## Appendix B. Choice of $(\rho, \lambda, \rho)$ based on a case study

Our previous papers [29, 30] showed the sensitivity of tentative adversarial images to the choice of the degrading and enlarging functions. In the present appendix B, we, therefore, want to find out which degrading and enlarging functions  $\rho$  and  $\lambda$ , and which combination  $(\rho, \lambda, \rho)$ , used in Scheme 11, provide the best outcome in terms of image quality and of adversity. For this purpose, we perform a case study.

Based on the results of [29, 30], the study is limited to the consideration of the "Lanczos" (L) and "Nearest" (N) functions, either for the degrading function  $\rho$  or for the enlarging function  $\lambda$ . This leads to 8 combinations for  $(\rho, \lambda, \rho)$ , namely (with obvious notations) L-L-L, L-L-N, L-N-L, N-L-L, L-N-N, N-L-N, N-N-L and N-N-N.

For each such combination  $(\rho, \lambda, \rho)$ , the study is performed on the 100 clean images  $\mathcal{A}_q^p$  represented in Figure A1, with the EA-based targeted attack against the CNN  $\mathcal{C} = \mathcal{C}_9 = \text{VGG-16}$ , according to the pairs  $(c_a, c_t)$  specified in Table 2.

However, although the images  $\mathcal{A}_q^p$  are picked from the ImageNet validation set in the categories  $c_{a_q}$ , VGG-16 does not systematically classify all of them in the "correct" category  $c_{a_q}$  in the process of Steps 1 and 2 of Scheme 11. Indeed, Tables A2 and A3 in Appendix A show that VGG-16 classifies "correctly" only 93 clean images  $\mathcal{A}_q^p$ , and classifies "wrongly" 7 when the degrading function used in Step 1 is  $\rho = L$  or is  $\rho = N$ . Let us observe that although the number of "correctly" classified and of "wrongly" classified images are the same independently on the  $\rho$  function used, the actual such images  $\mathcal{A}_q^p$  are not necessarily the same. In any case, the "wrongly" classified clean images are from now on disregarded (a symbol "XXX" represents this fact in the tables), since they introduce a native bias. The rest of the experiments are therefore performed on the set  $\mathcal{S}_{clean}^{VGG-16}(\rho) = 93$  of "correctly" classified clean images.

With this setting, the targeted attack aims at creating 0.55-strong adversarial images in the  $\mathcal{R}$  domain (hence meaning that it aims at creating images for which  $\tilde{\tau}_t \geq 0.55$ ).

As explained in Section 4.4, the attack succeeds when a 0.55-strong adversarial image in the  $\mathcal{R}$  domain is obtained within 10,000 generations. In the present case study, we also keep track of the unsuccessful such attacks. More precisely, for the  $\mathcal{S}_{clean}^{VGG-16}(\rho) = 93$  images considered, we also report the cases where either the best tentative adversarial image in the  $\mathcal{R}$  domain, obtained after 10,000 generations, is classified in  $c_t$  but with a label value < 0.55, or is classified in a category  $c \neq c_a, c_t$ , or is classified back to  $c = c_a$ .

Note en passant that, although unsuccessful for the 0.55-target scenario, the attack in the  $\mathcal{R}$  domain is successful at creating good enough adversarial images in the first case considered in the previous paragraph, respectively for the untarget scenario in the second case.

To summarize, we denote by  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}}$  the quadruplet composed (from left to right) of the number of 0.55-strong adversarial images, the number of good enough adversarial images, the number of tentative adversarial images classified in a category  $c \neq c_a, c_t$ , and the number of those classified back to  $c_a$ . Clearly, since all possibilities are considered, the sum of these quantities is equal to the number of clean images

considered, namely  $\#\mathcal{S}_{clean}^{\text{VGG-16}}(\rho) = 93$  (independently on whether  $\rho = L$  or  $\rho = N$ ). In the present study, we report the  $c_t$ -label values  $\tilde{\tau}_t$  only for the images contributing to  $\gamma_{st}$  and to  $\gamma_{qe}$ .

In the present study, Scheme 11 continues with Steps 4 to 8 only for the adversarial images that correspond to the first or the second component of the quadruplet in  $\mathcal{R}$ , namely those obtained in Step 3 that are classified in  $c_t$ . Note that we compute the average of the  $\tilde{\tau}_c = \tilde{\tau}_t$  for these images (a symbol "XXX" represents in the tables the disregarded images).

At the end of Step 8, we report the following indicators. From left to right, the triplet  $(\Gamma, \Theta, \Phi)_{\mathcal{H}}$  counts the number of HR tentative adversarial images classified in  $c_t$  (hence adversarial for the target scenario), classified in  $c \neq c_a, c_t$  (hence adversarial for the untarget scenario), classified back in  $c_a$  (not adversarial at all). For the images that contribute either to  $\Gamma$  or to  $\Theta$ , we report their  $c_t$ -label values  $\tau_t$ , the value of the loss function, and the values of the two  $L_2$  distances (written as  $L_{2,\mathcal{R}}$  and  $L_{2,\mathcal{H}}$  to simplify the notations) as specified in Section 3.2.

Remarks: Firstly, one has  $\Gamma + \Theta + \Phi = \gamma_{st} + \gamma_{ge} \leq \#\mathcal{S}_{clean}^{\text{VGG-16}}(\rho) = 93$ . Secondly, let us recall that the value of the loss function  $\mathcal{L}$  is systematically computed on the  $c_t$ -label values for the specific target category  $c_t$ . This holds in particular for the images that contribute to  $\Theta$ , namely those where ultimately (at the end of Step 8) the dominant category is not  $c_t$ .

The outcomes of these experiments are given in Table B1 for the L-L-L combination, in Table B2 for the L-L-N combination, in Table B3 for the L-N-L combination, in Table B4 for the N-L-L combination, in Table B5 for the L-N-N combination, in Table B6 for the N-L-N combination, in Table B7 for the N-N-L combination, and in Table B8 for the N-N-N combination.

| 1 2 3 4 5 6 7 8 9 10 1 1   | $ \begin{array}{c c} \tilde{\tau}_c \\ c_t \\ c_t \\ c_t \end{array} $ | Dominant<br>Category<br>0.5502 | $\tau_c$ | Dominant | r norm,adv | $L_0^{norm}$        |                     |                  |                     |                     |                                  |                  |                     |                     |                          |                          |                          |               |
|--|--|--------------------------------|----------|----------|------------|---------------------|---------------------|------------------|---------------------|---------------------|----------------------------------|------------------|---------------------|---------------------|--------------------------|--------------------------|--------------------------|---------------|
| $ \begin{array}{c c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} $ | $c_t$  |                                | $\tau_c$ | l ~ .    |            | r norm,adv          | r norm,clean        | $_{T}$ norm, adv | $_{T}$ norm, adv    | r norm,clean        | $L_{1,\mathcal{H}}^{norm,adv}$   | $_{T}$ norm, adv | r norm,adv          | r norm,clean        | r norm,adv               | r norm,adv               | $_{T}$ norm,clean        | $\mathcal{L}$ |
| $ \begin{array}{c c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} $      | $c_t$  | 0.5502                         |          | Category | $L_{0,R}$  | $L_{0,\mathcal{H}}$ | $L_{0,\mathcal{H}}$ | $L_{1,R}$        | $L_{1,\mathcal{H}}$ | $L_{1,\mathcal{H}}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $L_{2,R}$        | $L_{2,\mathcal{H}}$ | $L_{2,\mathcal{H}}$ | $L_{\infty,\mathcal{R}}$ | $L_{\infty,\mathcal{H}}$ | $L_{\infty,\mathcal{H}}$ |               |
| 1 5 6 7 8 9 10   | -  |                                | $c_t$    | 0.5137   | 0.962      | 0.960               | 0.999               | 0.035            | 0.032               | 0.023               | 1.391                            | 1.14E-04         | 8.31E-06            | 7.92E-06            | 53                       | 54                       | 161                      | 0.037         |
| $ \begin{array}{c c} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} $                | $c_t$  | 0.5505                         | $c_t$    | 0.5223   | 0.943      | 0.931               | 0.999               | 0.022            | 0.019               | 0.026               | 0.731                            | 7.17E-05         | 1.32E-05            | 2.55E-05            | 33                       | 33                       | 158                      | 0.028         |
| 1 5<br>6<br>7<br>8<br>9<br>10  |  | 0.5509                         | $c_t$    | 0.5289   | 0.958      | 0.954               | 0.999               | 0.029            | 0.027               | 0.002               | 13.500                           | 9.67E-05         | 7.41E-05            | 9.25E-06            | 44                       | 39                       | 18                       | 0.022         |
| 6<br>7<br>8<br>9<br>10   | $c_t$  | 0.5505                         | $c_t$    | 0.4912   | 0.971      | 0.967               | 0.999               | 0.042            | 0.039               | 0.010               | 3.900                            | 1.38E-04         | 6.72E-05            | 2.38E-05            | 64                       | 60                       | 71                       | 0.059         |
| 6<br>7<br>8<br>9<br>10   | $c_t$  | 0.5514                         | $c_t$    | 0.5224   | 0.959      | 0.953               | 1.000               | 0.029            | 0.027               | 0.017               | 1.588                            | 9.69E-05         | 4.58E-05            | 3.14E-05            | 44                       | 43                       | 95                       | 0.029         |
| 8<br>9<br>10   | $c_t$  | $0.2947~\mathrm{ge}$           | $c_t$    | 0.2734   | 0.974      | 0.970               | 0.913               | 0.046            | 0.042               | 0.011               | 3.818                            | 1.48E-04         | 7.32E-05            | 3.29E-05            | 68                       | 70                       | 85                       | 0.021         |
| 9  | $c_t$  | 0.5503                         | $c_t$    | 0.5091   | 0.967      | 0.961               | 0.999               | 0.039            | 0.036               | 0.028               | 1.286                            | 1.29E-04         | 6.13E-05            | 6.77E-05            | 61                       | 58                       | 128                      | 0.041         |
| 10   | $c_t$  | 0.5507                         | $c_t$    | 0.4339   | 0.957      | 0.942               | 0.999               | 0.035            | 0.033               | 0.038               | 0.868                            | 1.18E-04         | 5.16E-05            | 8.35E-05            | 54                       | 55                       | 192                      | 0.117         |
|  | $c_t$  | 0.5505                         | $c_t$    | 0.4925   | 0.964      | 0.958               | 0.999               | 0.034            | 0.031               | 0.029               | 1.069                            | 1.12E-04         | 4.61E-05            | 6.02E-05            | 50                       | 51                       | 149                      | 0.058         |
| 1  | $c_t$  | 0.5500                         | $c_t$    | 0.5357   | 0.958      | 0.953               | 0.999               | 0.030            | 0.027               | 0.025               | 1.080                            | 9.73E-05         | 7.03E-06            | 8.39E-06            | 49                       | 50                       | 192                      | 0.014         |
|  | $c_t$  | 0.5523                         | $c_t$    | 0.5209   | 0.932      | 0.923               | 0.999               | 0.018            | 0.016               | 0.021               | 0.762                            | 5.89E-05         | 2.80E-05            | 5.40E-05            | 27                       | 27                       | 122                      | 0.031         |
| 2  | $c_t$  | 0.5512                         | $c_t$    | 0.5281   | 0.928      | 0.921               | 0.999               | 0.017            | 0.015               | 0.018               | 0.833                            | 5.61E-05         | 2.39E-05            | 3.53E-05            | 32                       | 32                       | 133                      | 0.023         |
| 3  | $c_t$  | 0.5507                         | $c_t$    | 0.5434   | 0.938      | 0.932               | 1.000               | 0.020            | 0.018               | 0.020               | 0.900                            | 6.51E-05         | 3.09E-05            | 4.25E-05            | 30                       | 31                       | 124                      | 0.007         |
| 4  | $c_t$  | 0.5508                         | $c_t$    | 0.4929   | 0.946      | 0.940               | 0.999               | 0.023            | 0.021               | 0.009               | 2.333                            | 7.48E-05         | 3.56E-05            | 2.51E-05            | 34                       | 36                       | 88                       | 0.058         |
| 2 5  | $c_t$  | 0.5511                         | $c_t$    | 0.5362   | 0.958      | 0.952               | 0.999               | 0.029            | 0.026               | 0.021               | 1.238                            | 9.55E-05         | 3.92E-05            | 4.73E-05            | 47                       | 47                       | 119                      | 0.015         |
| 6  | $c_t$  | 0.5515                         | $c_t$    | 0.5260   | 0.951      | 0.944               | 1.000               | 0.026            | 0.023               | 0.026               | 0.885                            | 8.52E-05         | 3.49E-05            | 6.05E-05            | 42                       | 40                       | 162                      | 0.026         |
| 7  | $c_t$  | 0.5505                         | $c_t$    | 0.5014   | 0.943      | 0.941               | 0.996               | 0.023            | 0.021               | 0.005               | 4.200                            | 7.62E-05         | 3.65E-05            | 1.56E-05            | 33                       | 34                       | 86                       | 0.049         |
| 8  | $c_t$  | 0.5513                         | $c_t$    | 0.5051   | 0.948      | 0.943               | 0.999               | 0.024            | 0.022               | 0.007               | 3.143                            | 7.85E-05         | 3.73E-05            | 1.91E-05            | 37                       | 34                       | 69                       | 0.046         |
| 9  | $c_t$  | 0.5510                         | $c_t$    | 0.4714   | 0.943      | 0.938               | 0.999               | 0.022            | 0.020               | 0.008               | 2.500                            | 7.20E-05         | 3.09E-05            | 1.97E-05            | 33                       | 36                       | 73                       | 0.080         |
| 10   | $c_t$  | 0.5507                         | $c_t$    | 0.4992   | 0.940      | 0.931               | 1.000               | 0.021            | 0.019               | 0.020               | 0.950                            | 6.97E-05         | 3.51E-05            | 5.18E-05            | 31                       | 32                       | 127                      | 0.052         |
| 1  | $c_t$  | 0.5508                         | $c_t$    | 0.5164   | 0.960      | 0.954               | 0.999               | 0.033            | 0.030               | 0.015               | 2.000                            | 1.08E-04         | 4.79E-05            | 3.17E-05            | 52                       | 48                       | 87                       | 0.034         |
| 2  | $c_t$  | 0.5503                         | $c_t$    | 0.5068   | 0.955      | 0.949               | 0.999               | 0.028            | 0.025               | 0.004               | 6.250                            | 9.16E-05         | 7.87E-05            | 3.44E-05            | 45                       | 43                       | 78                       | 0.044         |
| 3  | $c_t$  | 0.5501                         | $c_t$    | 0.5214   | 0.962      | 0.957               | 0.999               | 0.032            | 0.029               | 0.013               | 2.231                            | 1.04E-04         | 7.41E-06            | 5.39E-06            | 47                       | 49                       | 172                      | 0.029         |
| 4  | $c_t$  | 0.5511                         | $c_t$    | 0.5220   | 0.940      | 0.929               | 1.000               | 0.023            | 0.021               | 0.014               | 1.500                            | 7.75E-05         | 3.87E-05            | 4.52E-05            | 36                       | 35                       | 132                      | 0.029         |
| 3 5  | $c_t$  | 0.5504                         | $c_t$    | 0.4479   | 0.960      | 0.954               | 0.999               | 0.036            | 0.033               | 0.007               | 4.714                            | 1.21E-04         | 8.18E-05            | 2.32E-05            | 59                       | 60                       | 61                       | 0.103         |
| 3 6  | $c_t$  | 0.5518                         | $c_t$    | 0.5082   | 0.952      | 0.945               | 0.999               | 0.026            | 0.024               | 0.010               | 2.400                            | 8.71E-05         | 5.76E-05            | 4.53E-05            | 39                       | 41                       | 66                       | 0.044         |
| 7  | $c_t$  | 0.5511                         | $c_t$    | 0.5060   | 0.938      | 0.939               | 0.996               | 0.022            | 0.021               | 0.008               | 2.625                            | 7.54E-05         | 3.51E-05            | 2.67E-05            | 37                       | 38                       | 108                      | 0.045         |
| 8  | $c_t$  | 0.5502                         | $c_t$    | 0.5008   | 0.953      | 0.945               | 1.000               | 0.027            | 0.025               | 0.018               | 1.389                            | 9.10E-05         | 4.46E-05            | 4.92E-05            | 44                       | 41                       | 131                      | 0.049         |
| 9  | $c_t$  | 0.5515                         | $c_t$    | 0.4953   | 0.963      | 0.958               | 1.000               | 0.033            | 0.030               | 0.023               | 1.304                            | 1.09E-04         | 5.16E-05            | 5.98E-05            | 48                       | 53                       | 164                      | 0.056         |
| 10   | $c_t$  | 0.5501                         | $c_t$    | 0.4851   | 0.972      | 0.964               | 1.000               | 0.047            | 0.043               | 0.014               | 3.071                            | 1.55E-04         | 9.67E-05            | 3.93E-05            | 77                       | 73                       | 115                      | 0.065         |
| 1  | XXX  | XXX                            | XXX      | XXX      | XXX        | XXX                 | XXX                 | XXX              | XXX                 | XXX                 | XXX                              | XXX              | XXX                 | XXX                 | XXX                      | XXX                      | XXX                      | XXX           |
| 2  | $c_t$  | 0.5502                         | $c_t$    | 0.5204   | 0.952      | 0.961               | 0.949               | 0.034            | 0.033               | 0.006               | 5.163                            | 1.13E-04         | 6.45E-05            | 2.00E-05            | 54                       | 55                       | 47                       | 0.0298        |
| 3  | $c_t$  | 0.5506                         | $c_t$    | 0.5332   | 0.944      | 0.939               | 0.997               | 0.023            | 0.022               | 0.015               | 1.467                            | 7.78E-05         | 3.87E-05            | 5.41E-05            | 34                       | 36                       | 109                      | 0.017         |
| 4  | $c_t$  | 0.5501                         | $c_t$    | 0.5391   | 0.935      | 0.947               | 0.903               | 0.026            | 0.026               | 0.005               | 5.200                            | 9.10E-05         | 6.61E-05            | 2.54E-05            | 44                       | 42                       | 33                       | 0.011         |
| 4 5  | $c_t$  | 0.5512                         | $c_t$    | 0.5266   | 0.967      | 0.961               | 0.999               | 0.036            | 0.033               | 0.006               | 5.500                            | 1.18E-04         | 5.08E-05            | 1.84E-05            | 53                       | 56                       | 80                       | 0.025         |
| 6  | $c_t$  | 0.5531                         | $c_t$    | 0.4371   | 0.966      | 0.957               | 1.000               | 0.039            | 0.035               | 0.114               | 0.307                            | 1.27E-04         | 6.04E-05            | 2.02E-04            | 59                       | 58                       | 197                      | 0.116         |
| 7  | $c_t$  | 0.5507                         | $c_t$    | 0.5136   | 0.956      | 0.953               | 0.999               | 0.031            | 0.029               | 0.010               | 2.900                            | 1.05E-04         | 5.40E-05            | 2.91E-05            | 44                       | 44                       | 58                       | 0.037         |
| 8  | $c_t$  | 0.5503                         | $c_t$    | 0.5316   | 0.952      | 0.942               | 0.999               | 0.025            | 0.022               | 0.032               | 0.688                            | 8.17E-05         | 4.02E-05            | 8.58E-05            | 38                       | 37                       | 175                      | 0.019         |
| 9  | $c_t$  | 0.5513                         | $c_t$    | 0.5307   | 0.946      | 0.947               | 0.966               | 0.027            | 0.026               | 0.010               | 2.600                            | 9.02E-05         | 4.58E-05            | 3.81E-05            | 41                       | 40                       | 88                       | 0.021         |
| 10   | $c_t$  | 0.5501                         | $c_t$    | 0.5270   | 0.952      | 0.956               | 0.977               | 0.034            | 0.032               | 0.008               | 4.000                            | 1.12E-04         | 4.28E-05            | 2.24E-05            | 51                       | 54                       | 124                      | 0.023         |
| 1  | $c_t$  | 0.5502                         | $c_t$    | 0.5228   | 0.962      | 0.957               | 1.000               | 0.033            | 0.030               | 0.023               | 1.316                            | 1.06E-04         | 3.79E-05            | 4.23E-05            | 48                       | 45                       | 152                      | 0.027         |
| 2  | XXX  | XXX                            | XXX      | XXX      | XXX        | XXX                 | XXX                 | XXX              | XXX                 | XXX                 | XXX                              | XXX              | XXX                 | XXX                 | XXX                      | XXX                      | XXX                      | XXX           |
| 3  | $c_t$  | 0.5503                         | $c_t$    | 0.5144   | 0.961      | 0.956               | 0.995               | 0.033            | 0.031               | 0.028               | 1.107                            | 1.09E-04         | 5.55E-05            | 6.28E-05            | 48                       | 53                       | 128                      | 0.036         |
| 4  | $c_t$  | 0.5525                         | $c_t$    | 0.4846   | 0.963      | 0.957               | 1.000               | 0.032            | 0.029               | 0.019               | 1.526                            | 1.06E-04         | 5.32E-05            | 4.19E-05            | 48                       | 46                       | 82                       | 0.068         |
| 5 5  | $c_t$  | 0.5516                         | $c_t$    | 0.5278   | 0.957      | 0.947               | 1.000               | 0.028            | 0.026               | 0.043               | 0.605                            | 9.30E-05         | 3.11E-05            | 6.70E-05            | 40                       | 41                       | 193                      | 0.024         |
| 6  | $c_t$  | 0.5506                         | $c_t$    | 0.4893   | 0.969      | 0.964               | 1.000               | 0.039            | 0.035               | 0.034               | 1.029                            | 1.27E-04         | 6.34E-05            | 7.54E-05            | 58                       | 57                       | 104                      | 0.061         |
| 7  | $c_t$  | 0.5513                         | $c_t$    | 0.5374   | 0.949      | 0.943               | 1.000               | 0.024            | 0.022               | 0.017               | 1.294                            | 8.03E-05         | 3.07E-05            | 2.68E-05            | 37                       | 39                       | 77                       | 0.014         |
| 8  | $c_t$  | 0.5503                         | $c_t$    | 0.5277   | 0.971      | 0.966               | 1.000               | 0.041            | 0.037               | 0.025               | 1.480                            | 1.34E-04         | 6.36E-05            | 5.12E-05            | 59                       | 57                       | 90                       | 0.023         |
| 9  | $c_t$  | 0.5501                         | $c_t$    | 0.5012   | 0.970      | 0.966               | 1.000               | 0.041            | 0.037               | 0.043               | 0.860                            | 1.34E-04         | 6.69E-05            | 9.12E-05            | 61                       | 61                       | 132                      | 0.049         |
| 10   | $c_t$  | 0.5541                         | $c_t$    | 0.5152   | 0.955      | 0.950               | 1.000               | 0.028            | 0.025               | 0.024               | 1.042                            | 9.18E-05         | 3.64E-05            | 4.34E-05            | 43                       | 43                       | 182                      | 0.039         |

