1) Delete from the B+-tree of order 4 you produced in task 1 of Sheet 8 the keys 25, 45, 24, 38, 32, 8, 27, 46, 13, 42, 5, 22, 26, 7 in this order.

Solution:
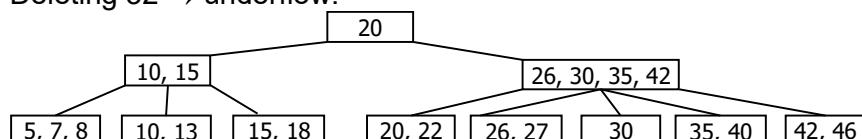
Deleting 25, 45, 24, 38 → underflow:
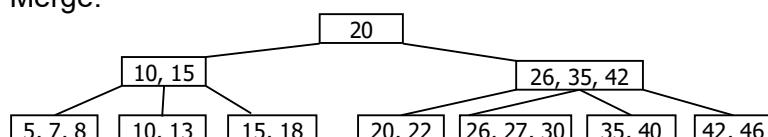

Transfer of 35:
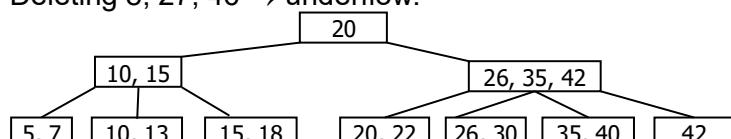

Deleting 32 → underflow:
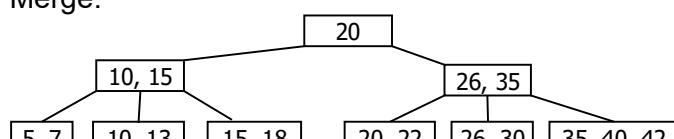
Merge:
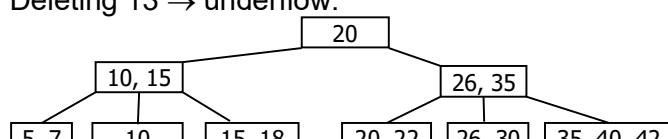
Deleting 8, 27, 46 → underflow:
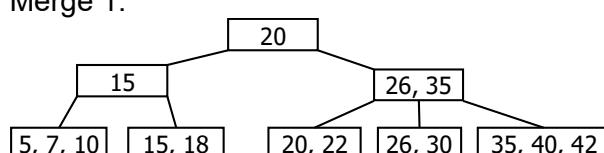
Merge:
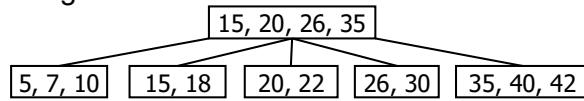
Deleting 13 → underflow:


Merge 1:

Merge 2:
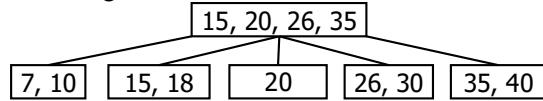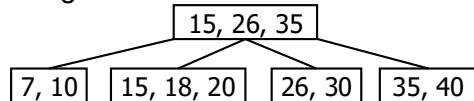
```
                    | 15, 20, 26, 35 |
        ┌──────┬────────┼────────┬──────────┐
| 5, 7, 10 | | 15, 18 | | 20, 22 | | 26, 30 | | 35, 40, 42 |
```
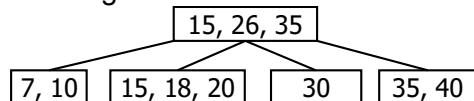
Deleting 42, 5, 22 → underflow:

```
                    | 15, 20, 26, 35 |
      ┌───────┬─────────┼─────────┬─────────┐
| 7, 10 | | 15, 18 | | 20 | | 26, 30 | | 35, 40 |
```

Merge:

```
                  | 15, 26, 35 |
      ┌───────┬───────┼───────┬───────┐
| 7, 10 | | 15, 18, 20 | | 26, 30 | | 35, 40 |
```

Deleting 26 → underflow:

```
                  | 15, 26, 35 |
      ┌───────┬───────┼───────┬───────┐
| 7, 10 | | 15, 18, 20 | | 30 | | 35, 40 |
```

