M T W T F S DATE: ___/___/______

Linear Assignment
Properties of Determinants

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1) Reflection Property :- the determinant remains the same if columns change to rows and vice versa.

2) All-zero property :- determinant is zero if all elements of a row (or column) are zero.

3) Repitition property : if all elements of some row are equal to another row (column to column as well), then the determinant is zero).

4) Switching property : the interchange of any two rows or columns of the determinant changes its sign.

5) Scalar multiple property: if all elements of a row or column are multiplied by a non-zero constant, then the determinant gets multip-lied by the same constant.

6) Sum Property;

$$\begin{vmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$$

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7) Property of Invariance :

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+\alpha b_1+\beta c_1 & b_1 & c_1 \\ a_2+\alpha b_2+\beta c_2 & b_2 & c_2 \\ a_3+\alpha b_3+\beta c_3 & b_3 & c_3 \end{vmatrix}$$

$\therefore$ Determinant remains unaltered under an operation of the form.

$$C_i \to C_i + \alpha C_j + \beta C_k,$$

where

$j, k \neq i$

or an operation of the form

$$R_i \to R_i + \alpha R_j + \beta R_k$$

where

$j, k \neq i$

8) Factor Property : if a determinant Δ becomes zero when we put $x = \alpha$, then $(x-\alpha)$ is a factor of Δ.

9) Triangle Property : if all the elements of a determinant above or below the main diagonal consist of zeroes, then the determinant is equal to the product of the diagonal elements.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

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10) Determinant of cofactor matrix :

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{then,}$$

$$\Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{23} & C_{33} \end{vmatrix}$$

where C_{ij} denotes the cofactor of the element a_{ij} in Δ

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