Introduction to Bayes' Theorem

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To define a Probabilistic Model, we need

- A sample space Ω: set of all possible *outcomes* of an experiment

  A subspace of the sample space is called *event* (the empty event is $\emptyset$)

- A probability law assigns to each event A a number $\mathcal{P}(A)$ such that
  - $0 \leq \mathcal{P}(A) \leq 1$
  - The Probability of the entire sample space is $\mathcal{P}(\Omega) = 1$
  - If A and B two disjoint events $A \cap B = \emptyset$ then

$$\mathcal{P}(A \cup B) = \mathcal{P}(A) + \mathcal{P}(B)$$

From these axioms, we can deduce some important properties

- The Probability of the empty event is $\mathcal{P}(\emptyset) = 0$
- If A and B are two events, then

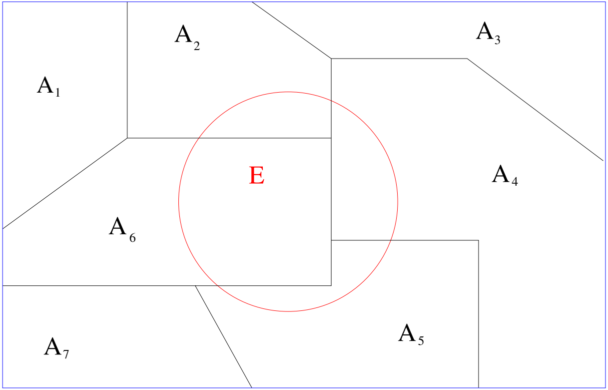
$$\mathcal{P}(A \cup B) = \mathcal{P}(A) + \mathcal{P}(B) - \mathcal{P}(A \cap B)$$

Partition of a sample space

- Suppose we can completely partition Ω into n disjoint events $A_1, A_2, \ldots A_n$
- Any event E can be decomposed by

$$\mathcal{P}(E) \quad = \quad \mathcal{P}(E \cap A_1) + \mathcal{P}(E \cap A_2) + \ldots + \mathcal{P}(E \cap A_n)$$

- This can easiliy be understood from a Venn diagram



Conditional probability

Appears when partial information about outcome is given.

Example: We roll a fair die, with the sample space $\{1, 2, 3, 4, 5, 6\}$.

Q : What is the probability that the outcome is 4 ?

Since there are 6 different outcomes (all equally likely) and 4 is one of them, the answer is

$$\mathcal{P}(4) = \frac{\text{number of elements in } \{4\}}{\text{number of elements in } \Omega} = \frac{1}{6}$$

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For two events A and B, we define the conditional probability

$$\mathcal{P}(A|B) = \frac{\mathcal{P}(A \cap B)}{\mathcal{P}(B)}$$

$\Rightarrow$ Express $P(A|B)$ in terms of $P(B|A)$

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Bayes' theorem

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First example: Detecting aircraft

A radar is designed to detect aircraft. If an aircraft is present, it is detected with probability 0.99. When no aircraft is present, the radar generates an alarm probability 0.02 (false alarm). We assume that an aircraft is present with probability 0.05. If the radar generates an alarm, what is the probability than an aircraft is present ?

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So we obtain

$$\begin{aligned} \mathcal{P}(aircraft|alarm) &= \frac{\mathcal{P}(alarm|aircraft) \times \mathcal{P}(aircraft)}{\mathcal{P}(alarm|aircraft) \times \mathcal{P}(aircraft) + \mathcal{P}(alarm|no\ aircraft) \times \mathcal{P}(no\ aircraft)} \\ &= \frac{0.99 \times 0.05}{0.99 \times 0.05 + 0.02 \times 0.95} \sim 0.72 \end{aligned}$$

Answer: 72%

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  - $\mathcal{P}(C_1|E) = \dfrac{\mathcal{P}(E|C_1)}{\mathcal{P}(E)} \times \mathcal{P}(C_1)$
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- In science, very often we have access to $\mathcal{P}(A|B)$ (for example by some experiments) but what we really want to know is $\mathcal{P}(B|A)$

- We can then use Bayes' theorem, provided we also know $\mathcal{P}(A)$ and $\mathcal{P}(B)$

$$\mathcal{P}(B|A) = \frac{\mathcal{P}(A|B)}{\mathcal{P}(B)} \times \mathcal{P}(A)$$

- This is illustrated by the next example

Second example: The False Positive Puzzle

We want to know if a clinical test for a given rare disease is reliable. One person per 1,000 is affected by this disease
The test results are assumed to be correct 95% *of the time:*
if a person has the disease, the test results are positive with probability 0.95,
and if the person does not have the disease, the test results are negative with probability 0.95.

