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BEng Individual Project

LOST: The Logic Semantics Tutor

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Abstract

The aim of this project was to develop a software tool that can teach the semantics of first order predicate logic to students by helping them visualise the process of sentence evaluation. Thus, the focus was on developing an intuitive and engaging user interface to show and allow modification of structures, signatures and sentences, as well as provide relevant exercises for the student to practice with. The latter is arguably the most important feature of this tool and an addition to the functionality of the previous LOST. The user can now ask to see a number of questions. Completing each question is an actual achievement and provides real confirmation of understanding the semantics of first order logic.

I believe these are firm grounds for many possible extensions (such as a Hintikka game) and can be of real use, standalone or alongside the first year predicate logic course. This report will provide further detail of its implementation and purpose.

Acknowledgements

I would like to express gratitude and appreciation to my supervisor, *Ian Hodkinson*, for his continuous support and motivation that have guided me towards completing this project, as well as to my second marker, *Fariba Sadri*, for her useful remarks and suggestions.

Altogether, the resources, staff and students of the Department of Computing have made working on this project enjoyable every day.

Contents

Chapter 1

Introduction

LOST stands for LOgic Semantics Tutor and aims at providing students with a helpful tool for learning the semantics of first order predicate logic.

1.1 Motivation and relevance

First order logic is a powerful tool, of great importance in computing, mathematics and really any system relying on proofs. For students it is a way of practising and developing important logical skills, it provides a formal mean for studying and understanding common mathematical structures and it forms a rigorous mind. However it is difficult for most students to learn the semantics of first order logic and with this being the key to assigning a truth value to any first-order logical sentence, the issue is important. This project attempts to provide a simple software solution and a solid base for future extensions.

On the current market, there are a considerable number of tools designed to help teach natural deduction or equivalences, but very few attempts have been made to design something that will accompany the student in learning the semantics of first order logic and even fewer of them are free. Feedback is currently restricted to lectures and tutorials which means this kind of instant guidance provided by computer software would make a great impact in the way students learn, improving both their and the teacher's experience. Designing such a tool raises interesting questions of what would make an interface intuitive and what does it need to provide the user with, such that they can learn and experiment with the semantics of first order predicate logic.

1.2 A brief, non-technical description

After a bit of thought and research into current solutions, I realised it would not be easy to implement in an attractive way and this will become more obvious after discussing the existing work. Thus one of my guidelines shaped up to be engaging the student as much as possible which software can do if it is intuitive, bug free and most importantly rewarding. The first of these would be achieved if the user is allowed to naturally interact with the application. Because in reality no one likes to read manuals, I aimed to make my program safe to use with a "click and see what happens" approach. This required reasoning about as many occuring events and exceptions as possible. As for the actual content point of view, it would be useful if the user could visualize structures in such a way that they could easily guess the meaning of a possible sentence within it. They should be able to use a toolbox-like signature representation to add and remove objects and relations as well as have a way of introducing and evaluating sentences. At this point it became clear that the interface would have three main aspects to handle: structures, signatures and sentences, which will be discussed further in the implementation details section.

Next, I aimed for a solid back end that could handle even a completely different interface implementation. This meant for it to be able to correctly evaluate the semantics of a logic formula given a structure, regardless of the input or output method. Parsing the input sentence (be it from a terminal or a GUI) would therefore have to be done reliably and its representation in memory be clear, effectively accessible and easy to evaluate.

Features described so far would constitute a minimum viable product, however making it such that the student feels their time was spent worthily was just as important. I aimed to achieve this by implementing a simple quiz system with the possibility of further development. This particular functionality would provide the confirmation a student needs when asked to learn anything.

1.3 Report Overview

In the following chapters the reader can expect a walk through the process of developing the final product, from gathering background information to implementation details, testing procedures and, an as much as possible honest evaluation of the outcome. Though previous knowledge of first order predicate logic and Java would help in understanding the effort that went into this project and its relevance, it is not compulsory. I will revise the essential aspects of the theoretical side[8], however basic knowledge of logic operators and their meaning (i.e. and, or, implies etc.) is assumed.

Chapter 2

Background

2.1 First order predicate logic semantics

In order to understand the product I am aiming for we should first take a look at what first order predicate logic is and why its semantics can be tricky. As an extension of propositional logic, it expresses statements such as *Socrates is a man* in much more detail. While propositional logic would regard this sentence as atomic and simply assign it a truth value, predicate logic provides a way of describing its internal structure and of evaluating it within a relevant context. I will use this sentence to briefly introduce the key concepts used in predicate logic to make "splitting the atom" possible. These are:

- *Constants* - which name the objects inside a context (e.g. Socrates). One constant can name exactly one object.
- *Relation symbols* - which describe properties of the objects they take as arguments or, in the case of nullary relations (which take no arguments), general properties of the structure (e.g. *man* is a unary relation, it represents a property that Socrates may or may not have).

In order to keep track of these two new concepts, we use a *signature*. This represents the syntactic side of evaluating a sentence and provides the necessary tools: constant names and relations symbols. For a computer scientist it may be easier to look at it as a collection of abstract classes that can be instantiated to form the structure and give it meaning.

Using just these concepts, we can now rewrite the sentence we discussed as *man(Socrates)*. However we still cannot decide its truth value and at this point two questions arise: First, which is the object that Socrates describes? Second, what does it mean for something to be a man?

This is where the *structure* comes in. It is defined to be a non-empty set of objects that the signature knows about. If we take our structure to be an imaginary world of hobbits and name one of them Socrates, our sentence would be false, as Socrates would not be a man, he would be a hobbit. However in the context of the real world where Socrates names the famous philosopher, the sentence is true.

Next, if we want to express something like *All men are mortal* we must introduce the two quantifiers. Again, these can be viewed as a way to iterate over the objects that form a structure. These are:

- $\exists$ *(Exists)* - which checks that there is at least one object in the structure that satisfies the sentence it refers to and makes it true.
- $\forall$ *(For all)* - which checks that all of the objects in the structure satisfy the sentence it refers to and each make it true.

The sentence can now be written as $\forall(man(x) \rightarrow mortal(x))$ and we refer to x as a bound variable because it is pinned down by a quantifier, in this case $\forall$. A more precise reading of the above formula would be: "If something is a man than it is mortal."

Another aspect that the user must understand is that sentences containing unbound variables are also valid but cannot be evaluated to a truth value. Saying "men(x) $\rightarrow$ mortal(x)" makes no sense until we decide what x refers to. If x were a constant than it would refer to the object it names, however common practice dictates that we reserve the last letters of the alphabet for naming variables. It is this kind of subtlety that I am hoping to make clearer with the help of an interface which shows exactly which objects are named by constants and which can only be referred to by using quantified variables.

The theoretical side is obviously what forms the base of this project. It is therefore natural that the implementation follows its key aspects: the structure, the signature and the logic formulas. Throughout the report I will inevitably make references to the concepts summarised in this section whilst possibly making further additions and clarifications.

2.2 Existing solutions

As mentioned before, previous attempts have been made to provide software solutions to teaching predicate logic semantics. In fact one student provided a solution for this same project specification in 2007. Furthermore, the Openproof Project at Stanford's Center for the Study of Language and Information (CSLI) is concerned precisely with the application of software in logic and they have developed Tarski's World for this purpose.

2.2.1 LOST 2007

This application was previously available on the DoC's lab machines. Currently however, the only available resource is the user manual. As it is a solution to the same project specification I have carefully studied it and picked up what I thought were several good ideas, whist taking note of things I should avoid.

