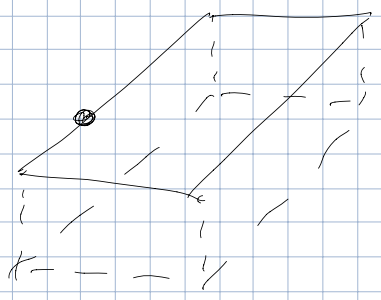
Hessenberg Variety in the Flag Variety

Flag is a sequence of nested subspaces



$$\begin{bmatrix} & \\ & \end{bmatrix}$$

$$\text{pt} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} = F_0$$

$$\text{line} = \text{span}\left\{ \begin{bmatrix} v_1 \\ ? \\ ? \end{bmatrix} \right\} = F_1$$

$$\text{plane} = \text{span}\{ v_1, v_2 \} = F_2$$

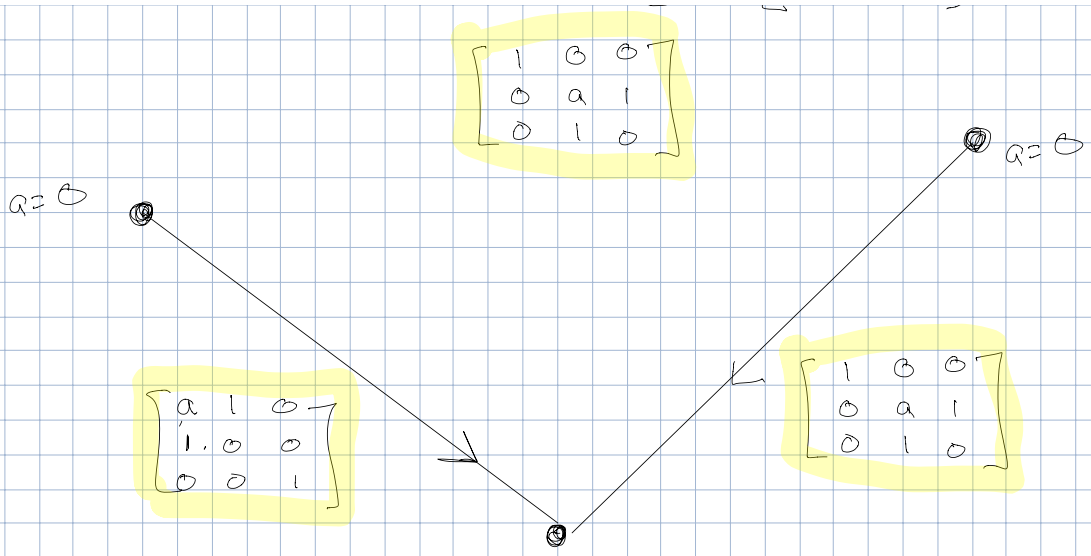
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 6 \\ 0 & 5 & 7 \\ 0 & 0 & 8 \end{bmatrix}$$

$\uparrow$ same flag

Identify matrices with the same flag

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}, \quad \begin{bmatrix} a & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} a & b & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

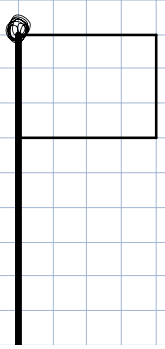
$$\begin{bmatrix} a & 1 & 0 \\ b & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} a & c & 1 \\ b & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$



$\lim_{a \to \infty}$

$$\begin{bmatrix} a & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}, \quad \begin{bmatrix} a & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$



g_1 and g_2 represent the same flag if for some upper $\triangle$ matrix b

$$g_1 = g_2 b$$