Is this a reliable test ? given that a person just tested positive, what is the probability of having the disease ?

Second example: The False Positive Puzzle

- When developing the test in a lab, we take certain persons who are known to have the disease and run the test.

  $\Rightarrow$ Therefore the scientists are measuring $\mathcal{P}(\textit{positive}|\textit{disease}), \mathcal{P}(\textit{negative}|\textit{disease})$.

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- Now we use the test in an practise. If the results are positive, we want to know the probability that the patient really has the disease, so we want to know $\mathcal{P}(\textit{disease}|\textit{positive})$.

  If the results are negative, what is the probability that the patient is infected (and that the test failed) ? $\mathcal{P}(\textit{disease}|\textit{negative})$

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We want to find $\mathcal{P}(disease|positive)$ knowing $\mathcal{P}(positive|disease)$ $\Rightarrow$ Bayes' theorem

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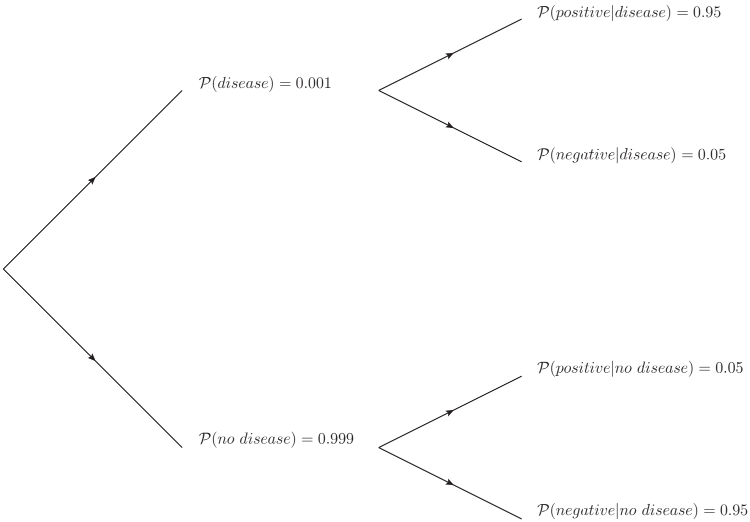

Bayes' theorem

$$\mathcal{P}(\textit{disease}|\textit{positive}) = \frac{\mathcal{P}(\textit{positive}|\textit{disease}) \times \mathcal{P}(\textit{disease})}{\mathcal{P}(\textit{positive})}$$

We need $\mathcal{P}(\textit{positive})$, we use the fact that *disease* and *no disease* form a partition of Ω

$$\begin{aligned} \mathcal{P}(\textit{positive}) &= \mathcal{P}(\textit{positive} \cap \textit{disease}) + \mathcal{P}(\textit{positive} \cap \textit{no disease}) \\ &= \mathcal{P}(\textit{positive}|\textit{disease}) \times \mathcal{P}(\textit{disease}) + \mathcal{P}(\textit{positive}|\textit{no disease}) \times \mathcal{P}(\textit{no disease}) \end{aligned}$$

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Therefore, using
$\mathcal{P}(\textit{positive}|\textit{disease}) = 0.95$, $\mathcal{P}(\textit{positive})|\textit{no disease}) = 0.05$, $\mathcal{P}(\textit{disease}) = 0.001$, we find

$$\mathcal{P}(\textit{disease}|\textit{positive}) = \frac{0.95 \times 0.001}{0.95 \times 0.001 + 0.05 \times 0.999} \sim 0.01866$$

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The probability that the patient has the disease given that the test is positive is less than 2% !!