The application provides a good representation of the logical structure. It displays objects as circles, filled with colours corresponding to the unary relation symbols that apply. Binary relation symbols are represented as arrows, also colour coded. The user can drag objects to rearrange them as he seems fit. This representation is quite clear and easy to interpret. For this reason my own implementation is similar, with a few changes aimed at smoothing the experience even further. First I decided objects should have a clear base colour displaying only their name in the case of constants. Then, according to which unary relations apply to it, coloured borders would be added or removed. This eliminates the risk of a confusing pie chart when an object has many relations referring to it. Next, the arrows would be labelled with a list of names of the relations they represent, such that if multiple relations exist amongst two objects there is no need for multiple arrows rendering better space efficiency.

Furthermore the application allows the user to interact with the structure with several buttons, a signature tree and a number of forms for creating structures and introducing formulas for evaluation. These are offered to the user according to their intention. The general purpose of all this is important and the functionality essential to the application. However in my own implementation I tried to minimise the number of buttons and additional windows or forms, keeping it simple and in one place.

Finally the application offers the possibility to play a Hintikka game, which is a wonderful way of practising logic semantics skills. It also makes an attempt at logic to English translation. These would make very useful extensions to my own application as I chose to focus on the tutorial aspect.

2.2.2 Tarski's World

The Tarski's World[7] application is based on the same principle. Its representation of the structure is a 3D world of blocks. It uses an interpreted first-order language which allows users to write sentences about the world and evaluate their truth. A Henkin-Hintikka game is also provided, along side the main game which consists of questions about the active sentences. This was useful when implementing my own tutorial questions.

My single reservation was once more the number of menus, panels and buttons that can become quite irritating since only a few are truly relevant and most don't seem worth the effort of figuring out. From the point of view of a student this can be quite off putting and might drive them towards the classic pen and paper approach. Otherwise it is generally very professionally made and I did not encounter any major flaws. The learning experience is complete if we take into account the extensions including text books and an online evaluation system that grades the student's performance which I particularly appreciated.

2.3 Ideas shaping the final approach

After looking into the above mentioned solutions and others similar to them it became obvious that the main difficulty with providing a teaching tool for logic semantics is in fact teaching the student to use it. When working with these tools I got the feeling they let the back end lead the implementation of the interface instead of using the latter to hide the inner works. For example, just because there are say 20 relation symbols active in the signature does not mean the user wants to see them at all times in a big grid of buttons that takes up half of the work space. I believe it is important to let the user focus on semantics without having to worry too much about the syntactic aspect, which should be provided and hidden as much as possible.

Another remark should be made on the tendency of developers to forget the user does not know as much as they do. It is impossible to develop a teaching tool without understanding first order predicate logic inside out, which makes it is even harder to empathise with a student that is just starting to learn it. It is therefore easy to unconsciously overestimate the user's knowledge and forget to provide explanations that might in fact be useful to them. For example it might seem counter intuitive that a sentence such as man(Socrates) is false. But if the signature's only unary relation symbol is *hobbit* this may seem obvious to a logician. However the user who tried to input the above sentence might benefit from a message such as “The unary relation symbol man is not defined in this structure”, instead of their sentence just being rejected as invalid.

Finally, I fixed the two features that would make this project unique. First of all I would keep it as simple as possible, making it a priority to minimise the number of buttons and choices the user has to make. This had to be done without sacrificing any of the essential functionality. Secondly, I would make it rewarding, by giving the user an opportunity of testing their skills. As several students have worked on this project specification across the years, I figured this would be a useful and fresh addition.

2.4 Software choices

Having sufficiently stressed the importance of the interface I had to make a choice as to which programming environment would best fit its requirements. I wanted to be able to focus on building something attractive without having to worry too much about the lower level issues like memory assignments, garbage collection or optimal for loop nesting. This, as well as my previous programming experience, perused me to choose Java 7[2] and Swing[3], which proved to be at a high enough level for my interface whilst offering me the freedom to design the back end for the sentence evaluation from scratch. Thus I had complete control and understanding of every step that takes a sentence from it's raw form to an outcome. And understanding made it easier to track bugs, make changes and pursue a steady development. Java also ensures a decent compatibility across computer platforms, has extensive documentation and is a reliable language that addresses a large audience.

Of further help with the the interface was the IDE. After starting off with Eclipse, I soon realised the simple task of arranging buttons on the panel was uselessly tedious. So after a little research I chose to use NetBeans for its GUI Builder. It took a bit of getting used to, especially with learning to get around the read-only generated code, but eventually everything proved to be 100% customisable and certainly worth switching to. Overall this saved me significant time and helped me achieve my desired esthetic standard.

Java proved to be the right choice again when it came to writing a parser for the logic formulas. It made it easy to link an automated parser, reducing my task to writing a comprehensive grammar. At this point, another choice was to favour Antlr4[1] over the previous, better known version. What convinced me was its ability to deal with left recursive grammars. Although removing recursivity is a straight forward algorithm[5], it makes the grammar quite ugly and a bit more difficult to read. Antlr4 also uses an impressive adaptive parsing strategy, making it significantly more time efficient than its static ancestors.

For version control I used Git[4]. It is an easy and reliable way of keeping backups, working remotely, tracking progress and proved essential for a project this size as it happened more than once that I had to revert to previous commits and slowly add in changes to discover bugs. It also proved useful for its branches, allowing me to experiment with external Java packages such as mxgraph, and make other temporary changes.

For the report I used LaTeX[6] with Vim, which greatly facilitated the text formatting and made it easy to keep track of the different sections. The Unix shell was also of great use with compiling Antlr and LaTeX and using Git as well as with occasional remote connections. Finally, the entire project was developed on a Ubuntu 12.04 system.

2.5 Design Patterns

As developing proceeded, the size of the code became overwhelming. The separations packages and classes provided was insufficient. When it came to implementing the smallest changes, dependencies inside the code made it a tedious and time consuming task. Therefore, when the back end was finished and I began developing the user interface, I took a step back and revised some software design patterns, in order to choose a system that would best fit my requirements.

It was the **Model-View-Controller** pattern that helped the most. By completely separating the back end from the display I was able to keep better track of the whole picture:

- Model - this represents everything in the back-end. It handles all the information concerning sentences, signatures, structures and evaluation.
- View - this represents the interface. It contains all the component types needed to represent logic objects. It also handles the display layout and appearance and contains the project's main method.
- Controller - this is the class that links the model to the view. To do this it pairs up logic objects with an appropriate graphical component. It also interprets user input such that it can be sent back to the model for interpretation.

Having separated the information from the representation in this way, it became much easier to change the interface since I no longer had to worry about the compatibility with the model. This was essential as designing the GUI meant repeatedly using the application and applying slight variations to see what would be most comfortable for the user.

I also used the **Adaptor** structural design pattern, which allowed me to separate the parser generating from the sentence structure. I then used an the Sentence class constructor to convert the parse tree into my own logic tree, which I had designed with an integrated evaluator.

Chapter 3

Approach and Implementation details

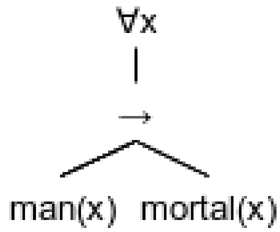
This logic semantics tutoring tool is naturally shaped around the theoretical elements that provide logic formulas with meaning. It contains three packages which I will discuss in the same order they were developed:

- Evaluator - that holds the logic tree structure along with an adaptor for the parse tree and an evaluator.
- Parser - which handles the translation of raw logic formulas input as strings into a parse tree.
- GUI - which contains the user interface and controller.

3.1 Evaluator

As it can be deducted from the brief description above, this is the main and largest part of the software. I will start with how logic formulas are supported then move onto the evaluation process and finally the adaptor.

The starting point of my project was representing sentences in an efficient computer readable format. It was also important to remain close to a human friendly representation. The solution came once from logic theory, where sentences are often represented with the help of logic trees. These trees have logic operators or quantifiers for nodes and simple atomic formulas for leaves. For example, the sentence $\forall$*(men(x)* $\rightarrow$ *mortal(x))* is represented by the following tree:



So with this in mind the first step was to build the classes to represent the logic elements that would be used to form the nodes of the logic tree.

