Aggregatable Subvector Commitments for Stateless Cryptocurrencies

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Motivation

Stateful transaction validation (for account-based cryptocurrencies)

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- 5. Proof serving nodes would like to compute all π_i 's fast

Contributions

Table 1: Asymptotic comparison to previous (aS)VCs: n is the size of \vec{v} and b is the # of proofs to aggregate.

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^{*}All schemes can (1) verify a proof π_i in $O(|\pi_i|)$ time and (2) update digests in O(1) time.

Outline

Motivation

Contributions

Techniques: VCs from Univariate Polynomial Commitments

Committing to Vectors (and Updating Digests)

Computing and Updating Proofs

Aggregating Proofs into Subvector Proofs

Conclusion

Fix *n*-SDH public parameters $(g^{\tau^i})_{0 \le i \le n}$ such that trapdoor τ is unknown.

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Can interpolate polynomial from n points $(x_i, \phi(x_i))_{i \in [n]}$ in $O(n \log^2 n)$ field operations.

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$$= (g^{\tau^n})^{\phi_n} (g^{\tau^{n-1}})^{\phi_{n-1}} \dots (g^{\tau})^{\phi_1} (g)^{\phi_0}$$
 (2)

$$=g^{\phi_n\tau^n}g^{\phi_{n-1}\tau^{n-1}}\dots g^{\phi_1\tau}g^{\phi_0} \tag{3}$$

$$= g^{\phi_n \tau^n + \phi_{n-1} \tau^{n-1} + \dots + \phi_1 \tau + \phi_0} \tag{4}$$

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Can interpolate polynomial from n points $(x_i, \phi(x_i))_{i \in [n]}$ in $O(n \log^2 n)$ field operations. Time to commit is an O(n)-sized multi-exponentiation.

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8

Outline

Motivation

Contributions

Techniques: VCs from Univariate Polynomial Commitments

Committing to Vectors (and Updating Digests)

Computing and Updating Proofs

Aggregating Proofs into Subvector Proofs

Conclusion

Proof π_i **for** v_i **Refresher**

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New technique: O(n) time w/o interpolating $\phi(X)$ via upk_i 's (see [TAB+20, Appendix D.7]).

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Big problem: To update π_i after a change to any j, need $upk_j = \{u_{i,j}, \forall i \neq j\} \Rightarrow |upk_j| = O(n)$. **New technique:** Compute $u_{i,j}$ in O(1) time from upk_j and upk_j .

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Outline

Motivation

Contributions

Techniques: VCs from Univariate Polynomial Commitments

Committing to Vectors (and Updating Digests)

Computing and Updating Proofs

Aggregating Proofs into Subvector Proofs

Conclusior

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$$=\phi(X)\sum_{i=l}\frac{1}{A_{l}'(i)(X-i)}-\left(\sum_{i=l}v_{i}\cdot\frac{A_{l}(X)}{A_{l}'(i)(X-i)}\right)\cdot\frac{1}{A_{l}(X)}$$

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(40)

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(37)

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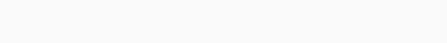
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Job done: Can aggregate π_i 's for $(v_i)_{i \in I}$ into constant-sized π_I .



Conclusion

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Other goodies (not in this talk):

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- Aggregating multiple *I*-subvector proofs across different commitments [GRWZ20].

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- Since g^{τ^i} 's are updatable, our public parameters are *updatable*.
- Can precompute all *n* proofs in *O*(*n* log *n*) time via FK technique [FK20].
- Can remove A'(i) from upk_i.

Questions?

Outline

Decomposition of 1/((X-i)(X-j))

Decomposition of $1/A_I(X)$

Decomposition of A(X)/((X-i)(X-j))

Note that:

$$\frac{1}{i-j} \cdot \frac{A(X)}{X-i} + \frac{1}{j-i} \cdot \frac{A(X)}{X-j} = \frac{1}{i-j} \cdot \frac{A(X)(X-j)}{(X-i)(X-j)} + \frac{1}{j-i} \cdot \frac{A(X)(X-i)}{(X-j)(X-i)}$$

$$= \frac{\frac{1}{i-j}A(X)(X-j) - \frac{1}{i-j}A(X)(X-i)}{(X-i)(X-j)}$$

$$= \frac{\frac{1}{i-j}A(X)[(X-j) - (X-i)]}{(X-i)(X-j)}$$

$$= \frac{\frac{1}{i-j}A(X)(-j+i)}{(X-i)(X-j)}$$

$$= \frac{A(X)}{(X-i)(X-j)}$$
(45)

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Decomposition of 1/((X-i)(X-j))

Decomposition of $1/A_I(X)$

Partial Fraction Decomposition From Lagrange Interpolation

It is well-known that Lagrange coefficients can be rewritten as [BT04, vzGG13]:

$$L_{i}(X) = \prod_{i \in I, i \neq i} \frac{X - j}{i - j} = \frac{A_{i}(X)}{A_{i}'(i)(X - i)}, \text{ where } A_{i}(X) = \prod_{i \in I} (X - i)$$
 (50)

Here, $A'_{i}(X)$ is the derivative of $A_{i}(X)$ and has the (non-obvious) property that $A'_{i}(i) = \prod_{j \in l, j \neq i} (i - j)$. Next, consider the Lagrange interpolation of $\phi(X) = 1$:

$$\phi(X) = \sum_{i=1}^{n} v_i L_i(X) \Leftrightarrow \tag{51}$$

$$1 = A_I(X) \sum_{i \in [0, n]} \frac{V_i}{A_I'(i)(X - i)} \Leftrightarrow$$
 (52)

$$\frac{1}{A_i(X)} = \sum_{i=1}^{n} \frac{1}{A_i'(i)(X-i)} \Leftrightarrow \tag{53}$$

$$\frac{1}{A_{l}(X)} = \sum_{i=1}^{l} \frac{1}{A_{l}'(i)} \cdot \frac{1}{(X-i)} \Rightarrow \tag{54}$$

$$c_i = \frac{1}{A'(i)} \tag{55}$$

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