TTK4150 Nonlinear Control Systems
Lecture 11

Passivity-based control design

NTNU
Innovation and Creativity

Previous lecture

Previous lecture:

Passivity

- How to analyze the passivity properties of a system by using the definitions of passivity for
  - Memoryless functions
  - Dynamical systems
- Understand the relations between passivity and
  - Lyapunov stability
  - $\mathcal{L}_2$ stability (IOS)
- The passivity theorems
  (for feedback connections)

Previous lecture

Previous lecture:

- Analysis of an autonomous system under the influence of a weak periodic perturbation
- Learn how to apply Averaging method

Outline I

1. Introduction
   - Previous lecture
   - Today's goals
   - Literature
2. Energy-based Lyapunov control design
   - Lyapunov control design
   - Example: Dynamic positioning system for ships
   - Example: Two-link robot manipulator
3. Control design using passivity theorems
   - 2 useful results: Passivity of LTI systems
   - PID feedback control
   - Example: Motor control
   - A new passivity theorem for stabilization: Theorem 14.4
4. How to achieve passivity
   - Choice of y

Outline II

- Choice of u (Feedback passivation)

5. Summary

6. Next lecture

Today's goals

After today you should...

Be able to design a passivity-based feedback control law

Literature

Today's lecture is based on

Khalil	**Chapter 6** Sections 6.4 - 6.5 (Pages 254-259, incl. Ex. 6.12, is additional material) **Chapter 14** Section 14.4
Lozano et al.	**Dissipative Systems Analysis and Control** Section 2.3 - 2.4

Lyapunov Control Design

Lyapunov Control Design

- Alternative A)

  Propose a control law $u = g(t,x)$

  Analyze the resulting system by Lyapunov's Direct Method (incl. La Salle/Barbalat)

- Alternative B)

  Propose a Lyapunov function candidate

  Find a control law $u = g(t,x)$ that makes this LFC a (strict) Lyapunov function

Energy-based Lyapunov Control Design

Energy-based Lyapunov Control Design

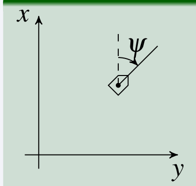
- Alternative B)

  Propose a Lyapunov function candidate
  = *Desired energy of the closed-loop system*

  Find a control law $u = g(t,x)$ that makes this LFC a (strict) Lyapunov function

Example: Dynamic positioning system for ships

Dynamic positioning system for ships



$$\eta = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}$$

System model:

$$M(\eta)\ddot{\eta} + C(\eta,\dot{\eta})\dot{\eta} + D(\eta)\dot{\eta} = \tau$$

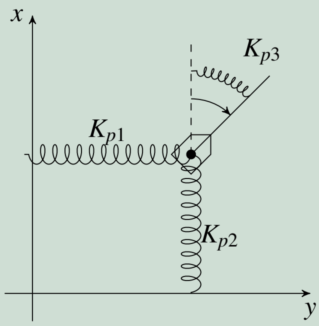
System properties:

$$M = M^T > 0$$
$$z^T D z > 0 \quad z \neq 0$$
$$z^T(\tfrac{1}{2}\dot{M} - C)z = 0 \quad \forall z \in \mathbb{R}^3$$

Find a control law $\tau = g(t,(\eta,\dot{\eta}))$ that makes the origin $(\eta,\dot{\eta}) = (0,0)$ an asymptotically stable equilibrium point.

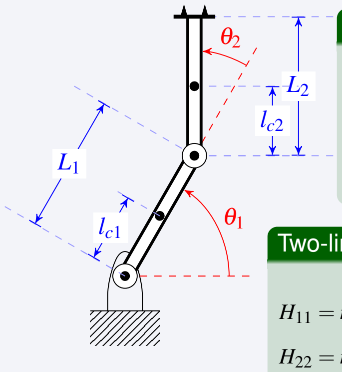
Example cont.

Desired energy of the closed-loop system



- We shape the (potential) energy
- We add virtual spring forces

Example: Two-link robot manipulator



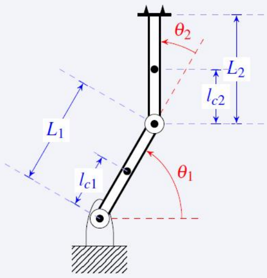
Control problem:

Find a feedback control law that stabilizes a constant desired configuration q_d. (Without loss of generality: Assume that $q_d = 0$, i.e. stabilize $(q,\dot{q}) = (0,0)$)

