Aggregatable Subvector Commitments for Stateless Cryptocurrencies

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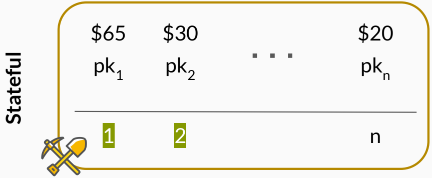
September 14th, 2020

Motivation

Miners rely on state to validate transactions and blocks.



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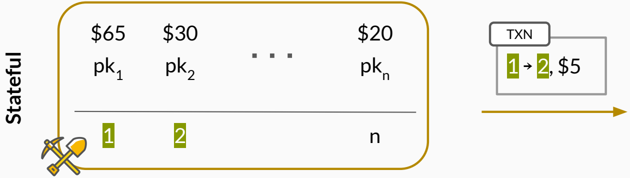

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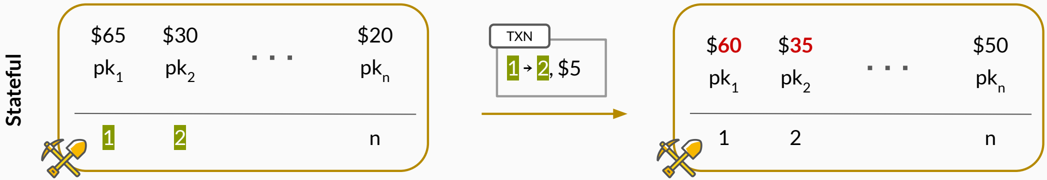

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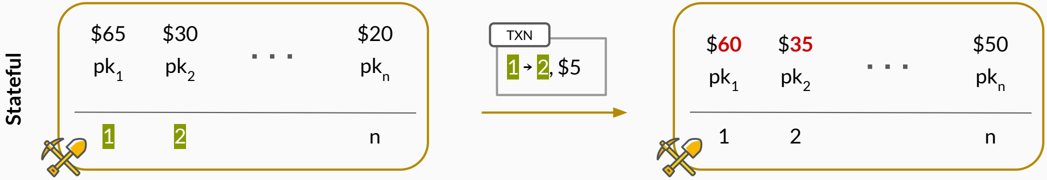

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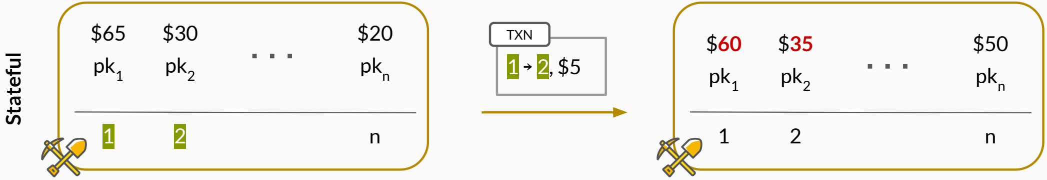

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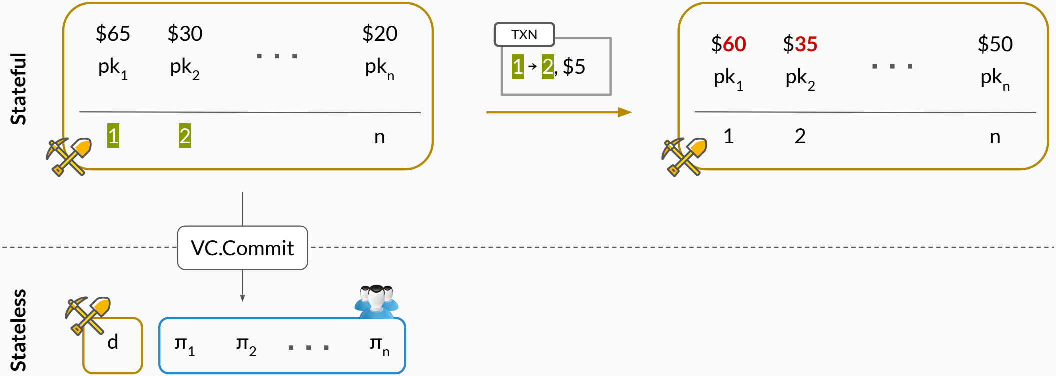

