Errata List For
Nonlinear Systems – Third Edition
Updated on August 12, 2014
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Preface

1. Page xiv, Line 5: Change "books" to "book"

Chapter 1

1. Page 10, Line 3: Change "Coulombs plus" to "Coulomb plus"
2. Page 24, Second line of Exercise 1.4: Change "++" to "+"
3. Page 27, Exercise 1.13(c): Change "Section 1.3" to "Section 1.2.3"
4. Page 27, Exercise 1.14: Change "v the" to"v is the"

Chapter 2

1. Page 52, Last line: change "3" to "4"
2. Page 61, Last line: insert "and $V(x) < c$ inside the curve" after "differentiable"
3. Page 67, Second line of footnote: Change "is a also" to "is also"
4. Page 81, Exercise 2.14: Change " $\ddot{y}$ " to " $m\ddot{y}$ "
5. Page 82, Exercise 2.18: Change "$zg(z) > 0$" to "$zg(z) > 0$ for $z \neq 0$"

Chapter 3

1. Page 105, Exercise 3.2(5): Change "Section 1.2.5" to "Section 1.2.6"
2. Page 109, Exercise 3.27: Change "$f((t, x_i(t))$" to "$f(t, x_i(t))$"

Chapter 4

1. Page 113, Line 17: Change "for any ε" to "for any sufficiently small ε"
2. Page 123: Remove the six-line segment from "For $\Omega_c \ldots$" to "$\ldots$ only for $c < 1$.", which start at Line 9 from the bottom. The discussion is not valid.
3. Page 125, Figure 4.5: At the bottom right corner change "$x_2 = x_1$" to "$x_2 = -x_1$"
4. Page 128, Line 21: At the end of the sentence "When $V(x)$ is positive definite, Ω_c is bounded for sufficiently small c." insert the footnote "Ω_c may have more than one component. This sentence refers to the bounded component that contains the origin."
5. Page 145, Lemma 4.2: See Lemma 4.1 of H.K. Khalil, Nonlinear Control, 2015, for better definitions of domains.
6. Page 145, Lemma 4.3: Replace the last two sentences "If $D = R^n$ ...... class $\mathcal{K}_\infty$." by "If $D = R^n$ and $V(x)$ is radially unbounded, then there exist class $\mathcal{K}_\infty$ functions α_1 and α_2 such that the foregoing inequality holds for all $x \in R^n$."
7. Page 156, Equation (4.29): Change $A(t)x$ to $A(t)x(t)$
8. Page 163, Line 4: Change "a function V" to "a continuously differentiable function V"
9. Page 167, Line 2: Change "Theorem 4.15" to "Theorem 4.14"
10. Page 170, Line after (4.37): Insert "for some $\mu > 0$."

11. Page 175, Second line of Definition 4.7: Change "for any initial state" to "for any initial time t_0, any initial state"
12. Page 176, Line 17: Change "consequences" to "consequence"
13. Page 176, Line 3 of the proof of Lemma 4.6: Change "satisfies (4.10) through (4.12)" to "satisfies the inequalities of the theorem"
14. Page 182, Exercise 4.6: change "for all $z \in R$" to "for all $z \neq 0$"
15. Page 184, Exercise 4.19: change "roots" to "root"
16. Page 185, Exercise 4.21, part (b): Take $D = R^n$
17. Page 186, Line 4: Change "nonsingular" to "positive definite"
18. Page 186, Line 4 from the bottom: Change "point asymptotically" to "point is asymptotically"
19. Page 188, Exercise 4.35, change"$\forall\ r_1, r_2 \in [0, a)$" to "$\forall\ r_1, r_2 \in [0, a/2)$"
20. Page 190, Exercise 4.49, change "exponentially" to "asymptotically" and "Y_1^2" to "y_1^2"

