Aggregatable Subvector Commitments for Stateless Cryptocurrencies

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While you're at it, feel free to read our blogpost too.

Motivation

Stateless Cryptocurrencies

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Stateful transaction validation (for account-based cryptocurrencies)

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5. Proof serving nodes would like to compute all π_i's fast

Contributions

Our Contribution: Aggregatable Subvector Commitments with Scalable Updates

Table 1: Asymptotic comparison to previous (aS)VCs: n is the size of $\vec{v}$ and b is the # of proofs to aggregate.

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**All schemes can (1) verify a proof π_i in $O(\lvert \pi_i \rvert)$ time and (2) update digests in $O(1)$ time.*

Techniques: VCs from Univariate Polynomial Commitments

Outline

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Outline

Proof π_i for v_i Refresher

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(15)

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New technique: $O(n)$ time w/o interpolating $\phi(X)$ via upk_i's (see [TAB+20, Appendix D.7]).

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Outline

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Computing I-subvector Proofs From Scratch

Can use KZG batch proofs to compute a constant-sized I-subvector proof π_I for all $(v_i)_{i\in I}$:

$$A_I(X) = \prod_{i\in I}(X - i) \tag{33}$$

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Note: *vrk* contains $(g^{\tau^i})_{i\in[0,|I|]}$

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Job done: Can aggregate π_i's for $(v_i)_{i\in I}$ into constant-sized π_I.

Conclusion

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- Can remove $A'(i)$ from upk_i.

Questions?

Outline

Decomposition of $1/((X-i)(X-j))$

Decomposition of $1/A_I(X)$

Decomposition of $A(X)/((X-i)(X-j))$

Note that:

$$
\begin{aligned}
\frac{1}{i-j}\cdot\frac{A(X)}{X-i}+\frac{1}{j-i}\cdot\frac{A(X)}{X-j} &= \frac{1}{i-j}\cdot\frac{A(X)(X-j)}{(X-i)(X-j)}+\frac{1}{j-i}\cdot\frac{A(X)(X-i)}{(X-j)(X-i)} && (45)\\
&= \frac{\frac{1}{i-j}A(X)(X-j)-\frac{1}{i-j}A(X)(X-i)}{(X-i)(X-j)} && (46)\\
&= \frac{\frac{1}{i-j}A(X)[(X-j)-(X-i)]}{(X-i)(X-j)} && (47)\\
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Partial Fraction Decomposition From Lagrange Interpolation

It is well-known that Lagrange coefficients can be *rewritten* as [BT04, vzGG13]:

$$L_i(X) = \prod_{j \in I, j \neq i} \frac{X - j}{i - j} = \frac{A_I(X)}{A_I{}'(i)(X - i)}, \text{ where } A_I(X) = \prod_{i \in I}(X - i) \tag{50}$$

Here, $A_I'(X)$ is the derivative of $A_I(X)$ and has the (non-obvious) property that $A_I'(i) = \prod_{j \in I, j \neq i}(i - j)$.
Next, consider the Lagrange interpolation of $\phi(X) = 1$:

$$\phi(X) = \sum_{i \in I} v_i L_i(X) \Leftrightarrow \tag{51}$$

$$1 = A_I(X) \sum_{i \in [0,n)} \frac{v_i}{A_I{}'(i)(X - i)} \Leftrightarrow \tag{52}$$

$$\frac{1}{A_I(X)} = \sum_{i \in I} \frac{1}{A_I{}'(i)(X - i)} \Leftrightarrow \tag{53}$$

$$\frac{1}{A_I(X)} = \sum_{i \in I} \frac{1}{A_I{}'(i)} \cdot \frac{1}{(X - i)} \Rightarrow \tag{54}$$

$$c_i = \frac{1}{A_I{}'(i)} \tag{55}$$

References i

J. Berrut and L. Trefethen.
Barycentric Lagrange Interpolation.
SIAM Review, 46(3):501–517, 2004.

Vitalik Buterin.
The stateless client concept.
ethresear.ch, 2017.
https://ethresear.ch/t/the-stateless-client-concept/172.

Vitalik Buterin.
Using polynomial commitments to replace state roots.
https://ethresear.ch/t/using-polynomial-commitments-to-replace-state-roots/7095, 2020.

