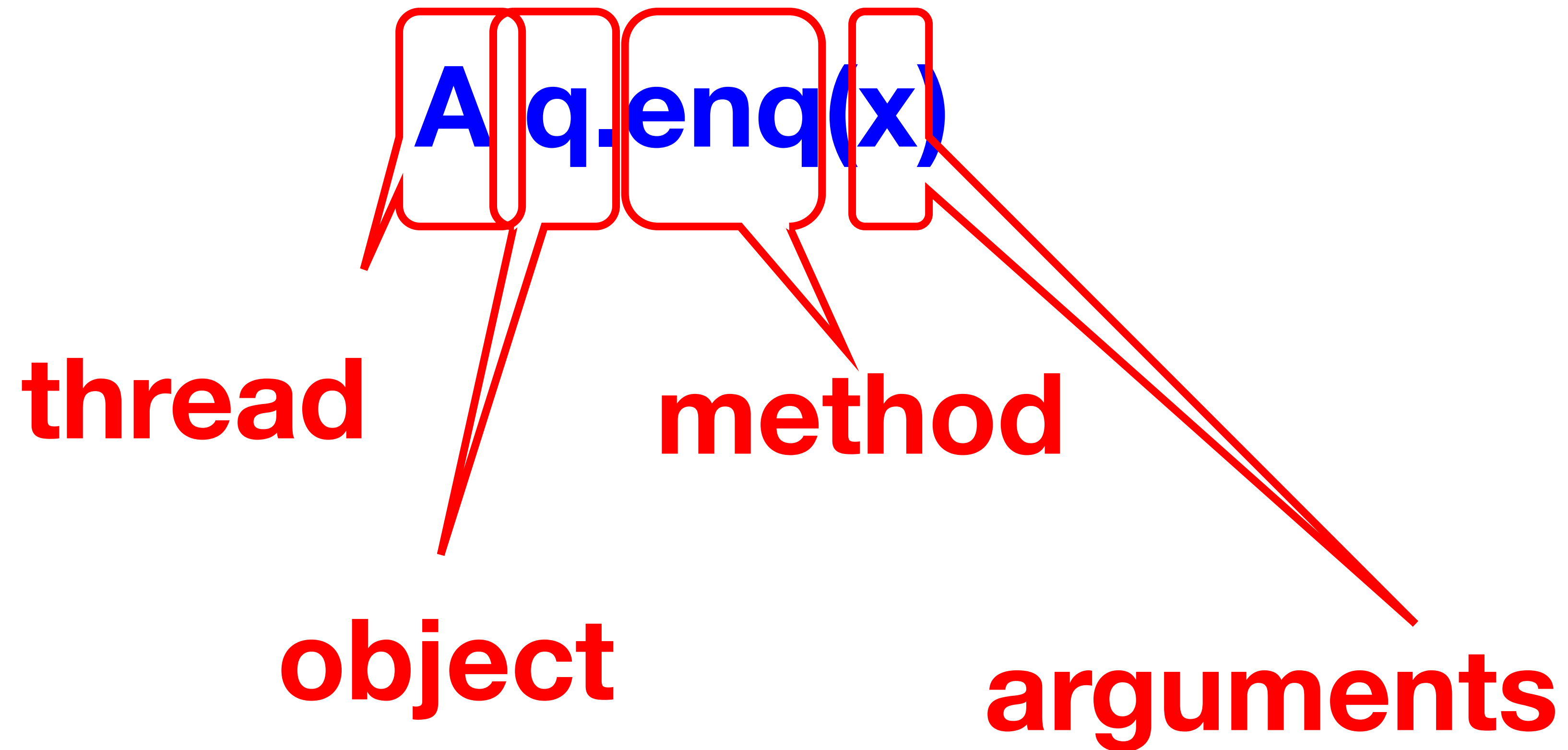
Invocation Notation



Invocation Notation



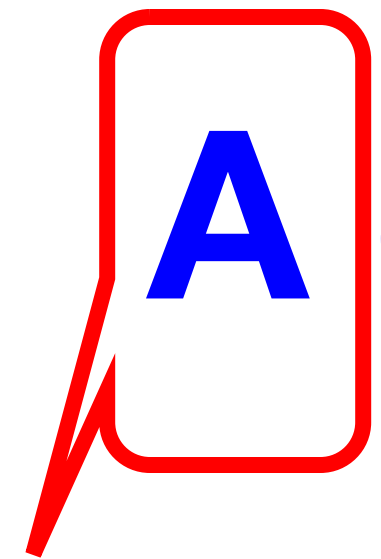
Invocation Notation



Response Notation

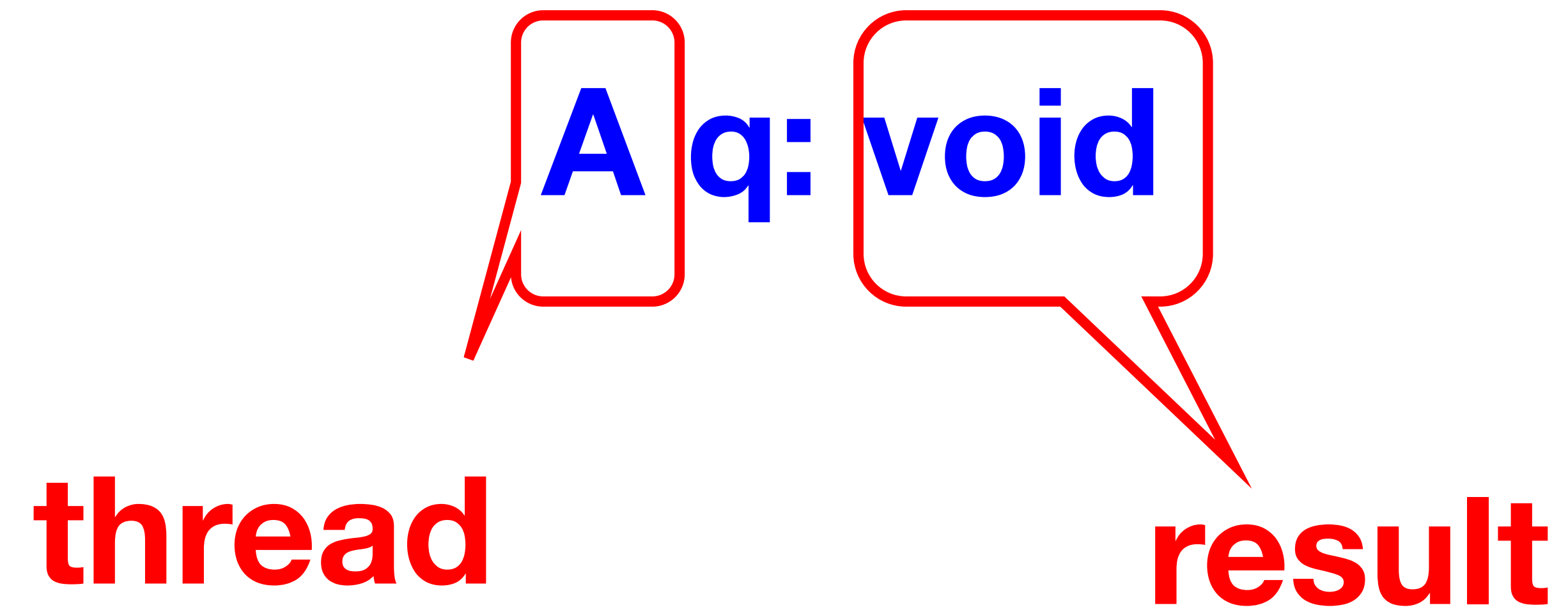
A q: void

Response Notation

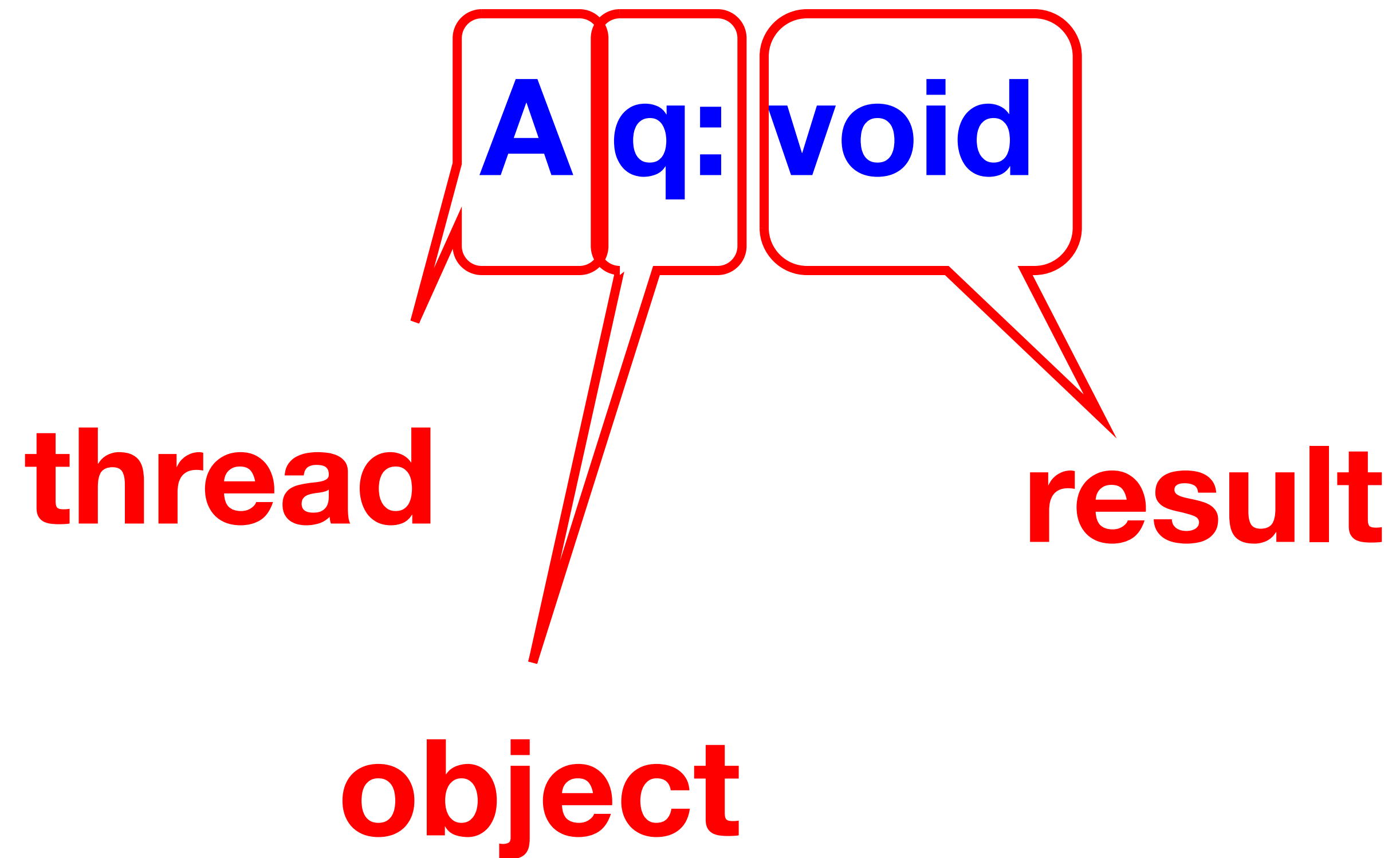
 **Aq: void**

thread

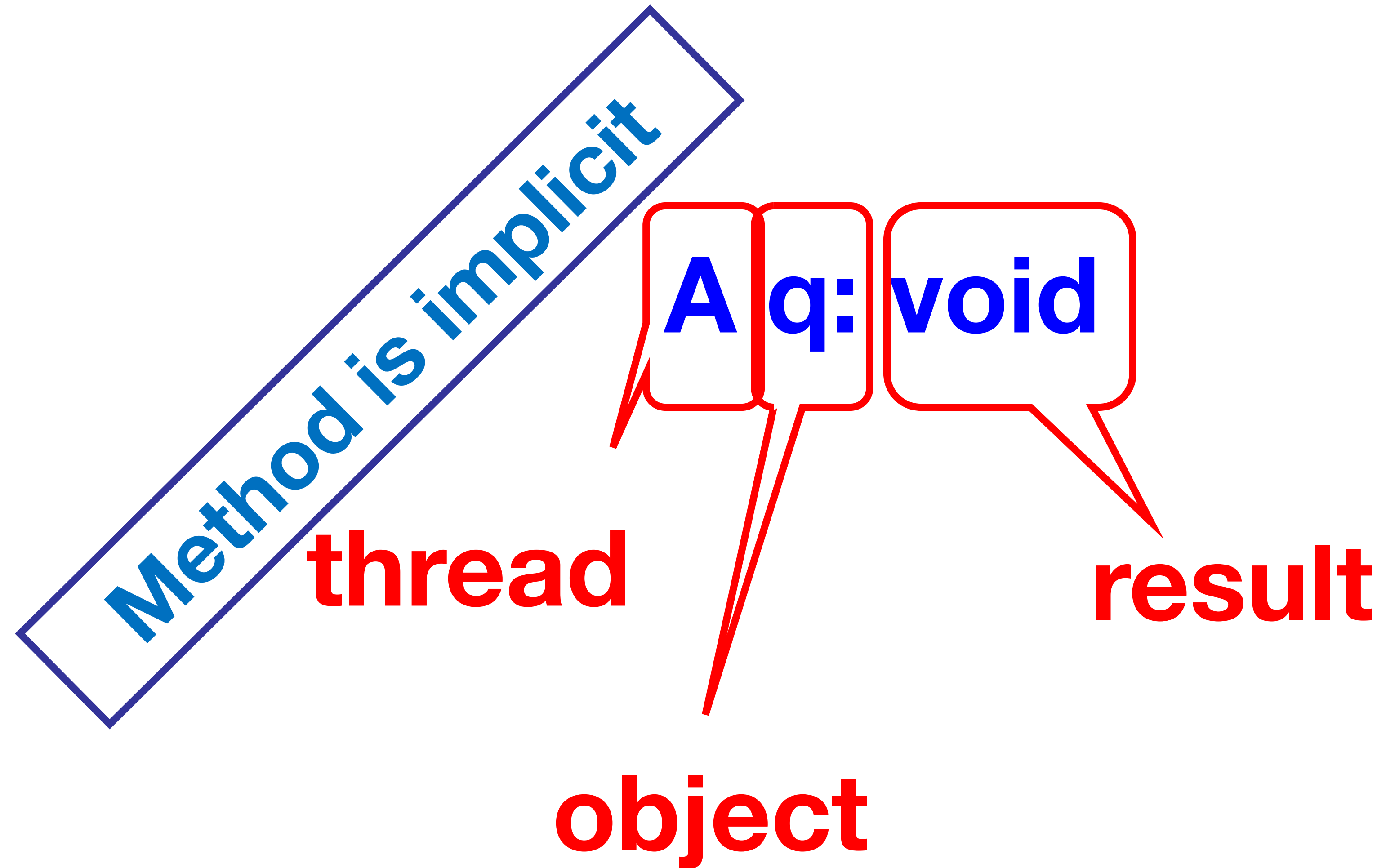
Response Notation



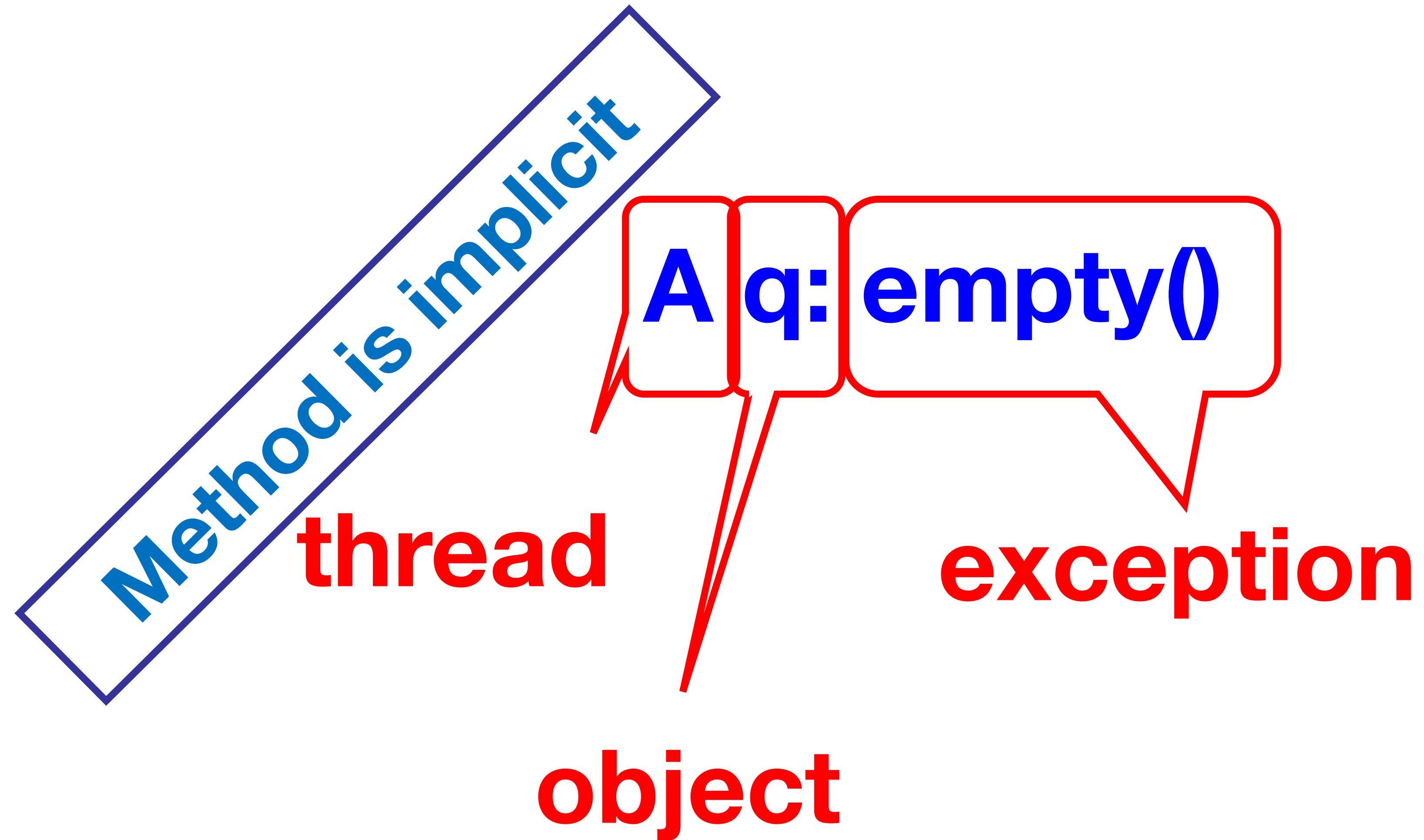
Response Notation



Response Notation



Response Notation



History — Describing an execution

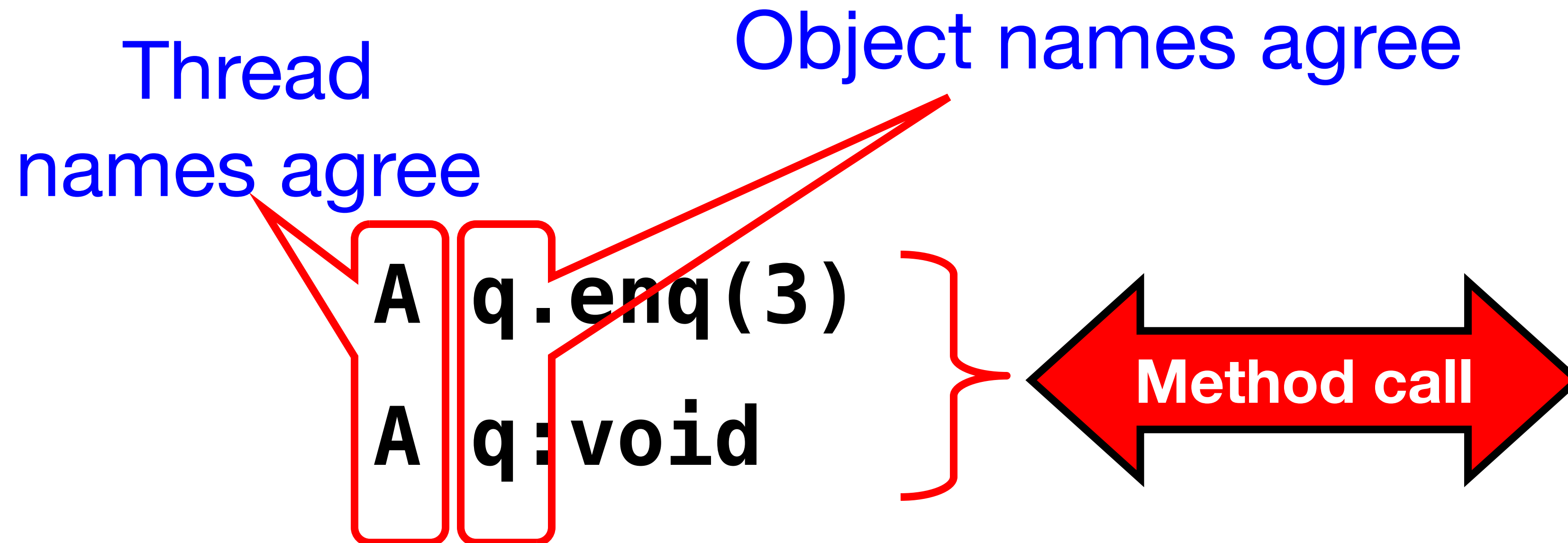
H =

- A q.enq(3)
- A q:void
- A q.enq(5)
- B p.enq(4)
- B p:void
- B q.deq()
- B q:3

**Sequence of
invocations and
responses**

History — Describing an execution

- Invocation & response *match* if



Object Projections

H =

A	q.enq(3)
A	q:void
B	p.enq(4)
B	p:void
B	q.deq()
B	q:3

Object Projections

A q . enq (3)
A q : void

H|q =

B q . deq ()
B q : 3

Thread Projections

H =

A	q . enq (3)
A	q : void
B	p . enq (4)
B	p : void
B	q . deq ()
B	q : 3

Thread Projections

$H|B =$

- $B \quad p.enq(4)$
- $B \quad p:void$
- $B \quad q.deq()$
- $B \quad q:3$

Complete Subhistory

H =

A	q.enq(3)
A	q:void
A	q.enq(5)
B	p.enq(4)
B	p:void
B	q.deq()
B	q:3



**An invocation is
pending if it has no
matching response**

Complete Subhistory

H =

A	q.enq(3)
A	q:void
A	q.enq(5)
B	p.enq(4)
B	p:void
B	q.deq()
B	q:3

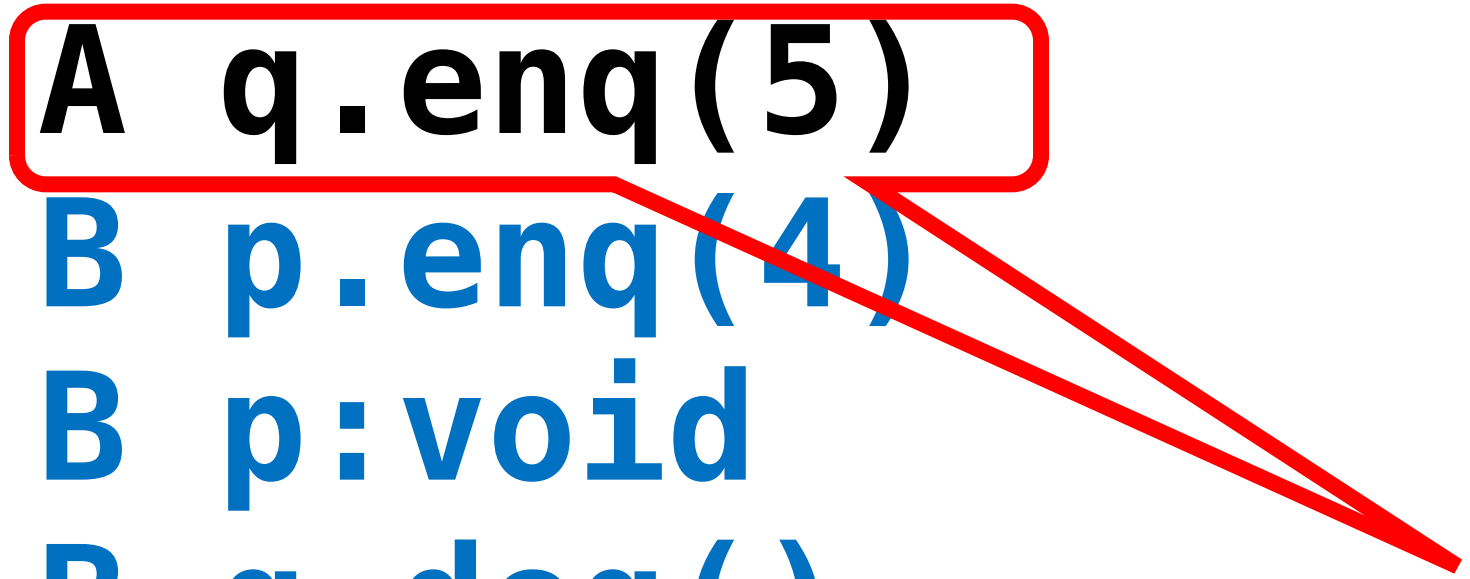
May or may not
have taken effect

Complete Subhistory

H =

A	q.enq(3)
A	q:void
A	q.enq(5)
B	p.enq(4)
B	p:void
B	q.deq()
B	q:3

may discard
pending invocations



Complete Subhistory

A q.enq(3)
A q:void

Complete(H) = B p.enq(4)
B p:void
B q.deq()
B q:3

Sequential Histories

A q.enq(3)

