CALIFORNIA STATE UNIVERSITY **LONG BEACH**
COLLEGE OF ENGINEERING
COMPUTER ENGINEERING COMPUTER SCIENCE DEPARTMENT

Digital Signal Processing

Project-1

Due: March 13, 2019

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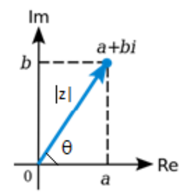
The labs and projects materials are taken from Dr. Thomas Johnson

Part 1: Complex Numbers.



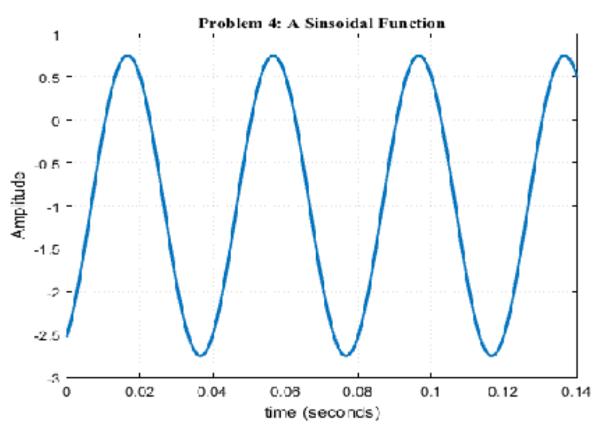
Most DSP is done in the complex plane. A complex number can be visually represented as a pair of numbers (a, b) forming a vector on a diagram representing the complex plane. "Re" is the real axis, "Im" is the imaginary axis, and i or j is the imaginary unit which satisfies $i^2 = j^2 = -1$. The point z=a+bi can also be represented in polar form: z=|z| ∠ θ where $|z| = = \sqrt{a^2 + b^2}$ and $\theta = \arctan(b/a)$.
Sinusoids are also represented in the complex plane as rotating vectors. If x(t)=Acos(ωt+θ), then in the complex plane we represent x(t) by a vector of length |A| initially lying at angle θ rotating at angular frequency ω. You cannot tell the angular frequency of a phasor by examining its plot since the information is not shown. Phasors are always derived from a ***cosine*** trigonometric function. If you have a ***sine*** function, change it to a cosine by subtracting a phase angle of 90 degrees: sin(θ) = cos(θ - π/2) before expressing it as a phasor in the complex plane.

1. Perform the following complex operations and verify using Matlab.
   (a) Let x=1-2j and y=-1+1j, then ***by hand*** find the quantities a=(x+y), b=(x-y), c=(x*y), d=(x/y) and p=x^y . Verify your results using Matlab as very smart calculator.
   (b) Show the collection of individual complex points, i.e. x, y, a, b, c, d, and p, of part (a) on the complex plane in figure(1) in Matlab using ***plot*** and ***hold on*** to compose the complex plane diagram.
   (c) If the points are interpreted as representing vectors, what is the magnitude of each and at what angle in degrees does each vector lie with respect to the positive real axis? In other words, what is the polar coordinate representation of each of the points (vectors) in part (a)? Add to figure(1) by again using plot, straight lines from the origin to each of the points in part (a) to represent the points as complex vectors.
   (d) In figure(2), plot as vectors four non-zero complex points using only random integer values in interval [-5,5] for both coordinate points. Have only one vector in each quadrant. Connect the points with straight lines to form a quadrilateral. Find its area and then put the result of the area calculation into the title of the plot.
2. Represent these two sinusoids as phasors in the complex plane:

   x(t) = 3 cos(25t + 45°) and y(t) = 2 sin(25t − 150°)