Chapter 5

1. Page 198, Line 6: Change "u_τ" to "u"
2. Page 201, Definition 5.2: Change "with $\sup_{0 \leq t \leq \tau} \|u(t)\| \leq r$" to "and $\tau \in [0, \infty)$ with $\sup_{t \geq 0} \|u(t)\| \leq r$"
3. Page 206, Line 16: Change "$x_0 \in R$" to "$x_0 \in R^2$"
4. Page 209, four lines before Section 5.3: Change "$W_3(x) = -(1-\theta)\|x\|_2^4$" to "$W_3(x) = (1-\theta)\|x\|_2^4$"
5. Page 211, Line 4: Change "$f(x)$ is locally Lipschitz, and $G(x)$, $h(x)$ are" to "$f(x)$, $G(x)$ are locally Lipschitz and $h(x)$ is"
6. Page 218, Line 2 from the bottom: Change "competed" to "completed"
7. Page 219, Example 5.13: Change "$e_2 = \psi(t, y_2)$" to "$y_2 = \psi(t, e_2)$"
8. Page 221, Line 10 from the bottom: Change "is $\mathcal{L}$ stable." to "is $\mathcal{L}$ stable if $\varepsilon\gamma_1\gamma_f < 1$.
9. Page 224, Exercise 5.16, part (a): remove "finite-gain"

Chapter 6

1. Page 228, Line 10 of Section 6.1: Change "$uy = Gy^2$" to "$uy = Gu^2$"
2. Page 232, Line 10 from the bottom: Change "Ly" to "Lu"
3. Page 238, Line 8: Change "of of" to "of"
4. Page 238, Lemma 6.1: Change the third bullet to: either $G(\infty) + G^T(\infty)$ is positive definite or it is positive semidefinite and $\lim_{\omega\to\infty} \omega^{2(p-q)} \det[G(j\omega) + G^T(-j\omega)] > 0$, where $q = \text{rank}[G(\infty) + G^T(\infty)]$. The proof of the corrected lemma is given in the paper: Corless, M. and Shorten, R., "On the Characterization of Strict Positive Realness for General Matrix Transfer Functions", *IEEE Transactions on Automatic Control*, Vol. 55, No. 8, pp. 1899–1904, 2010.
5. Page 239, Line 7 from the bottom: Change the (1,1) element of the matrix[1] from "s" to "$s+1$"
6. Page 239, Line 3 from the bottom: Change the (1,1) element of the matrix from "ω^2" to "$\omega^2 + 1$"

[1]This $G(s)$ and the one given at the end of the page need to be corrected because $G(0) + G^T(0)$ is singular.

7. Page 239, Last line: Change the (1,1) element of the matrix from "$s/(s+1)$" to "$(s+2)/(s+1)$"
8. Page 240, Line 4: Change the (1,1) element of the matrix from "$2\omega^2/(1+\omega^2)$" to "$2(2+\omega^2)/(1+\omega^2)$"
9. Page 241: First line of proof of Lemma 6.4: Change "$V(s)$" to "$V(x)$"
10. Page 244, Example 6.5: Change "f is locally Lipschitz, G and h are continuous" to "f and G are locally Lipschitz, h is continuous"
11. Page 244, Line 6 from the bottom: Change "$-ky^Ty$" to "ky^Ty"
12. Page 245, Section 6.5: In describing the feedback connection of Figure 6.11 it should be noted that u_1, y_1, u_2, and y_2 could be vectors of the same dimension.
13. Page 257, Line 4 to 6 from the bottom: Change "the condition

$$Re\left[\frac{1+j\omega a}{1-\omega^2+j\omega}\right] = \frac{1+(a-1)\omega^2}{(1-\omega^2)^2+\omega^2} > 0, \quad \forall\, \omega \in R$$

if $a \geq 1$. Thus, choosing $a \geq 1$, we can apply Lemmas 6.3 and 6.4 to conclude that" to "the conditions

$$Re\left[\frac{1+j\omega a}{1-\omega^2+j\omega}\right] = \frac{1+(a-1)\omega^2}{(1-\omega^2)^2+\omega^2} > 0, \quad \forall\, \omega \in R$$

and

$$\lim_{\omega\to\infty} \omega^2 \ Re\left[\frac{1+j\omega a}{1-\omega^2+j\omega}\right] = a-1 > 0$$

if $a > 1$. Thus, choosing $a > 1$, we can apply Lemmas 6.1 and 6.4 to conclude that"
14. Page 260, Exercise 6.9, Line 4: Change V to $\dot{V}$

Chapter 7

1. Page 285, Line 5 from the bottom should read as "Equation (7.29) can be written as" while Line 3 from the bottom should start with "Since the describing function $\Psi(a)$ is real, this equation ..."
2. Page 299, Line 2: Change "with $G(s)$" to "with output e, zero input, and $G(s)$"