A q:void

B p.enq(4)

B p:void

B q.deq()

B q:3

A q:enq(5)

Sequential Histories

A q.enq(3)

A q:void

B p.enq(4)

B p:void

B q.deq()

B q:3

A q:enq(5)

match

Sequential Histories

A q.enq(3)

A q:void

match

B p.enq(4)

B p:void

match

B q.deq()

B q:3

A q:enq(5)

Sequential Histories

A q.enq(3)

A q:void

match

B p.enq(4)

B p:void

match

B q.deq()

B q:3

match

A q:enq(5)

Sequential Histories

A q.enq(3)

A q:void

match

B p.enq(4)

B p:void

match

B q.deq()

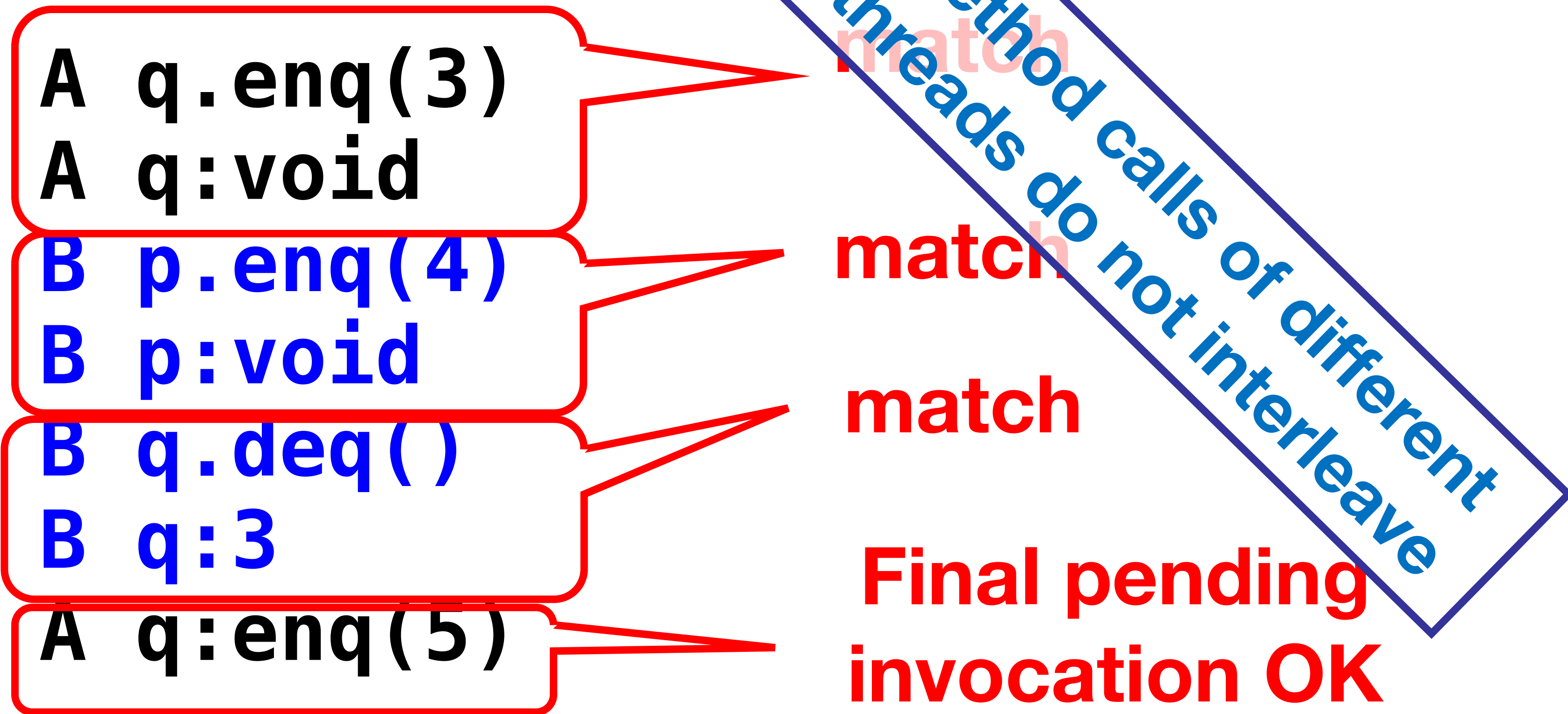
B q:3

match

A q:enq(5)

**Final pending
invocation OK**

Sequential Histories



Well-formed Histories

H=

A	q . enq (3)
B	p . enq (4)
B	p : void
B	q . deq ()
A	q : void
B	q : 3

Well-formed Histories

Per-thread projections
sequential

H=

A	q . enq (3)
B	p . enq (4)
B	p : void
B	q . deq ()
A	q : void
B	q : 3

H | B=

B	p . enq (4)
B	p : void
B	q . deq ()
B	q : 3

Well-formed Histories

Per-thread projections
sequential

H=

A	q . enq (3)
B	p . enq (4)
B	p : void
B	q . deq ()
A	q : void
B	q : 3

H | B=

B	p . enq (4)
B	p : void
B	q . deq ()
B	q : 3

H | A=

A	q . enq (3)
A	q : void

Equivalent Histories

Threads see the same
thing in both

$$\left\{ \begin{array}{l} H|A = G|A \\ H|B = G|B \end{array} \right.$$

H=

```
A q.enqueue(3)
B p.enqueue(4)
B p:void
B q.dequeue()
A q:void
B q:3
```

G=

```
A q.enqueue(3)
A q:void
B p.enqueue(4)
B p:void
B q.dequeue()
B q:3
```

Sequential Specifications

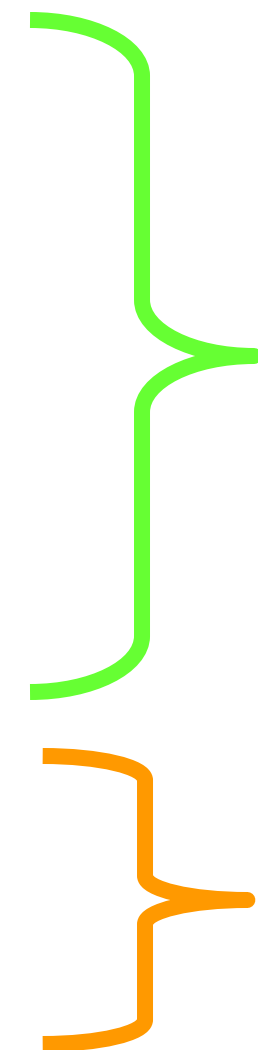
- A sequential *specification* is some way of telling whether a
 - Single-thread, single-object history
 - Is *legal*
- For example:
 - Pre and post-conditions
 - But plenty of other techniques exist ...

