Aggregatable Subvector Commitments for Stateless Cryptocurrencies

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While you're at it, feel free to read our blogpost too.

Motivation

Stateless Cryptocurrencies

Theorem [CPZ18]

Vector commitment (VC) $\Rightarrow$ "stateless," payment-only, cryptocurrency.

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2. Lower barrier to entry for both miners and P2P nodes
3. Faster sharding

VC API for Stateless Cryptocurrencies

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Contributions

Our Contribution: Aggregatable Subvector Commitments with Scalable Updates

Table 1: Asymptotic comparison to previous (aS)VCs: n is the size of $\vec{v}$ and b is the # of proofs to aggregate.

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All schemes can (1) verify a proof π_i in $O(\lvert \pi_i \rvert)$ time and (2) update digests in $O(1)$ time, except Merkle trees.

Techniques: VCs from Univariate Polynomial Commitments

Outline

KZG Constant-sized Polynomial Commitments [KZG10]

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Time to commit is an $O(n)$-sized multi-exponentiation.

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VCs from Lagrange Basis Polynomials [CDHK15]

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$$\phi(X) = \sum_{i=0}^{n-1} v_i \cdot L_i(X), \text{ where } L_i(X) = \prod_{\substack{j\in[0,n)\\ j\neq i}} \frac{X-j}{i-j} \tag{4}$$

Setup Lagrange basis polynomial commitments $\ell_i = g^{L_i(\tau)}, \forall i \in [0,n)$ (speed?). Then, commit to $\vec{v}$ as [CDHK15]:

$$\begin{aligned} c &= \prod_{i\in[0,n)} \ell_i^{v_i} && (5)\\ &= g^{\sum_{i\in[0,n)} L_i(\tau) v_i} && (6)\\ &= g^{\phi(\tau)} && (7) \end{aligned}$$

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Outline

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Conclusion

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- Aggregating multiple I-subvector proofs *across different commitments* [GRWZ20].

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- Can remove $A'(i)$ from upk_i.

Questions?

Outline

Decomposition of $1/((X-i)(X-j))$

Decomposition of $1/A_I(X)$

Decomposition of $A(X) / ((X - i)(X - j))$

Note that:

$$\frac{1}{i-j} \cdot \frac{A(X)}{X-i} + \frac{1}{j-i} \cdot \frac{A(X)}{X-j} = \frac{1}{i-j} \cdot \frac{A(X)(X-j)}{(X-i)(X-j)} + \frac{1}{j-i} \cdot \frac{A(X)(X-i)}{(X-j)(X-i)} \tag{40}$$

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Partial Fraction Decomposition From Lagrange Interpolation

It is well-known that Lagrange coefficients can be *rewritten* as [BT04, vzGG13]:

$$L_i(X) = \prod_{j\in I, j\neq i} \frac{X-j}{i-j} = \frac{A_I(X)}{A_I'(i)(X-i)}, \text{ where } A_I(X) = \prod_{i\in I}(X-i) \tag{45}$$

Here, $A_I'(X)$ is the derivative of $A_I(X)$ and has the (non-obvious) property that $A_I'(i) = \prod_{j\in I, j\neq i}(i-j)$.
Next, consider the Lagrange interpolation of $\phi(X) = 1$:

$$\phi(X) = \sum_{i\in I} v_i L_i(X) \Leftrightarrow \tag{46}$$

$$1 = A_I(X) \sum_{i\in[0,n)} \frac{v_i}{A_I'(i)(X-i)} \Leftrightarrow \tag{47}$$

$$\frac{1}{A_I(X)} = \sum_{i\in I} \frac{1}{A_I'(i)(X-i)} \Leftrightarrow \tag{48}$$

$$\frac{1}{A_I(X)} = \sum_{i\in I} \frac{1}{A_I'(i)} \cdot \frac{1}{(X-i)} \Rightarrow \tag{49}$$

$$c_i = \frac{1}{A_I'(i)} \tag{50}$$

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5. Proof serving nodes would like to compute all π_i's fast

Aggregating I-subvector Proof π_I From $(\pi_i)_{i \in I}$ (Continued)

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Step 1: Interpolate $A_I(X) = \prod_{i \in I}(X - i)$ in $O(|I| \log^2 |I|)$ field operations.

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$$\pi_I = \prod_{i\in I} \pi_i^{1/A_I'(i)} \tag{51}$$

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