Homework 2: Q2

Name: Ali Morshid, alimorsh

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1 Proof Idea

For this question, I will use direct proof by example to reach the conclusion that there are instances where the GS algorithm runs in Omega(n) time. Let n = 2, we have 2 women and 2 men, there is a pattern in their respective preference lists such that it takes the algorithm more than n loops on its instructions to reach a stable matching. This pattern can be shown below:

w1	m1>m2
w2	m1>m2
m1	w2>w1
m2	w2>w1

If we follow the GS algorithm we can see the following matching steps being taken:
1. (w1,m1)
2. (w2,m1)
3. (w1,m2)

Since n = 2, and as it is clear that the number of loops in GS was 3, we can safely reach the conclusion that it takes more than n loops to reach a stable matching, thus the runtime is at least omega(n^2).

2 Proof Details

For every n>=1, we will try a few instances that demonstrate our proof idea.

Let n = 1

w1	w1
m1	m1

We get:
1. (w1, m1)

There is no other possible way to arrange this pair such that they don't match in n time, but since n=1, we can safely say that it also runs in omega(n^2) because 1 is also 1^2.

Let n = 3

w1	m1>m2>m3
w2	m1>m2>m3
w3	m1>m2>m3
m1	w3>w2>w1
m2	w3>w2>w1
m3	w3>w2>w1

We get:

1.

In this current arrangement of preferences, we get:

1. (w1,m1)
2. (w2,m1)
3. (w1,m2)
4. (w3,m1)
5. (w2, m2)
6. (w1, m3)

Again, it is apparent that the algorithm took more than n loops to get a stable matchings, and here we can see a pattern shaping.

Given n women and n men, if the preference list of both was arranged like so:

w1	m1>…>mn
. . .	m1>…>mn
wn	m1>…>mn
m1	wn>…>w1
. . .	wn>…>w1
mn	wn>…>w1

We will always have more loops than n to find a stable matching. Thus the runtime is omega(n^2).