Legal Histories

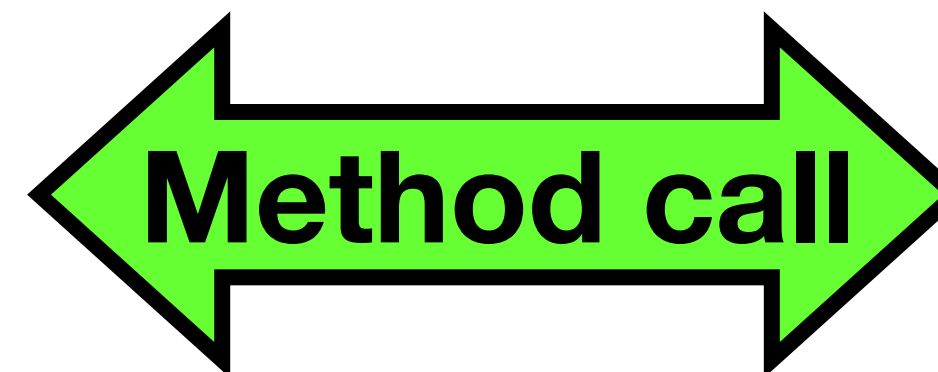
- A sequential *(multi-object)* history H is *legal* if
 - For every object x
 - $H|x$ is in the sequential spec for x

Precedence

A q.enq(3)
B p.enq(4)
B p.void
A q:void
B q.deq()
B q:3

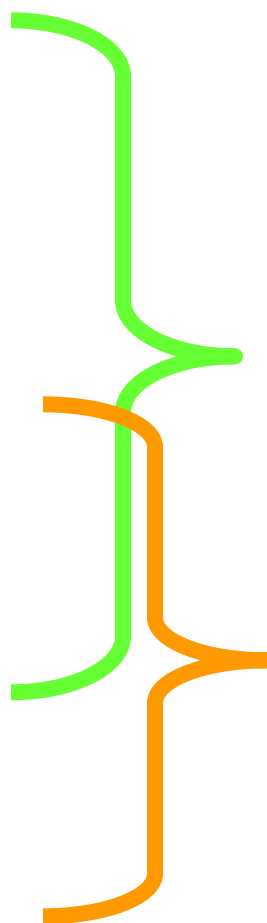


A method call **precedes**
another if response event
precedes invocation
event

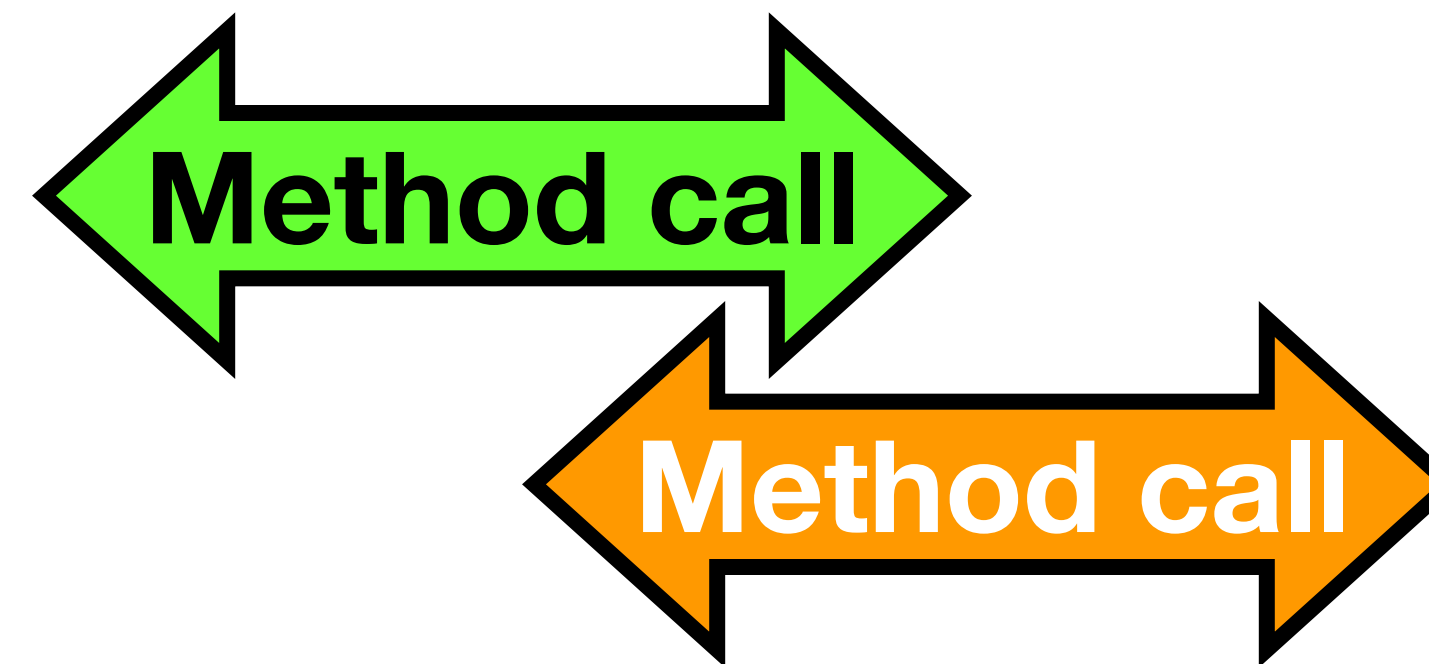


Non-Precedence

```
A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q: void
B q: 3
```

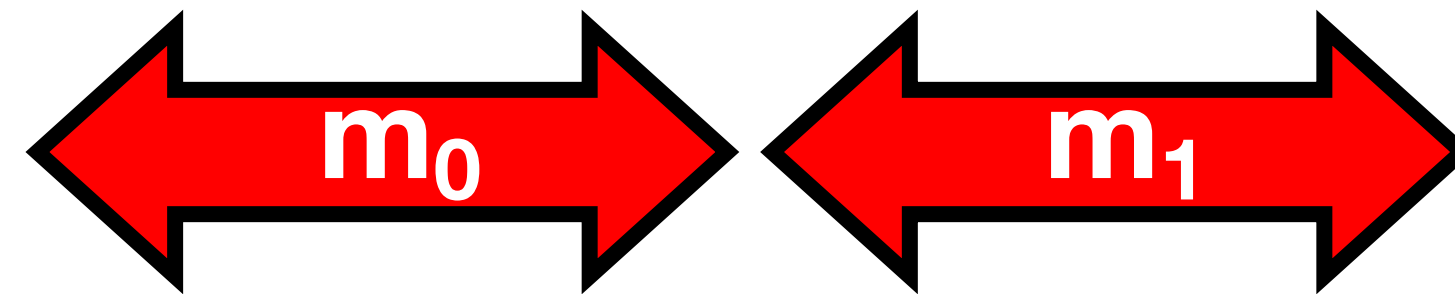


Some method calls
overlap one another



Notation

- Given
 - History **H**
 - method executions **m₀** and **m₁** in **H**
- We say **m₀ →_H m₁**, if
 - **m₀** precedes **m₁**
- Relation **m₀ →_H m₁** is a
 - Partial order
 - Total order if **H** is sequential



Linearizability

- History H is *linearizable* if it can be extended to \mathbf{G} by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that G is equivalent to
 - Legal sequential history \mathbf{S}
 - where $\rightarrow_{\mathbf{G}} \subset \rightarrow_{\mathbf{S}}$

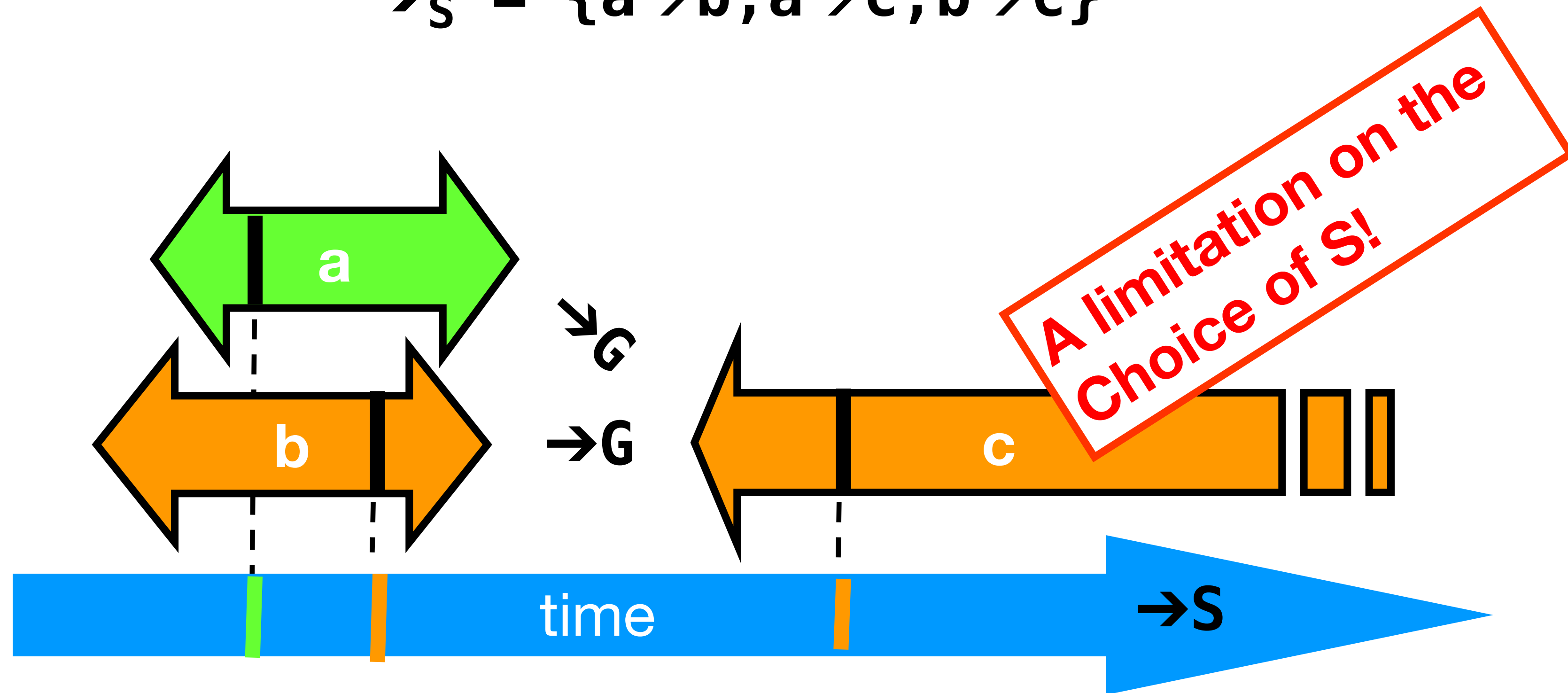
Remarks on Linearizability

- Some pending invocations
 - Took effect, so keep them
 - Discard the rest
- Condition $\rightarrow_G \subset \rightarrow_S$
 - Means that **S** respects “real-time order” of **G**

Ensuring $\rightarrow_G \subset \rightarrow_S$

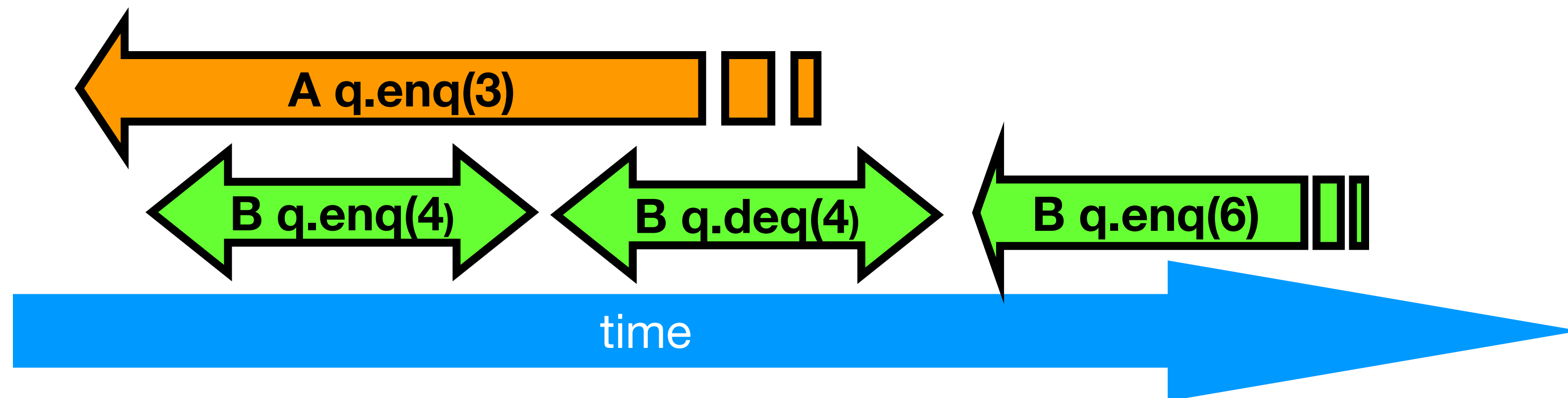
$$\rightarrow_G = \{a \rightarrow c, b \rightarrow c\}$$

$$\rightarrow_S = \{a \rightarrow b, a \rightarrow c, b \rightarrow c\}$$



Example

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)



Example

A q.enq(3)

B q.enq(4)

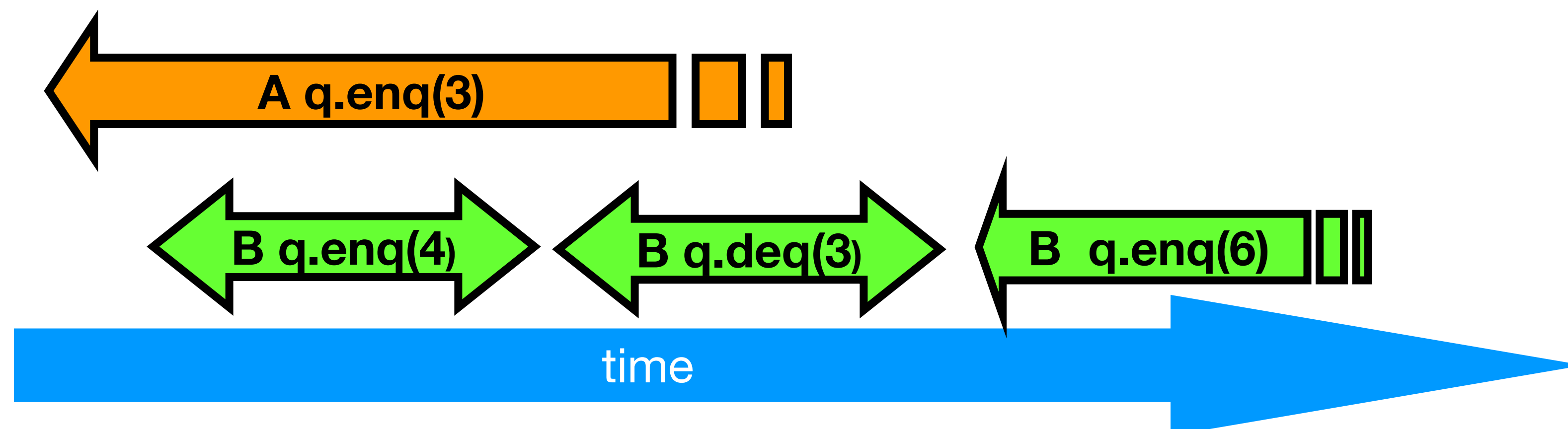
B q:void

B q.deq()

