1a)

Suppose that keys are hashed to three-bit sequences and that blocks can hold two records. Start with a hash table with two empty blocks (corresponding to 0 and 1). Show how the hash table evolves if we insert records with the following hash values:
000,001,...,111 using linear hashing with a capacity threshold of 75%.

1b)

Suppose that keys are hashed to five-bit sequences and that blocks can hold four records. Start with a hash table with three empty blocks (corresponding to 0, 1, and 2). Show how the hash table evolves if we insert records with the following hash values:
00000,00001,...,11111 using linear hashing with a capacity threshold of 50%.

2a)

Given the set of strings: {apple, apricot, banana, bandana, cat, caterpillar},
construct:
a) a **patricia tree** b) a **prefix tree**

2b)

Given the set of strings: {algorithm, align, algebra, alchemy, binary, biology, bioinformatics},
construct:
a) a **patricia tree** b) a **prefix tree**

Q.3

1. **Question 1**:
   We assume that a projection (like in SQL) does not remove duplicates. Provide an example to show that projection cannot be pushed below the intersection $\cap$. That is, give relations R and S such that:

$$\pi_A(R \cap S) \neq \pi_A(R) \cap \pi_A(S).$$

2. **Question 2**:
   Assume projection does not remove duplicates. Show that projection cannot be pushed below the difference $-$. Provide relations R and S such that:

$$\pi_A(R - S) \neq \pi_A(R) - \pi_A(S).$$

3a) We assume that selection (σ) in relational algebra does not remove duplicates. Give an example to show that selection cannot be pushed below set union (no duplicates!). Specifically, provide relations **R** and **S** along with a selection condition **σ_C** such that:

$$\sigma_C(R \cup S) \neq \sigma_C(R) \cup \sigma_C(S)$$

3b) Assume that **natural join (⋈)** does not remove duplicates. Give an example where projection **π** cannot be pushed below natural join. That is, provide relations **R(A, B)** and **S(B, C)** such that:

$$\pi_A(R \backslash \bowtie S) \neq \pi_A(R) \backslash \bowtie \pi_A(S)$$

3c) Suppose **set difference (−)** in relational algebra does not remove duplicates. Give an example to show that projection cannot be pushed below set difference. Provide relations **R(A, B)** and **S(A, B)** such that:

$$\pi_A(R - S) \neq \pi_A(R) - \pi_A(S)$$