Jan Camenisch, Maria Dubovitskaya, Kristiyan Haralambiev, and Markulf Kohlweiss.
Composable and Modular Anonymous Credentials: Definitions and Practical Constructions.
In Tetsu Iwata and Jung Hee Cheon, editors, *Advances in Cryptology – ASIACRYPT 2015*, pages 262–288, Berlin, Heidelberg, 2015. Springer Berlin Heidelberg.

Dario Catalano and Dario Fiore.
Vector Commitments and Their Applications.
In Kaoru Kurosawa and Goichiro Hanaoka, editors, *Public-Key Cryptography – PKC 2013*, pages 55–72, Berlin, Heidelberg, 2013. Springer Berlin Heidelberg.

Alexander Chepurnoy, Charalampos Papamanthou, and Yupeng Zhang.
Edrax: A Cryptocurrency with Stateless Transaction Validation.
Cryptology ePrint Archive, Report 2018/968, 2018.
`https://eprint.iacr.org/2018/968`.

Dankrad Feist and Dmitry Khovratovich.
Fast amortized Kate proofs, 2020.
https://github.com/khovratovich/Kate.

Sergey Gorbunov, Leonid Reyzin, Hoeteck Wee, and Zhenfei Zhang.
Pointproofs: Aggregating Proofs for Multiple Vector Commitments.
Cryptology ePrint Archive, Report 2020/419, 2020.
https://eprint.iacr.org/2020/419.

Aniket Kate, Gregory M. Zaverucha, and Ian Goldberg.
Constant-Size Commitments to Polynomials and Their Applications.
In Masayuki Abe, editor, *ASIACRYPT '10*, pages 177–194, Berlin, Heidelberg, 2010. Springer Berlin Heidelberg.

References iv

Russell W. F. Lai and Giulio Malavolta.
Subvector Commitments with Application to Succinct Arguments.
In Alexandra Boldyreva and Daniele Micciancio, editors, *Advances in Cryptology – CRYPTO 2019*, pages 530–560, Cham, 2019. Springer International Publishing.

Ralph C. Merkle.
A Digital Signature Based on a Conventional Encryption Function.
In Carl Pomerance, editor, *CRYPTO '87*, pages 369–378, Berlin, Heidelberg, 1988. Springer Berlin Heidelberg.

Andrew Miller.
Storing UTXOs in a balanced Merkle tree (zero-trust nodes with O(1)-storage).
BitcoinTalk Forums, 2012.
`https://bitcointalk.org/index.php?topic=101734.msg1117428`.

References v

Leonid Reyzin, Dmitry Meshkov, Alexander Chepurnoy, and Sasha Ivanov.
Improving Authenticated Dynamic Dictionaries, with Applications to Cryptocurrencies.
In Aggelos Kiayias, editor, *Financial Cryptography and Data Security*, pages 376–392, Cham, 2017. Springer International Publishing.

Alin Tomescu, Ittai Abraham, Vitalik Buterin, Justin Drake, Dankrad Feist, and Dmitry Khovratovich.
Aggregatable Subvector Commitments for Stateless Cryptocurrencies.
Cryptology ePrint Archive, Report 2020/527, 2020.
`https://eprint.iacr.org/2020/527`.

Alin Tomescu, Robert Chen, Yiming Zheng, Ittai Abraham, Benny Pinkas, Guy Golan Gueta, and Srinivas Devadas.
Towards Scalable Threshold Cryptosystems.
In *2020 IEEE Symposium on Security and Privacy (SP)*, May 2020.

Peter Todd.
Making utxo set growth irrelevant with low-latency delayed txo commitments, 2016.
https://petertodd.org/2016/delayed-txo-commitments.

Alin Tomescu.
How to Keep a Secret and Share a Public Key (Using Polynomial Commitments).
PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 2020.

Madars Virza.
On Deploying Succinct Zero-Knowledge Proofs.
PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 2017.

Joachim von zur Gathen and Jurgen Gerhard.
Fast polynomial evaluation and interpolation.
In *Modern Computer Algebra*, chapter 10, pages 295–310. Cambridge University Press, New York, NY, USA, 3rd edition, 2013.