Two-link robot manipulator

$$H_{11} = m_1 l_{c_1}^2 + I_1 + m_2\left[l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2}\cos q_2\right] + I_2$$
$$H_{22} = m_2 l_{c_2}^2 + I_2$$
$$H_{12} = H_{21} = m_2 l_1 l_{c_2}\cos q_2 + m_2 l_{c_2}^2 + I_2$$
$$h = m_2 l_1 l_{c2}\sin q_2$$
$$g_1 = m_1 l_{c1} g\cos q_1 + m_2 g\left[l_{c_2}\cos(q_1+q_2) + l_1\cos q_1\right]$$
$$g_2 = m_2 l_{c_2} g\cos(q_1+q_2)$$

Example: Two-link robot manipulator



Dynamic model:

General robot manipulator:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

System properties:

$$M(q) = M^T(q) > 0 \quad \forall q \in \mathbb{R}^m$$
$$z^T(\tfrac{1}{2}\dot{M} - C)z = 0 \quad \forall z, q, \dot{q} \in \mathbb{R}^m$$

Dynamic model:

Two-link robot manipulator:

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_1 - h\dot{q}_2 \\ h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

Control design using passivity theorems

Two useful results to include linear systems/controllers:

Result 1 (Theorem 2.3, Lozano et al.)

LTI system $y(s) = h(s)u(s)$ $\quad h(s)$ rational transfer function
$\quad$ Re$(p_i) < 0, \forall i$

1) The system is passive $\Leftrightarrow$ Re$[h(j\omega)] \geq 0, \quad \forall\omega$

2) The system is input strictly passive ($\varphi(u) = \delta u$)
$\Updownarrow$
Re$[h(j\omega)] \geq \delta > 0, \quad \forall\ \omega$

3) The system is output strictly passive ($\rho(y) = \varepsilon y$)
$\Updownarrow$
$\exists\varepsilon > 0$ s.t. Re$[h(j\omega)] \geq \varepsilon|h(j\omega)|^2$

Control design using passivity theorems

Example: Time constant

Example: Time constant

Consider the system $y(s) = \frac{1}{1+Ts}u(s)$

Analyze the passivity properties of this LTI system

Control design using passivity theorems

Two useful results to include linear systems/controllers:

Result 2 (Proposition 2.1, Lozano et al.)

Let

$$h(s) = \frac{(s-z_1)(s-z_2)\cdots}{s(s-p_1)(s-p_2)\cdots} \qquad \begin{array}{l} \mathrm{Re}(z_i) < 0 \\ \mathrm{Re}(p_i) < 0 \end{array}$$

The system $y(s) = h(s)u(s)$ is passive

$$\Updownarrow$$

$$\mathrm{Re}[h(j\omega)] \geq 0 \quad \forall\ \omega$$

Control design using passivity theorems

Example: PID controllers

Example: PID controllers

PID controller
with bounded derivative action:

$$h_{r1}(s) = K_p \frac{1+T_i s}{T_i s} \cdot \frac{1+T_d s}{1+\alpha T_d s} \qquad \begin{array}{ll} 0 \leq T_d < T_i & K_p > 0 \\ 0 \leq \alpha \leq 1 & \end{array}$$

PID controller
with bounded integral action and bounded derivative action:

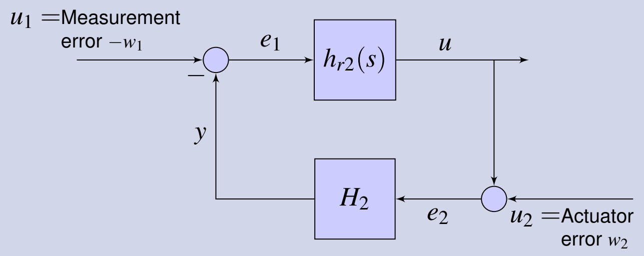
$$h_{r2}(s) = K_p \beta \frac{1+T_i s}{1+\beta T_i s} \cdot \frac{1+T_d s}{1+\alpha T_d s} \qquad \begin{array}{ll} 0 \leq T_d < T_i & K_p > 0 \\ 0 < \alpha \leq 1 & \\ 1 \leq \beta < \infty & \end{array}$$

Analyze the passivity properties of these two PID controllers

PID feedback control

Analysis/Design using passivity theorems

PID feedback control: Analysis/Design using passivity theorems



Example: Motor control
Analysis/Design using passivity theorems

Example: Motor control

Motor and load with elastic transmission:

$$J_m\ddot{\theta}_m = \phi_K(\Delta\theta) + \phi_D(\Delta\dot{\theta}) + T_m + F(\dot{\theta}_m)$$
$$J_L(\ddot{\theta}_m + \Delta\ddot{\theta}) = -(\phi_K(\Delta\theta) + \phi_D(\Delta\dot{\theta}))$$