- NullaryRel, UnaryRel, BinaryRel
  Classes to represent relations symbols with arities from 0 to 2. Although it is possible to have higher arities they are not common in predicate logic and would over complicate the representation too much. Once a stundents understands these three, relations of higher arities are merely an inductive step away.

- Term
  A class to represent the parameters relation symbols take. It has a name and is extended by Constant and Variable. The latter adds boolean fields indicating whether it is bound by a certain quantifier.

- BinOp
  An enum type containing all the binary logic operators along with definitions of their behaviour (i.e. functions overriding the abstract function evaluate using the Java logic operators).

To ensure the correct result when comparing instances of these classes, I made sure to to override their default equals method appropriately.

3.1.1 Logic Tree

Next I build the actual nodes. Each extends the abstract LogicTreeNode class, overriding the evaluate function and providing a pointer to the next node (or nodes in the case of binary operators). They are of course encapsulated by a LogicTree class which has the head of the tree to nicely call evaluate upon. Let us see how each node works in decreasing order of their complexity:

- ForAllNode
  This node's evaluation function returns the boolean value representing whether each of the objects in the structures make the sentence true. And it is very important to understand the difference between each object making the sentence true and all objects making the sentence true. For

example, to verify the sentence $\forall x$ *(men(x)* $\rightarrow$ *mortal(x))* (All men are mortal) we would take each man in turn and verify that he is mortal, disregarding any animals or other things. It would be wrong to first verify that everything is a man and if so, then verify that everything is mortal. Such a sentence would be written as $\forall x$ *men(x)*$\rightarrow$ $\forall x$ *mortal(x)*. To make this even more clear, let us assume the structure contains an immortal man. This would make the first sentence false because not all men are mortal. However in the case of the second sentence, when verifying that everything is a man, we would find that is not true since there are also animals and other things in the structure. And since falsity implying anything is always true, the second sentence is true. Hence the two sentences have different outcomes and mean different things.
In order to handle this correctly, the ForAll node passes an assignment down the tree to be evaluated with the entire sentence in scope. An assignment represents an object from the structure's domain. The class Assignment contains a Term and a Variable field to pair them. For each object in turn, the node adds one assignment to an array list of assignments in case other quantifiers are encountered down the tree. This is used as a paramater by the evaluation function. Only when it has finished iterating over the structure's objects, it puts a final result together. If all of them verify the sentence then the final result is returned as true, otherwise as false.

- ExistsNode
  This node follows the same principle. The difference is outcomes of each assignment are put together with an or operator, such that as soon as one is found to be true, the evaluation stops and returns true. If however it reaches the end of the structure domain without finding a term to satisfy the sentence, it returns false.
  At this point it must be clarified that there are in fact two abstract evaluation functions as follows:

  ```
  abstract boolean evaluate(Signature s);
  abstract boolean evaluate(Signature s, ArrayList<Assignment> assignments);
  ```

  This is because it may be the case that no quantifiers exist in the sentence. For example a sentence may only refer to constants.

- BinaryRelNode and UnaryRelNode
  This is a leaf node class and contains a BinaryRel field. The evaluation method checks that the structure contains the relation held in this field. If the relation's arguments are constant no assignments are involved. If however variables are involved, the second evaluation method is called and it creates a new relation with the relevant terms from the relevant assignments as arguments. This new relation is then checked against the structure. I believe the evaluation functions of the UnaryRelationNode class summarises this best:

```java
@Override
boolean evaluate(Structure s) {
    return s.unaryRels.contains(rel);
}

@Override
boolean evaluate(Structure s, ArrayList<Assignment> assignemnts) {
    if (rel.arg instanceof Variable) {
        for (Assignment a : assignemnts) {
            if (rel.arg.equals(a.boundVar)) {
                UnaryRel r = new UnaryRel(rel.name, a.assignedTerm);
                return s.unaryRels.contains(r);
            }
        }
    }
    return this.evaluate(s);
}
```

- NullaryRelNode
  This node has a NullaryRel field and always return's that relation's boolean value.

- EqualsNode
  Although it may seem like it could be part of the BinOpNode, this node is actually a leaf as the equals operator takes terms for arguments. So really it is more similar to the BinOpNode. If the arguments are constants it checks that they are in fact the same object. If any of the arguments are variables it uses the assignments to look for appropriate terms (i.e. it replaces the variable with the assignedTerm field value of the assignment that has the same variable in its var field). It will help to see the Assignment class:

```java
class Assignment {
    Term assignedTerm;
    Variable boundVar;

    public Assignment(Term t, Variable var) {
        this.boundVar = var;
        this.assignedTerm = t;
    }
}
```

- BinOpNode
  This is an internal node containing one of the types of operators defined in the enum type described before, as well as pointers to the nodes it takes as arguments. Its evaluation function calls the evaluation functions

of the these two nodes in the was described by the operator definition. For example AND will make sure the evaluation functions return true for both arguments.

- NotNode
  Also an internal node that points to the rest of the sentence through a LogicTreeNode field and simply returns its negated outcome.
- TruthNode and FalsityNode
  These node always return true and false respectively. They represent leaf nodes.
- DummyNode
  This node is used by the adaptor to translate parse trees an will be further discussed in the relevant section.

3.1.2 Signature

In order to understand the symbols used in the logic formula, a signature is needed. In a series of ArrayList fields it holds Strings representing names of the Constant, NullaryRel, UnaryRel and BinaryRel instances. In order to fill these in, its constructor takes a signature object as a parameter.

The choice for an array list (here and in other classes) was based on the fact that, being based on a dynamically resizeable array, it would greatly facilitate adding and deletion of objects as well as locating and iterating over its elements.

Using just the relation symbols' and constants' names it provides another useful check of the formula's syntax before passing it on to the evaluator. If any symbol used in the sentence is missing from the signature and is not a quantified variable, the user will be prompted with the appropriate message. However this will be discussed further in the section regarding interface implementation where the signature's contribution is greater. In the evaluation context its role is simple: it checks that all the symbols used are declared and valid.

3.1.3 Structure

Now that we have seen the representation of a logic formula and signature, we can proceed to understanding the format of the context in which it will be interpreted. Objects constructed by the Structure class are passed as a parameter to the evaluation method such that an interpretation can be made.

The fields of this class are array lists of terms and relations. In order to fill them in, the constructor calls a generate method. This in turn uses four enum types with names for constants and the three types of relations. Each enum type has a random chooser method to return one of its members. The generate method uses this and ensures no duplicates are allowed into the final content of the structure. It generates a random number of actual terms and relations using the Term, Constant, and relation class constructors described before.

Every time the program starts up it creates a new structure which the user can modify, thus ensuring a non empty set of elements at all times to remain consistent with the theoretical definition. The details of this will be discussed in the user interface section.

3.2 Parser

I have already discussed the choice of Antlr4 for the purpose of parsing a logic formula from plain string input. In order to generate a parse tree, Antlr needs two things: a grammar and lexer rules.

The grammar has proved quite tricky to write as I wanted to offer the user as much input freedom as possible. I also managed to preserve the operator precedence such that it would make it easier for the adaptor to interpret the parse tree. The final and best solution was once more the simple one, with the main rule shaping to be comprehensive and easy to read:

```
formula
  : TRUTH
  | FALSITY
  | term EQUALS term
  | relation
  | quantifier formula
  | NOT formula
  | formula AND formula
  | formula OR formula
  | formula IMPLIES formula
  | formula EQUIV formula
  | LPAREN formula RPAREN
```

Please refer to the anex for the complete grammar file.