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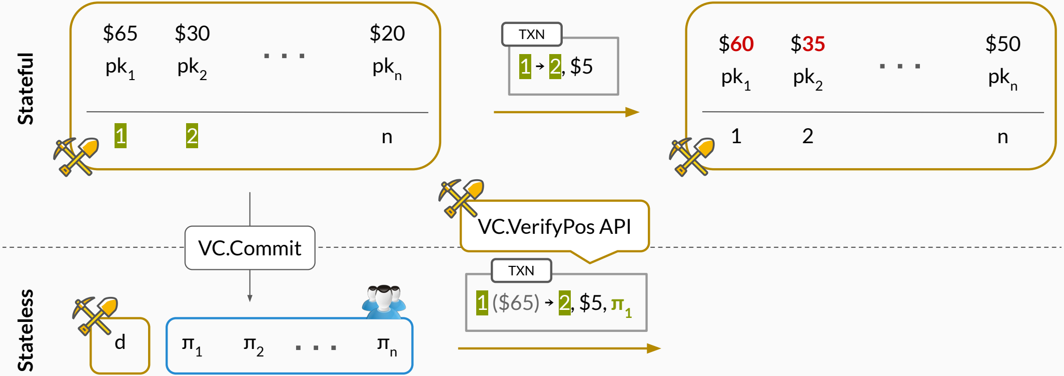

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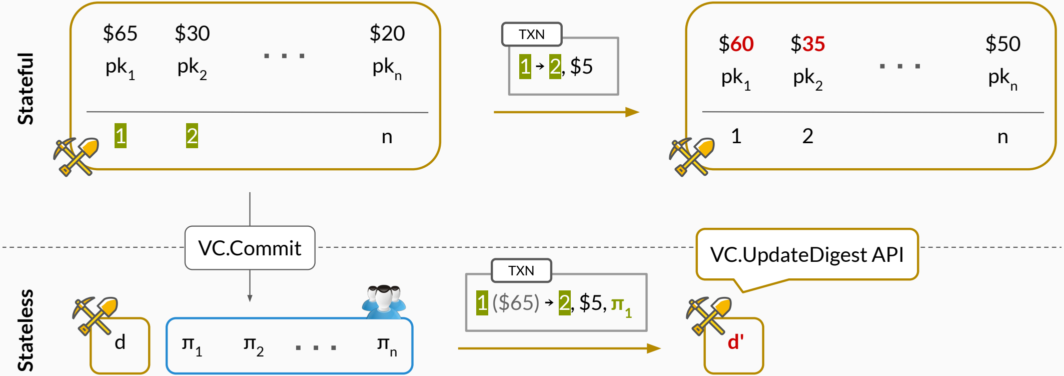

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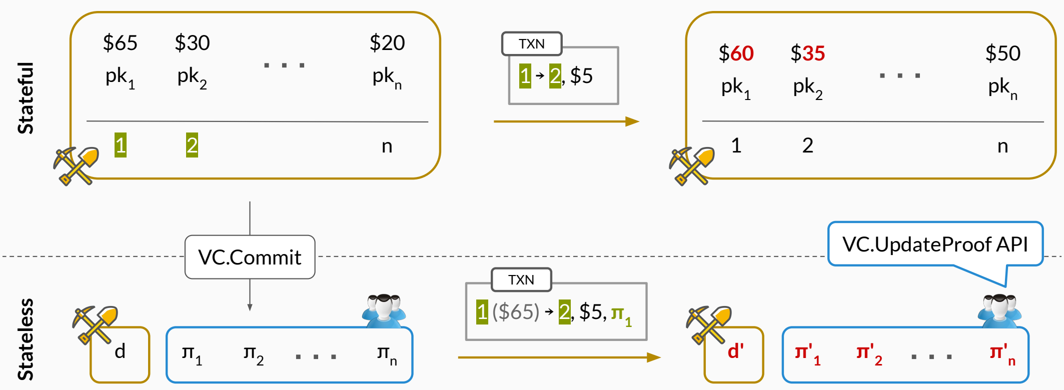