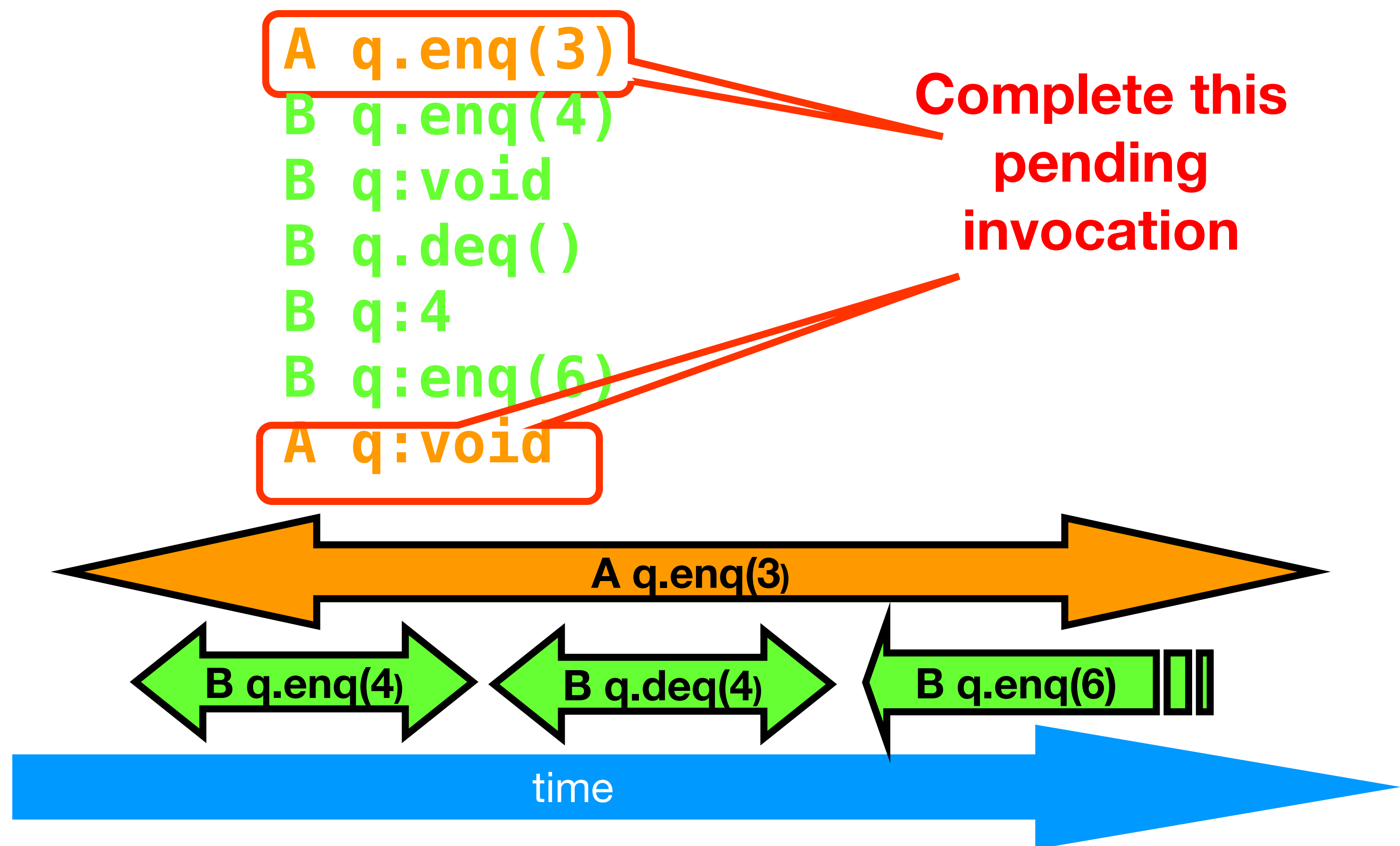
B q:4

B q:enq(6)

Complete this
pending
invocation



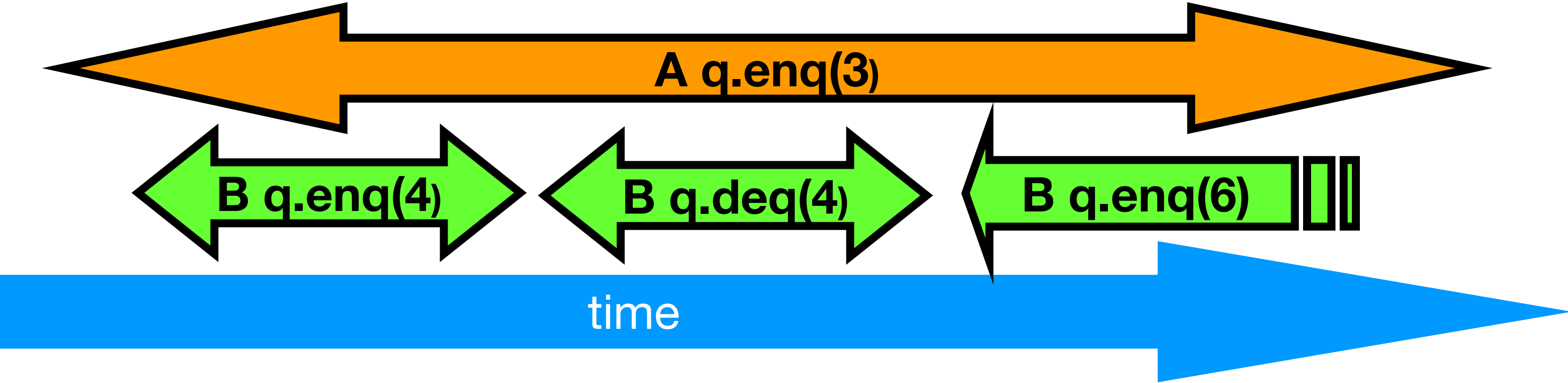
Example



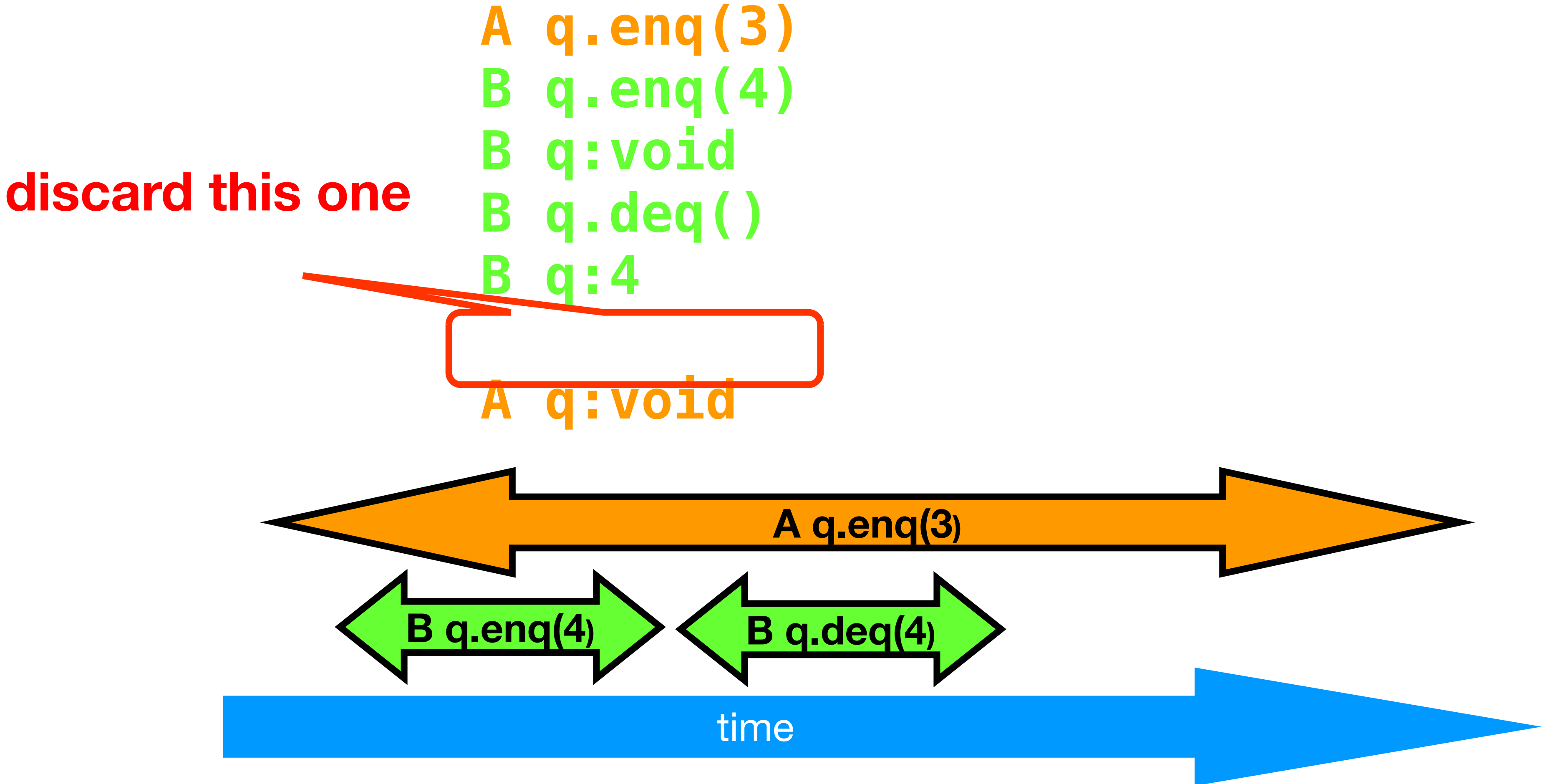
Example

discard this one

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void

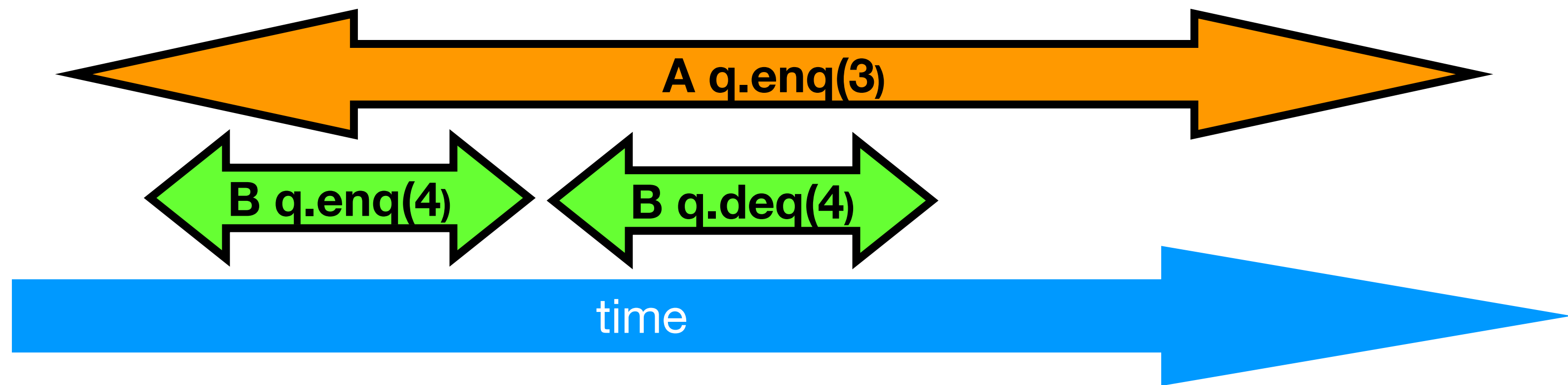


Example



Example

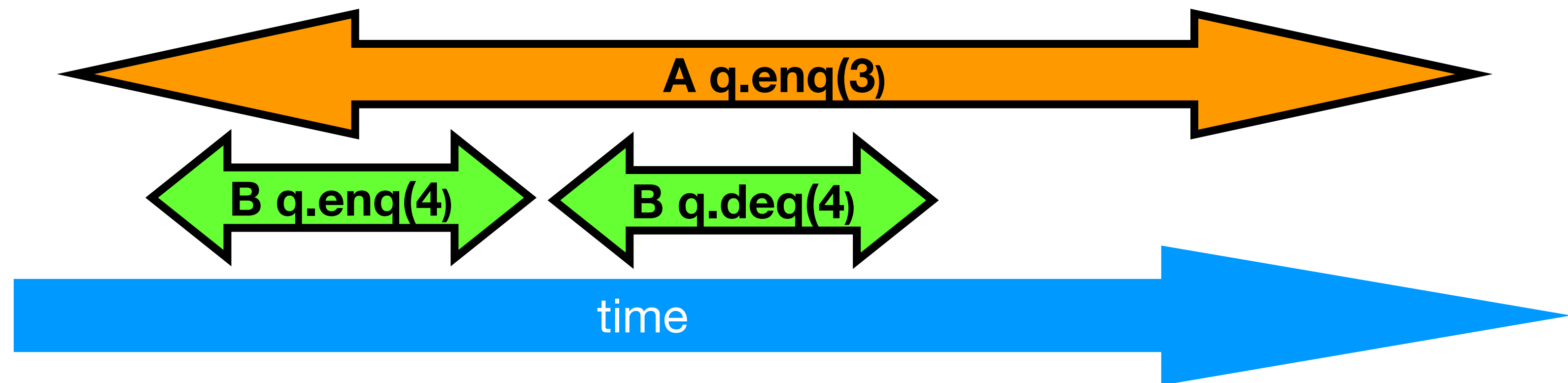
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void



Example

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void

B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4

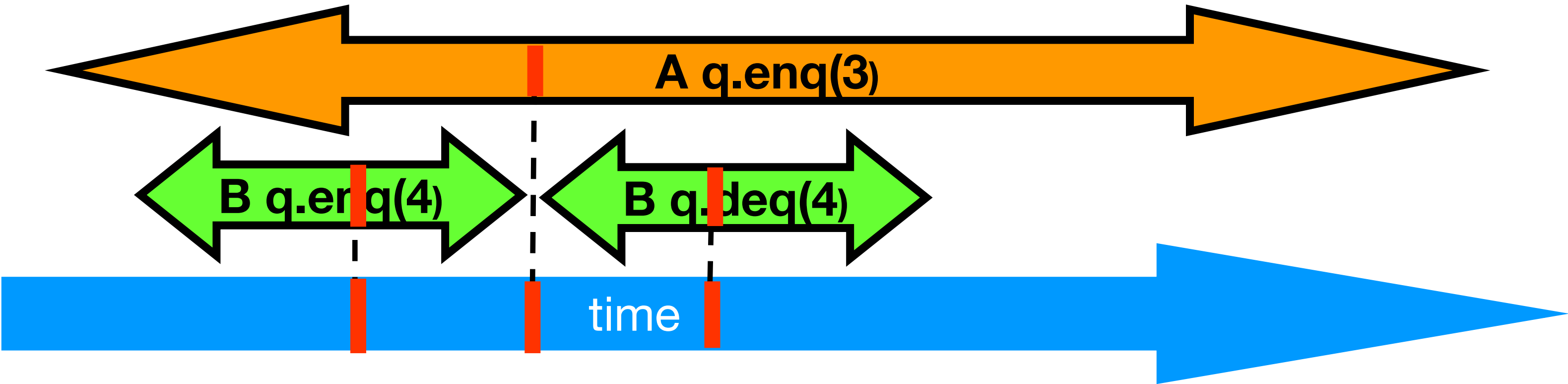


Example

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void

Equivalent sequential history

B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4



Composability Theorem

- History H is linearizable if and only if
 - For every object x
 - $H|x$ is linearizable
- We care about objects only!
 - (Materialism?)