Chapter 8

1. Page 311, Last line: Change "$-\|y\|_2^4$" to "$-\frac{1}{2}\|y\|_2^4$"
2. Page 312, Line 2: Change to "$\dot{V} \leq -\ \frac{1}{4}\|y\|_2^4, \ \ \text{for } \|y\|_2 < \frac{1}{4k}$"
3. Page 313, Line 2: Change "x_{pr} in Suppose" to "x_{pr} in Figure 8.1. Suppose"
4. Page 313, Line 8 from the bottom: Change "find t_c by Figure 8.1. integrating" to "find t_c by integrating"

Chapter 9

1. Page 342, Line 11 from the bottom: Change "$\dot{V}(x)$" to "$\dot{V}(t,x)$"

Chapter 11

1. Page 434, Equation (11.21) and Page 439, Equation (11.24): Change "$\hat{y}(t/\varepsilon)$" to "$\hat{y}((t-t_0)/\varepsilon)$"

Chapter 12

1. Page 474, Line 12 from the bottom: Change "contain" to "contains"
2. Page 482, Line 5: Change "$\mathcal{K} = \left[\begin{array}{cc} K_1 & K_2 \end{array}\right]$" to "$\mathcal{K} = \left[\begin{array}{cc} K_1 + K_3C & K_2 \end{array}\right]$"

3. Page 482, Line 15: Change "We note $\ldots K_3 = 0$" to "We note that in the stabilization of (x_{ss}, u_{ss}) we can take $K_3 = 0$"
4. Page 483, Line before Equation (12.24): Change "ρ_3" to "ρ_2"
5. Page 484, First line after the figure: Change "feeback" to "feedback"
6. Page 490, Line 10: Change "$v = \alpha$" to "$\rho = \alpha$"
7. Page 492, Line 22: Change "from ψ to u" to "from ψ to $u - M_3(\alpha)e$"
8. Page 495, second line after (12.57): Change "initited" to "initiated"

Chapter 13

1. Page 517, Line 5 from the bottom: Change "reduces" to "reduces to"
2. Page 527, Line 8 from the bottom: Change "span$(g, ad_f g, ad_f^2)$" to "span$(g, ad_f g, ad_f^2 g)$"
3. Page 533, five lines before Example 13.17: Change "$\dot{\xi} = f_0(\eta, \xi)$" to "$\dot{\eta} = f_0(\eta, \xi)$"
4. Page 535, Line 13: Change "Chapter 10" to "Chapter 9"

Chapter 14

1. Page 553, Line 5: It should be noted that $u = -\beta(x)\text{sgn}(s)$ is used only for $s \neq 0$ since in ideal sliding mode control u is not defined on the sliding surface $s = 0$. Alternatively, we can write $u = -\beta(x)\text{sgn}(s)$ for all s if $\text{sgn}(s)$ is not defined at $s = 0$. The same remark applies throughout the chapter to ideal sliding mode control.
2. Page 553, Line 10: Change "$W = \sqrt{V}$" to "$W = \sqrt{2V}$"
3. Page 553, Line 15: Change "$\dot{V} \leq -2g_0\beta_0|s|$" to "$\dot{V} \leq -g_0\beta_0|s|$"
4. Page 562: Line 2 from the bottom: Change "$A_0 P_0^T$" to "$A_0^T P_0$"
5. Page 568, Line 3 of Theorem 14.2: Change "stale" to "stable"
6. Page 573, last line and Page 574, Lines 4, 7, and 9 from the bottom: Change "r^ρ" to "$r^{(\rho)}$"
7. Page 575, Line 7: Change "exits" to "exists"
8. Page 577, Line 10: Change "$L_f^\rho(x)$" to "$L_f^\rho h(x)$"
9. Page 578: To show the inequality satisfied by $\dot{V}_0$, we need the additional condition:

$$\left| \frac{\Delta(x_{\text{ss}}, v_1, w, r) - \Delta(x_{\text{ss}}, v_2, w, r)}{L_g L_f^{\rho-1} h(x_{\text{ss}}, w)} \right| \leq \ell |v_1 - v_2|, \quad 0 \leq \ell < 1$$

for all $(v_1, v_2, w, r) \in R \times R \times D_w \times D_r$.