   by representing them as Matlab vectors x=3*exp(1j*45*pi/180) and y= x=2*exp(1j*(-150-90)*pi/180).
   Continuous rotation of these vectors (phasors) about the origin is implied but is not shown on a plot since the plotted vector representing the phasor is stationary. Plot in figure(3) these two phasor vectors as straight lines in the complex plane with the x(t) phasor colored blue and the y(t) phasor colored red. Use the ***hold on*** function for the plot since you will add another vector to the plot in the next part.
3. Addition of phasors is easy and is done by adding the real parts together and the imaginary parts together (simple complex addition) to get the resultant vector which is the phasor representation of the ***cosine*** that is their sum. Perform this operation on the phasors of (2) and plot the resulting phasor on the same plot as (2) and color it green. What is the resulting sinusoid's amplitude and phase angle (in degrees)? Put the information into the ***legend*** on the plot as magnitude and phase (polar coordinates) of each of the colored vectors. Note the trigonometric identity which is very useful in the future: sin(x) = cos(x-90°).
4. Examine this plot of a sinusoidal function. From the plot, determine the cosine's DC offset, amplitude A, angular frequency ω, and phase angle θ in degrees. Express the function in the form

   x(t) = A cos (ωt + θ) + DC Offset.



Part 2: Simple Signals

Introduction: A discrete signal y[n]=f(x[n]) where x[n] and thus y[n] are only defined at a set of discrete points and nowhere in between. The function x[n] is usually shortened to just n, where the relationship between n and x[n] is obvious from context. Unfortunately most DSP signals are written using the dependent variable x[n] and not y[n]. It is usually the case in DSP that signals are functions of time, so we write x(t)=f(t) where t occurs only at discrete times and we end up with the discrete relationship x[n]=f(n) instead of y[n]=f(x[n]) .

1. Consider the discrete-time signal with n being an arbitrary integer

$$x_M[n] = \sin(\frac{2\pi M n}{N})$$

   and assume N = 12. For M = 4, 5, 7, 10 and 15, plot $x_M[n]$ on the interval $0 \le n \le 2N-1$ or 2M-1 if M>N. Use stem to create your plots in separate figures 1 to 4, and be sure to appropriately title the figures and label your axes. In the title of each figure, indicate the fundamental period of the signal. In general, please state how the fundamental period can be determined from arbitrary integer values of M and N.

   Hints: (1) Use ***figure(k)*** for k a positive integer to number the plots.
   (2) Use "[P,Q]=rat(M/N)" to get integers P and Q as P/Q which is M/N reduced to lowest terms.
   (3) Use "str=sprintf('Fundamental Period=%d",P); title(str)" to put value of variable P into the title of the plot.

2. Consider the discrete-time signal

$$x_k[n] = \sin(\omega_k n)$$

   where $\omega_k = 2\pi k/5$. Over the interval $0 \le n \le 9$, plot each signal $x_k[n]$ for k=1, 2, 4 and 6 using stem. All of the signals should be plotted with separate axes in the same figure 5 using subplot. Identify which signals are identical and explain how signals with differing values of ω_k can result in the same signal.

3. Now consider the following three signals and find the period T of each term in the expressions assuming N=6:

$$x_1[n] = \cos\left(\frac{2\pi n}{N}\right) + 2\cos\left(\frac{3\pi n}{N}\right) = \cos\left(\frac{2\pi n}{T_1}\right) + 2\cos\left(\frac{2\pi n}{T_2}\right) =$$

$$x_2[n] = 2\cos\left(\frac{2n}{N}\right) + \cos\left(\frac{3n}{N}\right) = 2\cos\left(\frac{2\pi n}{T_3}\right) + \cos\left(\frac{2\pi n}{T_4}\right)$$

$$x_3[n] = \cos\left(\frac{2\pi n}{N}\right) + 3\sin\left(\frac{5\pi n}{2N}\right) = \cos\left(\frac{2\pi n}{T_5}\right) + 3\sin\left(\frac{2\pi n}{T_6}\right)$$

   Determine the periodicity in index n of each signal by noting this about a periodic x[n]:

   x[n]=x[n+k Tx] for integer k and period Tx. Suppose a periodic y[n]=y[n+mTy] for integer m and period Ty. Then a periodic z[n]=x[n]+y[n] = x[n+kTx]+y[n+mTy]=z[n+Tz] and Tz=kTx=mTy so that Tx/Ty=m/k, a rational fraction and Tz is the ***lcm(k,m).***

   If a signal is periodic, plot the signal for two periods starting at n=0. Put its period in the title. If the signal is aperiodic (not periodic), plot the signal for $0 \le n \le 12N-1$ and explain why it is not periodic. Use figure numbers 6, 7, and 8 for the three plots.