|    | $A_q^p$ | Step           | 1 - Step 3         | Step     | 4 - Step 8       |                      | $L_0^{norm}$                   |                                  |                      | $L_1^i$                        | norm                   |   |                      | $L_2^{norm}$                   |                                  |                           | $L_{\infty}$             |                                       |               |
|----|---------|----------------|--------------------|----------|------------------|----------------------|--------------------------------|----------------------------------|----------------------|--------------------------------|------------------------|---|----------------------|--------------------------------|----------------------------------|---------------------------|--------------------------|---------------------------------------|---------------|
| q  | p       | $	ilde{	au}_c$ | Dominant           | $\tau_c$ | Dominant         | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,2}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $_{I}$ $norm, adv$       | $L_{\infty,\mathcal{H}}^{norm,clean}$ | $\mathcal{L}$ |
|    |         |                | Category<br>0.5527 |          | Category         |                      |                                |                                  |                      |                                | L <sub>1,H</sub>       |   |                      |                                |                                  |                           | $L_{\infty,\mathcal{H}}$ |                                       |               |
|    | 1 2     | $c_t$          | 0.5527             | $c_t$    | 0.5248<br>0.4822 | 0.946<br>0.969       | 0.936<br>0.966                 | 1.000<br>1.000                   | 0.023                | 0.020                          | 0.048                  | 0.417<br>1.423  | 7.46E-05<br>1.33E-04 | 3.75E-05<br>5.25E-05           | 9.85E-05<br>4.52E-05             | 36<br>58                  | 34<br>55                 | 119<br>122                            | 0.028         |
|    | 3       | $c_t$          | 0.5501             | $c_t$    | 0.4622           | 0.963                | 0.957                          | 1.000                            | 0.041                | 0.037                          | 0.020                  | 0.714   | 1.09E-04             | 5.13E-05                       | 8.29E-05                         | 55                        | 53                       | 144                                   | 0.008         |
|    | 4       | $c_t$          | 0.5505             | $c_t$    | 0.4509           | 0.953                | 0.946                          | 1.000                            | 0.033                | 0.030                          | 0.042                  | 0.714   | 9.15E-05             | 4.32E-05                       | 5.38E-05                         | 40                        | 40                       | 118                                   | 0.100         |
|    | 5       | _              | 0.5502             | $c_t$    | 0.4305           | 0.946                | 0.940                          | 1.000                            | 0.023                | 0.023                          | 0.020                  | 1.167   | 7.53E-05             | 3.09E-05                       | 5.08E-05                         | 34                        | 35                       | 185                                   | 0.100         |
| 6  | 6       | $c_t$          | 0.5502             | $c_t$    | 0.4811           | 0.954                | 0.950                          | 1.000                            | 0.023                | 0.021                          | 0.021                  | 1.190   | 8.91E-05             | 3.73E-05                       | 4.07E-05                         | 40                        | 41                       | 92                                    | 0.020         |
|    | 7       | $c_t$          | 0.5504             | $c_t$    | 0.5150           | 0.963                | 0.959                          | 0.999                            | 0.027                | 0.025                          | 0.021                  | 1.033   | 1.10E-04             | 1.74E-05                       | 2.27E-05                         | 57                        | 58                       | 149                                   | 0.035         |
|    | 8       | $c_t$          | 0.5505             | $c_t$    | 0.4977           | 0.963                | 0.948                          | 0.999                            | 0.035                | 0.032                          | 0.028                  | 1.143   | 1.15E-04             | 5.77E-05                       | 8.20E-05                         | 55                        | 53                       | 181                                   | 0.053         |
|    | 9       | $c_t$          | 0.5503             | $c_t$    | 0.5255           | 0.943                | 0.934                          | 0.999                            | 0.022                | 0.020                          | 0.044                  | 0.455   | 7.16E-05             | 8.28E-06                       | 2.26E-05                         | 35                        | 35                       | 159                                   | 0.025         |
|    | 10      | $c_t$          | 0.5509             | $c_t$    | 0.4664           | 0.966                | 0.959                          | 0.997                            | 0.038                | 0.035                          | 0.052                  | 0.673   | 1.25E-04             | 5.91E-05                       | 1.12E-04                         | 55                        | 56                       | 168                                   | 0.085         |
|    | 1       | $c_t$          | 0.5503             | $c_t$    | 0.4698           | 0.959                | 0.943                          | 1.000                            | 0.031                | 0.028                          | 0.063                  | 0.444   | 1.03E-04             | 4.80E-05                       | 1.22E-04                         | 49                        | 49                       | 184                                   | 0.081         |
|    | 2       | $c_t$          | 0.5506             | $c_t$    | 0.5010           | 0.965                | 0.960                          | 0.999                            | 0.036                | 0.032                          | 0.029                  | 1.103   | 1.17E-04             | 1.72E-05                       | 2.28E-05                         | 56                        | 58                       | 191                                   | 0.050         |
|    | 3       | $c_t$          | 0.5510             | $c_t$    | 0.5160           | 0.959                | 0.955                          | 1.000                            | 0.031                | 0.028                          | 0.027                  | 1.037   | 1.00E-04             | 5.04E-05                       | 6.45E-05                         | 48                        | 46                       | 143                                   | 0.035         |
|    | 4       | XXX            | xxx                | XXX      | XXX              | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                    | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | xxx                      | XXX                                   | XXX           |
| 1_ | 5       | $c_t$          | 0.5515             | $c_t$    | 0.5164           | 0.959                | 0.953                          | 0.999                            | 0.029                | 0.027                          | 0.021                  | 1.286   | 9.57E-05             | 3.74E-05                       | 3.94E-05                         | 47                        | 48                       | 129                                   | 0.035         |
| 7  | 6       | $c_t$          | 0.5513             | $c_t$    | 0.4902           | 0.966                | 0.959                          | 1.000                            | 0.036                | 0.033                          | 0.033                  | 1.000   | 1.20E-04             | 5.66E-05                       | 6.77E-05                         | 59                        | 60                       | 145                                   | 0.061         |
|    | 7       | $c_t$          | 0.5502             | $c_t$    | 0.5088           | 0.955                | 0.951                          | 0.994                            | 0.028                | 0.026                          | 0.022                  | 1.182   | 9.29E-05             | 4.00E-05                       | 3.95E-05                         | 42                        | 44                       | 105                                   | 0.041         |
|    | 8       | $c_t$          | 0.5504             | $c_t$    | 0.5256           | 0.967                | 0.963                          | 0.999                            | 0.037                | 0.034                          | 0.023                  | 1.478   | 1.21E-04             | 5.52E-05                       | 4.24E-05                         | 54                        | 58                       | 79                                    | 0.025         |
|    | 9       | $c_t$          | 0.5504             | $c_t$    | 0.5159           | 0.965                | 0.960                          | 0.999                            | 0.037                | 0.034                          | 0.050                  | 0.680   | 1.21E-04             | 5.26E-05                       | 9.01E-05                         | 54                        | 55                       | 150                                   | 0.035         |
|    | 10      | $c_t$          | 0.5505             | $c_t$    | 0.5193           | 0.956                | 0.950                          | 1.000                            | 0.028                | 0.025                          | 0.020                  | 1.250   | 9.09E-05             | 4.54E-05                       | 4.69E-05                         | 42                        | 41                       | 144                                   | 0.031         |
|    | 1       | $c_t$          | 0.5502             | $c_t$    | 0.5296           | 0.955                | 0.950                          | 0.999                            | 0.028                | 0.025                          | 0.017                  | 1.471   | 9.17E-05             | 4.61E-05                       | 4.74E-05                         | 50                        | 48                       | 110                                   | 0.021         |
|    | 2       | $c_t$          | 0.5541             | $c_t$    | 0.5073           | 0.956                | 0.951                          | 0.999                            | 0.028                | 0.026                          | 0.018                  | 1.444   | 9.27E-05             | 3.54E-05                       | 3.65E-05                         | 41                        | 43                       | 140                                   | 0.047         |
|    | 3       | $c_t$          | 0.5506             | $c_t$    | 0.5236           | 0.962                | 0.956                          | 0.999                            | 0.033                | 0.030                          | 0.027                  | 1.111   | 1.09E-04             | 4.74E-05                       | 6.02E-05                         | 72                        | 74                       | 146                                   | 0.027         |
|    | 4       | $c_t$          | 0.5503             | $c_t$    | 0.5262           | 0.941                | 0.934                          | 0.999                            | 0.021                | 0.019                          | 0.006                  | 3.167   | 6.84E-05             | 4.53E-05                       | 2.57E-05                         | 29                        | 27                       | 62                                    | 0.024         |
| 8  | 5       | $c_t$          | 0.5512             | $c_t$    | 0.5021           | 0.961                | 0.957                          | 0.999                            | 0.032                | 0.029                          | 0.016                  | 1.813   | 1.06E-04             | 5.35E-05                       | 4.17E-05                         | 58                        | 52                       | 91                                    | 0.049         |
| 8  | 6       | $c_t$          | 0.5509             | $c_t$    | 0.5294           | 0.944                | 0.944                          | 0.991                            | 0.026                | 0.025                          | 0.012                  | 2.083   | 8.76E-05             | 4.51E-05                       | 3.71E-05                         | 44                        | 42                       | 175                                   | 0.022         |
|    | 7       | $c_t$          | 0.5511             | $c_t$    | 0.5082           | 0.962                | 0.952                          | 0.999                            | 0.033                | 0.030                          | 0.056                  | 0.536   | 1.10E-04             | 5.16E-05                       | 0.000110594                      | 56                        | 58                       | 152                                   | 0.043         |
|    | 8       | $c_t$          | 0.5518             | $c_t$    | 0.5078           | 0.955                | 0.947                          | 1.000                            | 0.027                | 0.025                          | 0.037                  | 0.676   | 8.86E-05             | 4.21E-05                       | 7.08E-05                         | 39                        | 38                       | 134                                   | 0.044         |
|    | 9       | $c_t$          | 0.5511             | $c_t$    | 0.4477           | 0.961                | 0.956                          | 1.000                            | 0.032                | 0.030                          | 0.016                  | 1.875   | 1.07E-04             | 6.97E-05                       | 5.23E-05                         | 52                        | 48                       | 66                                    | 0.103         |
|    | 10      | $c_t$          | 0.5537             | $c_t$    | 0.5067           | 0.957                | 0.953                          | 0.999                            | 0.030                | 0.027                          | 0.048                  | 0.563   | 9.72E-05             | 4.63E-05                       | 9.59E-05                         | 50                        | 49                       | 158                                   | 0.047         |
|    | 1       | XXX            | XXX                | XXX      | XXX              | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                    | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                      | XXX                                   | XXX           |
|    | 2       | $c_t$          | 0.5508             | $c_t$    | 0.5192           | 0.946                | 0.938                          | 0.999                            | 0.022                | 0.020                          | 0.025                  | 0.800   | 7.36E-05             | 3.49E-05                       | 5.40E-05                         | 32                        | 32                       | 122                                   | 0.032         |
|    | 3       | $c_t$          | 0.5512             | $c_t$    | 0.5407           | 0.957                | 0.952                          | 1.000                            | 0.028                | 0.026                          | 0.011                  | 2.364   | 9.32E-05             | 3.53E-05                       | 2.12E-05                         | 42                        | 43                       | 84                                    | 0.011         |
|    | 4       | XXX            | XXX                | XXX      | XXX              | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                    | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                      | XXX                                   | XXX           |
| 9  | 5       | $c_t$          | 0.5518             | $c_t$    | 0.5101           | 0.910                | 0.888                          | 1.000                            | 0.014                | 0.012                          | 0.026                  | 0.462   | 4.65E-05             | 2.66E-05                       | 7.70E-05                         | 21                        | 24                       | 117                                   | 0.042         |
| '  | 6       | XXX            | XXX                | XXX      | XXX              | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                    | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                      | XXX                                   | XXX           |
|    | 7       | $c_t$          | 0.5510             | $c_t$    | 0.5171           | 0.952                | 0.942                          | 0.999                            | 0.025                | 0.023                          | 0.055                  | 0.418   | 8.35E-05             | 1.93E-05                       | 5.22E-05                         | 41                        | 41                       | 175                                   | 0.034         |
|    | 8       | $c_t$          | 0.5521             | $c_t$    | 0.5343           | 0.924                | 0.911                          | 1.000                            | 0.016                | 0.014                          | 0.056                  | 0.250   | 5.34E-05             | 2.53E-05                       | 0.000118622                      | 23                        | 22                       | 171                                   | 0.018         |
|    | 9       | $c_t$          | 0.5512             | $c_t$    | 0.5172           | 0.947                | 0.941                          | 0.999                            | 0.024                | 0.022                          | 0.018                  | 1.222   | 7.88E-05             | 3.74E-05                       | 4.71E-05                         | 39                        | 39                       | 165                                   | 0.034         |
|    | 10      | $c_t$          | 0.5505             | $c_t$    | 0.4898           | 0.951                | 0.947                          | 1.000                            | 0.027                | 0.025                          | 0.008                  | 3.125   | 9.05E-05             | 4.27E-05                       | 2.17E-05                         | 43                        | 41                       | 133                                   | 0.061         |
|    | 1       | $c_t$          | 0.5506             | $c_t$    | 0.5126           | 0.954                | 0.950                          | 0.999                            | 0.027                | 0.025                          | 0.010                  | 2.500   | 8.95E-05             | 4.25E-05                       | 2.37E-05                         | 46                        | 44                       | 96                                    | 0.038         |
|    | 2       | $c_t$          | 0.5500             | $c_t$    | 0.4583           | 0.962                | 0.953                          | 0.998                            | 0.035                | 0.032                          | 0.065                  | 0.492   | 1.16E-04             | 5.48E-05                       | 0.000132496                      | 52                        | 58                       | 200                                   | 0.092         |
|    | 3       | $c_t$          | 0.5509             | $c_t$    | 0.4893           | 0.953                | 0.951                          | 0.987                            | 0.028                | 0.026                          | 0.031                  | 0.839   | 9.41E-05             | 4.50E-05                       | 7.48E-05                         | 43                        | 40                       | 154                                   | 0.062         |
|    | 4       | $c_t$          | 0.5531             | $c_t$    | 0.5066           | 0.955                | 0.951                          | 1.000                            | 0.028                | 0.026                          | 0.030                  | 0.867   | 9.29E-05             | 4.67E-05                       | 6.67E-05                         | 45                        | 41                       | 122                                   | 0.047         |
| 10 | 5       | $c_t$          | 0.5503             | $c_t$    | 0.4895           | 0.960                | 0.950                          | 0.989                            | 0.033                | 0.030                          | 0.062                  | 0.484   | 1.10E-04             | 5.21E-05                       | 0.000119982                      | 49                        | 49                       | 163                                   | 0.061         |
|    | ь       | $c_t$          | 0.5500             | $c_t$    | 0.5223           | 0.964                | 0.960                          | 1.000                            | 0.035                | 0.032                          | 0.009                  | 3.556   | 1.14E-04             | 6.72E-05                       | 2.67E-05                         | 51                        | 51                       | 73                                    | 0.028         |
|    | 7       | XXX            | XXX                | XXX      | XXX              | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                    | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                      | XXX                                   | XXX           |
|    | 8       | $c_t$          | 0.5503             | $c_t$    | 0.4978           | 0.968                | 0.964                          | 1.000                            | 0.039                | 0.035                          | 0.042                  | 0.833   | 1.27E-04             | 5.53E-05                       | 7.44E-05                         | 66                        | 59                       | 110                                   | 0.053         |
|    | 9       | $c_t$          | 0.5501             | $c_t$    | 0.4943           | 0.960                | 0.956                          | 0.980                            | 0.034                | 0.032                          | 0.050                  | 0.640   | 1.13E-04             | 5.41E-05                       | 9.87E-05                         | 57                        | 52                       | 143                                   | 0.056         |
|    | 10      | $c_t$          | 0.5503             | $c_t$    | 0.4507           | 0.967                | 0.960                          | 0.999                            | 0.040                | 0.036                          | 0.048                  | 0.750   | 1.30E-04             | 6.52E-05                       | 0.000102036                      | 67                        | 62                       | 150                                   | 0.100         |

Table B1.: Combination  $(\rho, \lambda, \rho) = \text{L-L-L}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 0, 0)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (93, 0, 0)$ .

| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $\begin{array}{c c} L_0^{orm} \\ L_0^{orm} \\ \\ L_0^{orm,adv} \\ L_0^{orm,clean}, \\ \\ L_0^{orm} \\ \\ \\ L_0^{orm} \\ \\ \\ L_0^{orm} \\ \\ \\ L_0^{orm} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$ | $\begin{array}{c} L_{1,\mathcal{R}}^{norm,adv} \\ 0.035 \\ 0.022 \\ 0.030 \\ 0.043 \\ 0.030 \\ 0.046 \\ 0.046 \\ 0.036 \\ 0.035 \\ 0.030 \end{array}$ | $\begin{array}{c} L_{1,\mathcal{H}}^{norm,adv} \\ 0.032 \\ 0.020 \\ 0.027 \\ 0.039 \\ 0.027 \\ 0.042 \\ 0.036 \\ 0.033 \end{array}$ | $L_{1,\mathcal{H}}^{norm,clean}$ $0.029$ $0.032$ $0.012$ $0.024$ $0.023$ $0.021$ $0.038$ | $\begin{array}{c} \frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}} \\ 1.127 \\ 0.623 \\ 2.366 \\ 1.601 \\ 1.193 \\ 2.017 \end{array}$ | $L_{2,\mathcal{R}}^{norm,adv}$ $1.14\text{E-}04$ $7.17\text{E-}05$ $9.67\text{E-}05$ $1.38\text{E-}04$ $9.69\text{E-}05$ | $L_{2,\mathcal{H}}^{norm,adv}$ $8.31\text{E}-06$ $1.32\text{E}-05$ $7.41\text{E}-05$ $6.72\text{E}-05$ $4.58\text{E}-05$ | $L^{norm,clean}_{2,\mathcal{H}}$ $1.09\text{E-}05$ $3.27\text{E-}05$ $4.65\text{E-}05$ $6.05\text{E-}05$ $4.84\text{E-}05$ | $L_{\infty,\mathcal{R}}^{norm,adv}$ $53$ $33$ $44$ $64$ $44$ | $L_{\infty}$ $L_{\infty,\mathcal{H}}^{norm,adv}$ $54$ $33$ $39$ $60$ $43$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ $219$ $210$ $138$ $142$ | L<br>0.546<br>0.521<br>0.493<br>0.349 |
|---|---|---|---|--|--|--|--|--|--|---|---|---------------------------------------|
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.960         0.901           0.932         0.888           0.955         0.791           0.967         0.825           0.953         0.928           0.970         0.493           0.961         0.834           0.943         0.777           0.958         0.938           0.954         0.924   | 0.035<br>0.022<br>0.030<br>0.043<br>0.030<br>0.046<br>0.040<br>0.036  | 0.032<br>0.020<br>0.027<br>0.039<br>0.027<br>0.042<br>0.036<br>0.033  | 0.029<br>0.032<br>0.012<br>0.024<br>0.023<br>0.021                                       | $L_{1,2}^{norm, clean}$ 1.127 0.623 2.366 1.601 1.193  | 1.14E-04<br>7.17E-05<br>9.67E-05<br>1.38E-04<br>9.69E-05   | 8.31E-06<br>1.32E-05<br>7.41E-05<br>6.72E-05<br>4.58E-05   | 1.09E-05<br>3.27E-05<br>4.65E-05<br>6.05E-05   | 53<br>33<br>44<br>64   | 54<br>33<br>39<br>60  | 219<br>210<br>138<br>142                                      | 0.546<br>0.521<br>0.493               |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 0.932         0.888           0.955         0.791           0.967         0.825           0.953         0.928           0.970         0.493           0.961         0.834           0.943         0.777           0.958         0.938           0.954         0.924   | 0.022<br>0.030<br>0.043<br>0.030<br>0.046<br>0.040<br>0.036<br>0.035  | 0.020<br>0.027<br>0.039<br>0.027<br>0.042<br>0.036<br>0.033   | 0.032<br>0.012<br>0.024<br>0.023<br>0.021  | 0.623<br>2.366<br>1.601<br>1.193   | 7.17E-05<br>9.67E-05<br>1.38E-04<br>9.69E-05   | 1.32E-05<br>7.41E-05<br>6.72E-05<br>4.58E-05   | 3.27E-05<br>4.65E-05<br>6.05E-05   | 33<br>44<br>64   | 33<br>39<br>60  | 210<br>138<br>142   | 0.521<br>0.493                        |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.955         0.791           0.967         0.825           0.953         0.928           0.970         0.493           0.961         0.834           0.943         0.777           0.958         0.938           0.954         0.924   | 0.030<br>0.043<br>0.030<br>0.046<br>0.040<br>0.036<br>0.035   | 0.027<br>0.039<br>0.027<br>0.042<br>0.036<br>0.033  | 0.012<br>0.024<br>0.023<br>0.021   | 2.366<br>1.601<br>1.193  | 9.67E-05<br>1.38E-04<br>9.69E-05   | 7.41E-05<br>6.72E-05<br>4.58E-05   | 4.65E-05<br>6.05E-05   | 44<br>64   | 39<br>60  | 138<br>142  | 0.493                                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.967         0.825           0.953         0.928           0.970         0.493           0.961         0.834           0.943         0.777           0.958         0.938           0.954         0.924   | 0.043<br>0.030<br>0.046<br>0.040<br>0.036<br>0.035  | 0.039<br>0.027<br>0.042<br>0.036<br>0.033   | 0.024<br>0.023<br>0.021  | 1.601<br>1.193   | 1.38E-04<br>9.69E-05   | 6.72E-05<br>4.58E-05   | 6.05E-05   | 64   | 60  | 142   |                                       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.953         0.928           0.970         0.493           0.961         0.834           0.943         0.777           0.958         0.938           0.954         0.924   | 0.030<br>0.046<br>0.040<br>0.036<br>0.035   | 0.027<br>0.042<br>0.036<br>0.033  | 0.023<br>0.021   | 1.193  | 9.69E-05   | 4.58E-05   |  | -  |   |   | 0.240                                 |
| $ \begin{array}{ c c c c c c c }\hline 1&6&c_t&{\bf 0.2947~ge}&c_a&0.8708&0.974&0\\\hline 7&c_t&0.5503&c_a&0.8634&0.967&0\\\hline 8&c_t&0.5507&c_a&0.9291&0.957&0\\\hline 9&c_t&0.5505&c_a&0.9997&0.964&0\\\hline 10&c_t&0.5500&c_a&0.8055&0.958&0\\\hline 1&c_t&0.5523&c\neq c_a,c_t&0.519&0.932&0\\\hline 2&c_t&0.5512&c_a&0.8416&0.928&0\\\hline \end{array} $ | 0.970     0.493       0.961     0.834       0.943     0.777       0.958     0.938       0.954     0.924   | 0.046<br>0.040<br>0.036<br>0.035  | 0.042<br>0.036<br>0.033   | 0.021  |  |  |  | 4.84E-05   | 44   | 43  |   |                                       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.961     0.834       0.943     0.777       0.958     0.938       0.954     0.924   | 0.040<br>0.036<br>0.035   | 0.036<br>0.033  |  | 2.017  |  |  |  |  |   | 187   | 0.485                                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.943 0.777<br>0.958 0.938<br>0.954 0.924   | 0.036<br>0.035  | 0.033   | 0.038  |  | 1.48E-04   | 7.32E-05   | 7.52E-05   | 68   | 70  | 193   | 0.276                                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.958 0.938<br>0.954 0.924  | 0.035   |   |  | 0.949  | 1.29E-04   | 6.13E-05   | 1.05E-04   | 61   | 58  | 231   | 0.544                                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.954 0.924   |   |   | 0.045  | 0.735  | 1.18E-04   | 5.16E-05   | 1.09E-04   | 54   | 55  | 227   | 0.545                                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   |   | 0.030   | 0.032   | 0.036  | 0.871  | 1.12E-04   | 4.61E-05   | 8.24E-05   | 50   | 51  | 238   | 0.551                                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 0.923   0.892   |   | 0.028   | 0.034  | 0.817  | 9.73E-05   | 7.03E-06   | 1.30E-05   | 49   | 50  | 193   | 0.517                                 |
|   |   | 0.018   | 0.017   | 0.033  | 0.507  | 5.89E-05   | 2.80E-05   | 7.76E-05   | 27   | 27  | 201   | 0.462                                 |
|   | 0.922 0.916   | 0.017   | 0.016   | 0.024  | 0.660  | 5.61E-05   | 2.39E-05   | 4.56E-05   | 32   | 32  | 172   | 0.551                                 |
|   | 0.933 0.948   | 0.020   | 0.018   | 0.035  | 0.529  | 6.51E-05   | 3.09E-05   | 6.93E-05   | 30   | 31  | 159   | 0.495                                 |
|   | 0.941 0.857   | 0.023   | 0.021   | 0.016  | 1.287  | 7.48E-05   | 3.56E-05   | 3.92E-05   | 34   | 36  | 157   | 0.515                                 |
|   | 0.952 0.660   | 0.029   | 0.027   | 0.026  | 1.034  | 9.55E-05   | 3.92E-05   | 5.91E-05   | 47   | 47  | 240   | 0.551                                 |
|   | 0.944 0.913   | 0.026   | 0.024   | 0.033  | 0.722  | 8.52E-05   | 3.49E-05   | 7.35E-05   | 42   | 40  | 223   | 0.546                                 |
| 7 47 5  | 0.941 0.747   | 0.023   | 0.022   | 0.012  | 1.818  | 7.62E-05   | 3.65E-05   | 3.22E-05   | 33   | 34  | 156   | 0.401                                 |
|   | 0.944 0.790   | 0.024   | 0.022   | 0.013  | 1.728  | 7.85E-05   | 3.73E-05   | 3.57E-05   | 37   | 34  | 139   | 0.426                                 |
|   | 0.938 0.826   | 0.022   | 0.020   | 0.017  | 1.232  | 7.20E-05   | 3.09E-05   | 3.70E-05   | 33   | 36  | 184   | 0.341                                 |
|   | 0.932 0.893   | 0.021   | 0.020   | 0.030  | 0.645  | 6.97E-05   | 3.51E-05   | 8.10E-05   | 31   | 32  | 204   | 0.504                                 |
|   | 0.954 0.849   | 0.033   | 0.031   | 0.027  | 1.121  | 1.08E-04   | 4.79E-05   | 6.66E-05   | 52   | 48  | 179   | 0.527                                 |
|   | 0.950 0.724   | 0.028   | 0.026   | 0.020  | 1.287  | 9.16E-05   | 7.87E-05   | 1.38E-04   | 45   | 43  | 255   | 0.442                                 |
|   | 0.957 0.879   | 0.032   | 0.029   | 0.018  | 1.601  | 1.04E-04   | 7.41E-06   | 7.83E-06   | 47   | 49  | 183   | 0.429                                 |
|   | 0.930 0.846   | 0.024   | 0.021   | 0.025  | 0.862  | 7.75E-05   | 3.87E-05   | 8.11E-05   | 36   | 35  | 215   | 0.547                                 |
| 3   | 0.955 0.797   | 0.037   | 0.034   | 0.028  | 1.193  | 1.21E-04   | 8.18E-05   | 9.87E-05   | 59   | 60  | 128   | 0.492                                 |
|   | 0.946 0.751   | 0.027   | 0.025   | 0.029  | 0.850  | 8.71E-05   | 5.76E-05   | 1.41E-04   | 39   | 41  | 217   | 0.445                                 |
| 7 0, 0  | 0.939 0.675   | 0.023   | 0.022   | 0.012  | 1.825  | 7.54E-05   | 3.51E-05   | 4.01E-05   | 37   | 38  | 165   | 0.363                                 |
|   | 0.945 0.862   | 0.028   | 0.025   | 0.026  | 0.981  | 9.10E-05   | 4.46E-05   | 8.31E-05   | 44   | 41<br>53  | 226<br>238  | 0.529                                 |
| 1   | 0.958 0.923<br>0.964 0.907  | 0.034<br>0.048  | 0.031<br>0.043  | 0.034  | 0.914<br>1.327   | 1.09E-04   | 5.16E-05<br>9.67E-05   | 8.69E-05<br>1.02E-04   | 48<br>77   | 73  | 186   | 0.552                                 |
|   |   |   | 0.045<br>XXX  |  |  | 1.55E-04   |  |  |  |   |   | 0.469<br>XXX                          |
|   | xxx xxx<br>0.961 0.560  | 0.034   | 0.033   | 0.017  | xxx<br>1.994   | xxx<br>1.13E-04  | xxx<br>6.43E-05  | xxx<br>5.59E-05  | xxx<br>54  | xxx<br>55   | 169   | 0.286                                 |
|   | 0.940 0.505   | 0.034   | 0.033   | 0.017  | 0.986  | 7.78E-05   | 3.87E-05   | 7.90E-05   | 34   | 36  | 176   | 0.429                                 |
|   | 0.947 0.350   | 0.024   | 0.022   | 0.022  | 1.555  | 9.10E-05   | 6.61E-05   | 8.40E-05   | 44   | 42  | 130   | 0.429                                 |
|   | 0.961 0.497   | 0.027   | 0.027   | 0.017  | 3.870  | 1.18E-04   | 5.08E-05   | 3.23E-05   | 53   | 56  | 207   | 0.333                                 |
|   | 0.958 0.975   | 0.039   | 0.035   | 0.130  | 0.273  | 1.27E-04   | 6.04E-05   | 2.30E-04   | 59   | 58  | 255   | 0.553                                 |
|   | 0.954 0.783   | 0.033   | 0.030   | 0.130  | 1.265  | 1.05E-04   | 5.40E-05   | 6.71E-05   | 44   | 44  | 159   | 0.331                                 |
|   | 0.943 0.778   | 0.032   | 0.023   | 0.040  | 0.576  | 8.17E-05   | 4.02E-05   | 1.08E-04   | 38   | 37  | 242   | 0.505                                 |
|   | 0.947 0.430   | 0.023   | 0.026   | 0.016  | 1.647  | 9.02E-05   | 4.58E-05   | 5.88E-05   | 41   | 40  | 149   | 0.303                                 |
|   | 0.956 0.427   | 0.034   | 0.033   | 0.013  | 2.537  | 1.12E-04   | 4.28E-05   | 3.25E-05   | 51   | 54  | 160   | 0.466                                 |
|   | 0.957 0.872   | 0.033   | 0.030   | 0.029  | 1.038  | 1.06E-04   | 3.79E-05   | 5.32E-05   | 48   | 45  | 181   | 0.550                                 |
|   | xxx xxx   | XXX   | XXX   | XXX  | XXX  | XXX  | XXX  | XXX  | XXX  | XXX   | XXX   | XXX                                   |
|   | 0.957 0.902   | 0.034   | 0.031   | 0.038  | 0.814  | 1.09E-04   | 5.55E-05   | 8.43E-05   | 48   | 53  | 195   | 0.550                                 |
|   | 0.958 0.936   | 0.033   | 0.030   | 0.031  | 0.959  | 1.06E-04   | 5.32E-05   | 6.84E-05   | 48   | 46  | 197   | 0.553                                 |
|   | 0.948 0.909   | 0.029   | 0.026   | 0.050  | 0.519  | 9.30E-05   | 3.11E-05   | 7.75E-05   | 40   | 41  | 218   | 0.552                                 |
|   | 0.965 0.936   | 0.039   | 0.036   | 0.041  | 0.866  | 1.27E-04   | 6.34E-05   | 9.13E-05   | 58   | 57  | 146   | 0.551                                 |
|   | 0.943 0.928   | 0.025   | 0.023   | 0.026  | 0.857  | 8.03E-05   | 3.07E-05   | 4.27E-05   | 37   | 39  | 113   | 0.496                                 |
|   | 0.967 0.941   | 0.042   | 0.038   | 0.034  | 1.131  | 1.34E-04   | 6.36E-05   | 6.63E-05   | 59   | 57  | 118   | 0.550                                 |
|   | 0.966 0.962   | 0.041   | 0.038   | 0.064  | 0.590  | 1.34E-04   | 6.69E-05   | 1.33E-04   | 61   | 61  | 205   | 0.550                                 |
|   | 0.950 0.930   | 0.028   | 0.026   | 0.030  | 0.878  | 9.18E-05   | 3.64E-05   | 5.33E-05   | 43   | 43  | 212   | 0.554                                 |