Transfer of 20:

```
                | 15, 20, 35 |
      ┌──────┬──────┼──────┬──────┐
| 7, 10 | | 15, 18 | | 20, 30 | | 35, 40 |
```

Deleting 7 → underflow:

```
                | 15, 20, 35 |
      ┌──────┬──────┼──────┬──────┐
| 10 | | 15, 18 | | 20, 30 | | 35, 40 |
```

Merge:

```
                | 20, 35 |
        ┌───────────┼───────────┐
| 10, 15, 18 | | 20, 30 | | 35, 40 |
```
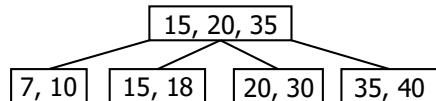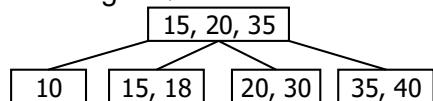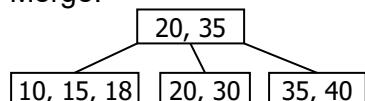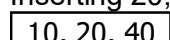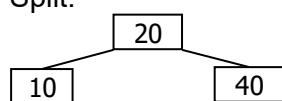
2) Insert the keys 20, 40, 10, 30, 15, 35, 7, 26, 18, 22, 29, 32 in this order into an initially empty B-tree with order 2.
Solution:
Inserting 20, 40, 10 → overflow:
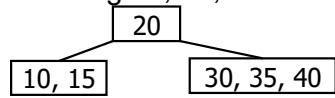
```
| 10, 20, 40 |
```

Split:

```
      | 20 |
   ┌────┴────┐
| 10 |     | 40 |
```
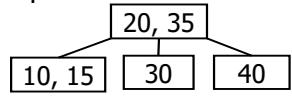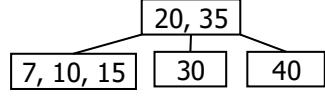
Inserting 30, 15, 35 → overflow:

```
      | 20 |
   ┌────┴──────┐
| 10, 15 |   | 30, 35, 40 |
```

Split:

```
        | 20, 35 |
   ┌───────┼───────┐
| 10, 15 | | 30 | | 40 |
```

Inserting 7 → overflow:

```
        | 20, 35 |
   ┌───────┼───────┐
| 7, 10, 15 | | 30 | | 40 |
```

Split 1:

```
              10, 20, 35
      ┌────┬─────┴─────┬────────┐
    [ 7 ] [ 15 ]    [ 30 ]    [ 40 ]
```

Split 2:

```
                  [ 20 ]
          ┌─────────┴─────────┐
        [ 10 ]              [ 35 ]
      ┌───┴───┐          ┌─────┴─────┐
    [ 7 ] [ 15 ]      [ 30 ]      [ 40 ]
```
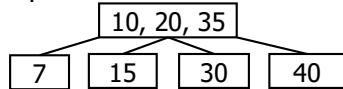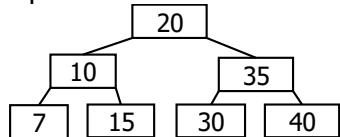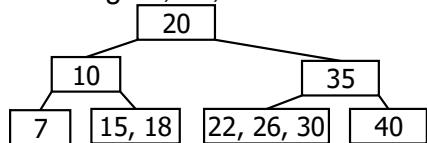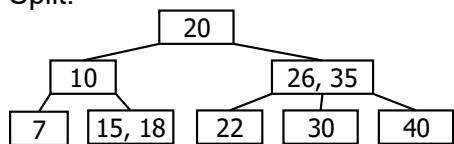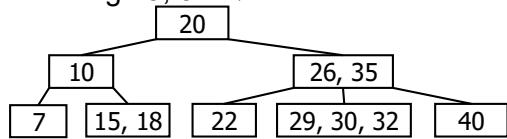
Inserting 26, 18, 22 → overflow:

```
                  [ 20 ]
          ┌─────────┴──────────┐
        [ 10 ]               [ 35 ]
      ┌───┴────┐          ┌─────┴─────┐
    [ 7 ] [ 15, 18 ]  [ 22, 26, 30 ] [ 40 ]
```