The lexer rules implied making a few decisions as to what the user should be allowed to use for naming the signature elements. For variable names, the general convention is to use letters towards the end of the alphabet, optionally follows by the character s and/or a number (e.g. x3). I stuck to this convention, however allowing all letters of the alphabet for a wider choice. The rule is as follows:

```
VARIABLE: [a-z] ’s’? [0-9]* ;
```

For naming relation symbols I allowed any combination of characters beginning with a lower case as long as its length is greater than 1 or beginning with an upper case letter in which case its length must only be greater than 0:

```
NAME: [A-Z] | [a-zA-Z] [a-zA-Z09’_’]+ ;
```

The lexer ignores white spaces or new line characters:

```
WS: [ \t\r\n]+ -> skip ;
```

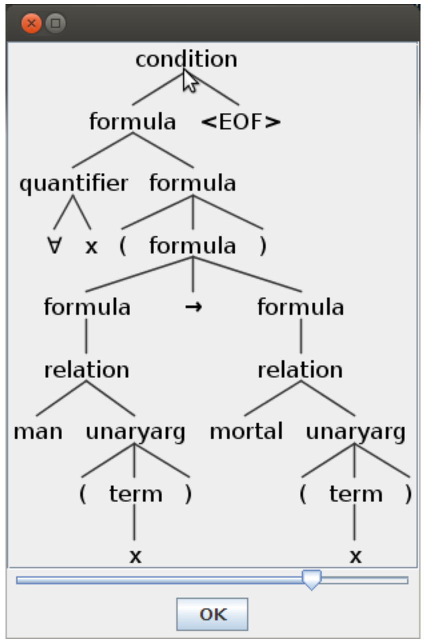
Finally, it provides rules for the logic symbol tokens:

```
LPAREN   : '('  ;
RPAREN   : ')'  ;
AND      : '∧'  ;
OR       : '∨'  ;
NOT      : '¬'  ;
IMPLIES  : '→'  ;
EXISTS   : '∃'  ;
FORALL   : '∀'  ;
TRUTH    : '⊤'  ;
FALSITY  : '⊥'  ;
EQUALS   : '='  ;
EQUIV    : '↔'  ;
```

Together, the rules above represent the entire lexer file.

3.3 Adaptor

At this point we can return to look at the LogicTree class constructor which works as an adaptor for the parse tree. Antlr provides a user basic interface which can be run from the terminal to visualise this tree. For the sentence *All men are mortal* the following tree will be generated:



Each node is created from one of the parser context classes. These are:

```
folParser$ConditionContext
folParser$FormulaContext
folParser$QuantifierContext
folParser$RelationContext
folParser$BinaryargContext
folParser$UnaryArgContext
folParser$TermContext
```

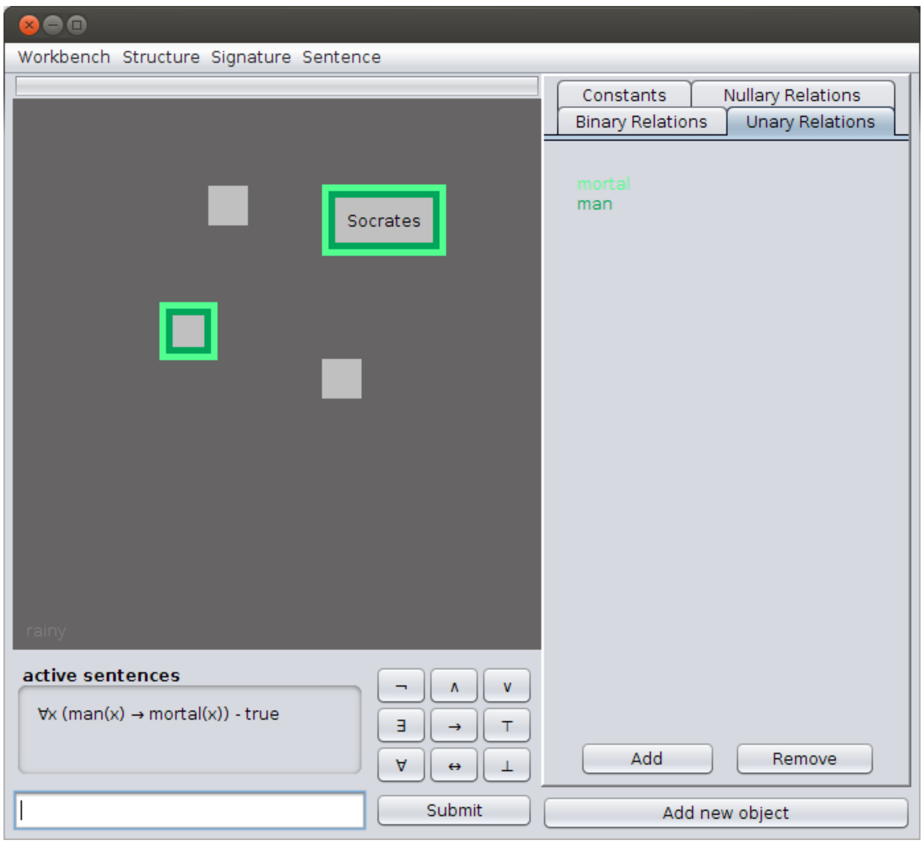
The first thing to notice is that the parser will generate a *formula* node that does not correspond a type of logic node but is just a rule that guides the parsing. For this reason I created the dummy node which simply points to the rest of the tree through a LogicTreeNode field and returns that node's outcome without altering it.

The adaptor provides a rule of interpretation for each of these classes through a switch statements. The default case is an error message for a complete picture, although it is not reachable, as each node of the parse must be an instance of one of these classes. The constructor performs a breadth first search and generates the appropriate LogicTreeNode for each node in the parse tree. Once finished it returns a pointer to the head of the tree.

It is important to mention that in order to create the unary and binary relation nodes the adaptor keeps track of quantified variables in order to prevent the creation of a relation with an unbound variable as argument. In this way the tree is created only if the formula does not contain any free variables and so is a sentence and can be evaluated. Otherwise the user will be prompted to revise their input.

3.4 User Interface

With the process of creating a logic sentence from a raw string input concluded, we can now take a look at how the user can use this to understand semantics. This is how the workbench looks like, with the designed such that the sentence *All men are mortal* evaluates to true:



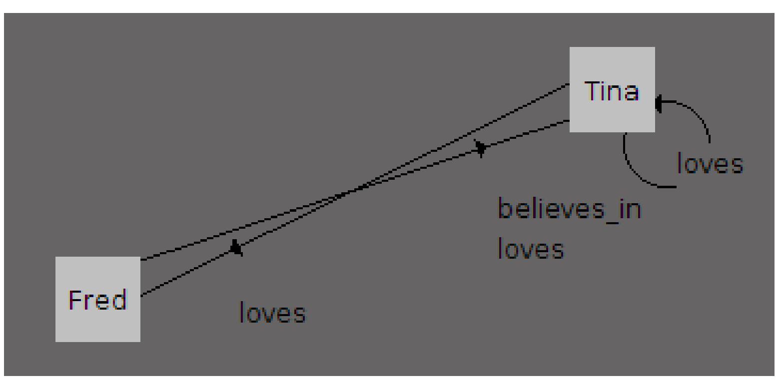
3.4.1 The View

The layout of the workbench is also designed around the three main elements involved in semantics:

1. Structure panel
   This represents the main working area, taking up most of the upper left

side, with a background of a darker grey than the rest of the workbench. It contains the following elements:

- Objects
  The light grey squares represent objects in the structure and are labelled with the name of the constant that refers to them, otherwise are left blank. In this case Socrates describes the upper right square. The squares can be dragged, deleted, renamed and different relation symbols can be applied to them. A new unlabelled object can be added using the button at the bottom right of the screen. Deleting an object refreshes the active sentence list, deletes any relation symbols related to it and if it is a constant any sentence that contains it is also deleted. At least one object must be present in the structure at all times. If there is only one left the user is not allowed to remove it. Objects are built from the Blob class.
- Unary relation symbols
  These relations are represented by coloured borders around the objects they apply to. Furthermore, with a right click on an object the user can see a list of the unary relations names with a tick indicating which ones apply. This menu allows the user to change if a unary relation applies to an object or not. The relations are colour coded. In the example above Socrates is described by the relations man and mortal and as such has two coloured borders. Unary relations are not created from a class, they are simply borders.
- Binary relation symbols
  These relations are represented by arrows. If an arrow exist between two object, a list of names labels the arrow indicating which relations exist between the two. The list is situated near the second argument, next to the arrow head, such that the user can clearly see two arrows if there are relations going both ways. In the following example the object named by Fred loves the object named by Tina and vice versa, Fred believes in Tina and Tina loves herself. Arrows are built from



the Arrow class, designed to override the paintComponent method such that they follow the objects around as they are being dragged.