| _  | 47)              | La               | 1 () 0               | I 0: 4                                 | Ct. O                |                      | r norm                         |                                  |                      |                                | norm                             |   |                      | I norm                         |                                  |                           | ,                                   |                                       |               |
|----|------------------|------------------|----------------------|--|----------------------|----------------------|--------------------------------|----------------------------------|----------------------|--------------------------------|----------------------------------|---|----------------------|--------------------------------|----------------------------------|---------------------------|-------------------------------------|---------------------------------------|---------------|
| 1  | $\mathbf{t}_q^r$ | Step             | 1 - Step 3           | Step 4                                 | - Step 8             |                      | $L_0^{norm}$                   |                                  |                      |                                | 1                                |   |                      | $L_2^{norm}$                   |                                  |                           | $L_{\infty}$                        |                                       |               |
| q  | p                | $\tilde{\tau}_c$ | Dominant<br>Category | $\tau_c$                               | Dominant<br>Category | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | $\mathcal{L}$ |
|    | 1                | $c_t$            | 0.5527               | $c_a$                                  | 0.3244               | 0.946                | 0.937                          | 0.949                            | 0.023                | 0.021                          | 0.063                            | 0.331   | 7.46E-05             | 3.75E-05                       | 1.34E-04                         | 36                        | 34                                  | 228                                   | 0.553         |
|    | 2                | $c_t$            | 0.5501               | $c_a$                                  | 0.4691               | 0.969                | 0.967                          | 0.937                            | 0.041                | 0.038                          | 0.036                            | 1.042   | 1.33E-04             | 5.25E-05                       | 6.31E-05                         | 58                        | 55                                  | 162                                   | 0.550         |
|    | 3                | $c_t$            | 0.5508               | $c_a$                                  | 0.5853               | 0.963                | 0.957                          | 0.952                            | 0.034                | 0.031                          | 0.052                            | 0.593   | 1.09E-04             | 5.13E-05                       | 1.07E-04                         | 55                        | 53                                  | 236                                   | 0.551         |
|    | 4                | $c_t$            | 0.5505               | $c_a$                                  | 0.2755               | 0.953                | 0.946                          | 0.913                            | 0.028                | 0.026                          | 0.035                            | 0.734   | 9.15E-05             | 4.32E-05                       | 7.94E-05                         | 40                        | 40                                  | 200                                   | 0.551         |
|    | 5                | $c_t$            | 0.5502               | $c_a$                                  | 0.1033               | 0.946                | 0.938                          | 0.888                            | 0.023                | 0.021                          | 0.027                            | 0.785   | 7.53E-05             | 3.09E-05                       | 7.51E-05                         | 34                        | 35                                  | 244                                   | 0.447         |
| 6  | 6                | $c_t$            | 0.5507               | $c_a$                                  | 0.493                | 0.954                | 0.950                          | 0.873                            | 0.028                | 0.025                          | 0.029                            | 0.872   | 8.91E-05             | 3.73E-05                       | 6.16E-05                         | 40                        | 41                                  | 161                                   | 0.551         |
|    | 7                | $c_t$            | 0.5504               | $c \neq c_a, c_t$                      | 0.3622               | 0.963                | 0.959                          | 0.929                            | 0.028                | 0.023                          | 0.040                            | 0.780   | 1.10E-04             | 1.74E-05                       | 2.92E-05                         | 57                        | 58                                  | 220                                   | 0.550         |
|    | 8                | $c_t$            | 0.5505               | $c \neq c_a, c_t$ $c_a$                | 0.2631               | 0.963                | 0.949                          | 0.898                            | 0.035                | 0.031                          | 0.043                            | 0.740   | 1.15E-04             | 5.77E-05                       | 1.23E-04                         | 55                        | 53                                  | 255                                   | 0.551         |
|    | 9                | $c_t$            | 0.5503               | $c \neq c_a, c_t$                      | 0.2031               | 0.943                | 0.934                          | 0.943                            | 0.033                | 0.032                          | 0.049                            | 0.412   | 7.16E-05             | 8.28E-06                       | 2.52E-05                         | 35                        | 35                                  | 205                                   | 0.550         |
|    | 10               | $c_t$            | 0.5509               | $c_{+}c_{a}, c_{t}$                    | 0.5704               | 0.966                | 0.959                          | 0.907                            | 0.022                | 0.020                          | 0.043                            | 0.523   | 1.25E-04             | 5.91E-05                       | 1.48E-04                         | 55                        | 56                                  | 221                                   | 0.551         |
| -  | 1                | $c_t$            | 0.5503               | $c_a$                                  | 0.5644               | 0.959                | 0.943                          | 0.969                            | 0.032                | 0.033                          | 0.007                            | 0.385   | 1.03E-04             | 4.80E-05                       | 1.46E-04                         | 49                        | 49                                  | 228                                   | 0.550         |
|    | 2                | $c_t$            | 0.5506               |  | 0.3718               | 0.965                | 0.943                          | 0.930                            | 0.032                | 0.028                          | 0.073                            | 0.365   | 1.03E-04<br>1.17E-04 | 1.72E-05                       | 2.85E-05                         | 56                        | 58                                  | 222                                   | 0.551         |
|    | 3                | -                | 0.5508               | $c_a$                                  | 0.6149               | 0.965                | 0.955                          | 0.930                            | 0.030                | 0.033                          | 0.039                            | 0.631   | 1.17E-04<br>1.00E-04 | 5.04E-05                       | 1.09E-04                         | 48                        | 46                                  | 237                                   | 0.551         |
|    | 4                | $c_t$            | 0.5506<br>XXX        | c <sub>a</sub>                         | 0.0149<br>XXX        | 0.959<br>XXX         | 0.955<br>XXX                   | 0.946<br>XXX                     | XXX                  | 0.026<br>XXX                   | XXX                              | XXX   | XXX                  | 3.04E-03<br>XXX                | XXX                              | XXX                       | XXX                                 | XXX                                   | 0.551<br>XXX  |
|    | 5                | _                | 0.5515               | $c_a$                                  | 0.9771               | 0.959                | 0.953                          | 0.917                            | 0.030                | 0.027                          | 0.027                            | 0.986   | 9.57E-05             | 3.74E-05                       | 5.46E-05                         | 47                        | 48                                  | 209                                   | 0.552         |
| 7  | 6                | $c_t$            | 0.5513               | $c_a$                                  | 0.9771               | 0.959                | 0.959                          | 0.917                            | 0.030                | 0.027                          | 0.027                            | 0.980   | 1.20E-04             | 5.66E-05                       | 9.50E-05                         | 59                        | 60                                  | 212                                   | 0.552         |
|    | 7                | $c_t$            | 0.5502               |  | 0.3366               | 0.955                | 0.952                          | 0.894                            | 0.037                | 0.034                          | 0.045                            | 0.743   | 9.29E-05             | 4.00E-05                       | 5.06E-05                         | 42                        | 44                                  | 190                                   | 0.550         |
|    | 8                | -                | 0.5504               | c <sub>a</sub>                         | 0.0946               | 0.967                | 0.964                          | 0.950                            | 0.023                | 0.020                          | 0.040                            | 0.850   | 1.21E-04             | 5.52E-05                       | 8.20E-05                         | 54                        | 58                                  | 199                                   | 0.456         |
|    | 9                | $c_t$            | 0.5504               | $c_t$                                  | 0.0940               | 0.965                | 0.960                          | 0.950                            | 0.037                | 0.034                          | 0.040                            | 0.608   | 1.21E-04<br>1.21E-04 | 5.26E-05                       | 1.02E-04                         | 54                        | 55                                  | 199                                   | 0.450         |
|    | 10               | $c_t$            | 0.5504               | c <sub>a</sub>                         | 0.8089               | 0.956                | 0.950                          | 0.932                            | 0.037                | 0.034                          | 0.030                            | 0.909   | 9.09E-05             | 4.54E-05                       | 6.73E-05                         | 42                        | 41                                  | 174                                   | 0.551         |
| -  | 1                | _                | 0.5503               | c <sub>a</sub>                         | 0.3065               | 0.955                | 0.951                          | 0.921                            | 0.028                | 0.026                          | 0.028                            | 0.847   | 9.03E-05<br>9.17E-05 | 4.61E-05                       | 7.51E-05                         | 50                        | 48                                  | 178                                   | 0.384         |
|    | 2                | $c_t$            | 0.5541               | $c_t$                                  | 0.1000               | 0.956                | 0.951                          | 0.908                            | 0.028                | 0.026                          | 0.031                            | 1.122   | 9.17E-05<br>9.27E-05 | 3.54E-05                       | 5.20E-05                         | 41                        | 43                                  | 239                                   | 0.554         |
|    | 3                | $c_t$            | 0.5506               | $c \neq c_a, c_t$                      | 0.089                | 0.962                | 0.952                          | 0.909                            | 0.029                | 0.020                          | 0.025                            | 0.813   | 9.27E-03<br>1.09E-04 | 4.74E-05                       | 8.39E-05                         | 72                        | 74                                  | 197                                   | 0.554         |
|    | 4                | $c_t$            | 0.5503               | $c \neq c_a, c_t$                      | 0.1003               | 0.962                | 0.935                          | 0.922                            | 0.034                | 0.031                          | 0.038                            | 1.155   | 6.84E-05             | 4.74E-05<br>4.53E-05           | 6.48E-05                         | 29                        | 27                                  | 122                                   | 0.331         |
|    | 5                | $c_t$            | 0.5512               | $c \neq c_a, c_t$                      | 0.3271               | 0.941                | 0.955                          | 0.767                            | 0.021                | 0.019                          | 0.017                            | 1.155   | 1.06E-04             | 4.55E-05<br>5.35E-05           | 7.37E-05                         | 58                        | 52                                  | 163                                   | 0.485         |
| 8  | 6                | $c_t$            | 0.5512               | $c \neq c_a, c_t$                      | 0.3044               | 0.901                | 0.937                          | 0.698                            | 0.033                | 0.030                          | 0.029                            | 1.174   | 8.76E-05             | 4.51E-05                       | 6.14E-05                         | 44                        | 42                                  | 215                                   | 0.351         |
|    | 7                | $c_t$            | 0.5511               | $c \neq c_a, c_t$                      | 0.124                | 0.944                | 0.943                          | 0.038                            | 0.027                | 0.023                          | 0.022                            | 0.468   | 1.10E-04             | 5.16E-05                       | 1.30E-04                         | 56                        | 58                                  | 222                                   | 0.451         |
|    | 8                | $c_t$            | 0.5511               | $c \neq c_a, c_t$                      | 0.7054               | 0.955                | 0.933                          | 0.962                            | 0.034                | 0.031                          | 0.003                            | 0.400   | 8.86E-05             | 4.21E-05                       | 1.01E-04                         | 39                        | 38                                  | 224                                   | 0.551         |
|    | 9                | -                | 0.5511               | c <sub>a</sub>                         | 0.7054               | 0.961                | 0.956                          | 0.865                            | 0.027                | 0.025                          | 0.031                            | 0.491   | 1.07E-04             | 6.97E-05                       | 1.01E-04<br>1.19E-04             | 52                        | 48                                  | 200                                   | 0.552         |
|    | 10               | $c_t$            | 0.5537               | $c_a$ $c \neq c_a, c_t$                | 0.3904               | 0.961                | 0.953                          | 0.905                            | 0.030                | 0.030                          | 0.057                            | 0.822   | 9.72E-05             | 4.63E-05                       | 1.19E-04<br>1.15E-04             | 50                        | 49                                  | 187                                   | 0.554         |
|    | 10               | XXX              | 0.5557<br>XXX        | $c \neq c_a, c_t$                      | XXX                  | XXX                  | 0.955<br>XXX                   | 0.905<br>XXX                     | XXX                  | 0.026<br>XXX                   | XXX                              | XXX   | 9.72E-03<br>XXX      | 4.03E-03                       | XXX                              | XXX                       | XXX                                 | XXX                                   | 0.554<br>XXX  |
|    | 2                | _                | 0.5508               |  | 0.3995               | 0.946                | 0.939                          | 0.919                            | 0.023                | 0.021                          | 0.032                            | 0.641   | 7.36E-05             | 3.49E-05                       | 7.51E-05                         | 32                        | 32                                  | 197                                   | 0.503         |
|    | 3                | $c_t$            | 0.5512               | $c \neq c_a, c_t$                      | 0.3953               | 0.940                | 0.959                          | 0.866                            | 0.023                | 0.021                          | 0.032                            | 1.400   | 9.32E-05             | 3.49E-05<br>3.53E-05           | 3.63E-05                         | 42                        | 43                                  | 120                                   | 0.354         |
|    | 4                | XXX              | XXX                  | c <sub>a</sub>                         | 0.5451<br>XXX        | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | 3.32E-03             | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX           |
|    | 5                |                  | 0.5518               | $c_t$                                  | 0.1439               | 0.910                | 0.889                          | 0.892                            | 0.014                | 0.013                          | 0.042                            | 0.308   | 4.65E-05             | 2.66E-05                       | 1.32E-04                         | 21                        | 24                                  | 235                                   | 0.408         |
| 9  | 6                | $c_t$            | XXX                  | XXX                                    | 0.1433<br>XXX        | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX           |
|    | 7                | $c_t$            | 0.551                | $c \neq c_a, c_t$                      | 0.2011               | 0.952                | 0.943                          | 0.973                            | 0.026                | 0.023                          | 0.069                            | 0.341   | 8.35E-05             | 1.93E-05                       | 6.63E-05                         | 41                        | 41                                  | 201                                   | 0.551         |
|    | 8                | $c_t$            | 0.5521               | $c_{\neq} c_{a}, c_{t}$                | 0.9589               | 0.924                | 0.912                          | 0.969                            | 0.026                | 0.025                          | 0.064                            | 0.236   | 5.34E-05             | 2.53E-05                       | 1.45E-04                         | 23                        | 22                                  | 247                                   | 0.552         |
|    | 9                | $c_t$            | 0.5512               | $c_a$                                  | 0.3155               | 0.947                | 0.941                          | 0.880                            | 0.010                | 0.013                          | 0.024                            | 0.230   | 7.88E-05             | 3.74E-05                       | 6.76E-05                         | 39                        | 39                                  | 237                                   | 0.332         |
|    | 10               | $c_t$            | 0.5505               | $c_a$                                  | 0.2941               | 0.951                | 0.947                          | 0.783                            | 0.024                | 0.022                          | 0.024                            | 1.899   | 9.05E-05             | 4.27E-05                       | 3.55E-05                         | 43                        | 41                                  | 161                                   | 0.256         |
| -  | 1                | _                | 0.5506               | $c \neq c_a, c_t$                      | 0.1846               | 0.954                | 0.951                          | 0.820                            | 0.027                | 0.025                          | 0.013                            | 1.536   | 8.95E-05             | 4.27E-05<br>4.25E-05           | 3.92E-05                         | 46                        | 44                                  | 145                                   | 0.390         |
|    | 2                | $c_t$            | 0.5500               | $c \neq c_a, c_t$ $c_a$                | 0.7668               | 0.962                | 0.954                          | 0.902                            | 0.026                | 0.023                          | 0.017                            | 0.413   | 1.16E-04             | 5.48E-05                       | 1.63E-04                         | 52                        | 58                                  | 247                                   | 0.550         |
|    | 3                | $c_t$            | 0.5509               | $c_a$                                  | 0.1959               | 0.954                | 0.951                          | 0.775                            | 0.030                | 0.033                          | 0.079                            | 0.413   | 9.41E-05             | 4.50E-05                       | 9.26E-05                         | 43                        | 40                                  | 234                                   | 0.551         |
|    | 4                | $c_t$            | 0.5531               | $c_a$                                  | 0.1939               | 0.954                | 0.951                          | 0.773                            | 0.029                | 0.027                          | 0.038                            | 0.710   | 9.41E-05<br>9.29E-05 | 4.67E-05                       | 1.04E-04                         | 45                        | 41                                  | 234                                   | 0.553         |
|    | 5                | $c_t$            | 0.5503               | $c_a$                                  | 0.1063               | 0.960                | 0.951                          | 0.895                            | 0.029                | 0.020                          | 0.043                            | 0.014   | 9.29E-03<br>1.10E-04 | 5.21E-05                       | 1.04E-04<br>1.48E-04             | 49                        | 49                                  | 245                                   | 0.550         |
| 10 | 6                | $c_t$            | 0.5500               | $c \neq c_a, c_t$                      | 0.3696               | 0.964                | 0.961                          | 0.899                            | 0.034                | 0.031                          | 0.074                            | 1.308   | 1.10E-04<br>1.14E-04 | 6.72E-05                       | 7.67E-05                         | 51                        | 51                                  | 160                                   | 0.330         |
|    | 7                | XXX              | XXX                  | $c \neq c_a, c_t$                      | XXX                  | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX           |
|    | 8                | $c_t$            | 0.5503               | $c_a$                                  | 0.226                | 0.969                | 0.964                          | 0.964                            | 0.039                | 0.036                          | 0.056                            | 0.645   | 1.27E-04             | 5.53E-05                       | 1.02E-04                         | 66                        | 59                                  | 216                                   | 0.550         |
|    | 9                | $c_t$            | 0.5501               | $c \neq c_a, c_t$                      | 0.220                | 0.960                | 0.957                          | 0.867                            | 0.035                | 0.030                          | 0.059                            | 0.544   | 1.13E-04             | 5.41E-05                       | 1.02E-04<br>1.18E-04             | 57                        | 52                                  | 172                                   | 0.550         |
|    | 10               | $c_t$            | 0.5503               | $c \neq c_a, c_t$<br>$c \neq c_a, c_t$ | 0.2331               | 0.968                | 0.960                          | 0.960                            | 0.033                | 0.032                          | 0.063                            | 0.544   | 1.30E-04             | 6.52E-05                       | 1.37E-04                         | 67                        | 62                                  | 241                                   | 0.550         |
|    | 10               | U                | 0.0000               | $\downarrow c \neq c_a, c_t$           | 0.0101               | 0.300                | 0.500                          | 0.500                            | 0.040                | 0.000                          | 0.000                            | 0.011   | 1.001-04             | 0.0211-00                      | 1.0115-04                        | L 01                      | 1 02                                | 271                                   | 1 0.000       |

Table B2.: Combination  $(\rho, \lambda, \rho) = \text{L-L-N}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 25, 58)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (10, 25, 58)$ .

|    | $\mathcal{A}_{q}^{p}$ | Step             | o 1 - Step 3 | Step 4            | - Step 8 |            | $L_0^{norm}$        |                     |            | $L^{i}$             | norm                   |                                  |            | $L_2^{norm}$        |                     |                | $L_{\infty}$             |                          |               |
|----|-----------------------|------------------|--------------|-------------------|----------|------------|---------------------|---------------------|------------|---------------------|------------------------|----------------------------------|------------|---------------------|---------------------|----------------|--------------------------|--------------------------|---------------|
|    | T                     | ~                | Dominant     |                   | Dominant | r norm,adv | r norm,adv          | r norm,clean        | r norm,adv | norm,adv            | r norm,clean           | $L_{1,\mathcal{H}}^{norm,adv}$   | 7 norm,adv | r norm,adv          | r norm,clean        | r norm,adv     | r norm,adv               | r norm,clean             |               |
| q  | p                     | $\tilde{\tau}_c$ | Category     | $\tau_c$          | Category | $L_{0,R}$  | $L_{0,\mathcal{H}}$ | $L_{0,\mathcal{H}}$ | $L_{1,R}$  | $L_{1,\mathcal{H}}$ | $L_{1,\mathcal{H}}^{}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $L_{2,R}$  | $L_{2,\mathcal{H}}$ | $L_{2,\mathcal{H}}$ | $L_{\infty,R}$ | $L_{\infty,\mathcal{H}}$ | $L_{\infty,\mathcal{H}}$ | $\mathcal{L}$ |
|    | 1                     | $c_t$            | 0.5502       | $c_t$             | 0.4225   | 0.962      | 0.961               | 0.901               | 0.035      | 0.035               | 0.029                  | 1.218                            | 1.14E-04   | 9.01E-06            | 1.09E-05            | 53             | 53                       | 219                      | 0.128         |
|    | 2                     | $c_t$            | 0.5505       | $c_t$             | 0.4325   | 0.943      | 0.937               | 0.888               | 0.022      | 0.022               | 0.032                  | 0.680                            | 7.17E-05   | 1.44E-05            | 3.27E-05            | 33             | 33                       | 210                      | 0.118         |
|    | 3                     | $c_t$            | 0.5509       | $c_a$             | 0.3201   | 0.958      | 0.958               | 0.791               | 0.030      | 0.030               | 0.012                  | 2.577                            | 9.67E-05   | 8.06E-05            | 4.65E-05            | 44             | 44                       | 138                      | 0.462         |
|    | 4                     | $c_t$            | 0.5505       | $c_a$             | 0.4918   | 0.971      | 0.969               | 0.825               | 0.043      | 0.043               | 0.024                  | 1.740                            | 1.38E-04   | 7.30E-05            | 6.05E-05            | 64             | 64                       | 142                      | 0.446         |
| ١, | 5                     | $c_t$            | 0.5514       | $c_t$             | 0.2473   | 0.959      | 0.957               | 0.928               | 0.030      | 0.030               | 0.023                  | 1.299                            | 9.69E-05   | 4.99E-05            | 4.84E-05            | 44             | 44                       | 187                      | 0.304         |
| 1  | 6                     | $c_t$            | 0.2947  ge   | $c_a$             | 0.821    | 0.974      | 0.974               | 0.493               | 0.046      | 0.046               | 0.021                  | 2.215                            | 1.48E-04   | 7.98E-05            | 7.52E-05            | 68             | 68                       | 193                      | 0.272         |
|    | 7                     | $c_t$            | 0.5503       | $c_a$             | 0.335    | 0.967      | 0.962               | 0.834               | 0.040      | 0.039               | 0.038                  | 1.023                            | 1.29E-04   | 6.63E-05            | 1.05E-04            | 61             | 61                       | 231                      | 0.327         |
|    | 8                     | $c_t$            | 0.5507       | $c_t$             | 0.2787   | 0.957      | 0.938               | 0.777               | 0.036      | 0.035               | 0.045                  | 0.771                            | 1.18E-04   | 5.51E-05            | 1.09E-04            | 54             | 54                       | 227                      | 0.272         |
|    | 9                     | $c_t$            | 0.5505       | $c_a$             | 0.7593   | 0.964      | 0.961               | 0.938               | 0.035      | 0.034               | 0.036                  | 0.947                            | 1.12E-04   | 5.01E-05            | 8.24E-05            | 50             | 50                       | 238                      | 0.447         |
|    | 10                    | $c_t$            | 0.5500       | $c_t$             | 0.4368   | 0.958      | 0.956               | 0.924               | 0.030      | 0.030               | 0.034                  | 0.890                            | 9.73E-05   | 7.66E-06            | 1.30E-05            | 49             | 49                       | 193                      | 0.113         |
|    | 1                     | $c_t$            | 0.5523       | $c_t$             | 0.2766   | 0.932      | 0.930               | 0.892               | 0.018      | 0.018               | 0.033                  | 0.550                            | 5.89E-05   | 3.04E-05            | 7.76E-05            | 27             | 27                       | 201                      | 0.276         |
| 1  | 2                     | $c_t$            | 0.5512       | $c_t$             | 0.2923   | 0.928      | 0.928               | 0.916               | 0.017      | 0.017               | 0.024                  | 0.717                            | 5.61E-05   | 2.59E-05            | 4.56E-05            | 32             | 32                       | 172                      | 0.259         |
|    | 3                     | $c_t$            | 0.5507       | $c_t$             | 0.3764   | 0.938      | 0.938               | 0.948               | 0.020      | 0.020               | 0.035                  | 0.576                            | 6.51E-05   | 3.36E-05            | 6.93E-05            | 30             | 30                       | 159                      | 0.174         |
|    | 4                     | $c_t$            | 0.5508       | $c_a$             | 0.2826   | 0.946      | 0.946               | 0.857               | 0.023      | 0.023               | 0.016                  | 1.400                            | 7.48E-05   | 3.87E-05            | 3.92E-05            | 34             | 34                       | 157                      | 0.393         |
|    | 5                     | $c_t$            | 0.5511       | $c_t$             | 0.2145   | 0.958      | 0.956               | 0.660               | 0.029      | 0.029               | 0.026                  | 1.129                            | 9.55E-05   | 4.26E-05            | 5.91E-05            | 47             | 47                       | 240                      | 0.337         |
| 2  | 6                     | $c_t$            | 0.5515       | $c_t$             | 0.3449   | 0.951      | 0.948               | 0.913               | 0.026      | 0.026               | 0.033                  | 0.788                            | 8.52E-05   | 3.80E-05            | 7.35E-05            | 42             | 42                       | 223                      | 0.207         |
|    | 7                     | $c_t$            | 0.5505       | $c_t$             | 0.2185   | 0.943      | 0.942               | 0.747               | 0.023      | 0.023               | 0.012                  | 1.955                            | 7.62E-05   | 3.94E-05            | 3.22E-05            | 33             | 33                       | 156                      | 0.332         |
|    | 8                     | $c_t$            | 0.5513       | $c_t$             | 0.2607   | 0.948      | 0.948               | 0.790               | 0.024      | 0.024               | 0.013                  | 1.885                            | 7.85E-05   | 4.07E-05            | 3.57E-05            | 37             | 37                       | 139                      | 0.291         |
|    | 9                     | $c_t$            | 0.5510       | $c_t$             | 0.1962   | 0.943      | 0.942               | 0.826               | 0.022      | 0.022               | 0.017                  | 1.337                            | 7.20E-05   | 3.36E-05            | 3.70E-05            | 33             | 33                       | 184                      | 0.355         |
|    | 10                    | $c_t$            | 0.5507       | $c_a$             | 0.7599   | 0.940      | 0.935               | 0.893               | 0.021      | 0.021               | 0.030                  | 0.700                            | 6.97E-05   | 3.82E-05            | 8.10E-05            | 31             | 31                       | 204                      | 0.459         |
|    | 1                     | $c_t$            | 0.5508       | $c_a$             | 0.2937   | 0.960      | 0.954               | 0.849               | 0.033      | 0.033               | 0.027                  | 1.205                            | 1.08E-04   | 5.17E-05            | 6.66E-05            | 52             | 52                       | 179                      | 0.373         |
|    | 2                     | $c_t$            | 0.5503       | $c_a$             | 0.8143   | 0.955      | 0.951               | 0.724               | 0.028      | 0.028               | 0.020                  | 1.392                            | 9.16E-05   | 8.54E-05            | 1.38E-04            | 45             | 45                       | 255                      | 0.515         |
|    | 3                     | $c_t$            | 0.5501       | $c_t$             | 0.4498   | 0.962      | 0.961               | 0.879               | 0.032      | 0.032               | 0.018                  | 1.750                            | 1.04E-04   | 8.09E-06            | 7.83E-06            | 47             | 47                       | 183                      | 0.100         |
|    | 4                     | $c_t$            | 0.5511       | $c_a$             | 0.7447   | 0.940      | 0.931               | 0.846               | 0.024      | 0.023               | 0.025                  | 0.938                            | 7.75E-05   | 4.23E-05            | 8.11E-05            | 36             | 36                       | 215                      | 0.456         |
|    | 5                     | $c_t$            | 0.5504       | $c_a$             | 0.8624   | 0.960      | 0.953               | 0.797               | 0.037      | 0.036               | 0.028                  | 1.278                            | 1.21E-04   | 8.84E-05            | 9.87E-05            | 59             | 59                       | 128                      | 0.542         |
| 3  | 6                     | $c_t$            | 0.5518       | $c_a$             | 0.2267   | 0.952      | 0.948               | 0.751               | 0.027      | 0.027               | 0.029                  | 0.924                            | 8.71E-05   | 6.28E-05            | 1.41E-04            | 39             | 39                       | 217                      | 0.454         |
|    | 7                     | $c_t$            | 0.5511       | $c_t$             | 0.2715   | 0.938      | 0.935               | 0.675               | 0.023      | 0.023               | 0.012                  | 1.924                            | 7.54E-05   | 3.75E-05            | 4.01E-05            | 37             | 37                       | 165                      | 0.280         |
|    | 8                     | $c_t$            | 0.5502       | $c_a$             | 0.4706   | 0.953      | 0.949               | 0.862               | 0.028      | 0.027               | 0.026                  | 1.072                            | 9.10E-05   | 4.89E-05            | 8.31E-05            | 44             | 44                       | 226                      | 0.352         |
|    | 9                     | $c_t$            | 0.5515       | $c_a$             | 0.1816   | 0.963      | 0.962               | 0.923               | 0.034      | 0.034               | 0.034                  | 0.999                            | 1.09E-04   | 5.64E-05            | 8.69E-05            | 48             | 48                       | 238                      | 0.411         |
|    | 10                    | $c_t$            | 0.5501       | $c_a$             | 0.5267   | 0.972      | 0.965               | 0.907               | 0.048      | 0.047               | 0.033                  | 1.431                            | 1.55E-04   | 1.05E-04            | 1.02E-04            | 77             | 77                       | 186                      | 0.502         |
|    | 1                     | XXX              | xxx          | xxx               | XXX      | xxx        | xxx                 | xxx                 | xxx        | xxx                 | xxx                    | XXX                              | XXX        | xxx                 | xxx                 | xxx            | xxx                      | xxx                      | XXX           |
|    | 2                     | $c_t$            | 0.5502       | $c_t$             | 0.2428   | 0.952      | 0.951               | 0.560               | 0.034      | 0.034               | 0.017                  | 2.042                            | 1.13E-04   | 6.75E-05            | 5.59E-05            | 54             | 54                       | 169                      | 0.307         |
|    | 3                     | $c_t$            | 0.5506       | $c_t$             | 0.323    | 0.944      | 0.941               | 0.505               | 0.024      | 0.024               | 0.022                  | 1.060                            | 7.78E-05   | 4.18E-05            | 7.90E-05            | 34             | 34                       | 176                      | 0.228         |
|    | 4                     | $c_t$            | 0.5501       | $c_a$             | 0.3089   | 0.935      | 0.932               | 0.350               | 0.027      | 0.026               | 0.017                  | 1.538                            | 9.10E-05   | 6.83E-05            | 8.40E-05            | 44             | 44                       | 130                      | 0.418         |
| ١, | 5                     | $c_t$            | 0.5512       | $c_t$             | 0.1865   | 0.967      | 0.965               | 0.497               | 0.037      | 0.036               | 0.009                  | 4.228                            | 1.18E-04   | 5.54E-05            | 3.23E-05            | 53             | 53                       | 207                      | 0.365         |
| 4  | 6                     | $c_t$            | 0.5531       | $c \neq c_a, c_t$ | 0.1288   | 0.966      | 0.959               | 0.975               | 0.039      | 0.038               | 0.130                  | 0.297                            | 1.27E-04   | 6.56E-05            | 2.30E-04            | 59             | 59                       | 255                      | 0.480         |
|    | 7                     | $c_t$            | 0.5507       | $c_t$             | 0.1812   | 0.956      | 0.952               | 0.783               | 0.032      | 0.032               | 0.023                  | 1.357                            | 1.05E-04   | 5.84E-05            | 6.71E-05            | 44             | 44                       | 159                      | 0.369         |
|    | 8                     | $c_t$            | 0.5503       | $c_t$             | 0.2103   | 0.952      | 0.948               | 0.778               | 0.025      | 0.025               | 0.040                  | 0.626                            | 8.17E-05   | 4.35E-05            | 1.08E-04            | 38             | 38                       | 242                      | 0.340         |
|    | 9                     | $c_t$            | 0.5513       | $c_t$             | 0.2608   | 0.946      | 0.944               | 0.430               | 0.027      | 0.027               | 0.016                  | 1.720                            | 9.02E-05   | 4.85E-05            | 5.88E-05            | 41             | 41                       | 149                      | 0.290         |
|    | 10                    | $c_t$            | 0.5501       | $c_t$             | 0.2789   | 0.952      | 0.950               | 0.427               | 0.034      | 0.034               | 0.013                  | 2.622                            | 1.12E-04   | 4.52E-05            | 3.25E-05            | 51             | 51                       | 160                      | 0.271         |
|    | 1                     | $c_t$            | 0.5502       | $c_t$             | 0.3211   | 0.962      | 0.961               | 0.872               | 0.033      | 0.033               | 0.029                  | 1.131                            | 1.06E-04   | 4.13E-05            | 5.32E-05            | 48             | 48                       | 181                      | 0.229         |
|    | 2                     | XXX              | XXX          | XXX               | xxx      | xxx        | XXX                 | xxx                 | XXX        | XXX                 | xxx                    | xxx                              | XXX        | XXX                 | xxx                 | XXX            | XXX                      | xxx                      | XXX           |
| 1  | 3                     | $c_t$            | 0.5503       | $c \neq c_a, c_t$ | 0.1371   | 0.961      | 0.958               | 0.902               | 0.034      | 0.033               | 0.038                  | 0.874                            | 1.09E-04   | 5.99E-05            | 8.43E-05            | 48             | 48                       | 195                      | 0.439         |
|    | 4                     | $c_t$            | 0.5525       | $c \neq c_a, c_t$ | 0.1321   | 0.963      | 0.963               | 0.936               | 0.033      | 0.033               | 0.031                  | 1.049                            | 1.06E-04   | 5.81E-05            | 6.84E-05            | 48             | 48                       | 197                      | 0.503         |
| -  | 5                     | $c_t$            | 0.5516       | $c_t$             | 0.3543   | 0.957      | 0.952               | 0.909               | 0.029      | 0.028               | 0.050                  | 0.566                            | 9.30E-05   | 3.39E-05            | 7.75E-05            | 40             | 40                       | 218                      | 0.197         |
| 5  | 6                     | $c_t$            | 0.5506       | $c_t$             | 0.1466   | 0.969      | 0.969               | 0.936               | 0.039      | 0.039               | 0.041                  | 0.946                            | 1.27E-04   | 6.91E-05            | 9.13E-05            | 58             | 58                       | 146                      | 0.404         |
| İ  | 7                     | $c_t$            | 0.5513       | $c_t$             | 0.2902   | 0.949      | 0.947               | 0.928               | 0.025      | 0.025               | 0.026                  | 0.934                            | 8.03E-05   | 3.35E-05            | 4.27E-05            | 37             | 37                       | 113                      | 0.261         |
| 1  | 8                     | $c_t$            | 0.5503       | $c_a$             | 0.2659   | 0.971      | 0.971               | 0.941               | 0.042      | 0.042               | 0.034                  | 1.236                            | 1.34E-04   | 6.94E-05            | 6.63E-05            | 59             | 59                       | 118                      | 0.434         |
|    | 9                     | $c_t$            | 0.5501       | $c_t$             | 0.1394   | 0.970      | 0.969               | 0.962               | 0.041      | 0.041               | 0.064                  | 0.644                            | 1.34E-04   | 7.29E-05            | 1.33E-04            | 61             | 61                       | 205                      | 0.411         |
|    | 10                    | $c_t$            | 0.5541       | $c_t$             | 0.2567   | 0.955      | 0.954               | 0.930               | 0.028      | 0.028               | 0.030                  | 0.959                            | 9.18E-05   | 3.96E-05            | 5.33E-05            | 43             | 43                       | 212                      | 0.297         |
| _  |                       |                  |              |                   |          |            |                     |                     |            | •                   |                        |                                  |            |                     |                     |                |                          |                          |               |