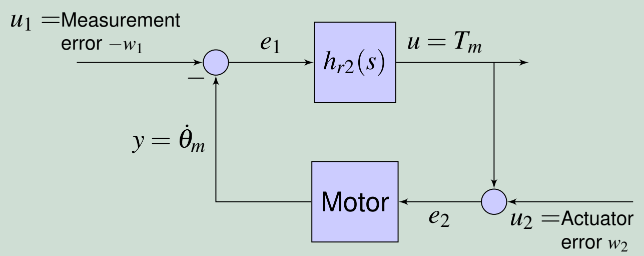
θ_m motor angle
$\Delta\theta$ angular deflection through spring
$\Delta\theta = \theta_L - \theta_m$

$\phi_K \in$ sector $[0,\infty)$
$\phi_D \in$ sector $[0,\infty)$

$$F(\dot{\theta}_m) = \begin{cases} F_0, & \dot{\theta}_m < 0 \\ -F_0, & \dot{\theta}_m > 0 \end{cases}$$

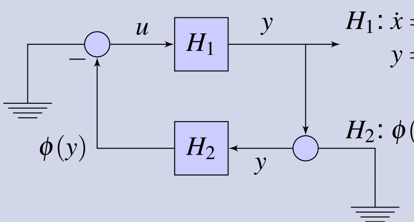
Example: Motor control
Analysis/Design using passivity theorems

Motor control: Analysis/Design using passivity theorems



A new passivity theorem for stabilization

Theorem 14.4



H_1: $\dot{x} = f(x,u)$ locally Lipschitz
$y = h(x)$ continuous
$f(0,0) = 0 \quad h(0) = 0$

H_2: $\phi(y)$ memoryless function
locally Lipschitz
$\phi(0) = 0$

If

i) H_1 is
- passive with V positive definite and radially unbounded
- zero-state observable

ii) H_2 satisfies $y^T\phi(y) > 0, \quad y \neq 0$

then the origin is *globally asymptotically stable*.

How to achieve passivity

Choice of y

Let
$$\dot{x} = f(x) + G(x)u \qquad \text{(affine system)}$$

If $\exists\, V(x)$

- C^1
- positive definite
- radially unbounded
- $\frac{\partial V}{\partial x} f(x) \le 0$

Choose $y = \left[\frac{\partial V}{\partial x} G(x)\right]^T$

Then $u \mapsto y$ is passive

How to achieve passivity

Choice of u (Feedback passivation)

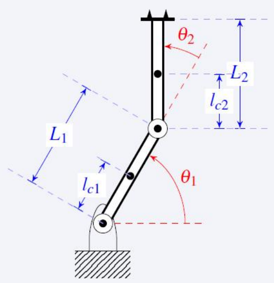
Let
$$\dot{x} = f(x) + G(x)u$$

Choose
$$\begin{aligned} u &= \alpha(x) + \beta(x)v \\ y &= h(x) \end{aligned}$$

such that
$$\begin{aligned} \dot{x} &= f(x) + G(x)\alpha(x) + G(x)\beta(x)v \\ y &= h(x) \end{aligned}$$

has desired passivity properties $v \mapsto y$

Example: Robot manipulator



Dynamic model:

General robot manipulator:
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

System properties:

$M(q) = M^T(q) > 0 \quad \forall q \in \mathbb{R}^m$
$z^T(\frac{1}{2}\dot{M} - C)z = 0 \quad \forall\, z, q, \dot{q} \in \mathbb{R}^m$

Control problem:

Find a feedback control law that stabilizes $(q,\dot{q}) = (0,0)$ using passivity based control design.

Summary

Passivity

- Relates nicely to Lyapunov stability and $\mathscr{L}_2$ stability
- Can provide a somewhat systematic way to build Lyapunov functions
  Choosing the LFC thinking in terms of energy of the controlled system. Typically potential energy shaping, possibly kinetic energy shaping (acceleration feedback). This is also denoted Energy-based control.
- Can give conclusions about properties of feedback connections based on the properties of each subsystem
  This allows for modular analysis and design, something which simplifies the design process. (Resembling the cascade results for ISS systems, page 179-180)

Summary

Passivity

- Robustness: If the model possesses the same passivity properties regardless of the numerical values of the physical parameters, and a controller is designed so that stability relies on the passivity properties only, the closed-loop system will be stable regardless of the values of the physical parameters
- A tool for choosing where to place sensors: Passivity considerations are helpful as a guide for the choice of a suitable variable y for output feedback. This is helpful for selecting where to place sensors for feedback control.
- A tool for choosing where to place actuators: A guide for choice of location of actuators

Next lecture

Next lecture:

- **Feedback linearization**
- Recommended reading
  Khalil **Chapter 13**
  Sections 13.1,13.2 and 13.4
  Example 13.16 - page 538 is additional material