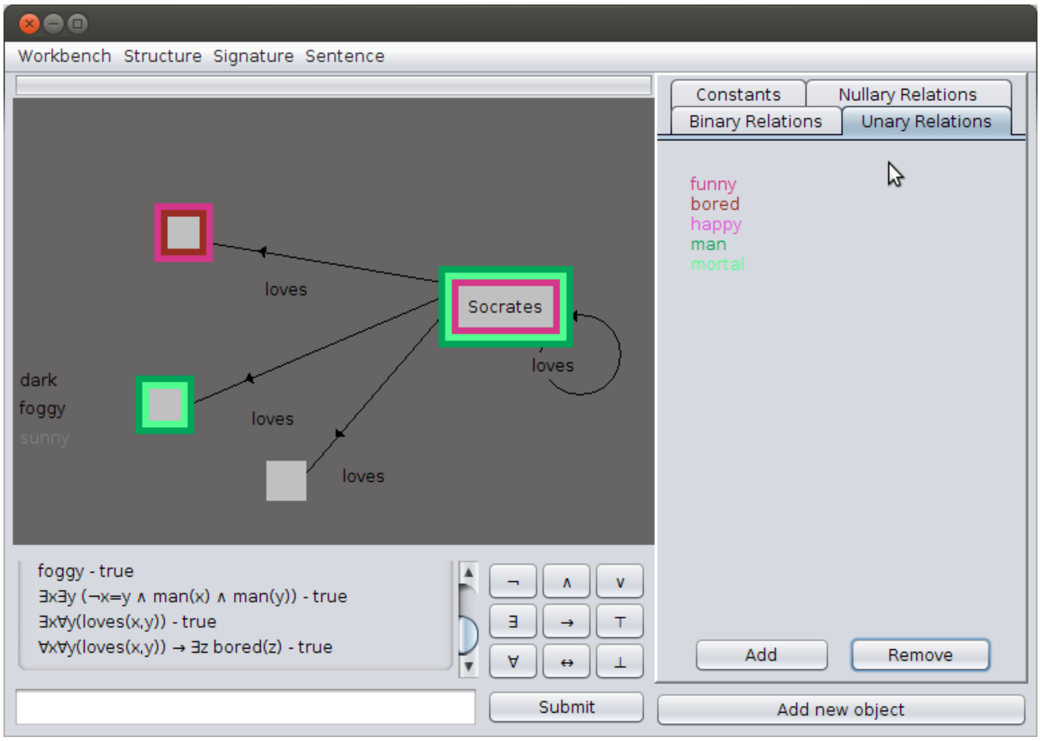
- Nullary relation symbols
These come up at the bottom left side of the structure panel as they are added. Their default value is false and this is indicated by their names being faded out. A tool tip text also indicates this. To toggle their value the user can simply click on their names. These are also dragable, such that the user can decide how to make best use of the space available. Nullary relations are built from the Blob class, with the right click functionality removed and the right click functionality added.

The structure panel is resizeable and scrollable for a more comfortable experience.

2. Signature panel
This is the tabbed panel at the right side of the workbench, with a tab for each type of element: constants, nullary, unary and binary relation symbols. Each tab also contains buttons for manipulating their content.
Constants can be removed, added or renamed. In the case of the second two actions, a text field appears such that the user can choose the desired name. Furthermore, in the case of renaming and removing, a constant must be selected from the list, otherwise the user is prompted accordingly.
Unary relations are the only ones colour coded. Their name colour in the signature corresponds to a border colour in the structure. To generate an unique colour for each I used their name's hash code. To display this in the unary relation tab, I wrote a cellRenderer class for the list to use.
Relations can be added or deleted. If the latter is performed, all corresponding borders are deleted along with any sentence that uses it.
In addition to this, the binary relation tab has got a *choose parameters* button. To use it, one of the relations in the list must be selected. Once pressed the button turns red and the user is prompted to click on two objects subsequently. To clear the selection the user can click on an empty area in the structure. Once the objects have been chosen an arrow is added (or a name to the list of labels if an arrow is already present) and the button returns to its original colour.
Finally, the nullary relations tab also provides buttons for adding, renaming or removing a relation from the structure.

3. Sentence panel Situated at the bottom of the window, it provides the means for introducing logic formulas and a scrollable list to store them and their outcome. Buttons are provided for the logic symbols along with a text field for the input and a submit button. The user will be prompted in case of bad input as accurately as possible. The sentence list is refreshed every time a change occurs in the structure, however the user can use the

menu to manually refresh it if they wish to do so. The list of active sentences ensures the last indexed sentence is visible. Here are a number of sentences evaluated in the Socrates structure.



3.4.2 The Controller

So far, the display contains a series of components. However, in order to have any meaning, they must be linked to the Structure and Signature described in the evaluator. This is the role of the controller.
To link the signature panel to the signature class is uses a list model for each of the tabs and ensures a consistency between the relations and constant names in displayed and the ones present in the signature object. List models can be modified dynamically so they serve this purpose well.
To link the structure panel to the structure class it uses two pairs:

```
Pair<Term, Blob>
Pair<BinaryRel, Arrow>
```

There is no pair for unary relations to borders as the link is simply the colour generated with the name's hash code. Along with a method to update the structure, the controller ensures a permanent consistency between the structure's fields' content and the panel.

To link the sentence panel to the logic tree, the controller uses the Antlr library to first parse the input and generate a parse tree, then uses the constructor in the LogicTree class to verify and build the sentence. After this it calls evaluate on the head of the logic tree and adds the outcome along with the sentence to the sentence list. When this is done it clears the quantified variables from the sentence scope such that the next sentence can start fresh. Finally the controller provides methods to link actions described by the buttons on the screen to their desired effect in the structure and signature objects.

3.5 Exception handling and user guidance

Even the most intuitive interface needs to be able to guide its user if necessary. This is done mainly with the help of useful error messages. There is also some user guidance provided through the menu bar at the top of the screen.