| A      | $l_q^p$ | Step             | 1 - Step 3         | Step 4                  | - Step 8           |                      | $L_0^{norm}$                   |                        |                      | $L^{i}$                        | norm                             |   |                      | $L_2^{norm}$                   |                        |                           | $L_{\infty}$                        |                                       |       |
|--------|---------|------------------|--------------------|-------------------------|--------------------|----------------------|--------------------------------|------------------------|----------------------|--------------------------------|----------------------------------|---|----------------------|--------------------------------|------------------------|---------------------------|-------------------------------------|---------------------------------------|-------|
| q      | p       | $\tilde{\tau}_c$ | Dominant           | $\tau_c$                | Dominant           | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,H}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,H}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | L     |
| H      | 1       | $c_t$            | Category<br>0.5527 | $c_t$                   | Category<br>0.2162 | 0.946                | 0.941                          | 0.949                  | 0.023                | 0.023                          | 0.063                            | 0.361   | 7.46E-05             | 4.08E-05                       | 1.34E-04               | 36                        | 36                                  | 228                                   | 0.337 |
|        | 2       | $c_t$            | 0.5521             | $c_t$                   | 0.2102             | 0.969                | 0.969                          | 0.937                  | 0.023                | 0.023                          | 0.036                            | 1.137   | 1.33E-04             | 5.73E-05                       | 6.31E-05               | 58                        | 58                                  | 162                                   | 0.353 |
|        | 3       | $c_t$            | 0.5508             | $c_t$                   | 0.1875             | 0.963                | 0.961                          | 0.952                  | 0.034                | 0.033                          | 0.052                            | 0.647   | 1.09E-04             | 5.59E-05                       | 1.07E-04               | 55                        | 55                                  | 236                                   | 0.363 |
|        | 4       | $c_t$            | 0.5505             | $c_t$                   | 0.1735             | 0.953                | 0.950                          | 0.913                  | 0.034                | 0.033                          | 0.035                            | 0.800   | 9.15E-05             | 4.72E-05                       | 7.94E-05               | 40                        | 40                                  | 200                                   | 0.377 |
|        | 5       | $c_t$            | 0.5502             | $c_t$                   | 0.3772             | 0.946                | 0.943                          | 0.888                  | 0.023                | 0.023                          | 0.027                            | 0.854   | 7.53E-05             | 3.36E-05                       | 7.51E-05               | 34                        | 34                                  | 244                                   | 0.173 |
| 6      | 6       | $c_t$            | 0.5507             | $c \neq c_a, c_t$       | 0.102              | 0.954                | 0.953                          | 0.873                  | 0.028                | 0.027                          | 0.029                            | 0.951   | 8.91E-05             | 4.06E-05                       | 6.16E-05               | 40                        | 40                                  | 161                                   | 0.551 |
|        | 7       | $c_t$            | 0.5504             | $c_t$                   | 0.3671             | 0.963                | 0.963                          | 0.929                  | 0.034                | 0.034                          | 0.040                            | 0.848   | 1.10E-04             | 1.89E-05                       | 2.92E-05               | 57                        | 57                                  | 220                                   | 0.183 |
|        | 8       | $c_t$            | 0.5505             | $c_a$                   | 0.1464             | 0.963                | 0.951                          | 0.898                  | 0.035                | 0.035                          | 0.043                            | 0.802   | 1.15E-04             | 6.27E-05                       | 1.23E-04               | 55                        | 55                                  | 255                                   | 0.551 |
|        | 9       | $c_t$            | 0.5503             | $c_t$                   | 0.4011             | 0.943                | 0.938                          | 0.943                  | 0.022                | 0.022                          | 0.049                            | 0.448   | 7.16E-05             | 8.99E-06                       | 2.52E-05               | 35                        | 35                                  | 205                                   | 0.149 |
|        | 10      | $c_t$            | 0.5509             | $c_a$                   | 0.1437             | 0.966                | 0.962                          | 0.907                  | 0.038                | 0.038                          | 0.067                            | 0.565   | 1.25E-04             | 6.40E-05                       | 1.48E-04               | 55                        | 55                                  | 221                                   | 0.551 |
|        | 1       | $c_t$            | 0.5503             | $c_t$                   | 0.0895             | 0.959                | 0.945                          | 0.969                  | 0.032                | 0.031                          | 0.073                            | 0.418   | 1.03E-04             | 5.22E-05                       | 1.46E-04               | 49                        | 49                                  | 228                                   | 0.461 |
|        | 2       | $c_t$            | 0.5506             | $c_t$                   | 0.3482             | 0.965                | 0.964                          | 0.930                  | 0.036                | 0.036                          | 0.039                            | 0.923   | 1.17E-04             | 1.88E-05                       | 2.85E-05               | 56                        | 56                                  | 222                                   | 0.202 |
|        | 3       | $c_t$            | 0.5508             | $c_t$                   | 0.0967             | 0.959                | 0.957                          | 0.948                  | 0.031                | 0.031                          | 0.045                            | 0.687   | 1.00E-04             | 5.49E-05                       | 1.09E-04               | 48                        | 48                                  | 237                                   | 0.454 |
| İ      | 4       | XXX              | XXX                | XXX                     | XXX                | XXX                  | XXX                            | XXX                    | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                    | XXX                       | XXX                                 | XXX                                   | XXX   |
| 7      | 5       | $c_t$            | 0.5515             | $c_t$                   | 0.1644             | 0.959                | 0.958                          | 0.917                  | 0.030                | 0.030                          | 0.027                            | 1.076   | 9.57E-05             | 4.07E-05                       | 5.46E-05               | 47                        | 47                                  | 209                                   | 0.387 |
| 1      | 6       | $c_t$            | 0.5513             | $c_a$                   | 0.2765             | 0.966                | 0.962                          | 0.951                  | 0.037                | 0.037                          | 0.045                            | 0.810   | 1.20E-04             | 6.17E-05                       | 9.50E-05               | 59                        | 59                                  | 212                                   | 0.469 |
|        | 7       | $c_t$            | 0.5502             | $c_t$                   | 0.1809             | 0.955                | 0.954                          | 0.894                  | 0.029                | 0.029                          | 0.027                            | 1.060   | 9.29E-05             | 4.34E-05                       | 5.06E-05               | 42                        | 42                                  | 190                                   | 0.369 |
|        | 8       | $c_t$            | 0.5504             | $c_t$                   | 0.2361             | 0.967                | 0.967                          | 0.950                  | 0.037                | 0.037                          | 0.040                            | 0.930   | 1.21E-04             | 6.02E-05                       | 8.20E-05               | 54                        | 54                                  | 199                                   | 0.314 |
|        | 9       | $c_t$            | 0.5504             | $c_t$                   | 0.2025             | 0.965                | 0.961                          | 0.952                  | 0.037                | 0.037                          | 0.056                            | 0.657   | 1.21E-04             | 5.70E-05                       | 1.02E-04               | 54                        | 54                                  | 190                                   | 0.348 |
| İ      | 10      | $c_t$            | 0.5505             | $c_a$                   | 0.3881             | 0.956                | 0.955                          | 0.927                  | 0.028                | 0.028                          | 0.028                            | 0.990   | 9.09E-05             | 4.94E-05                       | 6.73E-05               | 42                        | 42                                  | 174                                   | 0.426 |
|        | 1       | $c_t$            | 0.5502             | $c_t$                   | 0.3017             | 0.955                | 0.954                          | 0.908                  | 0.028                | 0.028                          | 0.031                            | 0.923   | 9.17E-05             | 5.03E-05                       | 7.51E-05               | 50                        | 50                                  | 178                                   | 0.248 |
|        | 2       | $c_t$            | 0.5541             | $c_t$                   | 0.2231             | 0.956                | 0.955                          | 0.909                  | 0.029                | 0.029                          | 0.023                            | 1.223   | 9.27E-05             | 3.86E-05                       | 5.20E-05               | 41                        | 41                                  | 239                                   | 0.331 |
|        | 3       | $c_t$            | 0.5506             | $c_t$                   | 0.2882             | 0.962                | 0.959                          | 0.922                  | 0.034                | 0.033                          | 0.038                            | 0.883   | 1.09E-04             | 5.15E-05                       | 8.39E-05               | 72                        | 72                                  | 197                                   | 0.262 |
|        | 4       | $c_t$            | 0.5503             | $c \neq c_a, c_t$       | 0.2012             | 0.941                | 0.940                          | 0.767                  | 0.021                | 0.021                          | 0.017                            | 1.260   | 6.84E-05             | 4.93E-05                       | 6.48E-05               | 29                        | 29                                  | 122                                   | 0.461 |
| 8      | 5       | $c_t$            | 0.5512             | $c_t$                   | 0.1306             | 0.961                | 0.960                          | 0.884                  | 0.033                | 0.032                          | 0.029                            | 1.100   | 1.06E-04             | 5.78E-05                       | 7.37E-05               | 58                        | 58                                  | 163                                   | 0.421 |
| "      | 6       | $c_t$            | 0.5509             | $c_t$                   | 0.2527             | 0.944                | 0.939                          | 0.698                  | 0.027                | 0.026                          | 0.022                            | 1.227   | 8.76E-05             | 4.79E-05                       | 6.14E-05               | 44                        | 44                                  | 215                                   | 0.298 |
|        | 7       | $c_t$            | 0.5511             | $c_t$                   | 0.1413             | 0.962                | 0.956                          | 0.966                  | 0.034                | 0.033                          | 0.065                            | 0.510   | 1.10E-04             | 5.62E-05                       | 1.30E-04               | 56                        | 56                                  | 222                                   | 0.410 |
|        | 8       | $c_t$            | 0.5518             | $c_t$                   | 0.2215             | 0.955                | 0.952                          | 0.962                  | 0.027                | 0.027                          | 0.051                            | 0.534   | 8.86E-05             | 4.57E-05                       | 1.01E-04               | 39                        | 39                                  | 224                                   | 0.330 |
|        | 9       | $c_t$            | 0.5511             | $c_a$                   | 0.1062             | 0.961                | 0.958                          | 0.865                  | 0.033                | 0.033                          | 0.037                            | 0.890   | 1.07E-04             | 7.56E-05                       | 1.19E-04               | 52                        | 52                                  | 200                                   | 0.551 |
|        | 10      | $c_t$            | 0.5537             | $c_t$                   | 0.2214             | 0.957                | 0.956                          | 0.905                  | 0.030                | 0.030                          | 0.057                            | 0.525   | 9.72E-05             | 5.02E-05                       | 1.15E-04               | 50                        | 50                                  | 187                                   | 0.332 |
|        | 1       | XXX              | XXX                | XXX                     | XXX                | XXX                  | XXX                            | XXX                    | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                    | XXX                       | XXX                                 | XXX                                   | XXX   |
|        | 2       | $c_t$            | 0.5508             | $c \neq c_a, c_t$       | 0.3609             | 0.946                | 0.943                          | 0.919                  | 0.023                | 0.023                          | 0.032                            | 0.698   | 7.36E-05             | 3.80E-05                       | 7.51E-05               | 32                        | 32                                  | 197                                   | 0.344 |
|        | 3       | $c_t$            | 0.5512             | $c_t$                   | 0.3062             | 0.957                | 0.956                          | 0.866                  | 0.029                | 0.029                          | 0.019                            | 1.528   | 9.32E-05             | 3.84E-05                       | 3.63E-05               | 42                        | 42                                  | 120                                   | 0.245 |
|        | 4       | XXX              | XXX                | XXX                     | XXX                | XXX                  | XXX                            | XXX                    | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                    | XXX                       | XXX                                 | XXX                                   | XXX   |
| 9      | 5       | $c_t$            | 0.5518             | $c_t$                   | 0.324              | 0.910                | 0.893                          | 0.892                  | 0.014                | 0.014                          | 0.042                            | 0.334   | 4.65E-05             | 2.88E-05                       | 1.32E-04               | 21                        | 21                                  | 235                                   | 0.228 |
|        | 6       | XXX              | XXX                | XXX                     | XXX                | XXX                  | XXX                            | XXX                    | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                    | XXX                       | XXX                                 | XXX                                   | XXX   |
|        | 7       | $c_t$            | 0.5510             | $c_t$                   | 0.374              | 0.952                | 0.947                          | 0.973                  | 0.026                | 0.026                          | 0.069                            | 0.371   | 8.35E-05             | 2.10E-05                       | 6.63E-05               | 41                        | 41                                  | 201                                   | 0.177 |
|        | 8       | $c_t$            | 0.5521<br>0.5512   | $c_t$                   | 0.3988<br>0.2818   | 0.924<br>0.947       | 0.918<br>0.943                 | 0.969<br>0.880         | 0.016<br>0.024       | 0.016                          | 0.064<br>0.024                   | 0.256<br>1.016  | 5.34E-05<br>7.88E-05 | 2.74E-05                       | 1.45E-04<br>6.76E-05   | 23<br>39                  | 23<br>39                            | 247<br>237                            | 0.153 |
|        | 10      | $c_t$            | 0.5512             | $c_a$                   | 0.2818             | 0.947                | 0.943                          | 0.880                  | 0.024                | 0.024<br>0.027                 | 0.024                            | 2.073   | 9.05E-05             | 4.06E-05<br>4.68E-05           | 3.55E-05               | 43                        | 43                                  | 161                                   | 0.345 |
|        |         | $c_t$            | 0.5506             | $c_t$                   | 0.2145             | 0.951                | 0.950                          | 0.783                  | 0.027                | 0.027                          |                                  | 1.672   |                      |                                |                        | 43                        | 45                                  | 145                                   |       |
| -      | 1 2     | $c_t$            | 0.5500             | $c \neq c_a, c_t$       | 0.1335             | 0.954                | 0.954                          | 0.820                  | 0.028                | 0.028                          | 0.017<br>0.079                   | 0.445   | 8.95E-05<br>1.16E-04 | 4.63E-05<br>5.93E-05           | 3.92E-05<br>1.63E-04   | 52<br>52                  | 52                                  | 247                                   | 0.467 |
|        | 3       | $c_t$            | 0.5500             | $c \neq c_a, c_t$       | 0.1663             | 0.962                | 0.954                          | 0.902                  | 0.036                | 0.035                          | 0.079                            | 0.445   | 9.41E-05             | 5.93E-05<br>4.86E-05           | 9.26E-05               | 43                        | 43                                  | 234                                   | 0.464 |
|        | 4       | $c_t$            | 0.5531             | $c_t$                   | 0.0809             | 0.954                | 0.953                          | 0.775                  | 0.029                | 0.029                          | 0.038                            | 0.763   | 9.41E-05<br>9.29E-05 | 4.86E-05<br>5.09E-05           | 9.26E-05<br>1.04E-04   | 45                        | 45                                  | 234                                   | 0.410 |
|        | 5       | $c_t$            | 0.5503             | $c \neq c_a, c_t$ $c_t$ | 0.1470             | 0.960                | 0.954                          | 0.942                  | 0.029                | 0.029                          | 0.043                            | 0.670   | 9.29E-05<br>1.10E-04 | 5.61E-05                       | 1.04E-04<br>1.48E-04   | 49                        | 49                                  | 234                                   | 0.410 |
| 10     | 6       | $c_t$            | 0.5500             | $c \neq c_a, c_t$       | 0.2011             | 0.964                | 0.952                          | 0.899                  | 0.034                | 0.035                          | 0.074                            | 1.424   | 1.10E-04<br>1.14E-04 | 7.31E-05                       | 7.67E-05               | 51                        | 51                                  | 160                                   | 0.349 |
|        | 7       | XXX              | XXX                | $c \neq c_a, c_t$ xxx   | XXX                | 0.304<br>XXX         | 0.304<br>XXX                   | XXX                    | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                    | XXX                       | XXX                                 | XXX                                   | XXX   |
|        | 8       | $c_t$            | 0.5503             | $c_t$                   | 0.2144             | 0.969                | 0.968                          | 0.964                  | 0.039                | 0.039                          | 0.056                            | 0.703   | 1.27E-04             | 6.03E-05                       | 1.02E-04               | 66                        | 66                                  | 216                                   | 0.336 |
|        | 9       | $c_t$            | 0.5501             | $c_t$                   | 0.2144             | 0.960                | 0.957                          | 0.867                  | 0.035                | 0.035                          | 0.059                            | 0.703   | 1.13E-04             | 5.82E-05                       | 1.02E-04<br>1.18E-04   | 57                        | 57                                  | 172                                   | 0.356 |
|        | 10      | $c_t$            | 0.5503             | $c \neq c_a, c_t$       | 0.1405             | 0.968                | 0.962                          | 0.960                  | 0.033                | 0.033                          | 0.063                            | 0.628   | 1.30E-04             | 7.10E-05                       | 1.16E-04<br>1.37E-04   | 67                        | 67                                  | 241                                   | 0.350 |
| $\Box$ | 10      | $\sim$ t         | 0.0000             | $c \neq c_a, c_t$       | 0.1400             | 0.300                | 0.502                          | 0.500                  | 0.040                | 0.040                          | 0.000                            | 0.020   | 1.001-04             | 1.101-00                       | 1.0115-04              | 01                        | 01                                  | 271                                   | 0.400 |