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is small compared to the probability of being a "false positive"

$$\mathcal{P}(\textit{positive} \cap \textit{no disease}) = 0.05 \times 0.999 = 0.04995$$

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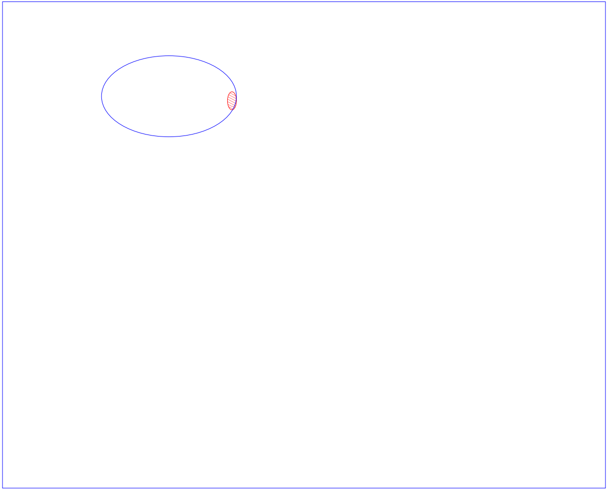
In other words: if somebody is tested positive, it is very likely that he is *false positive*

$$\mathcal{P}(no\ \ disease|positive) = \frac{0.04995}{0.0509} \sim 0.9811 \qquad \mathcal{P}(disease|positive) = \frac{0.00095}{0.0509} \sim 0.0186$$

Solving the False Positive Puzzle (cont.)



- Blue *positive*
- Red *infected*

If we pick up a random person detected *positive*, most likely it is a *false positive*

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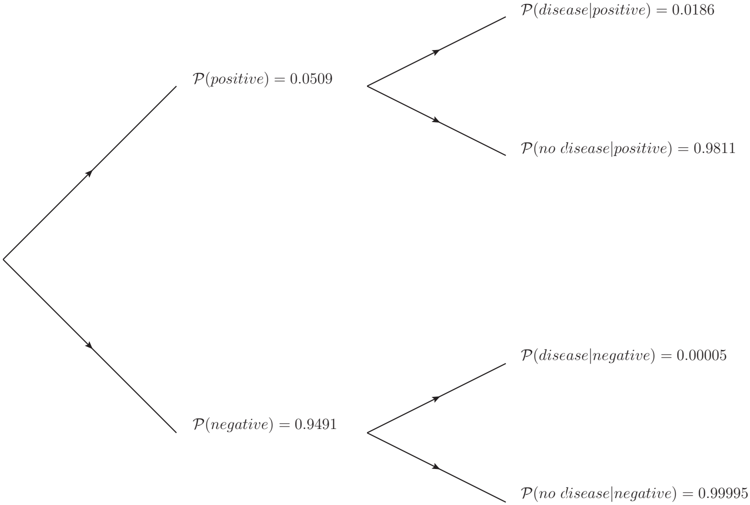

Let us check by changing the numbers

We want to know if a clinical test for a given rare disease is reliable. One person per 1,000 is affected by this disease
if a person has the disease, the test results are positive with probability 0.95,
and if the person does not have the disease, the test results are negative with probability 0.95.

given that a person just tested positive, what is the probability of having the disease ?

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$$\mathcal{P}(disease|positive) \quad = \quad \frac{0.99 \times 0.001}{0.99 \times 0.001 + \quad 0.05 \times 0.999} \sim 0.01943$$

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Homework: change $\mathcal{P}(disease)$ to $0.005, 0.01, 0.05, 0.1 \dots$

Conclusions

The last example shows the importance of a definition in probability: if a lab says a test has a success rates of 95%, one should ask if the number corresponds to $\mathcal{P}(\textit{positive}|\textit{disease})$ or to $\mathcal{P}(\textit{disease}|\textit{positive})$

- Bayes theorem plays a crucial part in probability and statistics
- We saw two simple applications
- Its demonstration is very simple but the results can be surprising
- Has a lot of important applications, in particular it "inverts" a probability diagram
- An example: for a desired $\mathcal{P}(\textit{disease}|\textit{positive})$ what $\mathcal{P}(\textit{positive}|\textit{disease})$ should we require ?