3.5.1 Menu bar

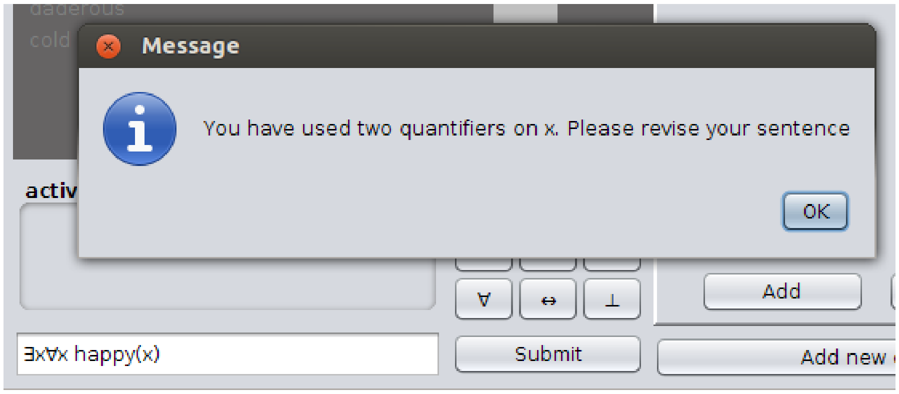
There are five menus at the top of the window:

- Workbench
  Provides the functionality needed for saving and loading an entire session comprising of the signature, structure, sentence list and progress made with the tutorial. It also allows choosing whether to display the question panels.
  The account and preferences items have not yet been implemented, however they are intended for the user to be able to customize their experience. This will be discussed in the evaluation section.
- Structure
  This menu provides functionality for saving and loading just the structure state. It offers an option to generate a new random structure which will replace the active one, as only one structure can be active at a time.
- Signature
  As it is strongly related to the structure and updated according to its content there are not a lot of things one can do with it. For this reason the only option is whether to show or hide the panel.
- Sentence
  This menu provides options for refreshing or clearing the sentence list, as well as for loading a sentence file. The format of such a file is plain text, with a sentence on each line. Once loaded they will be evaluated and placed in the active sentences list.
- Help
  This menu has an option to bring up a window with basic, concise instructions and a link to further first order logic information (i.e. the Wikipedia page for first order logic).

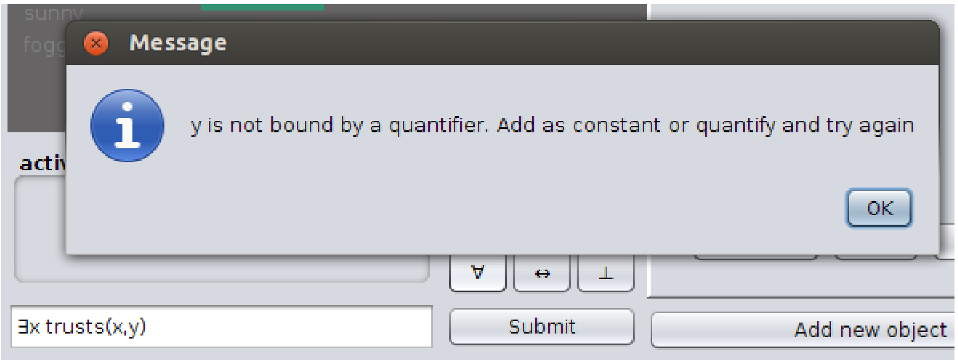
3.5.2 Error messages

In order to handle bad user input or illegal operations I designed a number of exception classes. These prompt the user with regards to their logic sentence input.

- DuplicateDefinitionException
  It is thrown during the creation of the logic tree if a second quantifier attempts to bind an already quantified variable.



- UnboundException
  It is thrown if the LogicTree constructor attempts to create a unary or binary relation node using an unbound variable as argument for the node's relation. It may also be thrown if the user tries to use the name of a Constant that does not exist.



- UndefinedRelationException
  It is thrown if the LogicTree constructor attempts to create a relation node with a name that the signature does not contain. The same type of notification is brought up on the screen.

Some further assistance is provided using default Java exceptions:

- Index out of bounds
  This can occur when a user presses buttons that require a list element to be selected so that the method can pass down an index. In such cases an error message is displayed only once to instruct the user that they must make a selection.

- I/O exception
  This can occur when the user tries to load a corrupted or otherwise inaccessible file. The error message informs accordingly.

Finally some guiding messages are displayed depending on the user's intention:

- If they want to add a binary relation a message tells them to click on two objects subsequent

- If they want to add or rename a constant a message tells them they need to choose a name

- If they want to load or generate a new signature a message asks them if they want to save the current signature

All these messages are only displayed once since it might otherwise become irritating for the user to keep having to click 'ok' when they want to, for example, add a constant.

Chapter 4

Evaluation

This section will try to provide an overview of what my solution to the project specification has achieved by relating it to existing solutions as well as my own expectations. I will use this opportunity to point out what is flawed or missing, as well as justify and explain why it is so.

4.1 Quantitative evaluation

My first priority was to make a minimum viable product that meets the project's compulsory requirements. These are as follows:

- Notation should be as used in the course: Yes.
  The formula field allows as valid input any sentence that respects first order logic syntax, making the brackets optional where possible. The lexer section describes how it allows the right names for relation, variable and constant names.

- All aspects of the software should be controlled via a graphical user interface. It should be attractive robust, and easy to learn and use: Yes.
  Although it is a subjective matter whether the interface is easy to use, the fact that it covers all aspects of the back end and provides complete functionality for modifying the structure and signature is verifiable. I have also received some good feedback from two of my colleagues. I asked them to modify the structure such that the sentence *All men are mortal* would be evaluated to true and then input the sentence for evaluation. They had no trouble in doing so. One tester did refer to the help menu, however just out of curiosity and admitted they would probably not use it. As for its appearance, they found the program's simplicity pleasing and said it is quite comfortable. However they both had a reservation regarding the Swing look and feel which they considered a bit outdated. I agree the program should be customizable to use more modern skins and fonts. This is the intended purpose for the preferences item in the Workbench

menu. However time did not allow it and it was quite low on my list of priorities.

- The user should be able to create and edit signatures with up to at least 6 constants, 3 unary relation symbols, and 3 binary relation symbols, and structures whose domains have up to at least 12 objects, all in the sense of the department's logic course: Yes.
  I managed to implement the ideal case, where there are no limits on the number of objects or symbols. The program also uses a minimum number of arrows such that the work area does not become confusing. Although it does not automatically arrange the objects to a best distribution across the surface, it allows them to be rearranged by dragging, such that the user can decide what is most comfortable to them. There is a basic alignment done across the diagonal of the structure area. Automatic uniform or otherwise better distribution would be a good addition, however the complexity of the algorithm was to high and could not be implemented within the time limit.

- Users should be able to save structures to a file and load saved structures for further editing: Yes. Structures can be saved and loaded locally on the user's computer. When loading a structure the signature is updated accordingly. In addition, the user can save the entire workbench consisting of the signature, structure, sentences and tutorial progress.

- The user should be able to create and edit first-order sentences. The software should display them correctly, with proper logical symbols for boolean connectives and quantifiers: Yes.
  The interface displays a user friendly sentence format which is consistent with first order predicate logic syntax rules. It hides at all times the tokens used in the parsing process.

- The software should be able to correctly evaluate any first-order sentence in any structure of appropriate signature: Yes.
  When developing the evaluator at the start of this project, testing was quite difficult as input was only possible via the terminal. My understanding of the bigger picture was not yet perfect. This only came after writing the interface and implementing the model-view-controller design. Separating the back end from the interface played a crucial role in allowing me to make changes, debug and simplify the back end code, whilst the interface allowed me to more easily input sentences for testing. It was only towards the end of the project that I managed to fix some of the evaluation bugs such as the asymmetry of the parameters for binary relations and the equals operator. In the end, I managed to half the size of the code for the logic tree and simplify the process of assignment passing to an easily readable implementation. I even removed one of the exception classes as it became obsolete. Although it might be very tempting for one to consider a part of their development finished, revising it after the entire

thing is done will inevitably improve it as it will interact better with the rest of the software.
Within a structure containing the binary relation R, the unary relation P, the nullary relations A, B, C and constants Fred and Tina, I evaluated the following sentences, changing the situation to make them true and false subsequently. The evaluator has performed correctly in every case:

```
∀x(R(Fred,x) → ∀y(R(x,y) → P(y)))
∃x∀y (A ∨ B) ∧ B ∧ ¬R(x,y) ∨ C
∃x∀y ((A ∨ B) → P(y) ↔ R(y,x))
R(Fred,Tina) ↔ P(Fred)
⊤
Fred = Fred
∀x Fred = x
∃x P(x) ∧ P(y) ¬x=y
∀x P(x) → ∃y R(x,y)
∃x(happy(x) → ¬A)
```

- The software should be able to lead the user interactively through the evaluation, along with comprehensive help facilities: No. Although the program provides a few basic tutorial questions, the tutorial tool cannot yet asses the student's performance. Unfortunately time was not sufficient to further develop this feature. As it is now, the student can read and answer the questions and mark the question as completed himself once he considers it is resolved.