Table B3.: Combination  $(\rho, \lambda, \rho) = \text{L-N-L}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 11, 23)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (59, 11, 23)$ .

| $\mathcal{A}_q^p$ | Ste             | p 1 - Step 3       | Step 4                       | - Step 8          |                      | $L_0^{norm}$                   |                        |                      | $L_1^r$                        | norm                             |   |                      | $L_2^{norm}$                   |                                  |                           | $L_{\infty}$                        |                                       |       |
|-------------------|-----------------|--------------------|------------------------------|-------------------|----------------------|--------------------------------|------------------------|----------------------|--------------------------------|----------------------------------|---|----------------------|--------------------------------|----------------------------------|---------------------------|-------------------------------------|---------------------------------------|-------|
| q p               | $\tilde{	au}_c$ | Dominant           | $\tau_c$                     | Dominant          | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,H}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | L     |
| 1                 | $c_t$           | Category<br>0.5504 | $c_a$                        | Category<br>0.616 | 0.965                | 0.962                          | 1.000                  | 0.037                | 0.034                          | 0.029                            | $\frac{L_{1,H}}{1.175}$   | 1.19E-04             | 8.70E-06                       | 1.03E-05                         | 53                        | 53                                  | 217                                   | 0.529 |
| 2                 | $c_t$           | 0.5519             | $c \neq c_a, c_t$            | 0.3144            | 0.948                | 0.940                          | 1.000                  | 0.037                | 0.034                          | 0.023                            | 0.700   | 8.07E-05             | 1.49E-05                       | 3.29E-05                         | 41                        | 42                                  | 227                                   | 0.409 |
| 3                 | $c_t$           | 0.5511             | $c_{\tau}$ $c_{a}$ , $c_{t}$ | 0.3165            | 0.957                | 0.954                          | 1.000                  | 0.030                | 0.027                          | 0.006                            | 4.619   | 9.56E-05             | 7.32E-05                       | 2.26E-05                         | 41                        | 41                                  | 56                                    | 0.235 |
| 4                 | $c_t$           | 0.5501             | $c_a$                        | 0.2669            | 0.970                | 0.968                          | 0.999                  | 0.043                | 0.040                          | 0.014                            | 2.734   | 1.40E-04             | 6.79E-05                       | 3.58E-05                         | 64                        | 68                                  | 127                                   | 0.385 |
| , 5               |                 | 0.5501             | $c_a$                        | 0.4416            | 0.958                | 0.953                          | 1.000                  | 0.030                | 0.027                          | 0.022                            | 1.247   | 9.70E-05             | 4.58E-05                       | 4.31E-05                         | 45                        | 45                                  | 140                                   | 0.466 |
| 1 6               | $c_t$           | 0.3507 ge          | $c_a$                        | 0.8674            | 0.974                | 0.971                          | 0.912                  | 0.048                | 0.043                          | 0.015                            | 2.972   | 1.53E-04             | 7.55E-05                       | 4.90E-05                         | 66                        | 67                                  | 160                                   | 0.326 |
| 7                 | $c_t$           | 0.5503             | $c_a$                        | 0.3487            | 0.968                | 0.964                          | 1.000                  | 0.043                | 0.039                          | 0.034                            | 1.149   | 1.38E-04             | 6.58E-05                       | 9.18E-05                         | 74                        | 67                                  | 259                                   | 0.477 |
| 8                 | $c_t$           | 0.5506             | $c_a$                        | 0.4767            | 0.957                | 0.948                          | 1.000                  | 0.036                | 0.034                          | 0.050                            | 0.678   | 1.19E-04             | 5.22E-05                       | 1.20E-04                         | 53                        | 54                                  | 263                                   | 0.551 |
| 9                 | $c_t$           | 0.5505             | $c_a$                        | 0.9943            | 0.965                | 0.960                          | 1.000                  | 0.037                | 0.033                          | 0.035                            | 0.942   | 1.18E-04             | 4.86E-05                       | 7.98E-05                         | 53                        | 53                                  | 242                                   | 0.550 |
| 10                |                 | 0.5504             | $c \neq c_a, c_t$            | 0.389             | 0.959                | 0.953                          | 1.000                  | 0.030                | 0.027                          | 0.031                            | 0.870   | 9.70E-05             | 7.00E-06                       | 1.07E-05                         | 48                        | 49                                  | 198                                   | 0.460 |
| 1                 | $c_t$           | 0.5504             | $c \neq c_a, c_t$            | 0.8499            | 0.928                | 0.922                          | 1.000                  | 0.017                | 0.016                          | 0.027                            | 0.592   | 5.67E-05             | 2.70E-05                       | 7.22E-05                         | 26                        | 30                                  | 208                                   | 0.538 |
| 2                 |                 | 0.5519             | $c_a$                        | 0.6437            | 0.929                | 0.921                          | 1.000                  | 0.017                | 0.016                          | 0.023                            | 0.672   | 5.53E-05             | 2.36E-05                       | 4.68E-05                         | 25                        | 25                                  | 215                                   | 0.536 |
| 3                 | $c_t$           | 0.5502             | $c_t$                        | 0.2026            | 0.940                | 0.935                          | 1.000                  | 0.021                | 0.019                          | 0.026                            | 0.727   | 6.80E-05             | 3.23E-05                       | 5.75E-05                         | 30                        | 31                                  | 153                                   | 0.348 |
| 4                 | $c_t$           | 0.5524             | $c_a$                        | 0.6941            | 0.945                | 0.939                          | 1.000                  | 0.022                | 0.021                          | 0.012                            | 1.673   | 7.25E-05             | 3.44E-05                       | 3.36E-05                         | 32                        | 30                                  | 125                                   | 0.540 |
| 5                 | $c_t$           | 0.5508             | $c \neq c_a, c_t$            | 0.2646            | 0.958                | 0.952                          | 1.000                  | 0.029                | 0.027                          | 0.026                            | 1.026   | 9.55E-05             | 3.92E-05                       | 6.20E-05                         | 43                        | 46                                  | 207                                   | 0.551 |
| 2 6               | $c_t$           | 0.5511             | $c_a$                        | 0.2045            | 0.952                | 0.946                          | 1.000                  | 0.027                | 0.024                          | 0.033                            | 0.733   | 8.67E-05             | 3.56E-05                       | 7.98E-05                         | 39                        | 41                                  | 256                                   | 0.388 |
| 7                 | $c_t$           | 0.5516             | $c_t$                        | 0.3282            | 0.946                | 0.943                          | 0.997                  | 0.024                | 0.022                          | 0.008                            | 2.941   | 7.85E-05             | 3.76E-05                       | 2.23E-05                         | 34                        | 36                                  | 98                                    | 0.223 |
| 8                 | $c_t$           | 0.5505             | $c_t$                        | 0.3085            | 0.951                | 0.946                          | 1.000                  | 0.025                | 0.023                          | 0.009                            | 2.550   | 8.17E-05             | 3.88E-05                       | 2.63E-05                         | 39                        | 37                                  | 114                                   | 0.242 |
| 9                 | $c_t$           | 0.5504             | $c_t$                        | 0.1607            | 0.943                | 0.938                          | 1.000                  | 0.022                | 0.021                          | 0.011                            | 1.842   | 7.26E-05             | 3.11E-05                       | 2.69E-05                         | 33                        | 32                                  | 107                                   | 0.390 |
| 10                |                 | 0.5511             | $c_a$                        | 0.9978            | 0.939                | 0.932                          | 1.000                  | 0.021                | 0.019                          | 0.026                            | 0.759   | 6.92E-05             | 3.48E-05                       | 7.04E-05                         | 33                        | 31                                  | 180                                   | 0.551 |
| 1                 | $c_t$           | 0.5507             | $c_a$                        | 0.3739            | 0.961                | 0.957                          | 1.000                  | 0.034                | 0.031                          | 0.019                            | 1.621   | 1.11E-04             | 4.92E-05                       | 4.43E-05                         | 56                        | 54                                  | 135                                   | 0.446 |
| 2                 | $c_t$           | 0.5504             | $c_t$                        | 0.3129            | 0.954                | 0.951                          | 0.999                  | 0.029                | 0.026                          | 0.011                            | 2.350   | 9.26E-05             | 7.96E-05                       | 7.61E-05                         | 47                        | 40                                  | 166                                   | 0.237 |
| 3                 | $c_t$           | 0.5500             | $c_a$                        | 0.3851            | 0.962                | 0.957                          | 1.000                  | 0.032                | 0.029                          | 0.017                            | 1.748   | 1.04E-04             | 7.40E-06                       | 6.85E-06                         | 47                        | 54                                  | 227                                   | 0.335 |
| 4                 | $c_t$           | 0.5503             | $c_a$                        | 0.9141            | 0.943                | 0.934                          | 1.000                  | 0.025                | 0.022                          | 0.019                            | 1.163   | 8.09E-05             | 4.04E-05                       | 6.27E-05                         | 37                        | 37                                  | 260                                   | 0.520 |
| 3 5               | $c_t$           | 0.5502             | $c_a$                        | 0.4545            | 0.961                | 0.956                          | 0.999                  | 0.037                | 0.034                          | 0.015                            | 2.221   | 1.21E-04             | 8.22E-05                       | 5.07E-05                         | 62                        | 58                                  | 86                                    | 0.334 |
| 3 6               | $c_t$           | 0.5509             | $c_a$                        | 0.2594            | 0.950                | 0.945                          | 1.000                  | 0.026                | 0.024                          | 0.018                            | 1.350   | 8.44E-05             | 5.59E-05                       | 7.80E-05                         | 42                        | 36                                  | 137                                   | 0.390 |
| 7                 | $c_t$           | 0.5518             | $c_a$                        | 0.8386            | 0.937                | 0.938                          | 0.997                  | 0.022                | 0.021                          | 0.010                            | 2.081   | 7.37E-05             | 3.43E-05                       | 3.61E-05                         | 32                        | 33                                  | 192                                   | 0.543 |
| 8                 | $c_t$           | 0.5503             | $c_a$                        | 0.9523            | 0.952                | 0.946                          | 1.000                  | 0.027                | 0.025                          | 0.022                            | 1.107   | 9.00E-05             | 4.41E-05                       | 6.76E-05                         | 40                        | 38                                  | 196                                   | 0.543 |
| 9                 | $c_t$           | 0.5500             | $c \neq c_a, c_t$            | 0.4469            | 0.964                | 0.959                          | 1.000                  | 0.034                | 0.031                          | 0.029                            | 1.071   | 1.11E-04             | 5.27E-05                       | 7.97E-05                         | 50                        | 51                                  | 289                                   | 0.535 |
| 10                | $c_t$           | 0.5505             | $c_a$                        | 0.3734            | 0.971                | 0.965                          | 1.000                  | 0.047                | 0.043                          | 0.021                            | 2.066   | 1.52E-04             | 9.48E-05                       | 6.20E-05                         | 70                        | 66                                  | 180                                   | 0.481 |
| 1                 | XXX             | XXX                | XXX                          | XXX               | XXX                  | XXX                            | XXX                    | XXX                  | XXX                            | XXX                              | xxx   | XXX                  | XXX                            | xxx                              | XXX                       | XXX                                 | XXX                                   | XXX   |
| 2                 | $c_t$           | 0.5503             | $c_t$                        | 0.4211            | 0.951                | 0.960                          | 0.952                  | 0.032                | 0.032                          | 0.010                            | 3.326   | 1.08E-04             | 6.14E-05                       | 3.12E-05                         | 53                        | 57                                  | 89                                    | 0.129 |
| 3                 | $c_t$           | 0.5501             | $c_t$                        | 0.2317            | 0.944                | 0.940                          | 0.997                  | 0.023                | 0.022                          | 0.019                            | 1.171   | 7.67E-05             | 3.82E-05                       | 7.20E-05                         | 35                        | 36                                  | 182                                   | 0.318 |
| 4                 | $c_t$           | 0.5501             | $c_a$                        | 0.667             | 0.939                | 0.951                          | 0.902                  | 0.028                | 0.028                          | 0.009                            | 3.024   | 9.56E-05             | 6.95E-05                       | 4.60E-05                         | 49                        | 45                                  | 73                                    | 0.486 |
| 4 5               | $c_t$           | 0.5507             | $c_t$                        | 0.2449            | 0.963                | 0.959                          | 1.000                  | 0.034                | 0.031                          | 0.007                            | 4.309   | 1.11E-04             | 4.77E-05                       | 2.48E-05                         | 50                        | 52                                  | 144                                   | 0.306 |
| 6                 | $c_t$           | 0.5503             | $c_a$                        | 0.9539            | 0.970                | 0.964                          | 1.000                  | 0.044                | 0.040                          | 0.137                            | 0.296   | 1.44E-04             | 6.89E-05                       | 2.65E-04                         | 71                        | 69                                  | 298                                   | 0.550 |
| 7                 | $c_t$           | 0.5513             | $c_a$                        | 0.1381            | 0.957                | 0.955                          | 0.999                  | 0.032                | 0.030                          | 0.015                            | 2.016   | 1.05E-04             | 5.40E-05                       | 4.22E-05                         | 51                        | 50                                  | 96                                    | 0.422 |
| 8                 | $c_t$           | 0.5504             | $c_a$                        | 0.2582            | 0.947                | 0.940                          | 1.000                  | 0.024                | 0.022                          | 0.038                            | 0.571   | 7.70E-05             | 3.79E-05                       | 1.14E-04                         | 40                        | 38                                  | 288                                   | 0.470 |
| 9                 | $c_t$           | 0.5512             | $c_a$                        | 0.5328            | 0.943                | 0.945                          | 0.966                  | 0.026                | 0.025                          | 0.013                            | 1.974   | 8.53E-05             | 4.33E-05                       | 5.04E-05                         | 42                        | 40                                  | 141                                   | 0.475 |
| 10                | -               | 0.5500             | $c_a$                        | 0.1564            | 0.952                | 0.955                          | 0.978                  | 0.033                | 0.032                          | 0.011                            | 2.937   | 1.09E-04             | 4.17E-05                       | 2.97E-05                         | 52                        | 50                                  | 174                                   | 0.450 |
| 1                 | $c_t$           | 0.551              | $c_a$                        | 0.5605            | 0.960                | 0.956                          | 1.000                  | 0.031                | 0.029                          | 0.028                            | 1.032   | 1.01E-04             | 3.63E-05                       | 5.54E-05                         | 46                        | 46                                  | 199                                   | 0.551 |
| 2                 |                 | XXX                | XXX                          | XXX               | XXX                  | XXX                            | XXX                    | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX   |
| 3                 | $c_t$           | 0.5506             | $c_a$                        | 0.9245            | 0.958                | 0.956                          | 0.996                  | 0.032                | 0.030                          | 0.034                            | 0.866   | 1.04E-04             | 5.31E-05                       | 8.37E-05                         | 51                        | 48                                  | 186                                   | 0.551 |
| 4                 | _               | 0.5510             | $c_a$                        | 0.1801            | 0.959                | 0.955                          | 1.000                  | 0.030                | 0.028                          | 0.025                            | 1.121   | 9.83E-05             | 4.94E-05                       | 5.66E-05                         | 49                        | 48                                  | 144                                   | 0.524 |
| 5 5               | $c_t$           | 0.5503             | $c \neq c_a, c_t$            | 0.4233            | 0.956                | 0.949                          | 1.000                  | 0.029                | 0.026                          | 0.054                            | 0.488   | 9.33E-05             | 3.12E-05                       | 8.85E-05                         | 42                        | 42                                  | 258                                   | 0.550 |
| 6                 | $c_t$           | 0.5505             | $c_a$                        | 0.7689            | 0.969                | 0.965                          | 1.000                  | 0.039                | 0.036                          | 0.041                            | 0.867   | 1.26E-04             | 6.32E-05                       | 1.00E-04                         | 58                        | 54                                  | 172                                   | 0.551 |
| 7                 | $c_t$           | 0.5501             | $c \neq c_a, c_t$            | 0.0537            | 0.949                | 0.944                          | 1.000                  | 0.025                | 0.023                          | 0.022                            | 1.050   | 8.13E-05             | 3.10E-05                       | 3.56E-05                         | 38                        | 39                                  | 107                                   | 0.516 |
| 8                 | $c_t$           | 0.5501             | $c_a$                        | 0.5562            | 0.970                | 0.966                          | 1.000                  | 0.041                | 0.037                          | 0.031                            | 1.186   | 1.32E-04             | 6.26E-05                       | 6.83E-05                         | 58                        | 60                                  | 153                                   | 0.550 |
| 9                 | $c_t$           | 0.5500             | $c_a$                        | 0.0835            | 0.970                | 0.967                          | 1.000                  | 0.042                | 0.038                          | 0.053                            | 0.710   | 1.34E-04             | 6.74E-05                       | 1.23E-04                         | 59<br>40                  | 57<br>41                            | 239<br>231                            | 0.550 |
| 10                | $c_t$           | 0.5518             | $c_a$                        | 0.8159            | 0.952                | 0.947                          | 1.000                  | 0.020                | 0.024                          | 0.031                            | 0.771   | 8.50E-05             | 3.37E-05                       | 5.78E-05                         | 40                        | 41                                  | 231                                   | 0.552 |