10. Page 587, Line 9: Change "$k_0 =$" to "$k_0 \geq$" and "$k =$" to "$k \geq$"
11. Page 596, Lines 3 and 4: Change "ϕ" to "ϕ_0"
12. Page 610, Last equation: Change "$k\zeta$" to "$k\xi$"
13. Page 611, Line 14: Change "designing C" to "designing H"
14. Page 622, Theorem 14.6: In the fourth bullet of the theorem add the requirement that $f(\mathcal{X})$ is continuously differentiable in some neighborhood of $\mathcal{X} = 0$.
15. Page 625, Line 18: Change "e_ρ" to "$\hat{e}_\rho$"

16. Page 644, Exercise 14.56(d): Change the units of I from "Kg/m^2" to "Kg m^2"

Appendix

1. Page 662, Line 14: Change "If $D = R^n$," to "If $D = R^n$ and $V(x)$ is radially unbounded,"
2. Page 662, Line 18: Change "If $V(x)$" to "Because $V(x)$"
3. Page 665: Change the second and third lines to
$$\|x(t)\| \le \min\{\alpha(\|x(t_0)\|), U_r(t-t_0)\}, \quad \forall\, t \ge t_0, \quad \forall\, \|x(t_0)\| < r$$
Thus, inequality (4.20) is satisfied with $\beta(r,s) = \min\{\alpha(r), U_r(s)\}$.[2]
4. Page 693: Replace lines 18 to 24 (starting with "This expression ..." and ending with "... parameterized in η.") by the following:[3]
This expression is valid for any $t \in R$. The limit of the integral term as $t \to -\infty$ is
$$\int_{-\infty}^{0} \exp(-Bs) G(\pi(s; y(\tau), \eta), \eta(\pi(s; y(\tau), \eta)))\; ds \qquad (C.60)$$
which is well defined because η is bounded, G is globally bounded in π, and B is Hurwitz. Let us rewrite the expression (C.60) with $y(\tau)$ replaced by y and denote it by $(P\eta)(y)$.
$$(P\eta)(y) = \int_{-\infty}^{0} \exp(-Bs) G(\pi(s; y, \eta), \eta(\pi(s; y, \eta)))\; ds \qquad (C.61)$$
With this definition, we can write
$$\begin{aligned} &\exp[B(t-\tau)]\,[z(\tau) - (P\eta)(y(\tau))] = \\ &\quad z(t) - \int_{-\infty}^{t-\tau} \exp[-B(s-t+\tau)] G(\pi(s; y(\tau), \eta), \eta(\pi(s; y(\tau), \eta)))\; ds \end{aligned}$$
Substituting $\xi = s - t + \tau$ in the integral and using $\pi(\xi + t - \tau; y(\tau), \eta) = \pi(\xi; y(t), \eta)$, we obtain
$$\exp[B(t-\tau)]\,[z(\tau) - (P\eta)(y(\tau))] = z(t) - (P\eta)(y(t))$$
which shows that if $z(\tau) = (P\eta)(y(\tau))$, then $z(t) = (P\eta)(y(t))$ for all $t \in R$. Hence, $z = (P\eta)(y)$ defines an invariant manifold for (C.58)–(C.59) parameterized in η.
5. Page 695, Line 2: Change "$, \eta((\pi(t; y, \eta)))$" to "$, \eta(\pi(t; y, \eta)))$"
6. Page 698, Equation (C.66): Change "$A_1\, +$" to "$A_1 y\, +$"
7. Page 716, Lines 17 and 18: Change "$U(x)$" to "$U(\mathcal{X})$"

Bibliography

1. Page 725, Ref.[9]: Change "Anstaklis" to "Antsaklis"
2. Page 731, Ref.[93]: Change "1995" to "1994"
3. Page 732, Ref.[109]: Change "Trans. Amer. Math. Soc., 24:19–17" to "Amer. Math. Soc. Transl., Ser. 2, 24:19–77"
4. Page 737, Ref.[185]: Change "tracking" to "torque"

Index

1. Page 744, Feedback Passivation: Change "607" to "606"

In the third print of the book, a new typo appeared on page 553, line 13: Change "*dotV*" to "$\dot{V}$"

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[2] This change was suggested by Professor E. Sontag because $U_r(s)$ is infinite at $s = 0$.

[3] The need for this change in the proof was pointed out by Professor H. Shim. The original proof implicitly used $\lim_{t\to-\infty} \exp[-B(t-\tau)]z(t) = 0$, which cannot be guaranteed at this point in the proof because we cannot guarantee that $z(t)$ is bounded as $t \to -\infty$.