- Several examples of structures and sentences should be provided: Yes. The program automatically generates random structures. There is also a file provided with a number of example sentences which the user can load in the active sentence list. However at this stage the program will only load the sentences that are valid in the structure. A better approach would be for it to automatically add any constants and relation symbols it uses to the structure.

4.2 Qualitative evaluation

Qualitative aspects I feel are important to mention mainly concern the interface experience. This is relative to each user, however there are a two things that I am certain are satisfactory:

- Speed - throughout testing and general use the program has behaved very well, without crashing or delays. The buttons and menus are responsive and the overall experience smooth

- Simplicity - counting the logic symbol buttons as a group, there are never more than 7 buttons visible on the screen at one time. This allows the user to focus on the structure and sentences, without having to worry about overwhelming toolbars.

Of course there are also a number of improvements that can be made. Amongst these, the ones I consider most important are as follows:

- The ability to rename objects from the structure and add new binary relations directly from the structure: this would virtually eliminate the need for a signature panel as they are the only two actions that cannot be performed without it. It would further simplify the interface and provide a clean interaction.
- Provide better user feedback: as mentioned before my tutorial implementation is very basic because of time limitations. However user feedback is extremely important for keeping the student engaged and interested.
- Progress bar: although it exists on the workbench, it is not correlated to the user activity.

4.3 Conclusions and Future Work

With a full bug proof evaluator and all the essential functionality provided through the interface, this program could be the base of beautiful extensions. As mentioned before, one of them could be an implementation of the Hintikka game, which would contribute to the sense of satisfaction of the student as well as to their motivation to use the tool.

I also consider it an important improvement that the application is made available online. This would would make it easier for the students to access the application and keep track of their progress, and would allow them to compare their performance to others.

For robustness, the tool should be linked to a lesson database accessible by students and lecturers alike. The former should also have permissions to add, edit or remove lessons and see their student's progress. I strongly believe these three improvements would make a robust and truly useful teaching tool.

Bibliography

[1] Terence Parr, *The Definitive ANTLR 4 Reference*, The Pragmatic Bookshelf, 2013

[2] *Java®Platform, Standard Edition 7, API Specification* Oracle, http://docs.oracle.com/javase/7/docs/api/index.html

[3] *The Java® Tutorials*, Oracle, http://docs.oracle.com/javase/tutorial/index.html

[4] Scott Chacon, *Pro Git* http://git-scm.com/book

[5] Naranker Dulay, *Compilers 221 lecture slides* https://www.doc.ic.ac.uk/ nd/compilers/

[6] *LaTeX manual*, WikiBooks, http://en.wikibooks.org/wiki/LaTeX

[7] Ian Hodkinson, *Logic 140 lecture slides*, http://www.doc.ic.ac.uk/ imh/

[8] *Tarski's World Support and Documentation*, OpenProof Software, http://ggweb.stanford.edu/support/manual/tarski

