TTK4150 Nonlinear Control Systems
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Fall 2016 - Assignment 6
Due date: Wednesday 30 November at 16.00.

1. Consider the third-order model of a synchronous generator connected to an infinite bus

$$\begin{aligned} M\ddot{\delta} &= P - D\dot{\delta} - \eta_1 E_q \sin(\delta) \\ \tau \dot{E}_q &= -\eta_2 E_q + \eta_3 \cos(\delta) + E_{FD} \end{aligned}$$

where δ is an angle in radians, E_q is voltage, P is mechanical input power, E_{FD} is field voltage (input), D is damping coefficient, M is inertial coefficient τ is time constant and η_1, η_2 and η_3 are constant parameters. Consider two possible choices of the output:

(1) $y = \delta$

(2) $y = \delta + \gamma\dot{\delta} \quad \gamma \neq 0$

In each case, study the relative degree of the system and transform it into the normal form. Specify the region over which transformation is valid. If there are nontrivial zero dynamics, find whether or not the system is minimum phase.

2. Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 - x_3 \\ \dot{x}_2 &= -x_1 x_3 - x_2 + u \\ \dot{x}_3 &= -x_1 + u \\ y &= x_3 \end{aligned}$$

(a) Is the system input-output linearizable?

(b) If yes, transform it into the normal form and specify the region over which the transformation is valid.

(c) Is the system minimum phase?

3. Given is the system

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} -x_1 + e^{x_2} u \\ x_1 x_2 + u \\ x_2 \end{bmatrix} \\ y &= x_3 \end{aligned}$$

(a) Find the relative degree of the system and specify the region on which this relative degree holds.

(b) Show that the system is input-output linearizable. Specify the region on which it is input-output linearizable.

(c) Find a coordinate transformation $z = T(x)$ that transforms the system into the normal form. (Note: $T(x)$ must be a *diffeomorphism*[1] over the region of interest and $T(0) = 0$)

(d) Express the system in normal form. Determine all functions and constants involved in the normal form. Which part of the normal form counts for the internal dynamics?

(e) Find the zero dynamics and show that it has a globally asymptotically stable equilibrium at the origin.

(f) Choose an input u to solve the stabilization problem for the entire system (asymptotically stable equilibrium in the origin).

(g) Choose an input u to solve the tracking problem for the entire system (asymptotically stable equilibrium at the origin).

4. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 + 2x_1^2\\ \dot{x}_2 &= x_3 + u\\ \dot{x}_3 &= x_1 - x_3\\ y &= x_1\end{aligned}$$

Design a state feedback control law such that the output y asymptotically tracks the reference signal $r(t) = \sin(t)$.

5. Using backstepping, design a state feedback controller to globally stabilize the system

$$\begin{aligned}\dot{x}_1 &= x_2 + a + (x_1 - a^{1/3})^3\\ \dot{x}_2 &= x_1 + u\end{aligned}$$

where a is a known constant.

6. Use backstepping method to stabilize

$$\begin{aligned}\dot{x}_1 &= x_1x_2 + x_1^2\\ \dot{x}_2 &= u\end{aligned}$$

[1] A function $y = f(x)$ is a *diffeomorphism* over a domain $\mathcal{D}$ if $f(x)$ and $\frac{\partial f}{\partial x}(x)$ are continuous over D and there exist the inverse of f, $f^{-1}(y)$ such that $f^{-1}(y)$ and $\frac{\partial f^{-1}}{\partial y}(y)$ are continuous over $\bar{\mathcal{D}} = \{y = f(x) | x \in \mathcal{D}\}$.
(Example: $f(x) = x^3$ is a diffeomorphism over $\mathbb{R}_+ = \{x : x > 0\}$ since $f(x)$ and $\frac{\partial f}{\partial x}(x) = 3x^2$ are continuous over $\mathbb{R}_+$, $f^{-1}(y) = \sqrt[3]{y}$ and $\frac{\partial f^{-1}}{\partial y}(y) = \frac{1}{3}y^{-\frac{2}{3}}$) are continuous over $\mathbb{R}_+$).