| 1  | $l_a^p$ | Step             | 1 - Step 3 | Step 4                                    | - Step 8 |            | $L_0^{norm}$        |                     |            | L!                  | norm                |   |            | $L_2^{norm}$        |                     |                | $L_{\infty}$             |                          |               |
|----|---------|------------------|------------|---|----------|------------|---------------------|---------------------|------------|---------------------|---------------------|---|------------|---------------------|---------------------|----------------|--------------------------|--------------------------|---------------|
|    | ľ       | _                | Dominant   | -   | Dominant | r norm,adv | r norm,adv          | τ norm,clean        | r norm,adv | r norm,adv          | r norm,clean        | $L_{1}^{norm,adv}$  | r norm,adv | r norm,adv          | τ norm,clean        | r norm,adv     | r norm,adv               | r norm,clean             | -             |
| q  | p       | $\tilde{\tau}_c$ | Category   | $	au_c$                                   | Category | $L_{0,R}$  | $L_{0,\mathcal{H}}$ | $L_{0,\mathcal{H}}$ | $L_{1,R}$  | $L_{1,\mathcal{H}}$ | $L_{1,\mathcal{H}}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}$  | $L_{2,\mathcal{H}}$ | $L_{2,\mathcal{H}}$ | $L_{\infty,R}$ | $L_{\infty,\mathcal{H}}$ | $L_{\infty,\mathcal{H}}$ | $\mathcal{L}$ |
|    | 1       | $c_t$            | 0.5532     | $c_a$                                     | 0.3832   | 0.944      | 0.935               | 1.000               | 0.022      | 0.020               | 0.059               | 0.342   | 7.12E-05   | 3.59E-05            | 1.31E-04            | 35             | 34                       | 198                      | 0.553         |
|    | 2       | $c_t$            | 0.5500     | $c_a$                                     | 0.2086   | 0.971      | 0.968               | 1.000               | 0.043      | 0.039               | 0.032               | 1.240   | 1.38E-04   | 5.45E-05            | 5.97E-05            | 62             | 65                       | 161                      | 0.550         |
|    | 3       | $c_t$            | 0.5501     | $c_a$                                     | 0.2026   | 0.965      | 0.962               | 1.000               | 0.037      | 0.034               | 0.051               | 0.662   | 1.21E-04   | 5.71E-05            | 1.10E-04            | 56             | 56                       | 228                      | 0.550         |
|    | 4       | $c_t$            | 0.5508     | $c_a$                                     | 0.3603   | 0.953      | 0.947               | 1.000               | 0.027      | 0.025               | 0.031               | 0.799   | 8.93E-05   | 4.22E-05            | 7.21E-05            | 40             | 41                       | 194                      | 0.551         |
| 6  | 5       | $c_t$            | 0.5509     | $c_t$                                     | 0.2139   | 0.947      | 0.941               | 1.000               | 0.024      | 0.022               | 0.022               | 0.982   | 7.81E-05   | 3.21E-05            | 6.65E-05            | 36             | 37                       | 300                      | 0.337         |
| "  | 6       | $c_t$            | 0.5510     | $c_a$                                     | 0.3909   | 0.956      | 0.952               | 1.000               | 0.029      | 0.026               | 0.026               | 1.013   | 9.27E-05   | 3.88E-05            | 5.52E-05            | 46             | 44                       | 139                      | 0.551         |
|    | 7       | $c_t$            | 0.5502     | $c \neq c_a, c_t$                         | 0.2458   | 0.964      | 0.960               | 1.000               | 0.034      | 0.032               | 0.038               | 0.833   | 1.11E-04   | 1.75E-05            | 2.95E-05            | 51             | 53                       | 196                      | 0.550         |
|    | 8       | $c_t$            | 0.5508     | $c_a$                                     | 0.6274   | 0.960      | 0.951               | 1.000               | 0.034      | 0.031               | 0.036               | 0.885   | 1.13E-04   | 5.64E-05            | 1.12E-04            | 54             | 51                       | 320                      | 0.551         |
|    | 9       | $c_t$            | 0.5501     | $c \neq c_a, c_t$                         | 0.1237   | 0.939      | 0.932               | 1.000               | 0.021      | 0.019               | 0.057               | 0.339   | 6.85E-05   | 7.93E-06            | 2.98E-05            | 32             | 33                       | 214                      | 0.550         |
|    | 10      | $c_t$            | 0.5505     | $c_a$                                     | 0.6966   | 0.964      | 0.959               | 0.997               | 0.037      | 0.034               | 0.062               | 0.549   | 1.21E-04   | 5.73E-05            | 1.49E-04            | 57             | 54                       | 284                      | 0.551         |
|    | 1       | $c_t$            | 0.5501     | $c_a$                                     | 0.7844   | 0.957      | 0.947               | 1.000               | 0.032      | 0.029               | 0.076               | 0.376   | 1.03E-04   | 4.84E-05            | 1.62E-04            | 48             | 47                       | 275                      | 0.550         |
|    | 2       | $c_t$            | 0.5519     | $c_a$                                     | 0.7073   | 0.965      | 0.960               | 1.000               | 0.036      | 0.032               | 0.037               | 0.887   | 1.15E-04   | 1.70E-05            | 2.92E-05            | 56             | 57                       | 236                      | 0.552         |
|    | 3       | $c_t$            | 0.5503     | $c_a$                                     | 0.611    | 0.961      | 0.956               | 1.000               | 0.032      | 0.029               | 0.034               | 0.844   | 1.03E-04   | 5.18E-05            | 8.90E-05            | 47             | 49                       | 238                      | 0.550         |
|    | 4       | $c_t$            | 0.5504     | $c_t$                                     | 0.1027   | 0.949      | 0.942               | 1.000               | 0.024      | 0.022               | 0.024               | 0.919   | 7.89E-05   | 4.47E-05            | 8.05E-05            | 35             | 37                       | 270                      | 0.448         |
| -  | 5       | $c_t$            | 0.5511     | $c_a$                                     | 0.2798   | 0.962      | 0.958               | 1.000               | 0.033      | 0.030               | 0.025               | 1.209   | 1.06E-04   | 4.12E-05            | 5.22E-05            | 48             | 49                       | 167                      | 0.551         |
| 7  | 6       | $c_t$            | 0.5519     | $c_a$                                     | 0.9747   | 0.962      | 0.956               | 1.000               | 0.033      | 0.030               | 0.041               | 0.736   | 1.07E-04   | 5.07E-05            | 9.03E-05            | 52             | 47                       | 238                      | 0.552         |
|    | 7       | $c_t$            | 0.5501     | $c \neq c_a, c_t$                         | 0.0952   | 0.956      | 0.953               | 0.995               | 0.030      | 0.027               | 0.028               | 0.984   | 9.59E-05   | 4.13E-05            | 5.24E-05            | 48             | 48                       | 212                      | 0.550         |
|    | 8       | $c_t$            | 0.5507     | $c_t$                                     | 0.0977   | 0.964      | 0.961               | 1.000               | 0.036      | 0.033               | 0.029               | 1.121   | 1.15E-04   | 5.25E-05            | 5.68E-05            | 54             | 52                       | 105                      | 0.453         |
|    | 9       | $c_t$            | 0.5501     | $c_a$                                     | 0.9592   | 0.968      | 0.964               | 1.000               | 0.040      | 0.037               | 0.062               | 0.602   | 1.31E-04   | 5.70E-05            | 1.19E-04            | 60             | 63                       | 237                      | 0.550         |
|    | 10      | $c_t$            | 0.5504     | $c_a$                                     | 0.9786   | 0.952      | 0.948               | 1.000               | 0.027      | 0.025               | 0.025               | 0.993   | 8.66E-05   | 4.33E-05            | 6.33E-05            | 40             | 39                       | 210                      | 0.550         |
|    | 1       | $c_t$            | 0.5501     | $c_t$                                     | 0.1076   | 0.953      | 0.949               | 1.000               | 0.027      | 0.025               | 0.023               | 1.077   | 8.70E-05   | 4.38E-05            | 6.49E-05            | 37             | 37                       | 173                      | 0.442         |
|    | 2       | $c_t$            | 0.5515     | $c \neq c_a, c_t$                         | 0.1275   | 0.957      | 0.953               | 1.000               | 0.029      | 0.027               | 0.022               | 1.224   | 9.43E-05   | 3.60E-05            | 4.98E-05            | 39             | 41                       | 241                      | 0.552         |
|    | 3       | $c_t$            | 0.5517     | $c_a$                                     | 0.108    | 0.961      | 0.958               | 1.000               | 0.034      | 0.031               | 0.034               | 0.923   | 1.10E-04   | 4.80E-05            | 7.89E-05            | 51             | 54                       | 234                      | 0.481         |
|    | 4       | $c_t$            | 0.5501     | $c_t$                                     | 0.1814   | 0.946      | 0.940               | 1.000               | 0.023      | 0.021               | 0.010               | 2.009   | 7.46E-05   | 4.94E-05            | 4.16E-05            | 33             | 32                       | 103                      | 0.369         |
| _  | 5       | $c_t$            | 0.5510     | $c_t$                                     | 0.1393   | 0.961      | 0.958               | 1.000               | 0.033      | 0.031               | 0.022               | 1.424   | 1.08E-04   | 5.47E-05            | 5.77E-05            | 59             | 58                       | 148                      | 0.412         |
| 8  | 6       | $c_t$            | 0.5505     | $c \neq c_a, c_t$                         | 0.114    | 0.943      | 0.944               | 0.992               | 0.026      | 0.025               | 0.016               | 1.488   | 8.48E-05   | 4.38E-05            | 5.09E-05            | 39             | 39                       | 222                      | 0.551         |
|    | 7       | $c_t$            | 0.5505     | $c_a$                                     | 0.7127   | 0.961      | 0.954               | 1.000               | 0.033      | 0.030               | 0.068               | 0.445   | 1.08E-04   | 5.10E-05            | 1.47E-04            | 58             | 59                       | 257                      | 0.551         |
|    | 8       | $c_t$            | 0.5533     | $c \neq c_a, c_t$                         | 0.1481   | 0.955      | 0.951               | 1.000               | 0.028      | 0.026               | 0.046               | 0.568   | 9.25E-05   | 4.40E-05            | 9.44E-05            | 45             | 45                       | 191                      | 0.490         |
|    | 9       | $c_t$            | 0.5520     | $c_a$                                     | 0.1177   | 0.962      | 0.959               | 1.000               | 0.035      | 0.032               | 0.024               | 1.350   | 1.12E-04   | 7.31E-05            | 8.04E-05            | 54             | 53                       | 131                      | 0.531         |
|    | 10      | $c_t$            | 0.5520     | $c_a$                                     | 0.5762   | 0.952      | 0.948               | 1.000               | 0.027      | 0.024               | 0.058               | 0.420   | 8.60E-05   | 4.10E-05            | 1.27E-04            | 43             | 38                       | 227                      | 0.552         |
|    | 1       | XXX              | XXX        | xxx                                       | XXX      | XXX        | XXX                 | XXX                 | XXX        | XXX                 | XXX                 | XXX   | XXX        | XXX                 | XXX                 | xxx            | XXX                      | XXX                      | XXX           |
|    | 2       | $c_t$            | 0.5507     | $c \neq c_a, c_t$                         | 0.1465   | 0.943      | 0.937               | 1.000               | 0.022      | 0.020               | 0.031               | 0.653   | 7.08E-05   | 3.36E-05            | 7.31E-05            | 35             | 33                       | 183                      | 0.551         |
|    | 3       | $c_t$            | 0.5500     | $c_a$                                     | 0.4201   | 0.955      | 0.952               | 1.000               | 0.028      | 0.026               | 0.014               | 1.871   | 9.13E-05   | 3.46E-05            | 2.81E-05            | 44             | 41                       | 160                      | 0.360         |
|    | 4       | XXX              | XXX        | XXX                                       | XXX      | XXX        | XXX                 | XXX                 | XXX        | XXX                 | XXX                 | XXX   | XXX        | XXX                 | XXX                 | xxx            | XXX                      | XXX                      | XXX           |
| _  | 5       | XXX              | XXX        | XXX                                       | XXX      | XXX        | XXX                 | XXX                 | XXX        | XXX                 | XXX                 | XXX   | XXX        | XXX                 | XXX                 | XXX            | XXX                      | XXX                      | XXX           |
| 9  | 6       | XXX              | XXX        | XXX                                       | XXX      | XXX        | XXX                 | XXX                 | XXX        | XXX                 | XXX                 | XXX   | XXX        | XXX                 | XXX                 | XXX            | XXX                      | XXX                      | XXX           |
|    | 7       | XXX              | XXX        | XXX                                       | XXX      | XXX        | XXX                 | XXX                 | XXX        | XXX                 | XXX                 | XXX   | XXX        | XXX                 | XXX                 | XXX            | XXX                      | XXX                      | XXX           |
|    | 8       | $c_t$            | 0.5508     | $c_a$                                     | 0.802    | 0.942      | 0.933               | 1.000               | 0.022      | 0.020               | 0.064               | 0.307   | 7.00E-05   | 3.31E-05            | 1.55E-04            | 34             | 32                       | 302                      | 0.481         |
|    | 9       | $c_t$            | 0.5509     | $c_a$                                     | 0.725    | 0.944      | 0.939               | 1.000               | 0.023      | 0.021               | 0.022               | 0.958   | 7.49E-05   | 3.56E-05            | 6.29E-05            | 34             | 36                       | 277                      | 0.551         |
|    | 10      | $c_t$            | 0.5512     | $c \neq c_a, c_t$                         | 0.1736   | 0.947      | 0.944               | 1.000               | 0.026      | 0.023               | 0.011               | 2.093   | 8.44E-05   | 3.98E-05            | 2.98E-05            | 43             | 44                       | 218                      | 0.474         |
|    | 1       | $c_t$            | 0.5512     | $c \neq c_a, c_t$                         | 0.2176   | 0.954      | 0.950               | 1.000               | 0.027      | 0.025               | 0.013               | 1.915   | 8.76E-05   | 4.17E-05            | 3.18E-05            | 41             | 42                       | 120                      | 0.459         |
|    | 2       | $c_t$            | 0.5509     | $c_{\tau} c_{a}, c_{t}$                   | 0.4947   | 0.960      | 0.954               | 0.999               | 0.034      | 0.031               | 0.079               | 0.397   | 1.12E-04   | 5.29E-05            | 1.75E-04            | 57             | 65                       | 305                      | 0.551         |
|    | 3       | $c_t$            | 0.5503     | $c_a$                                     | 0.1602   | 0.955      | 0.954               | 0.987               | 0.031      | 0.029               | 0.039               | 0.739   | 9.99E-05   | 4.78E-05            | 1.00E-04            | 45             | 46                       | 255                      | 0.390         |
|    | 4       | $c_t$            | 0.5500     | $c \neq c_a, c_t$                         | 0.2394   | 0.956      | 0.951               | 1.000               | 0.028      | 0.026               | 0.037               | 0.696   | 9.10E-05   | 4.58E-05            | 9.02E-05            | 42             | 41                       | 212                      | 0.550         |
|    | 5       | $c_t$            | 0.5510     | $c \neq c_a, c_t$ $c_a$                   | 0.3088   | 0.963      | 0.956               | 0.989               | 0.028      | 0.020               | 0.037               | 0.463   | 1.22E-04   | 5.81E-05            | 1.60E-04            | 54             | 60                       | 252                      | 0.551         |
| 10 | 6       | $c_t$            | 0.5506     | $c \neq c_a, c_t$                         | 0.4585   | 0.965      | 0.962               | 1.000               | 0.035      | 0.033               | 0.014               | 2.313   | 1.14E-04   | 6.76E-05            | 4.41E-05            | 52             | 50                       | 127                      | 0.403         |
|    | 7       | $c_t$            | 0.5515     | $c \neq c_a, c_t$ $c \neq c_a, c_t$       | 0.1027   | 0.957      | 0.949               | 1.000               | 0.030      | 0.028               | 0.055               | 0.501   | 9.88E-05   | 4.67E-05            | 1.31E-04            | 45             | 49                       | 227                      | 0.489         |
|    | 8       | $c_t$            | 0.5509     | $c \neq c_a, c_t$<br>$c \neq c_a, c_t$    | 0.4068   | 0.970      | 0.967               | 1.000               | 0.030      | 0.028               | 0.051               | 0.751   | 1.35E-04   | 5.88E-05            | 9.62E-05            | 65             | 63                       | 184                      | 0.507         |
|    | 9       | $c_t$            | 0.5504     | $c \neq c_a, c_t$ $c_a$                   | 0.4000   | 0.959      | 0.958               | 0.980               | 0.042      | 0.033               | 0.061               | 0.731   | 1.15E-04   | 5.50E-05            | 1.31E-04            | 56             | 59                       | 221                      | 0.550         |
|    | 10      | $c_t$            | 0.5504     | $c \neq c_a, c_t$                         | 0.3258   | 0.968      | 0.961               | 1.000               | 0.033      | 0.038               | 0.059               | 0.637   | 1.34E-04   | 6.72E-05            | 1.36E-04            | 58             | 59                       | 243                      | 0.550         |
|    | 1.0     |                  | 0.0002     | $rac{1}{rac} = rac{1}{rac} = rac{1}{rac}$ | 0.0200   | 0.500      | 0.001               | 1.000               | 0.011      | 0.000               | 0.000               | 0.001   | 1.011.04   | J.12E 00            | 1.001 04            | 1 00           |                          | 210                      | 1 3.000       |

Table B4.: Combination  $(\rho, \lambda, \rho) = \text{N-L-L}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 21, 56)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (16, 21, 56)$ .