Split:

```
                  [ 20 ]
          ┌─────────┴──────────┐
        [ 10 ]             [ 26, 35 ]
      ┌───┴────┐       ┌──────┼──────┐
    [ 7 ] [ 15, 18 ] [ 22 ] [ 30 ] [ 40 ]
```
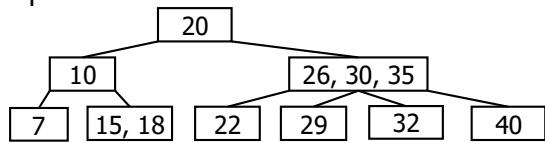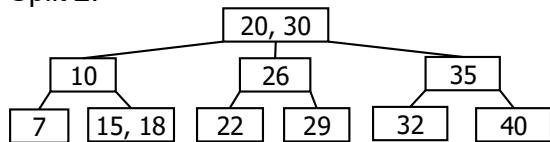
Inserting 29, 32 → overflow:

```
                  [ 20 ]
          ┌─────────┴──────────┐
        [ 10 ]             [ 26, 35 ]
      ┌───┴────┐       ┌──────┼──────────┐
    [ 7 ] [ 15, 18 ] [ 22 ] [ 29, 30, 32 ] [ 40 ]
```

Split 1:

```
                  [ 20 ]
          ┌─────────┴──────────┐
        [ 10 ]            [ 26, 30, 35 ]
      ┌───┴────┐      ┌─────┬──┴──┬──────┐
    [ 7 ] [ 15, 18 ] [ 22 ] [ 29 ] [ 32 ] [ 40 ]
```

Split 2:

```
                     [ 20, 30 ]
          ┌──────────────┼──────────────┐
        [ 10 ]         [ 26 ]         [ 35 ]
      ┌───┴────┐     ┌───┴───┐     ┌────┴────┐
    [ 7 ] [ 15, 18 ] [ 22 ] [ 29 ] [ 32 ]  [ 40 ]
```
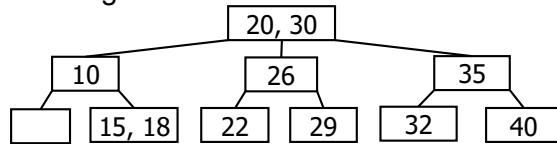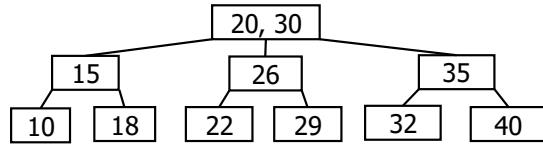
3) Delete from the B-tree of order 2 you produced in task 2 the keys 7, 30, 10 in this order.

Solution:

Deleting 7 → underflow:

```
                    20, 30
        10            26            35
    [  ]  15, 18   22    29     32       40
```

Transfer:

```
                    20, 30
        15            26            35
    10    18       22    29      32    40
```
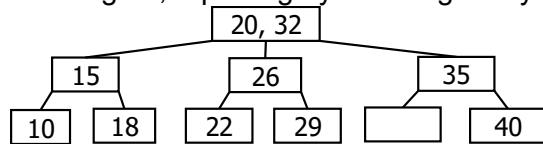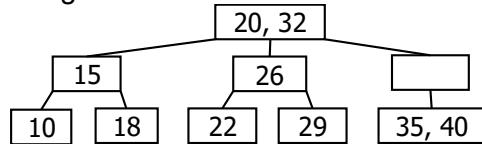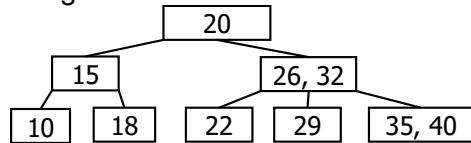
Deleting 30, replacing by next larger key in tree → underflow:

```
                    20, 32
        15            26            35
    10    18       22    29     [  ]    40
```

Merge 1:

```
                    20, 32
        15            26           [  ]
    10    18       22    29      35, 40
```

Merge 2:

```
                    20
        15                 26, 32
    10    18       22    29    35, 40
```

Deleting 10 → underflow:

```
                    20
        15                 26, 32
  [  ]    18      22    29    35, 40
```

Merge:

```
                    20
        [  ]              26, 32
   15, 18      22     29    35, 40
```

Transfer:

```
                    26
           20              32
     15, 18    22      29     35, 40
```