Why does composability matter?

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

Reasoning about Linearizability: Locking

```
let def q =  
  Mutex.lock q.lock;  
  try  
    if q.tail = q.head then  
      raise Empty;  
    match q.items.(q.head mod q.capacity) with  
    | None -> assert false  
    | Some x ->  
      q.head <- q.head + 1;  
      Mutex.unlock q.lock;  
      x  
  with e ->  
    Mutex.unlock q.lock;  
    raise e
```

Linearization points
are when locks are
released

More Reasoning — Wait-free

(** Enqueue – should be called by only ONE thread *)

```
let enq q x =  
  (* Check if queue is full *)  
  if q.tail - q.head = q.capacity then  
    raise Full;  
  (* Write to the array *)  
  q.items.(q.tail mod q.capacity) <- Some x;  
  (* Advance tail *)  
  q.tail <- q.tail + 1
```

(** Dequeue – should be called by only ONE thread *)

```
let deq q =  
  (* Check if queue is empty *)  
  if q.tail = q.head then  
    raise Empty;  
  (* Read from the array *)  
  match q.items.(q.head mod q.capacity) with  
  | None -> assert false (* Should never happen *)  
  | Some x ->  
    (* Advance head *)  
    q.head <- q.head + 1;  
    x
```

More Reasoning – Wait-free

```
(** Enqueue – should be called by only ONE thread *)
let enq q x =
  (* Check if queue is full *)
  if q.tail - q.head = q.capacity then
    raise Full;
  (* Write to the array *)
  q.items.(q.tail mod q.capacity) <- x;
  (* Advance tail *)
  q.tail <- q.tail + 1;

(** Dequeue – should be called by only ONE thread *)
let deq q =
  (* Check if queue is empty *)
  if q.tail = q.head then
    raise Empty;
  (* Read from the array *)
  match q.items.(q.head mod q.capacity) with
  | None -> assert false (* Should never happen *)
  | Some x ->
    (* Advance head *)
    q.head <- q.head + 1;
    x
```

**Remember that there
is only one enqueuer
and only one dequeuer**

Linearization order is
order head and tail
fields modified

Finding linearisation points

- Identify one atomic step where the method “happens”
 - Critical section
 - Machine instruction
- Doesn't always work
 - Might need to define several different steps for a given method
 - We will see this phenomenon in future lectures

Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being “atomic”
- Don't leave home without it

Alternative: Sequential Consistency

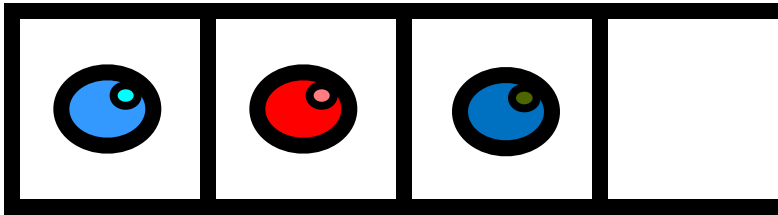
- History H is **Sequentially Consistent** if it can be extended to G by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that G is equivalent to
 - Legal sequential history S
 - ~~where $\rightarrow_G \subseteq \rightarrow_S$~~

*Differs from
Linearizability*

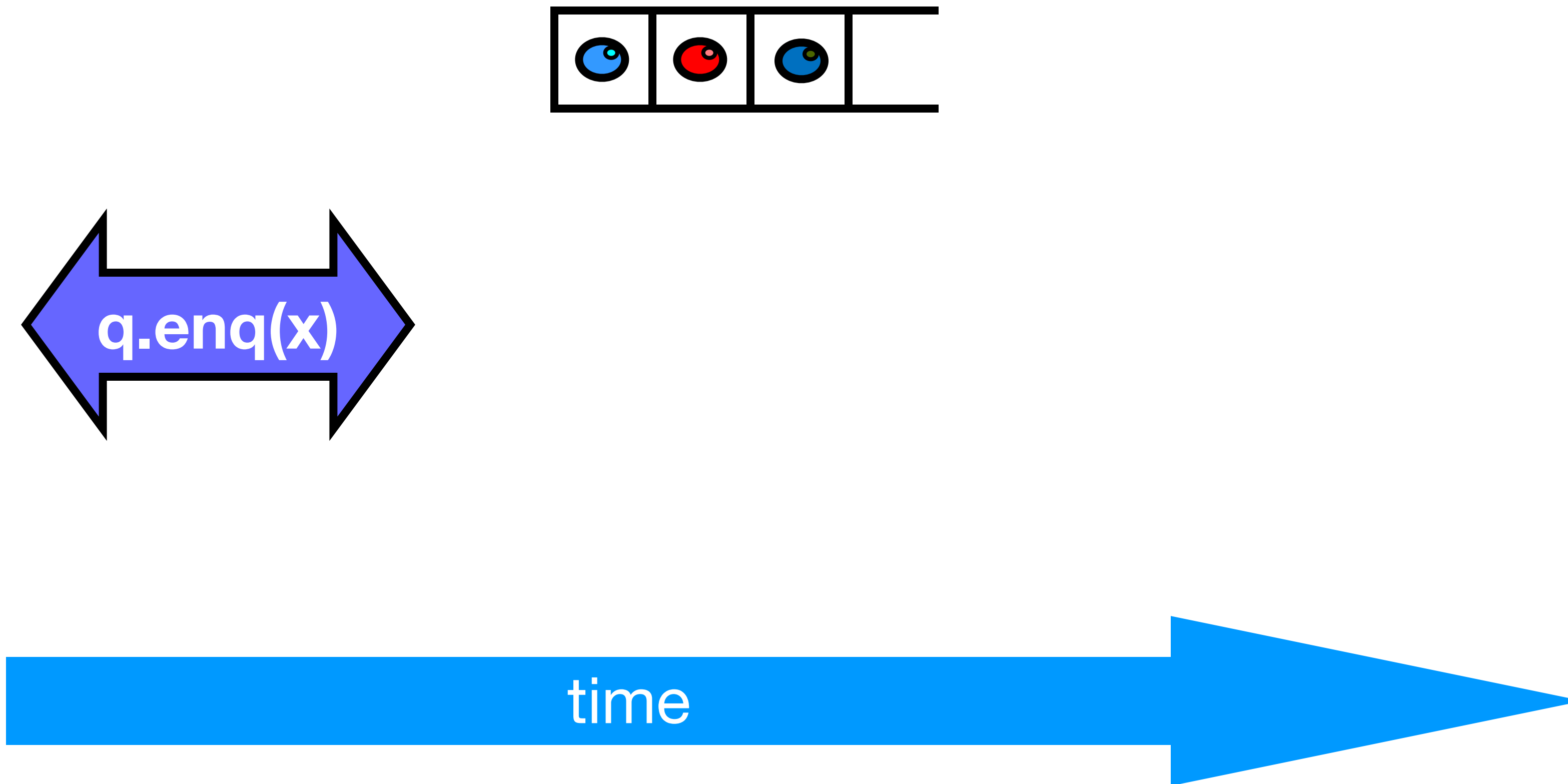
Sequential Consistency

- No need to preserve real-time order
 - **Cannot** re-order operations done by the same thread
 - **Can** re-order non-overlapping operations done by different threads
- Often used to describe multiprocessor memory architectures

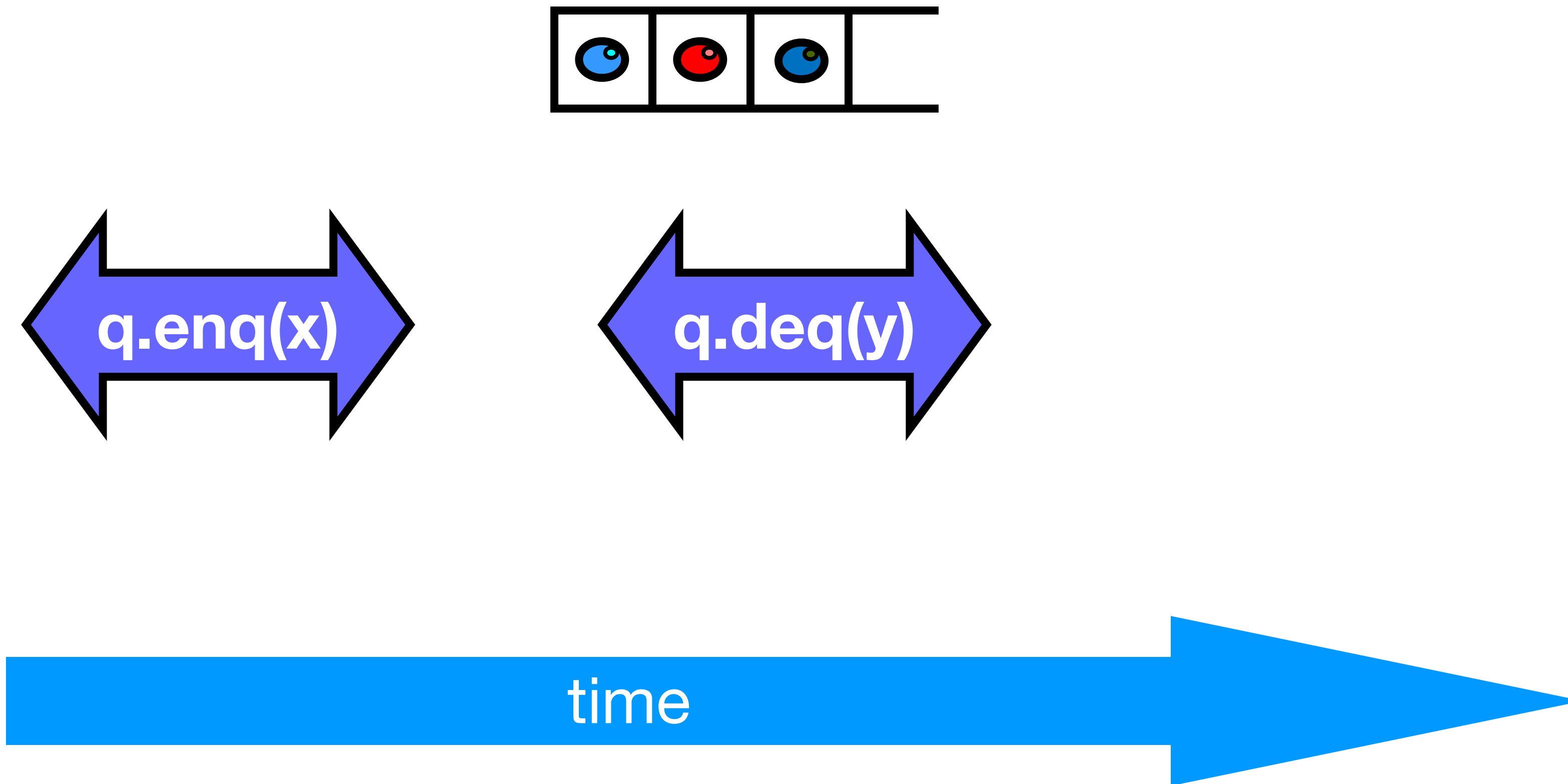
Example



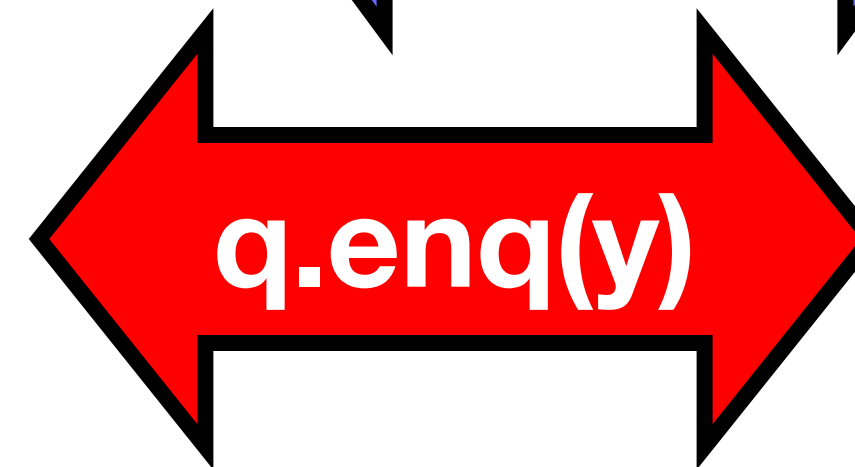
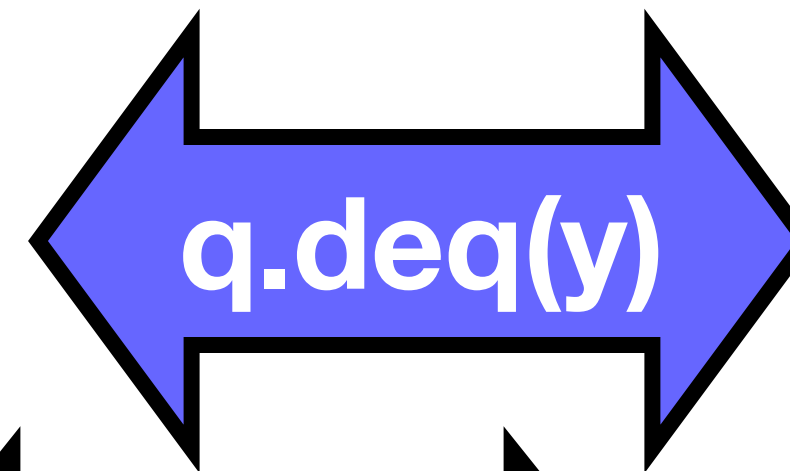
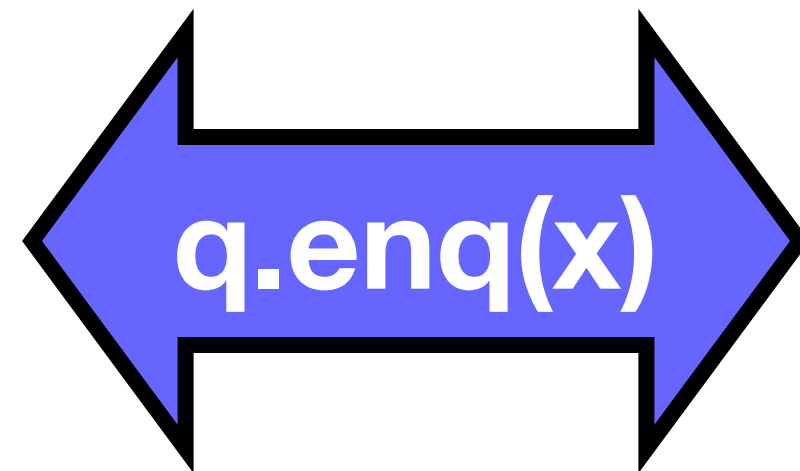
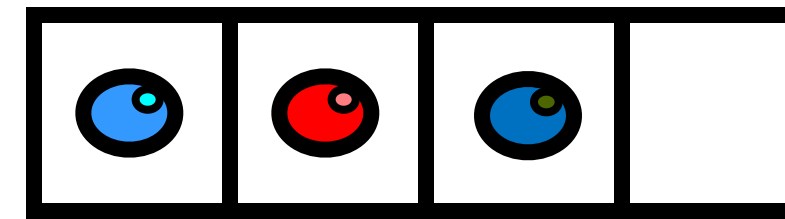
Example



