NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ENGINEERING CYBERNETICS

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Exam
TTK4150 Nonlinear Control Systems

Thursday December 11, 2014

Hours: 09.00 – 13.00

Aids:	D - No printed or written materials allowed. NTNU type approved calculator with an empty memory allowed.
Language:	English
No. of pages:	3
Grades available:	January 12, 2014

This exam counts for 100% of the final grade.

Problem 1 (14%)

Consider the system

$$\dot{x}_1 = (1 - x_1)x_1 - \frac{2x_1x_2}{1 + x_1}$$
$$\dot{x}_2 = \frac{(1 - x_2)x_2}{1 + x_1}$$

Find all equilibrium points and determine the type of each isolated equilibrium point.

Problem 2 (10%)

Consider the system

$$\dot{x}_1 = 10x_1x_2$$
$$\dot{x}_2 = 3x_1^7 + 9u$$

Use Lyapunov based methods to find a feedback control law $u = g(x)$ such that the origin becomes globally asymptotically stable. (*Hint: you may try* $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$)

Problem 3 (12%)

Consider the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 - (b + \cos t)x_1$$

with $V(t, x) = x_1^2 + \frac{1}{b+\cos t}x_2^2$.

- **a [5%]** Find the values of b for which the function $V(t, x)$ is positive definite and decrescent.
- **b [7%]** Find the values of b for which the origin is a uniformly stable equilibrium point.

Hint: $\sin t - 2\cos t \leq 2.24$

Problem 4 (25%)

Consider the system

$$\begin{aligned} \dot{x}_1 &= -3x_1 + 2x_2 \\ \dot{x}_2 &= -2\psi(x_1) - x_2 + \delta \\ y &= x_2 \end{aligned}$$

with $x = [x_1, x_2]^T \in \mathbb{R}^2$, where $k_1 z^2 \leq z\psi(z) \leq k_2 z^2$ for all $z \in \mathbb{R}$ and for some $k_1, k_2 > 0$. The time varying $\delta(t)$ is the disturbance to the system.

- **a [5%]** For zero disturbance (i.e. $\delta(t) = 0$ for all t) show that the origin is globally asymptotically stable using $V(x) = \int_0^{x_1} \psi(z)\,dz + \frac{1}{2}x_2^2$. Hint: $\frac{d}{dv}\int_0^v \psi(z)\,dz = \psi(v)$.

b [5%] Show that the system from δ to y is strictly passive (state strictly passive).

c [3%] Show that the system from δ to y is also output strictly passive.

d [7%] Show that the system is input to state stable when δ is viewed as the input.

e [5%] Show that the system is zero state observable when δ is viewed as the input.

Problem 5 (27%)

Consider the following system

$$\begin{aligned} \dot{x}_1 &= x_1^2 + x_2 \\ \dot{x}_2 &= x_3^2 + u \\ \dot{x}_3 &= x_2 - kx_3 \\ y &= x_1 \end{aligned}$$

where $k > 0$ is a constant.

a [4%] Find the relative degree of this system. Is the system input-output linearizable?

b [11%] Transform the system into the normal form

$$\begin{aligned} &\dot{\eta} = f_0\left(\eta, \xi\right) \\ &\dot{\xi} = A_c \xi + B_c \gamma\left(x\right)\left[u - \alpha\left(x\right)\right] \end{aligned}$$

Specify the diffeomorphism $z = T\left(x\right) = \begin{bmatrix} \eta & \xi \end{bmatrix}^T$, the functions $\gamma\left(x\right)$, $\alpha\left(x\right)$ and $f_0\left(\eta, \xi\right)$ and the matrices A_c and B_c. In which domain is the transformation valid?

c [4%] Find an input-output linearizing controller on the form $u = \alpha\left(x\right) + \beta\left(x\right) v$.

d [4%] Find a controller v such that the external dynamics ξ is asymptotically stable at the origin.

e [4%] Is the system minimum phase?

Hint: If you were not able to solve **b** you may use the following equations for the internal dynamics: (It is not the correct internal dynamics equation, but it has the same property with respect to minimum phase)

$$\dot{\eta} = -k\eta + \xi_1^2 + \xi_2^2$$

Problem 6 (12%)

Consider the following system:

$$\begin{aligned} \dot{x}_1 &= 5x_1x_2 + x_1^2 \\ \dot{x}_2 &= -4x_2^2 + u \end{aligned}$$

Use the backstepping method to design a controller to globally stabilize the origin of the system.