4. Plot each of the following signals on the interval $0 \le n \le 31$ in figures 9, 10 and 11 respectively:

$$x_1[n] = \sin\left(\frac{\pi n}{4}\right)\cos\left(\frac{\pi n}{4}\right) \qquad x_2[n] = cos^2\left(\frac{\pi n}{4}\right) \qquad x_3[n] = \sin\left(\frac{\pi n}{4}\right)\cos\left(\frac{\pi n}{8}\right)$$

   Put the period of each in its title. For each, know how to find the period without relying on Matlab's plots.
   Hint: A trigonometric identity is *sin(x)cos(y)=(sin(x+y)+sin(x-y))/2*.

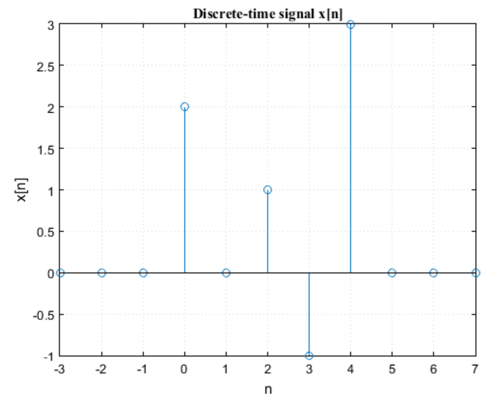
5. (a) Demonstrate that the addition of two periodic signals is not always periodic by giving a counter example.
   (b) Demonstrate that the product of two periodic signals is not always periodic by giving a counter example.

Part-3: Discrete Time Signals

1. Define a Matlab vector n to be the time indicies $-3 \le nx \le 7$ and the vector x to be the values of the signal x[n] at those samples given as

$$x[n] = \begin{cases} 2, & n = 0 \\ 1, & n = 2 \\ -1, & n = 3 \\ 3, & n = 4 \\ 0, & otherwise \end{cases}$$

Hint: Use "x(n==k) = x[k]". Plot the discrete time vector x[n] in figure(1) as ***stem(n,x)*** to get the following figure:



2. Define the vectors $y_k[n]$ with corresponding sample index vectors ny_k as the following discrete-time signals

$$y_1[n] = x[n-2]$$
$$y_2[n] = x[n+1]$$
$$y_3[n] = x[-n]$$
$$y_4[n] = x[-n+1]$$

These signals are shifted versions of the signal x[n], so to do this you must modify the sample index vector n of x[n] to reflect the shifting process. The resulting index vectors need not span the same space as n of x[n], but may need to be extended in either direction to accommodate the shifted signal. Take for example y[n]=x[n-1]. Then y[1]=x[0], y[2]=x[1], etc., and you see that all the sample values of x[n] will be shifted to the right by 1 sample index to get the value of y[n]. Hence the sample index vector ny would just be ny=n+1 and values of vector y are just the same as the values of vector x. Thus only the sample index needs to be changed. Write a Matlab function "[y,m]=sigshift(x,n,k)" to shift vector x with sample index n by an amount k to get vector y with sample index m.

3. Generate the vectors $y_k[n]$ and in figure(2). Using subplot and stem, plot x[n] and each $y_k[n]$ for k=1 to 4 over the sample interval $-10 \le n \le 10$. In each of the subplot's title include the discrete-time signal plotted (see signals in (2)) and label the axes of each plot appropriately.

Part 4: Linear Shift Invariant (LSI) Systems

1. The following systems violate a given LSI property. By using an input or set of inputs, the failure of the system to pass the LSI test often can be shown in Matlab plots. For each system, define the Matlab sequences for inputs and outputs and by referring to subsequent plots, explain how these figures show that the system violates the LSI property indicated.