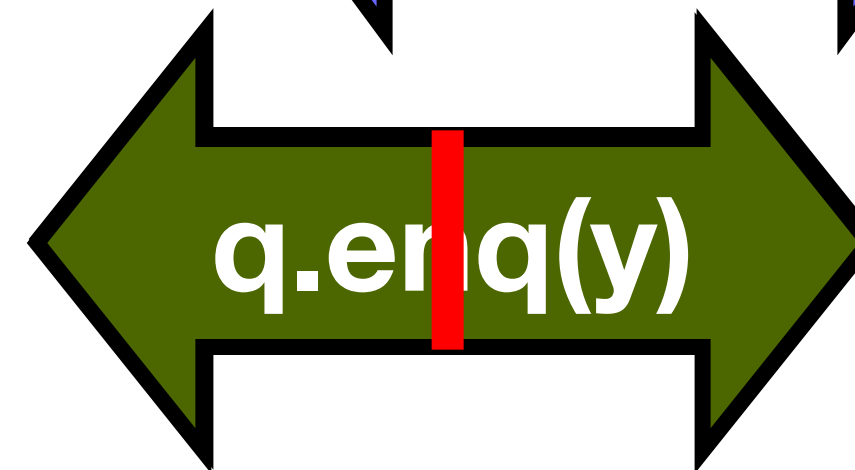
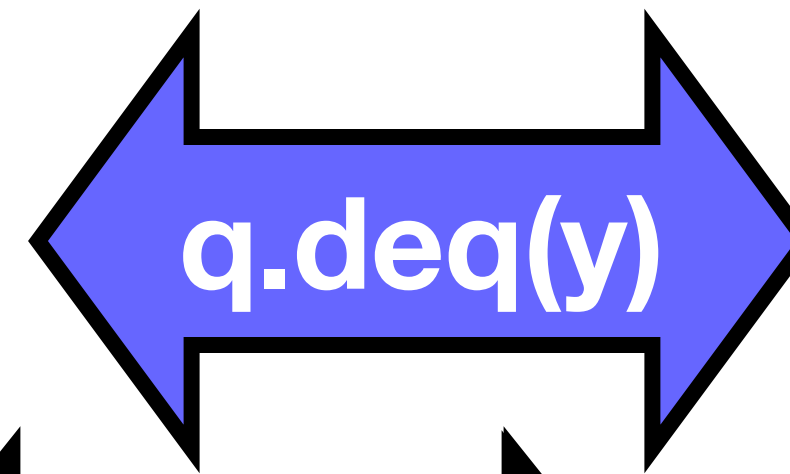
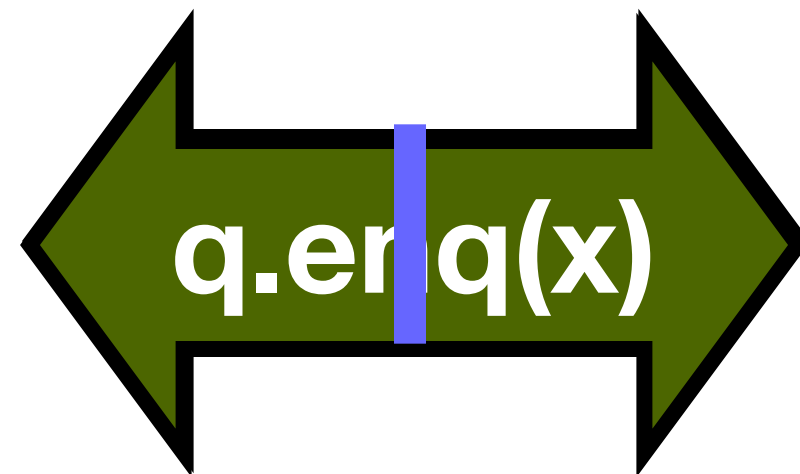
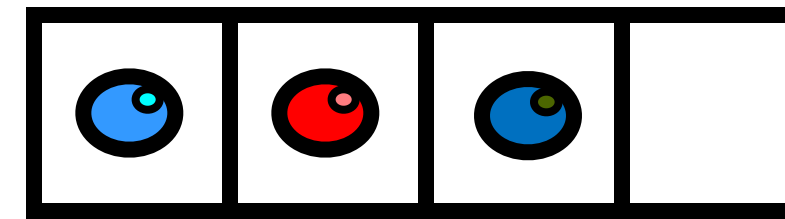
Example



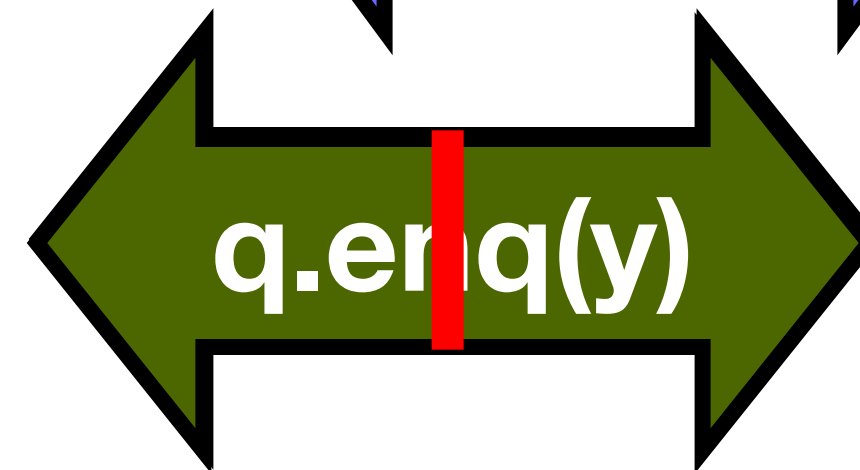
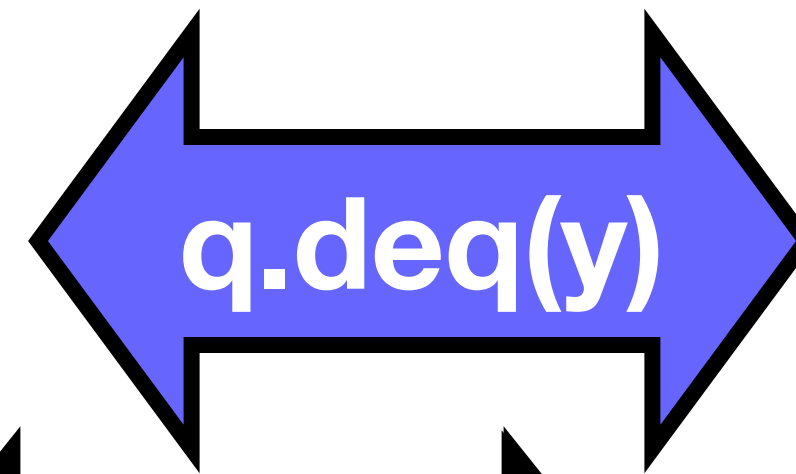
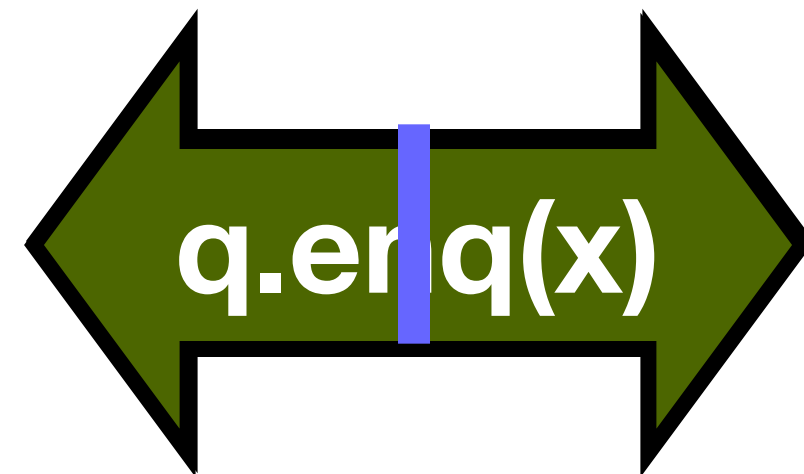
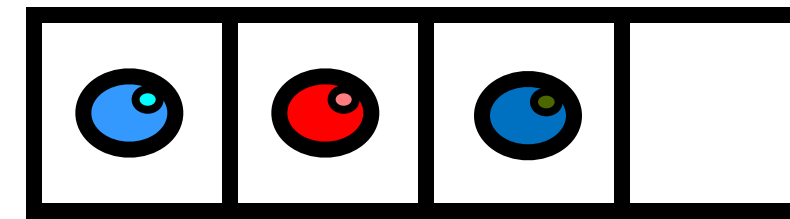
Example



Example

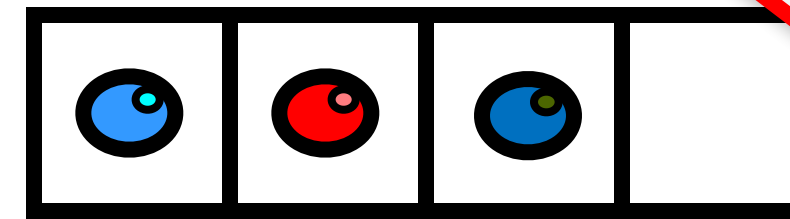


Example

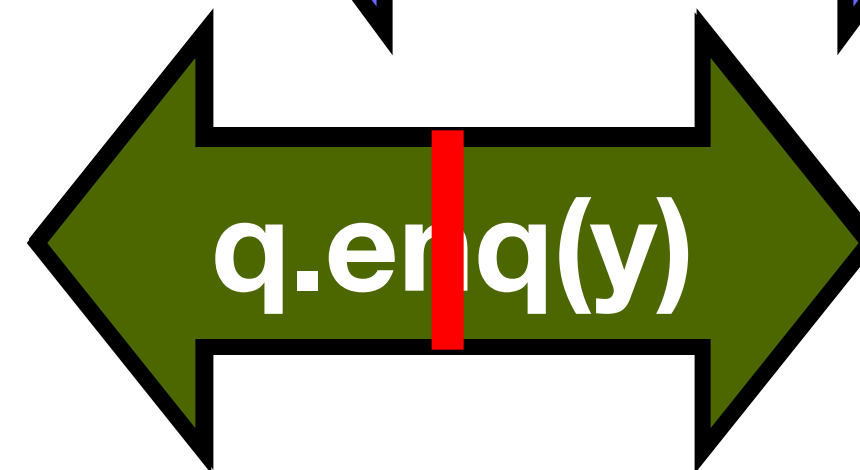
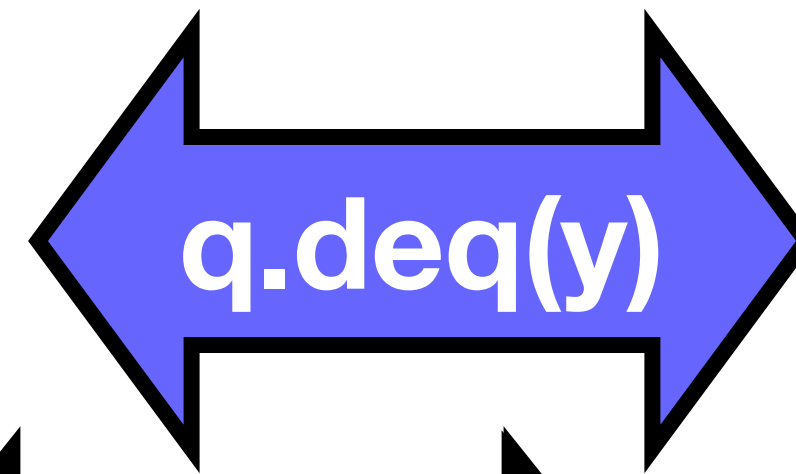
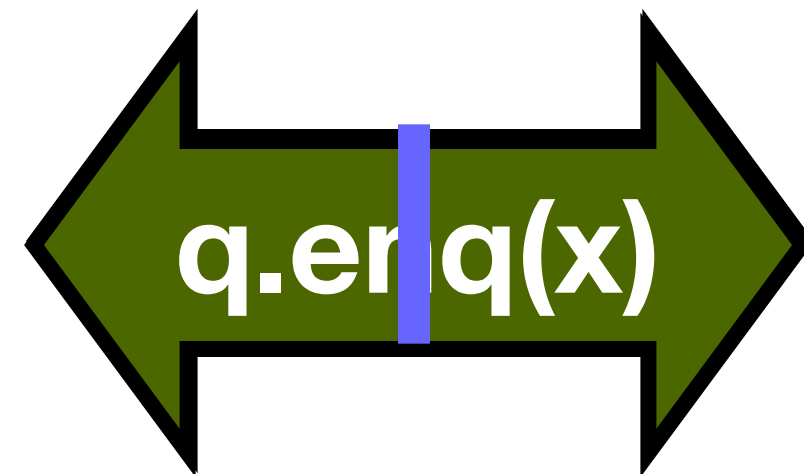


