Homework 1

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1.
    a. The best-case scenario of this bubble sort will go through every iteration of both for loops but not have to do any other line of code as the if statement will be false if it is all already sorted. This would be $O(n^2)$.
    b. The worst-case scenario would be if it is in reverse order, it would require going through every line of code for both of the for loops. This would not however increase the complexity as it would just be multiplied by a constant giving again $O(n^2)$.
2.
    a. The best-case scenario would be if the item being searched for is at the top, this would give $O(1)$.
    b. The worst-case would be if the item is at the bottom of the list, giving $O(n)$.
3.
    a. If f(n) has an upper bound of g(n) and g(n) has an upper bound of h(n) this therefor implies that f(n) must have an upper bound of h(n)
    b. If f(n) = n^2 and g(n) =n^3 it does not imply that they have an identical upper bound.
4.
    a.
        i. Optimization: Given a weighted graph, find a spanning tree of minimum cost
        ii. Decision: Given a weighted graph and integer L, is there a spanning tree with a cost less than or equal to L?
    b.
        i. Optimization: Given a graph, find the largest set of matching edges such that no two edges in the matching share an endpoint.
        ii. Decision: Given a graph and integer L, is there a set larger or equal to L of matching edges such that no two edges in the matching share an endpoint?
    c.
        i. Optimization: Given a weighted graph and two vertices, find the shortest path between the two vertices.
        ii. Decision: Given a weighted graph, two vertices, and an integer L, is there a path equal to or shorter than integer L?
5.
    a. While an exact algorithm always gives an optimal solution eventually, a heuristic is an exhaustive algorithm with a time limit. It can give a suboptimal solution.

b. A polynomial time algorithm is one whose time complexity function is O(p(n)) for some polynomial p(n)
c. An intractable problem is one where no polynomial time algorithm exists that can solve it.
d. The complexity class p is that of polynomial time.