   (It might be useful to have in your working directory the functions ***impseq***, ***stepseq***, ***sigshift***, ***sigfold***, ***sigadd*** and ***sigmult*** that are found presented in Chapter 2 of the text.)

   a. The system $y[n]=\sin((\pi/2)x[n])$ is not linear. Use the signals $x_1[n]=\delta(n)$ and $x_2[n]=2\delta(n)$ and show that the system y[n] violates linearity. The linearity property states $y[Ax_1[n] + Bx_2[n]]=Ay[x_1[n]] + By[x_2[n]]$ for arbitrary constants A and B. Let A = 2 and B = -1 here. $\delta(n)$ is the delta function. Subplot the right hand side (rhs) and the left hand side (lhs) of the linearity equation over interval $0\le n\le 10$ to show that they are not identical.

   b. The system $y[n]=nx[n]$ is not shift invariant. Use the signal $x[n]=\delta(n)$ over $0\le n\le 10$. Shift invariance says that the order of transformation and shifting is invertible, i.e. shifting the input and then transforming this sequence results in the same output as transforming the input sequence and then shifting this output. Use a shift amount k= -1 in your investigations.

   c. The system $y[n]=x[n]+x[n+1]$ is not causal. Use input $x[n]=u(n)$ where u(n) is the step function. Define the Matlab vector x to represent input on the sample interval $-5\le n\le 9$ and the output vector y on the sample interval $-6\le n\le 9$. Causality says there can be no output before there are any inputs to a system. Subplot the input and the output of the system over interval $-10\le n\le 10$ to show that there is output before there is any input to the system. Stem plot the input x[n] and the output y[n] using a 2x1 subplot in figure(1) to show your results. Use ***axis([-10,10,min(y)-1,max(y)+1])*** to set the axes of the stem plots.

Part 5: Constant Coefficient Difference Equations

1. Discrete-time systems are often implemented with constant-coefficient difference equations. Two examples of simple difference equations are

   $$y[n] = x[n] + b\, x[n-1] \quad (first - order\ moving - average: FOMA)$$

   $$y[n] = a\, y[n-1] + \ x[n] \quad (first - order\ auto - regression: FOAR)$$

   Write a function y=diffeqn(a,x,yn1) which will compute the output y[n] of the causal system of the FOAR equation. The input vector x contains the sequence x[n] over 0≤n≤N-1, the output vector y contains y[n] over the same interval, and yn1 supplies the value y[-1]. The first line of the M-file is ***function y=diffeqn(a,x,yn1).*** Hint: y[-1] is required to compute y[0] which is the initial step of the auto-reqression. Use a for-loop in your M-file for the rest of the indexed values of y[n], starting with n=1.

2. Using the FOAR equation, assume that a=1 and y[-1]=0 and that we are interested in the output over the range 0≤n≤N-1 for N=31. Use your function to find the output for inputs of (a) the unit pulse $x_1[n]=\delta[n]$ and (b) the unit step $x_2[n]=u[n]$. In figure(1) and figure(2), respectively plot each response using stem, titling and labeling each figure appropriately.

3. Using the FOAR equation, assume that a=1, but that y[-1] = -1. Test the FOAR for linearity, namely the equation:
   T[A x1[n]+ B x2[n]] = A T[x1[n]] + B T[x2[n]]
   For A=1 and B=1 and using $x_1[n]=u[n]$ and $x_2[n]=2u[n]$ over 0≤n≤N-1 for N=11, compute the left hand side (LHS) as ylhs[n] as the FOAR output for the input $x[n]=Ax_1[n]+Bx_2[n]$. Then find the output y_1 [n] as the FOAR output for input $x_1[n]$ and output $y_2[n]$ as the output for FOAR input $x_2[n]$. Now compute the right hand side (RHS) as $yrhs[n]=Ay_1[n] + By_2[n]$. Finally, for figure(3), use subplot and stem to plot the two sequences ylhs[n], yrhs[n] and the difference. Since the FOAR equation is a linear difference equation, why aren't the two results, namely the RHS and LHS, identical?

4. The FOAR causal system is BIBO (bounded-input bounded-output) stable whenever |a|< 1. A property of stable systems is that the effect of initial conditions becomes insignificant for sufficiently large n. Test this out by assuming a=3/4 and x[n]=u[n] for 0≤n≤N-1 for N=31. First let y[-1]=0 and compute the output $y_1[n]$ and then let y[-1]=-1 and compute the output $y_2[n]$. For figure(4), use stem and subplot to plot both outputs. Title and label them appropriately. How do they differ? What is the limiting value of the outputs as n gets very large?