Appendix A

Code UML Diagram

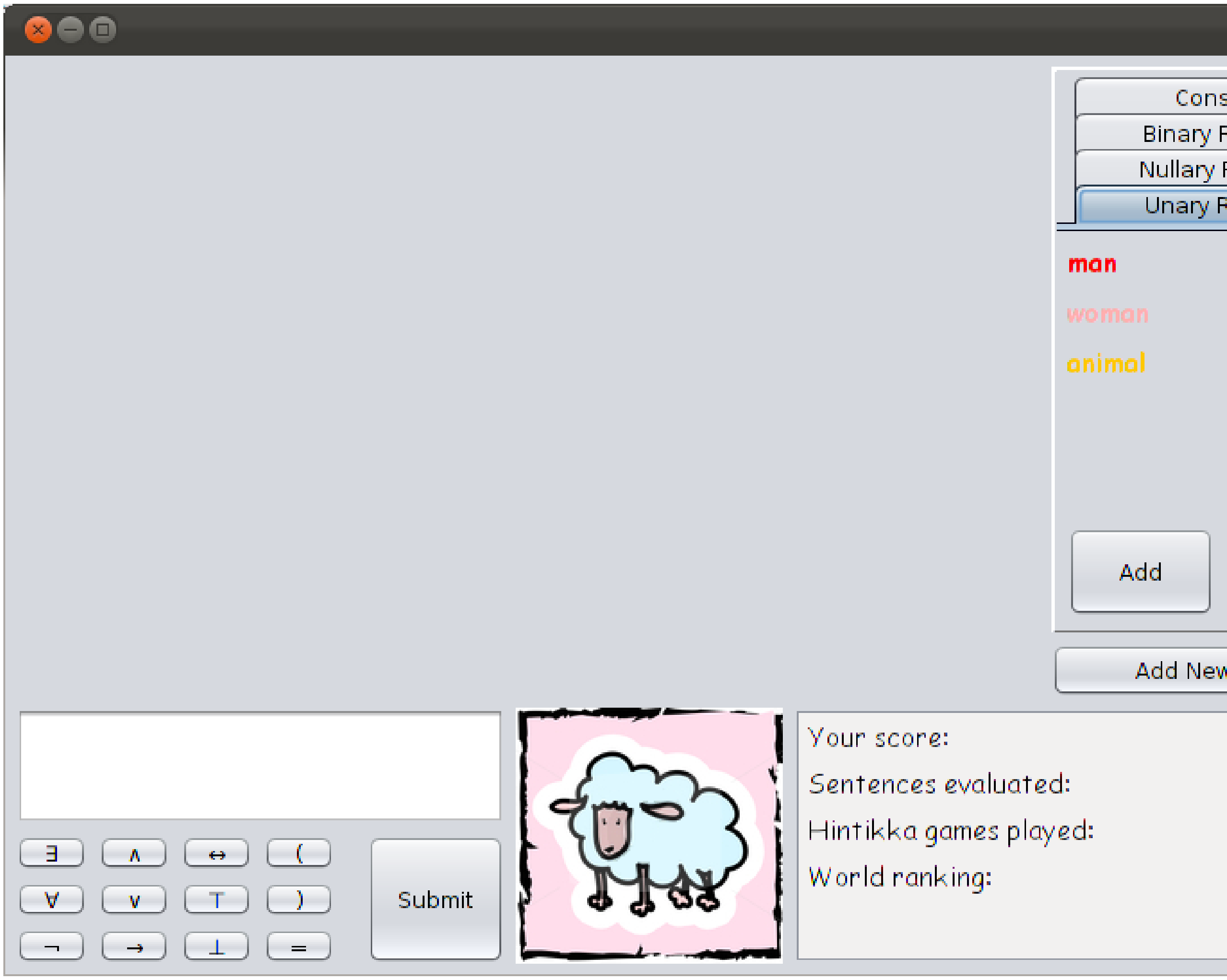


Figure A.1: Summarises the GUI package

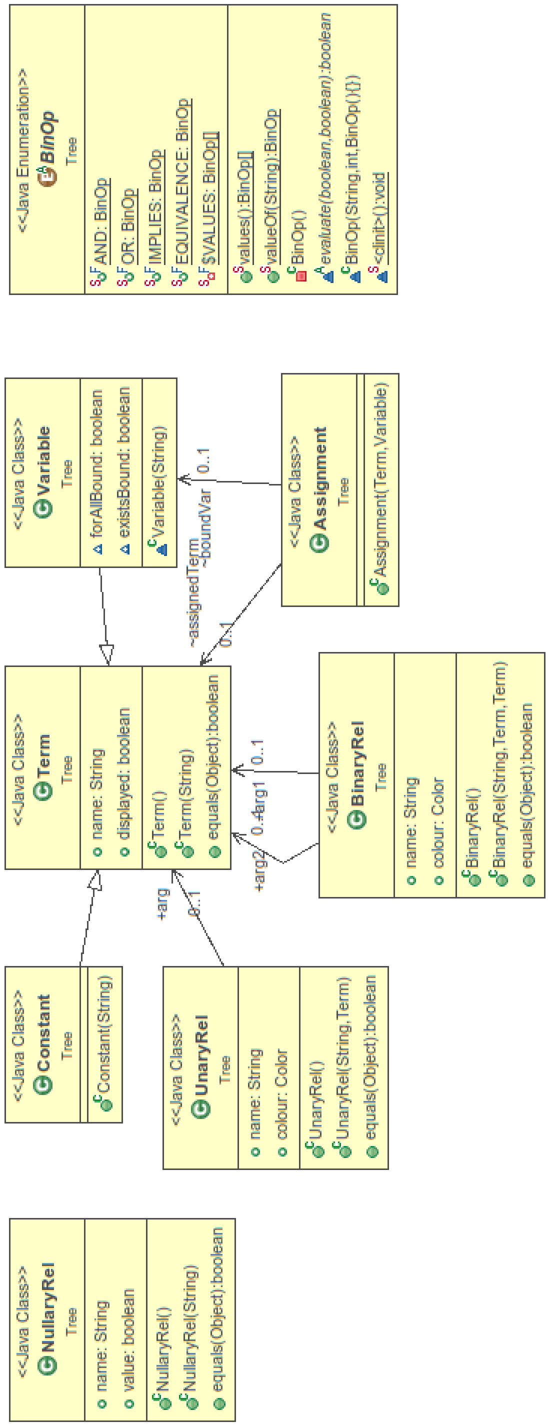


Figure A.2: Summarises the classes the logic tree is based on (fields of the logic tree nodes)

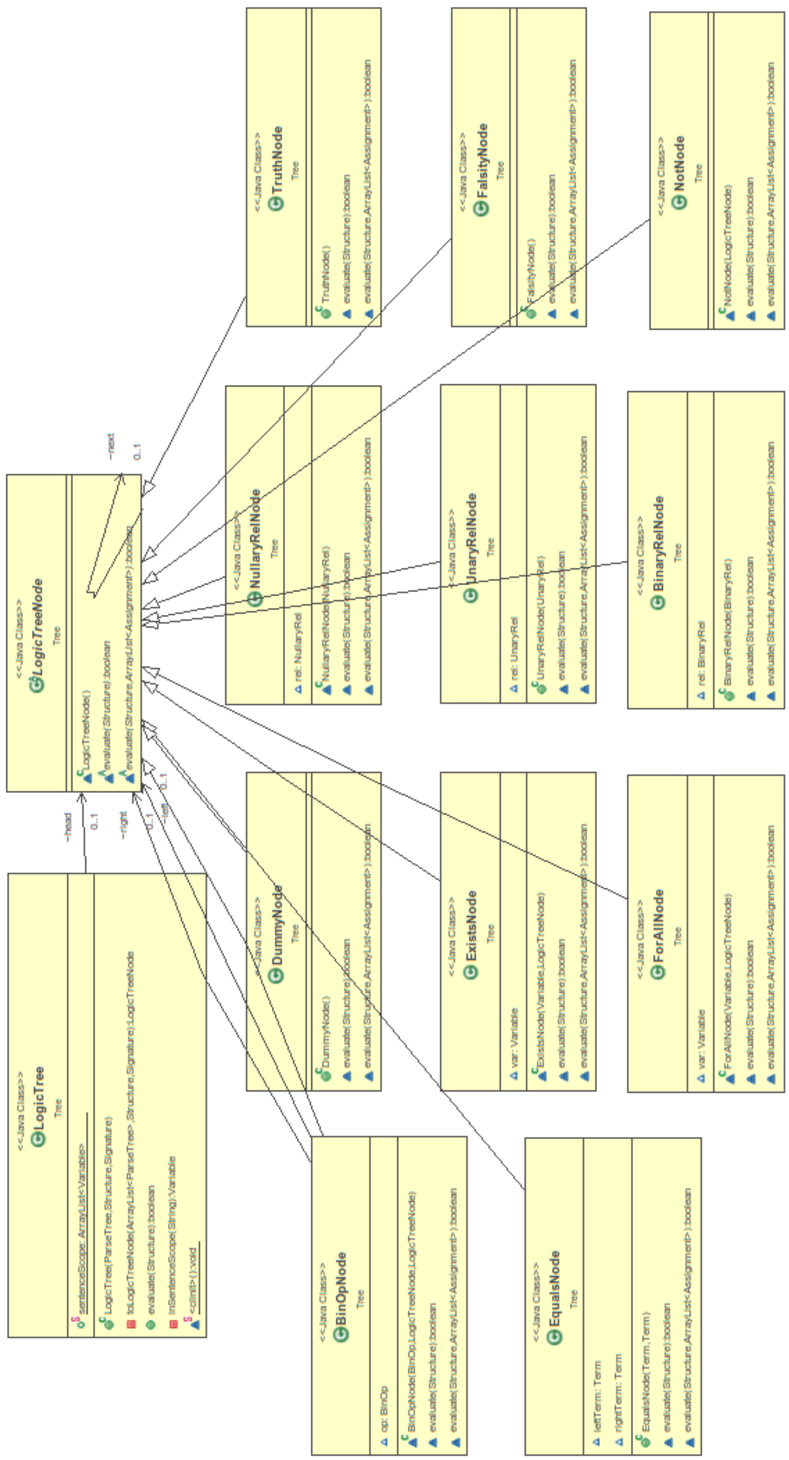


Figure A.3: Summarises the LogicTree structure

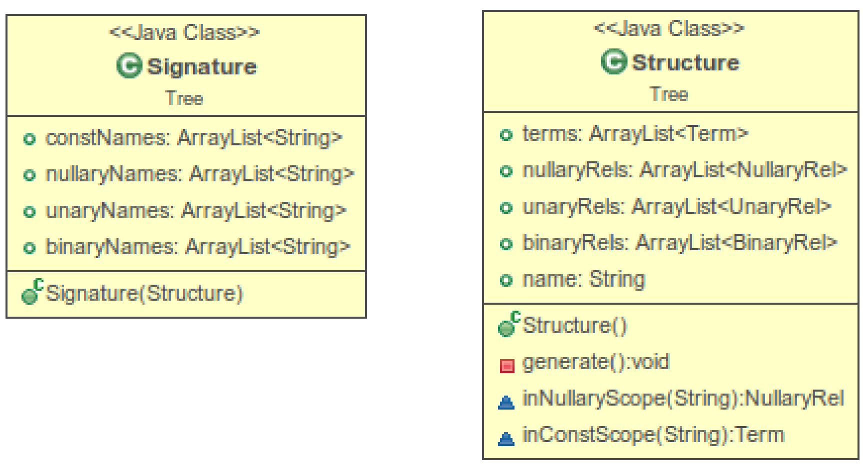


Figure A.4: Summarises the contents of Signatures and Structures

Appendix B

Parser grammar

```
grammar fol ;

condition
  : formula EOF ;

formula
  : TRUTH
  | FALSITY
  | term EQUALS term
  | relation
  | quantifier formula
  | NOT formula
  | formula AND formula
  | formula OR formula
  | formula IMPLIES formula
  | formula EQUIV formula
  | LPAREN formula RPAREN ;

term
  : VARIABLE
  | NAME ;

quantifier
  : FORALL VARIABLE
  | EXISTS VARIABLE ;

relation
    : NAME binaryarg
    | NAME unaryarg
    | NAME ;
```

```
binaryarg
    : LPAREN term ',' term RPAREN ;

unaryarg
    : LPAREN term RPAREN ;
```