not linearizable

Example



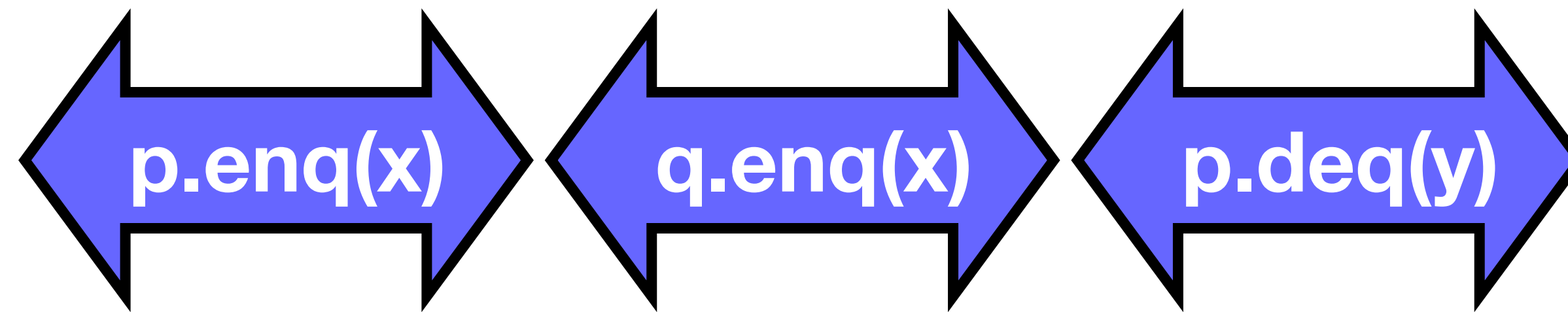
Yet Sequentially
Consistent



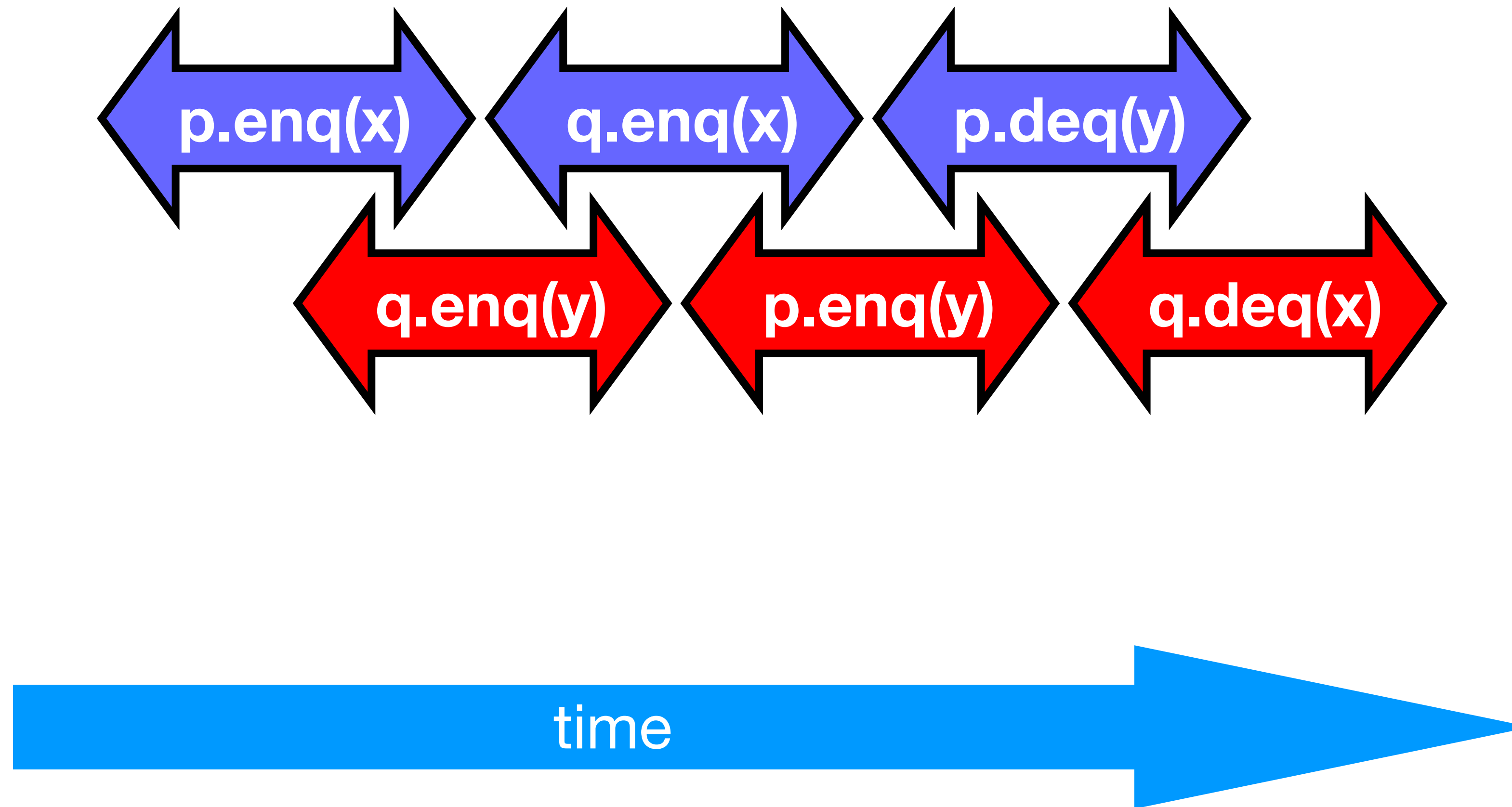
Theorem

Sequential Consistency is not Composable

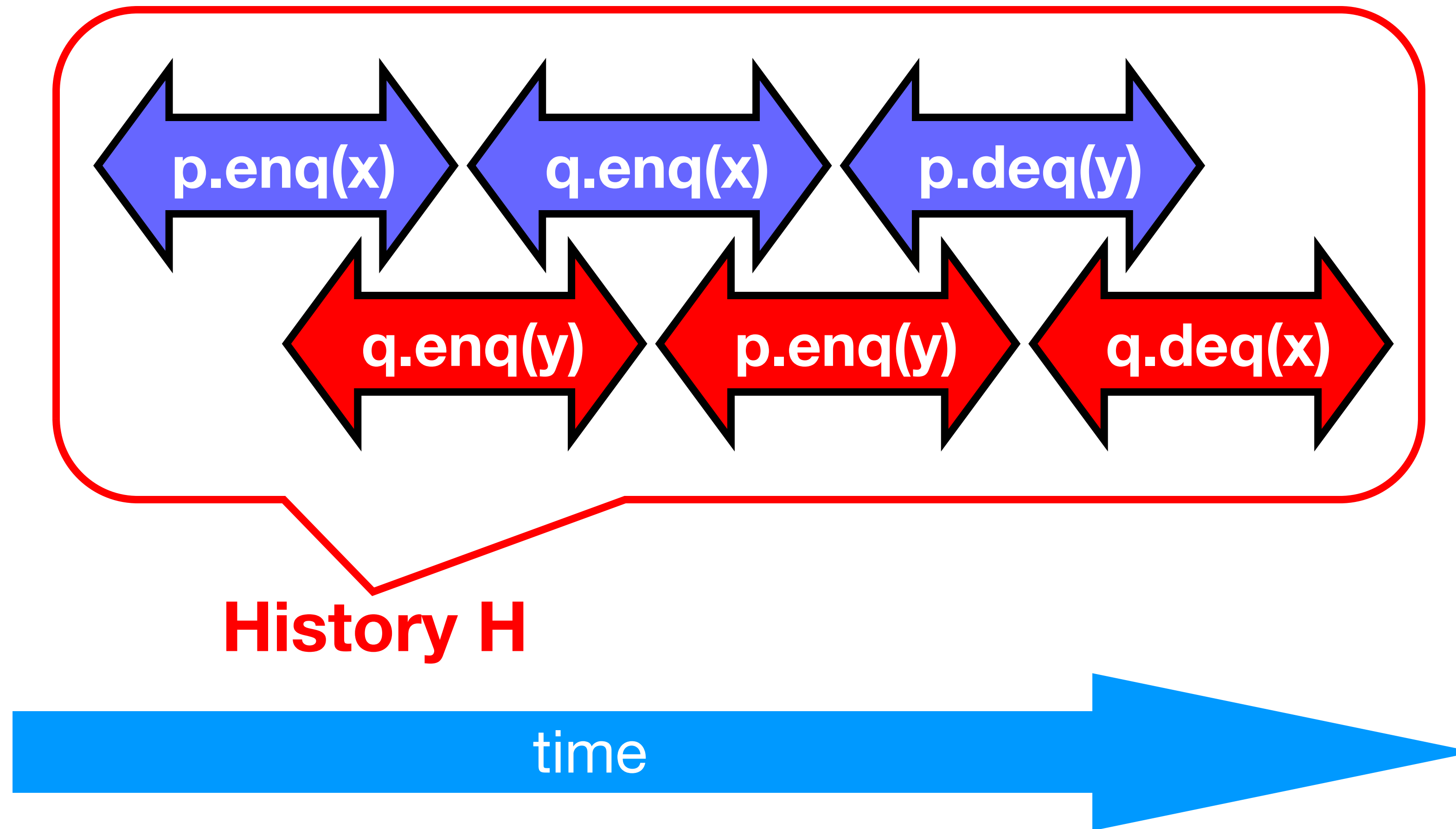
FIFO Queue Example



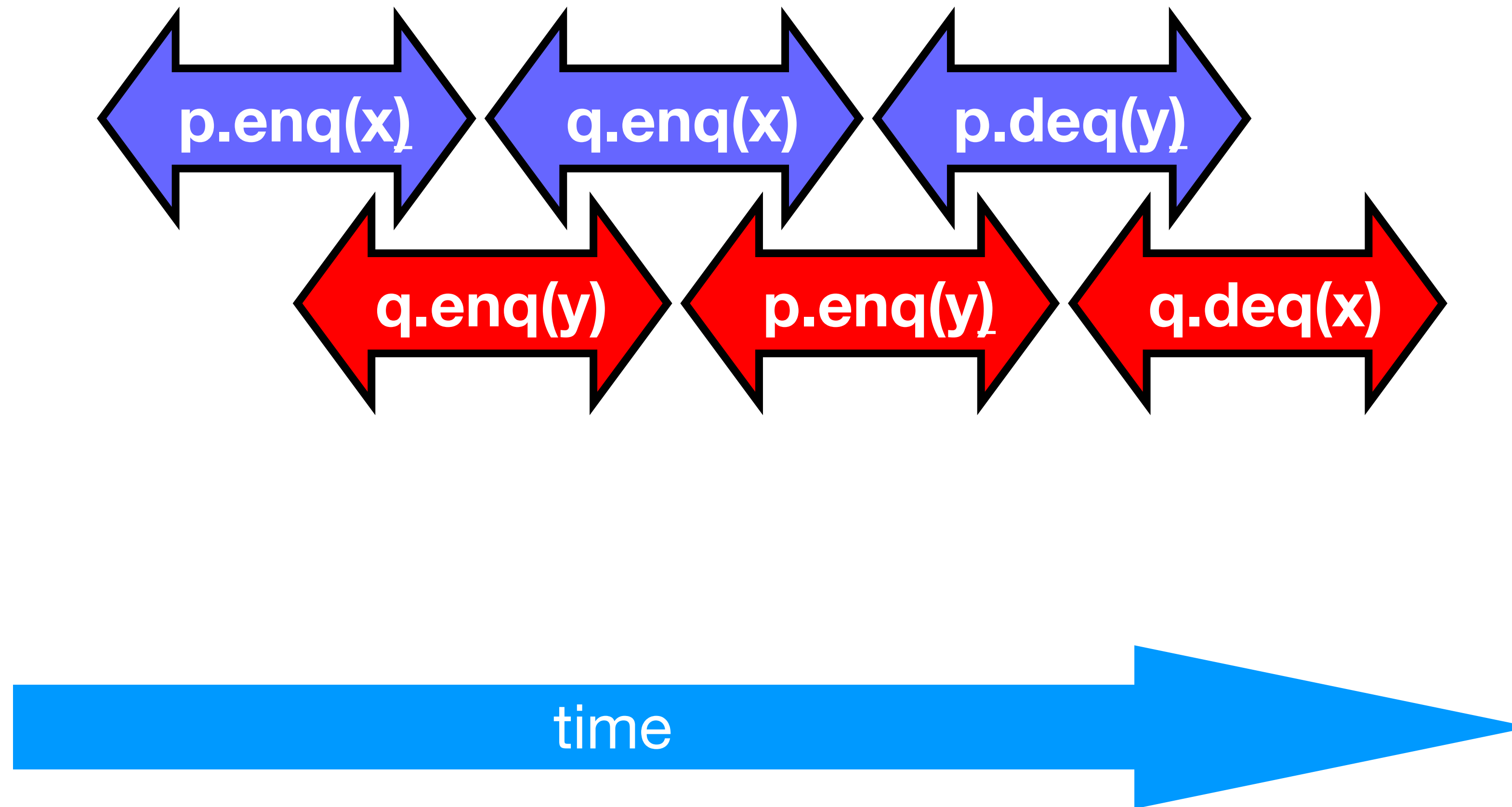
FIFO Queue Example



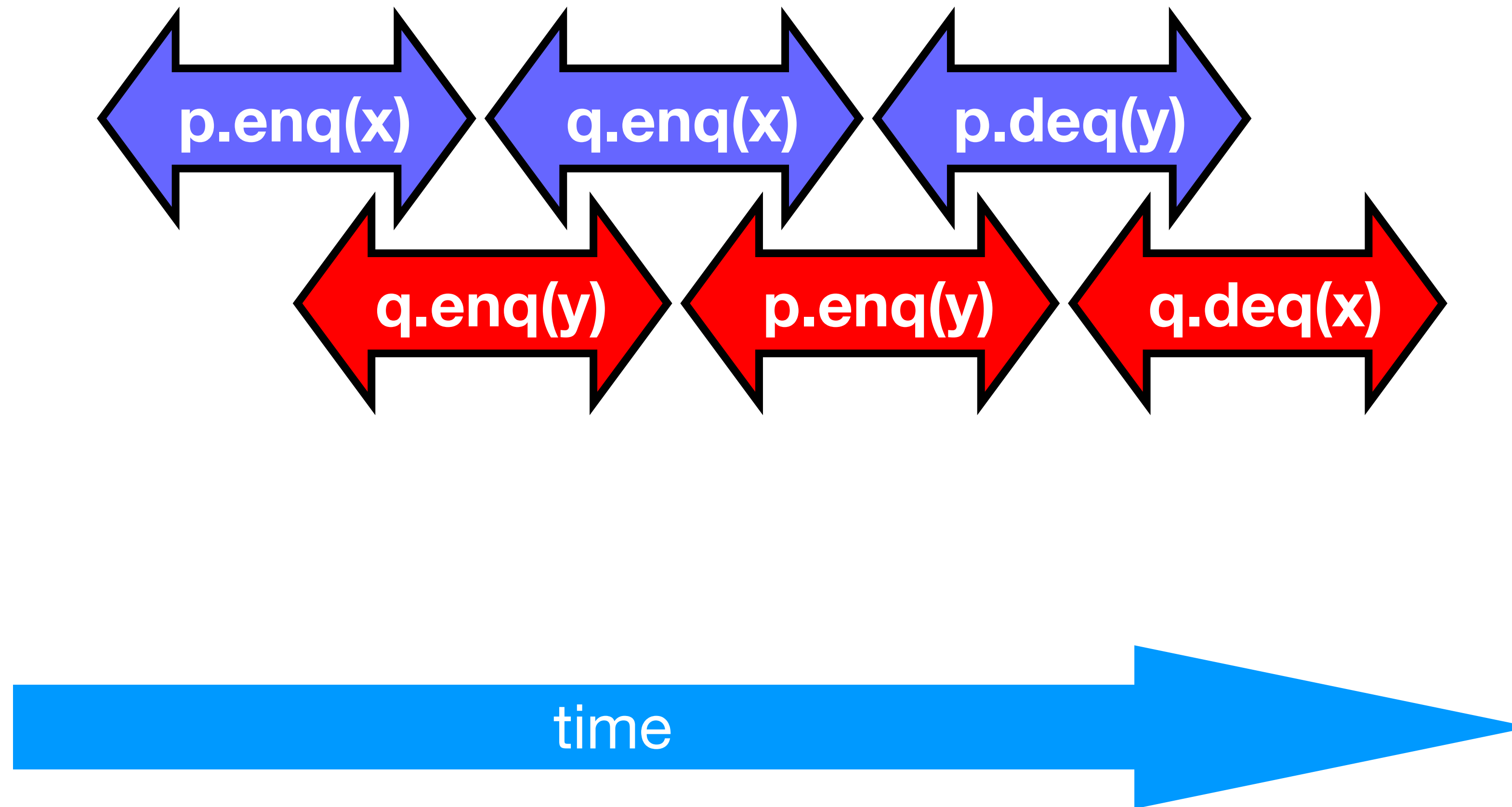
FIFO Queue Example



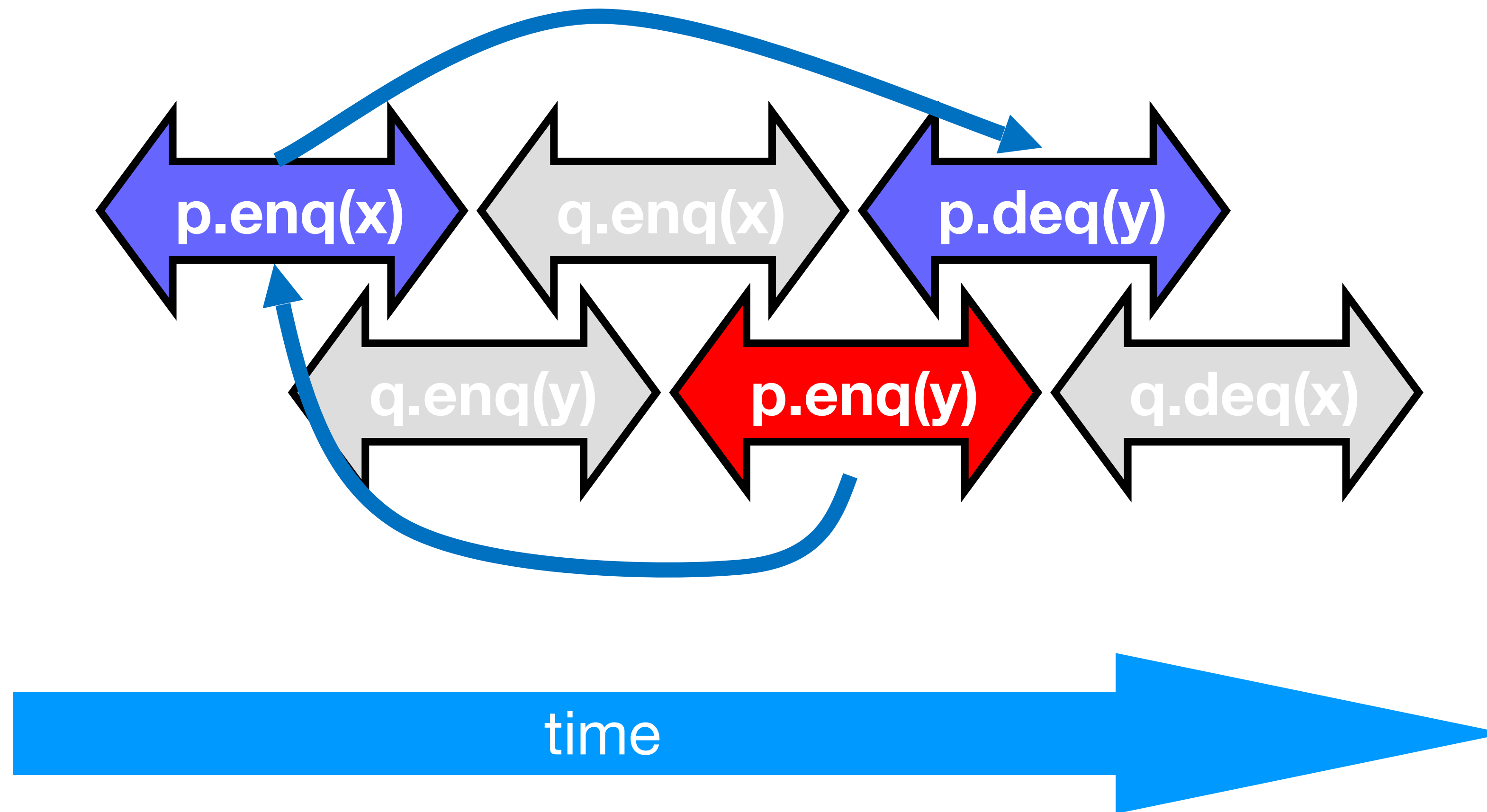
H/p Sequentially Consistent



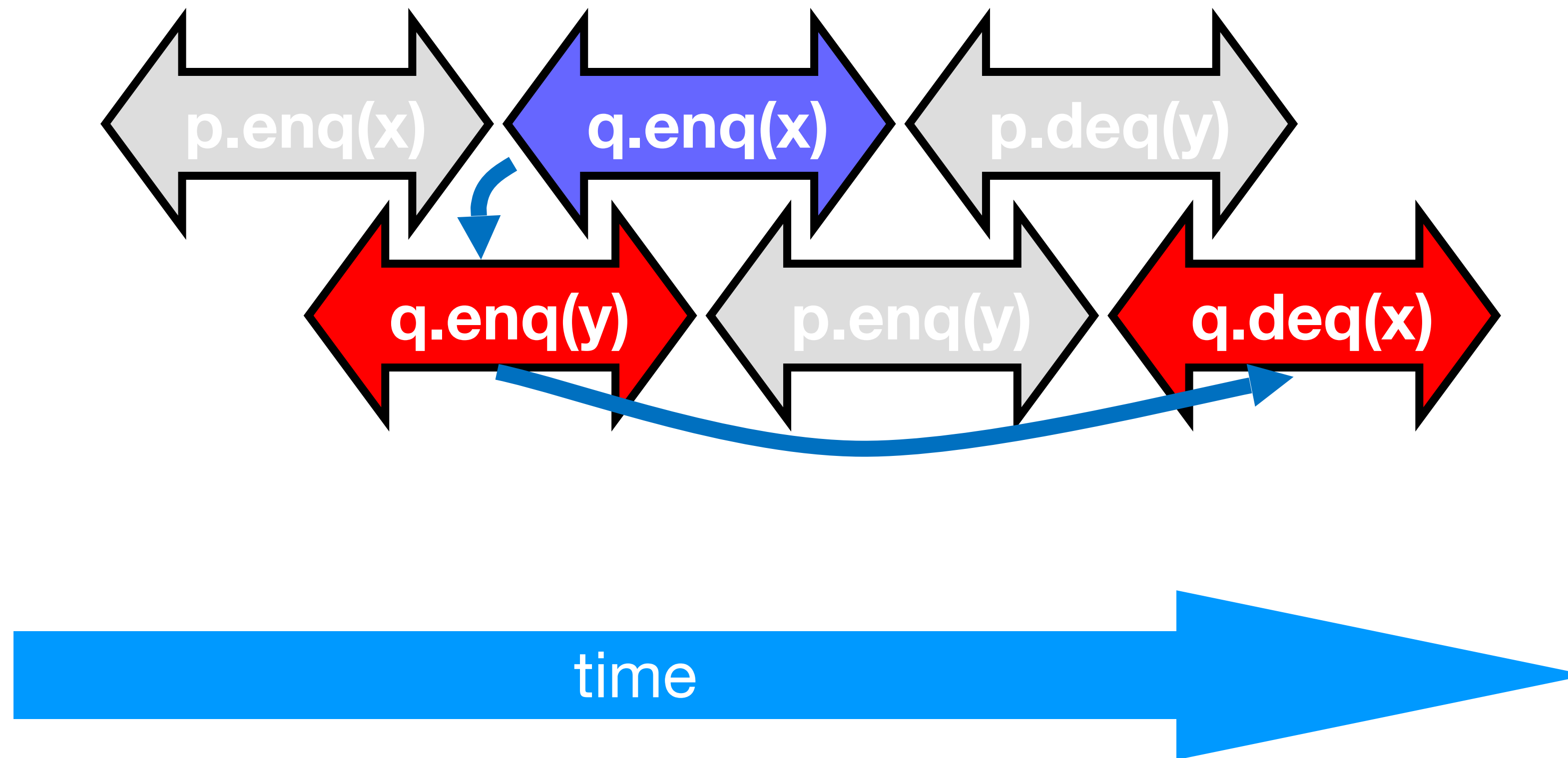
H/q Sequentially Consistent



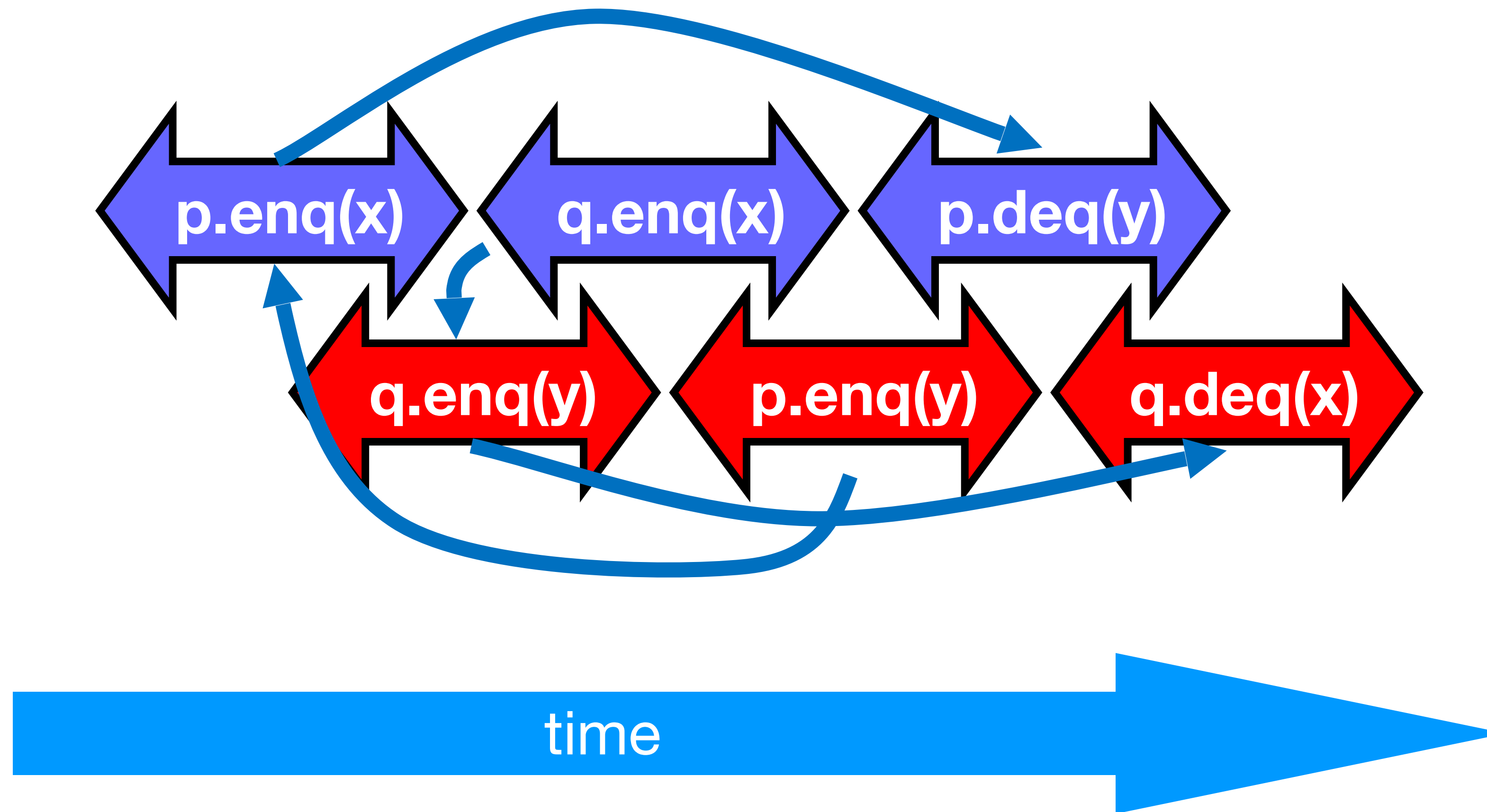
Ordering imposed by p



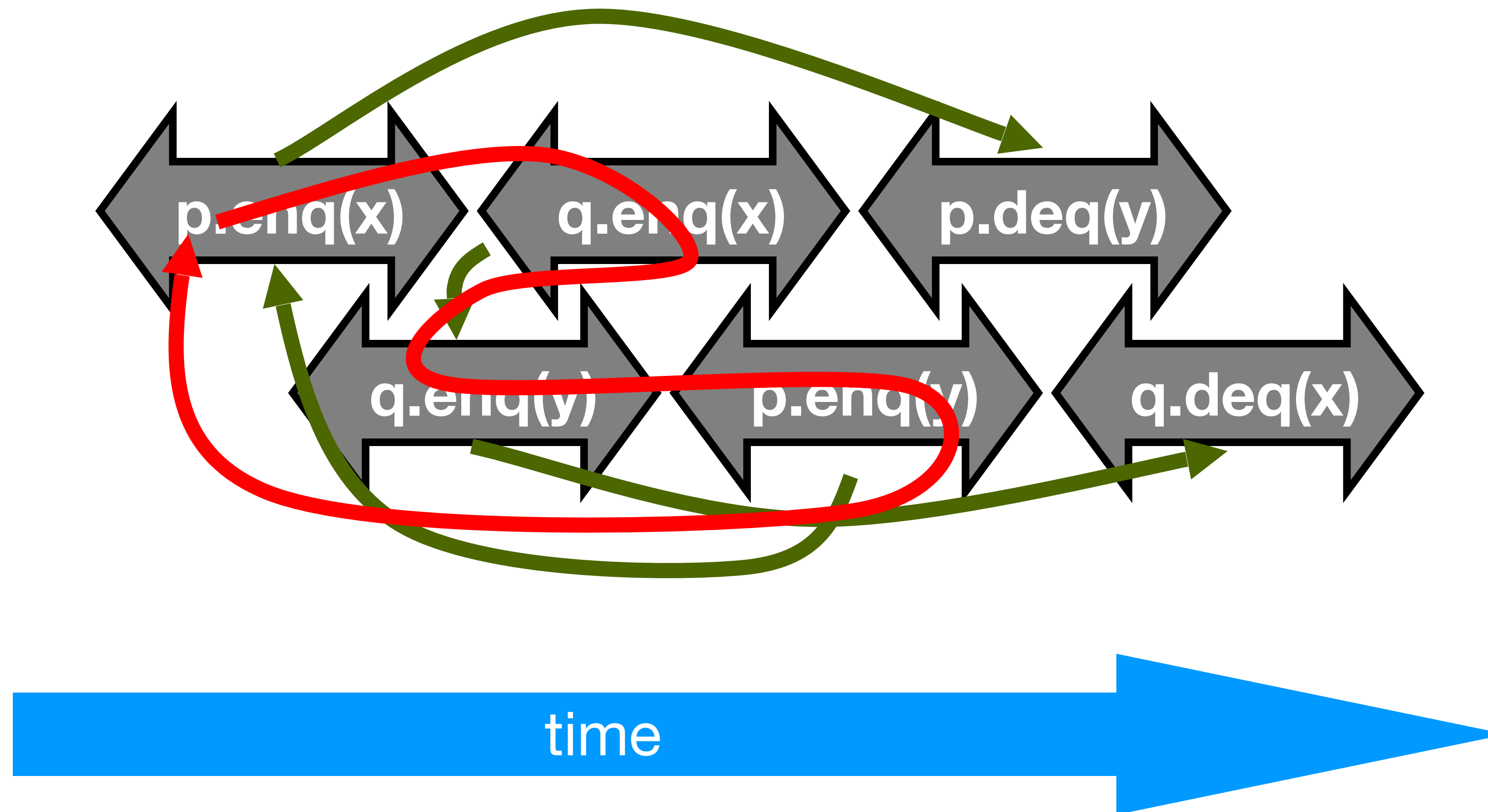
Ordering imposed by q



Ordering imposed by both



Combining orders



Concurrency Testing

- Linearizability and Sequential Consistency are good specifications for *testing* the correctness of concurrent data structures.
 - Any observed execution must match a sequential execution
 - Can be exploited for pragmatic testing
- See <https://github.com/ocaml-multicore/multicoretests>

qcheck-lin

- Checks for sequential consistency violations (despite what the name says)
- *Every sequential consistency violation is a linearizability violation*
- Check that the observed result of a parallel implementation can be observed with a sequential run

Demo

qcheck-stm

- In *qcheck-lin*, what if the implementation is wrong for the sequential program itself?
 - We're only comparing equivalence.
 - Sequential run of a buggy implementation \equiv Parallel run of a buggy implementation
 - *Is not useful!*
- qcheck-stm
 - Write a state-machine *model* of the concurrent object
 - Compare the sequential and parallel executions of the implementation against the state machine model
 - *More work!*

Demo

Summary

- ***Linearizability***
 - The operation takes effect instantaneously between the invocation and the response
 - Uses sequential specification, locality implies composability
- ***Sequential Consistency***
 - Linearizability without real-time ordering
 - Not composable
 - Harder to work with
 - Useful to reason about hardware models (next lecture)
- We will use ***linearizability*** as our consistency condition for reasoning about objects

Progress

- We saw an implementation whose methods were lock-based (deadlock-free)
- We saw an implementation whose methods did not use locks (lock-free)
- How do they relate?

Progress Conditions

- **Deadlock-free:** some thread trying to acquire the lock eventually succeeds.
- **Starvation-free:** every thread trying to acquire the lock eventually succeeds.
- **Lock-free:** some thread calling a method eventually returns.
- **Wait-free:** every thread calling a method eventually returns.

	Non-Blocking	Blocking
Everyone makes progress	Wait-free	Starvation-free
Someone makes progress	Lock-free	Deadlock-free

We will look at linearizable blocking and non-blocking implementations of objects.



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