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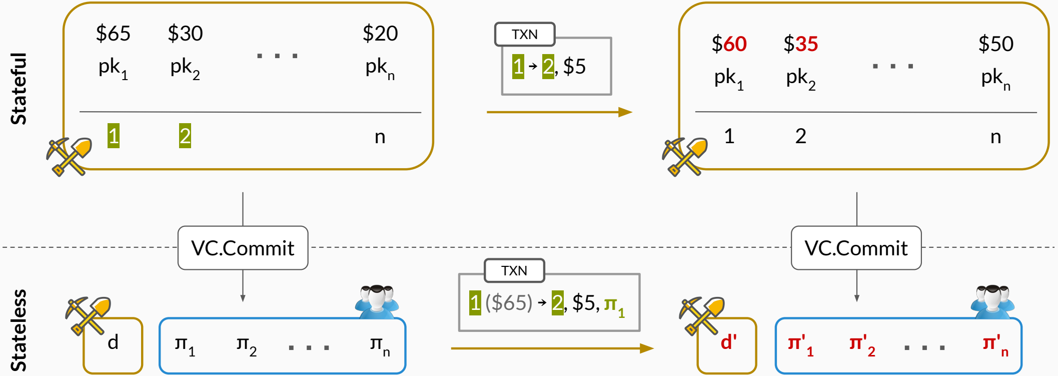

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Thank you!

Paper is too long? **Read our blogpost!**

https://alinush.github.io/2020/05/06/aggregatable-subvector-commitments-for-stateless-cryptocurrencies.html

Appendix

Outline

Aggregatable Subvector Commitments (aSVCs) versus Previous Work

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- No space-time trade-off for proof pre-computation [BBF19, CFG+20]

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$$L_i(X) = \prod_{\substack{j \in [0,n) \\ j \neq i}} \frac{X - j}{i - j} \tag{7}$$

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Thus, for our purposes, each upk_i will include $c\left(L_i\right)$.

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Outline

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Solution: Compute $c\left(\frac{L_j(X)}{X-i}\right)$ in $O(1)$ time from information in upk_i and upk_j.

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New Technique: Compute $c\left(\frac{L_j(X)}{X-i}\right)$ in $O(1)$ Time

Let $A(X) = \prod_{i\in[0,n)}(X - i)$. Then:

$$\frac{L_j(X)}{X - i} = \frac{1}{A'(j)} \cdot \frac{A(X)}{(X - j)(X - i)} \tag{26}$$

Next, use **partial fraction decomposition** to rewrite:

$$\frac{A(X)}{(X - i)(X - j)} = \frac{1}{i - j} \cdot \frac{A(X)}{X - i} + \frac{1}{j - i} \cdot \frac{A(X)}{X - j} \tag{27}$$

Now, replacing Equation (27) into Equation (26):

$$\frac{L_j(X)}{X - i} = \frac{1}{A'(j)} \cdot \left(\frac{1}{i - j} \cdot \frac{A(X)}{X - i} + \frac{1}{j - i} \cdot \frac{A(X)}{X - j}\right) \tag{28}$$

As a result:

$$c\left(\frac{L_j(X)}{X - i}\right) = \left(c\left(\frac{A(X)}{X - i}\right)^{\frac{1}{i-j}} \cdot c\left(\frac{A(X)}{X - j}\right)^{\frac{1}{j-i}}\right)^{\frac{1}{A'(j)}} \tag{29}$$

Thus, each upk_i must include $c\left(\frac{A(X)}{X-i}\right)$ and $A'(i)$. *Can derive from g^{τ^i}'s!*

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- Thus, $\pi_I = \prod_{i \in I} \pi_i^{c_i}$

Outline

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