|   | $A_q^p$ | Step            | o 1 - Step 3        | Step 4                  | - Step 8           |                   | $L_0^{norm}$      |                                   |                   | $L_1^r$           | norm                             |   |                       | $L_2^{norm}$          |                       |                     | $L_{\infty}$                |                              |               |
|---|---------|-----------------|---------------------|-------------------------|--------------------|-------------------|-------------------|-----------------------------------|-------------------|-------------------|----------------------------------|---|-----------------------|-----------------------|-----------------------|---------------------|-----------------------------|------------------------------|---------------|
| q | p       | $\tilde{	au}_c$ | Dominant            | $\tau_c$                | Dominant           | Lnorm,adv         | r norm,adv        | L'norm,clean                      | L'norm,adv        | L'norm,adv        | $L_{1,\mathcal{H}}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | L'norm,adv            | L'norm,adv            | L'norm,clean          | $L^{norm,adv}$      | L <sup>norm,adv</sup>       | L <sup>norm,clean</sup>      | $\mathcal{L}$ |
| 1 | 1       | $c_t$           | Category<br>0.5502  | $c_a$                   | Category<br>0.9144 | $L_{0,R} = 0.962$ | $L_{0,H} = 0.961$ | $\frac{L_{0,\mathcal{H}}}{0.901}$ | $L_{1,R} = 0.035$ | $L_{1,H} = 0.035$ | 0.029                            | $L_{1,\mathcal{H}}^{norm,elean}$ $1.218$                              | $L_{2,R}$<br>1.14E-04 | $L_{2,H}$<br>9.01E-06 | $L_{2,H}$<br>1.09E-05 | $L_{\infty,R}$ $53$ | $L_{\infty,\mathcal{H}}$ 53 | $L_{\infty,\mathcal{H}}$ 219 | 0.546         |
|   | 2       | $c_t$           | 0.5505              | $c \neq c_a, c_t$       | 0.4347             | 0.943             | 0.937             | 0.888                             | 0.033             | 0.033             | 0.032                            | 0.680   | 7.17E-05              | 1.44E-05              | 3.27E-05              | 33                  | 33                          | 210                          | 0.519         |
|   | 3       | $c_t$           | 0.5509              | $c \neq c_a, c_t$ $c_a$ | 0.7448             | 0.958             | 0.958             | 0.791                             | 0.022             | 0.022             | 0.032                            | 2.577   | 9.67E-05              | 8.06E-05              | 4.65E-05              | 44                  | 44                          | 138                          | 0.513         |
|   | 4       | $c_t$           | 0.5505              |                         | 0.9092             | 0.971             | 0.969             | 0.825                             | 0.043             | 0.043             | 0.012                            | 1.740   | 1.38E-04              | 7.30E-05              | 6.05E-05              | 64                  | 64                          | 142                          | 0.551         |
|   | 5       | $c_t$           | 0.5514              | $c_a$                   | 0.4875             | 0.971             | 0.957             | 0.928                             | 0.043             | 0.043             | 0.024                            | 1.740   | 9.69E-05              | 4.99E-05              | 4.84E-05              | 44                  | 44                          | 187                          | 0.527         |
| 1 | 6       | $c_t$           | 0.2947 ge           | $c_a$                   | 0.4873             | 0.939             | 0.974             | 0.493                             | 0.030             | 0.030             | 0.023                            | 2.215   | 1.48E-04              | 7.98E-05              | 7.52E-05              | 68                  | 68                          | 193                          | 0.327         |
|   | 7       | $c_t$           | 0.2547 ge<br>0.5503 | $c_a$                   | 0.9215             | 0.967             | 0.962             | 0.433                             | 0.040             | 0.040             | 0.021                            | 1.023   | 1.48E-04<br>1.29E-04  | 6.63E-05              | 1.05E-04              | 61                  | 61                          | 231                          | 0.293         |
|   | 8       | $c_t$           | 0.5507              | $c_a$                   | 0.9334             | 0.957             | 0.938             | 0.777                             | 0.046             | 0.035             | 0.045                            | 0.771   | 1.18E-04              | 5.51E-05              | 1.09E-04              | 54                  | 54                          | 227                          | 0.545         |
|   | 9       | $c_t$           | 0.5505              | $c_a$                   | 0.9995             | 0.964             | 0.961             | 0.938                             | 0.035             | 0.034             | 0.036                            | 0.947   | 1.10E-04<br>1.12E-04  | 5.01E-05              | 8.24E-05              | 50                  | 50                          | 238                          | 0.550         |
|   | 10      | $c_t$           | 0.5500              | $c_a$                   | 0.8054             | 0.958             | 0.956             | 0.924                             | 0.030             | 0.030             | 0.034                            | 0.890   | 9.73E-05              | 7.66E-06              | 1.30E-05              | 49                  | 49                          | 193                          | 0.517         |
|   | 1       | $c_t$           | 0.5523              | $c \neq c_a, c_t$       | 0.4651             | 0.932             | 0.930             | 0.892                             | 0.018             | 0.018             | 0.033                            | 0.550   | 5.89E-05              | 3.04E-05              | 7.76E-05              | 27                  | 27                          | 201                          | 0.552         |
|   | 2       | $c_t$           | 0.5512              | $c_{\tau} c_{a}, c_{t}$ | 0.8367             | 0.928             | 0.928             | 0.916                             | 0.017             | 0.017             | 0.024                            | 0.717   | 5.61E-05              | 2.59E-05              | 4.56E-05              | 32                  | 32                          | 172                          | 0.551         |
|   | 3       | $c_t$           | 0.5507              | $c_a$                   | 0.3793             | 0.938             | 0.938             | 0.948                             | 0.020             | 0.020             | 0.035                            | 0.576   | 6.51E-05              | 3.36E-05              | 6.93E-05              | 30                  | 30                          | 159                          | 0.551         |
|   | 4       | $c_t$           | 0.5508              | $c_a$                   | 0.762              | 0.946             | 0.946             | 0.857                             | 0.023             | 0.023             | 0.016                            | 1.400   | 7.48E-05              | 3.87E-05              | 3.92E-05              | 34                  | 34                          | 157                          | 0.526         |
|   | 5       | $c_t$           | 0.5511              | $c_a$                   | 0.3514             | 0.958             | 0.956             | 0.660                             | 0.029             | 0.029             | 0.026                            | 1.129   | 9.55E-05              | 4.26E-05              | 5.91E-05              | 47                  | 47                          | 240                          | 0.551         |
| 2 | 6       | $c_t$           | 0.5515              | $c_a$                   | 0.9272             | 0.951             | 0.948             | 0.913                             | 0.025             | 0.025             | 0.033                            | 0.788   | 8.52E-05              | 3.80E-05              | 7.35E-05              | 42                  | 42                          | 223                          | 0.542         |
|   | 7       | $c_t$           | 0.5505              | $c \neq c_a, c_t$       | 0.2301             | 0.943             | 0.942             | 0.747                             | 0.023             | 0.023             | 0.012                            | 1.955   | 7.62E-05              | 3.94E-05              | 3.22E-05              | 33                  | 33                          | 156                          | 0.449         |
|   | 8       | $c_t$           | 0.5513              | $c_a$                   | 0.3276             | 0.948             | 0.948             | 0.790                             | 0.024             | 0.024             | 0.013                            | 1.885   | 7.85E-05              | 4.07E-05              | 3.57E-05              | 37                  | 37                          | 139                          | 0.446         |
|   | 9       | $c_t$           | 0.5510              | $c_t$                   | 0.249              | 0.943             | 0.942             | 0.826                             | 0.022             | 0.022             | 0.017                            | 1.337   | 7.20E-05              | 3.36E-05              | 3.70E-05              | 33                  | 33                          | 184                          | 0.302         |
|   | 10      | $c_t$           | 0.5507              | $c_a$                   | 0.8532             | 0.940             | 0.935             | 0.893                             | 0.021             | 0.021             | 0.030                            | 0.700   | 6.97E-05              | 3.82E-05              | 8.10E-05              | 31                  | 31                          | 204                          | 0.542         |
|   | 1       | $c_t$           | 0.5508              | $c_a$                   | 0.8559             | 0.960             | 0.954             | 0.849                             | 0.033             | 0.033             | 0.027                            | 1.205   | 1.08E-04              | 5.17E-05              | 6.66E-05              | 52                  | 52                          | 179                          | 0.543         |
|   | 2       | $c_t$           | 0.5503              | $c_a$                   | 0.9836             | 0.955             | 0.951             | 0.724                             | 0.028             | 0.028             | 0.020                            | 1.392   | 9.16E-05              | 8.54E-05              | 1.38E-04              | 45                  | 45                          | 255                          | 0.550         |
|   | 3       | $c_t$           | 0.5501              | $c_a$                   | 0.5412             | 0.962             | 0.961             | 0.879                             | 0.032             | 0.032             | 0.018                            | 1.750   | 1.04E-04              | 8.09E-06              | 7.83E-06              | 47                  | 47                          | 183                          | 0.428         |
|   | 4       | $c_t$           | 0.5511              | $c_a$                   | 0.9992             | 0.940             | 0.931             | 0.846                             | 0.024             | 0.023             | 0.025                            | 0.938   | 7.75E-05              | 4.23E-05              | 8.11E-05              | 36                  | 36                          | 215                          | 0.551         |
|   | 5       | $c_t$           | 0.5504              | $c_a$                   | 0.9669             | 0.960             | 0.953             | 0.797                             | 0.037             | 0.036             | 0.028                            | 1.278   | 1.21E-04              | 8.84E-05              | 9.87E-05              | 59                  | 59                          | 128                          | 0.550         |
| 3 | 6       | $c_t$           | 0.5518              | $c_a$                   | 0.1776             | 0.952             | 0.948             | 0.751                             | 0.027             | 0.027             | 0.029                            | 0.924   | 8.71E-05              | 6.28E-05              | 1.41E-04              | 39                  | 39                          | 217                          | 0.552         |
|   | 7       | $c_t$           | 0.5511              | $c_a$                   | 0.3701             | 0.938             | 0.935             | 0.675                             | 0.023             | 0.023             | 0.012                            | 1.924   | 7.54E-05              | 3.75E-05              | 4.01E-05              | 37                  | 37                          | 165                          | 0.484         |
|   | 8       | $c_t$           | 0.5502              | $c_a$                   | 0.9689             | 0.953             | 0.949             | 0.862                             | 0.028             | 0.027             | 0.026                            | 1.072   | 9.10E-05              | 4.89E-05              | 8.31E-05              | 44                  | 44                          | 226                          | 0.547         |
|   | 9       | $c_t$           | 0.5515              | $c_a$                   | 0.5288             | 0.963             | 0.962             | 0.923                             | 0.034             | 0.034             | 0.034                            | 0.999   | 1.09E-04              | 5.64E-05              | 8.69E-05              | 48                  | 48                          | 238                          | 0.552         |
|   | 10      | $c_t$           | 0.5501              | $c_a$                   | 0.9244             | 0.972             | 0.965             | 0.907                             | 0.048             | 0.047             | 0.033                            | 1.431   | 1.55E-04              | 1.05E-04              | 1.02E-04              | 77                  | 77                          | 186                          | 0.550         |
|   | 1       | XXX             | XXX                 | XXX                     | XXX                | XXX               | XXX               | XXX                               | XXX               | XXX               | XXX                              | XXX   | XXX                   | XXX                   | XXX                   | xxx                 | xxx                         | XXX                          | XXX           |
|   | 2       | $c_t$           | 0.5502              | $c_a$                   | 0.1122             | 0.952             | 0.951             | 0.560                             | 0.034             | 0.034             | 0.017                            | 2.042   | 1.13E-04              | 6.75E-05              | 5.59E-05              | 54                  | 54                          | 169                          | 0.523         |
|   | 3       | $c_t$           | 0.5506              | $c_a$                   | 0.774              | 0.944             | 0.941             | 0.505                             | 0.024             | 0.024             | 0.022                            | 1.060   | 7.78E-05              | 4.18E-05              | 7.90E-05              | 34                  | 34                          | 176                          | 0.551         |
|   | 4       | $c_t$           | 0.5501              | $c_a$                   | 0.9895             | 0.935             | 0.932             | 0.350                             | 0.027             | 0.026             | 0.017                            | 1.538   | 9.10E-05              | 6.83E-05              | 8.40E-05              | 44                  | 44                          | 130                          | 0.550         |
| 4 | 5       | $c_t$           | 0.5512              | $c_t$                   | 0.1498             | 0.967             | 0.965             | 0.497                             | 0.037             | 0.036             | 0.009                            | 4.228   | 1.18E-04              | 5.54E-05              | 3.23E-05              | 53                  | 53                          | 207                          | 0.401         |
| 4 | 6       | $c_t$           | 0.5531              | $c_a$                   | 0.949              | 0.966             | 0.959             | 0.975                             | 0.039             | 0.038             | 0.130                            | 0.297   | 1.27E-04              | 6.56E-05              | 2.30E-04              | 59                  | 59                          | 255                          | 0.553         |
|   | 7       | $c_t$           | 0.5507              | $c_a$                   | 0.4775             | 0.956             | 0.952             | 0.783                             | 0.032             | 0.032             | 0.023                            | 1.357   | 1.05E-04              | 5.84E-05              | 6.71E-05              | 44                  | 44                          | 159                          | 0.551         |
|   | 8       | $c_t$           | 0.5503              | $c_a$                   | 0.5207             | 0.952             | 0.948             | 0.778                             | 0.025             | 0.025             | 0.040                            | 0.626   | 8.17E-05              | 4.35E-05              | 1.08E-04              | 38                  | 38                          | 242                          | 0.550         |
|   | 9       | $c_t$           | 0.5513              | $c \neq c_a, c_t$       | 0.2242             | 0.946             | 0.944             | 0.430                             | 0.027             | 0.027             | 0.016                            | 1.720   | 9.02E-05              | 4.85E-05              | 5.88E-05              | 41                  | 41                          | 149                          | 0.551         |
|   | 10      | $c_t$           | 0.5501              | $c_a$                   | 0.3023             | 0.952             | 0.950             | 0.427                             | 0.034             | 0.034             | 0.013                            | 2.622   | 1.12E-04              | 4.52E-05              | 3.25E-05              | 51                  | 51                          | 160                          | 0.384         |
|   | 1       | $c_t$           | 0.5502              | $c_a$                   | 0.5789             | 0.962             | 0.961             | 0.872                             | 0.033             | 0.033             | 0.029                            | 1.131   | 1.06E-04              | 4.13E-05              | 5.32E-05              | 48                  | 48                          | 181                          | 0.550         |
|   | 2       | XXX             | XXX                 | XXX                     | XXX                | XXX               | XXX               | XXX                               | XXX               | XXX               | XXX                              | XXX   | XXX                   | XXX                   | XXX                   | XXX                 | XXX                         | XXX                          | XXX           |
|   | 3       | $c_t$           | 0.5503              | $c_a$                   | 0.452              | 0.961             | 0.958             | 0.902                             | 0.034             | 0.033             | 0.038                            | 0.874   | 1.09E-04              | 5.99E-05              | 8.43E-05              | 48                  | 48                          | 195                          | 0.550         |
|   | 4       | $c_t$           | 0.5525              | $c_a$                   | 0.3127             | 0.963             | 0.963             | 0.936                             | 0.033             | 0.033             | 0.031                            | 1.049   | 1.06E-04              | 5.81E-05              | 6.84E-05              | 48                  | 48                          | 197                          | 0.553         |
| 5 | 5       | $c_t$           | 0.5516              | $c \neq c_a, c_t$       | 0.5806             | 0.957             | 0.952             | 0.909                             | 0.029             | 0.028             | 0.050                            | 0.566   | 9.30E-05              | 3.39E-05              | 7.75E-05              | 40                  | 40                          | 218                          | 0.552         |
| " | 6       | $c_t$           | 0.5506              | $c \neq c_a, c_t$       | 0.2655             | 0.969             | 0.969             | 0.936                             | 0.039             | 0.039             | 0.041                            | 0.946   | 1.27E-04              | 6.91E-05              | 9.13E-05              | 58                  | 58                          | 146                          | 0.551         |
|   | 7       | $c_t$           | 0.5513              | $c \neq c_a, c_t$       | 0.1417             | 0.949             | 0.947             | 0.928                             | 0.025             | 0.025             | 0.026                            | 0.934   | 8.03E-05              | 3.35E-05              | 4.27E-05              | 37                  | 37                          | 113                          | 0.484         |
|   | 8       | $c_t$           | 0.5503              | $c_a$                   | 0.9007             | 0.971             | 0.971             | 0.941                             | 0.042             | 0.042             | 0.034                            | 1.236   | 1.34E-04              | 6.94E-05              | 6.63E-05              | 59                  | 59                          | 118                          | 0.550         |
|   | 9       | $c_t$           | 0.5501              | $c \neq c_a, c_t$       | 0.3178             | 0.970             | 0.969             | 0.962                             | 0.041             | 0.041             | 0.064                            | 0.644   | 1.34E-04              | 7.29E-05              | 1.33E-04              | 61                  | 61                          | 205                          | 0.550         |
|   | 10      | $c_t$           | 0.5541              | $c_a$                   | 0.5077             | 0.955             | 0.954             | 0.930                             | 0.028             | 0.028             | 0.030                            | 0.959   | 9.18E-05              | 3.96E-05              | 5.33E-05              | 43                  | 43                          | 212                          | 0.554         |

|    | A D               | Lou              | 1 Ct 9               | L Cu 4            | Ct 0                 |                      | T norm                         |                                  |                      | 7.0                            | norm                             |   |                      | Inorm                          |                                  |                           | 7                                   |                                       |       |
|----|-------------------|------------------|----------------------|-------------------|----------------------|----------------------|--------------------------------|----------------------------------|----------------------|--------------------------------|----------------------------------|---|----------------------|--------------------------------|----------------------------------|---------------------------|-------------------------------------|---------------------------------------|-------|
|    | $\mathcal{A}_q^p$ | Step             | 1 - Step 3           | Step 4            | - Step 8             |                      | $L_0^{norm}$                   |                                  |                      | $L_1^{\prime}$                 | ioi iii                          |   |                      | $L_2^{norm}$                   |                                  |                           | $L_{\infty}$                        |                                       |       |
| q  | p                 | $\tilde{\tau}_c$ | Dominant<br>Category | $\tau_c$          | Dominant<br>Category | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | L     |
|    | 1                 | $c_t$            | 0.5527               | $c_a$             | 0.5426               | 0.946                | 0.941                          | 0.949                            | 0.023                | 0.023                          | 0.063                            | 0.361   | 7.46E-05             | 4.08E-05                       | 1.34E-04                         | 36                        | 36                                  | 228                                   | 0.553 |
|    | 2                 | $c_t$            | 0.5501               | $c_a$             | 0.4744               | 0.969                | 0.969                          | 0.937                            | 0.041                | 0.041                          | 0.036                            | 1.137   | 1.33E-04             | 5.73E-05                       | 6.31E-05                         | 58                        | 58                                  | 162                                   | 0.550 |
|    | 3                 | $c_t$            | 0.5508               | $c_a$             | 0.6324               | 0.963                | 0.961                          | 0.952                            | 0.034                | 0.033                          | 0.052                            | 0.647   | 1.09E-04             | 5.59E-05                       | 1.07E-04                         | 55                        | 55                                  | 236                                   | 0.551 |
|    | 4                 | $c_t$            | 0.5505               | $c_a$             | 0.3366               | 0.953                | 0.950                          | 0.913                            | 0.028                | 0.028                          | 0.035                            | 0.800   | 9.15E-05             | 4.72E-05                       | 7.94E-05                         | 40                        | 40                                  | 200                                   | 0.551 |
|    | 5                 | $c_t$            | 0.5502               | $c_t$             | 0.1002               | 0.946                | 0.943                          | 0.888                            | 0.023                | 0.023                          | 0.027                            | 0.854   | 7.53E-05             | 3.36E-05                       | 7.51E-05                         | 34                        | 34                                  | 244                                   | 0.450 |
| 6  | 6                 | $c_t$            | 0.5507               | $c_a$             | 0.6549               | 0.954                | 0.953                          | 0.873                            | 0.028                | 0.027                          | 0.029                            | 0.951   | 8.91E-05             | 4.06E-05                       | 6.16E-05                         | 40                        | 40                                  | 161                                   | 0.551 |
|    | 7                 | $c_t$            | 0.5504               | $c \neq c_a, c_t$ | 0.3764               | 0.963                | 0.963                          | 0.929                            | 0.034                | 0.034                          | 0.040                            | 0.848   | 1.10E-04             | 1.89E-05                       | 2.92E-05                         | 57                        | 57                                  | 220                                   | 0.550 |
|    | 8                 | $c_t$            | 0.5505               | $c_a$             | 0.2895               | 0.963                | 0.951                          | 0.898                            | 0.035                | 0.035                          | 0.043                            | 0.802   | 1.15E-04             | 6.27E-05                       | 1.23E-04                         | 55                        | 55                                  | 255                                   | 0.551 |
|    | 9                 | $c_t$            | 0.5503               | $c \neq c_a, c_t$ | 0.0764               | 0.943                | 0.938                          | 0.943                            | 0.022                | 0.022                          | 0.049                            | 0.448   | 7.16E-05             | 8.99E-06                       | 2.52E-05                         | 35                        | 35                                  | 205                                   | 0.550 |
|    | 10                | $c_t$            | 0.5509               | $c_{\tau}$        | 0.6553               | 0.966                | 0.962                          | 0.907                            | 0.038                | 0.038                          | 0.067                            | 0.565   | 1.25E-04             | 6.40E-05                       | 1.48E-04                         | 55                        | 55                                  | 221                                   | 0.551 |
| -  | 1                 | $c_t$            | 0.5503               | $c_a$             | 0.485                | 0.959                | 0.945                          | 0.969                            | 0.032                | 0.031                          | 0.073                            | 0.418   | 1.03E-04             | 5.22E-05                       | 1.46E-04                         | 49                        | 49                                  | 228                                   | 0.550 |
|    | 2                 | $c_t$            | 0.5506               | $c_a$             | 0.455                | 0.965                | 0.964                          | 0.930                            | 0.032                | 0.031                          | 0.039                            | 0.923   | 1.17E-04             | 1.88E-05                       | 2.85E-05                         | 56                        | 56                                  | 222                                   | 0.551 |
|    | 3                 | $c_t$            | 0.5508               | $c_a$             | 0.8267               | 0.959                | 0.957                          | 0.948                            | 0.030                | 0.031                          | 0.045                            | 0.687   | 1.00E-04             | 5.49E-05                       | 1.09E-04                         | 48                        | 48                                  | 237                                   | 0.551 |
|    | 4                 | XXX              | XXX                  | XXX               | XXX                  | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX   |
|    | 5                 | $c_t$            | 0.5515               | $c_a$             | 0.9772               | 0.959                | 0.958                          | 0.917                            | 0.030                | 0.030                          | 0.027                            | 1.076   | 9.57E-05             | 4.07E-05                       | 5.46E-05                         | 47                        | 47                                  | 209                                   | 0.552 |
| 7  | 6                 | $c_t$            | 0.5513               | $c_a$             | 0.9994               | 0.966                | 0.962                          | 0.951                            | 0.037                | 0.037                          | 0.045                            | 0.810   | 1.20E-04             | 6.17E-05                       | 9.50E-05                         | 59                        | 59                                  | 212                                   | 0.551 |
|    | 7                 | $c_t$            | 0.5502               | $c_a$             | 0.2751               | 0.955                | 0.954                          | 0.894                            | 0.029                | 0.029                          | 0.045                            | 1.060   | 9.29E-05             | 4.34E-05                       | 5.06E-05                         | 42                        | 42                                  | 190                                   | 0.550 |
|    | 8                 | $c_t$            | 0.5504               | $c_a$             | 0.2787               | 0.967                | 0.967                          | 0.950                            | 0.023                | 0.023                          | 0.040                            | 0.930   | 1.21E-04             | 6.02E-05                       | 8.20E-05                         | 54                        | 54                                  | 199                                   | 0.472 |
|    | 9                 | _                | 0.5504               |                   | 0.9895               | 0.965                | 0.961                          | 0.952                            | 0.037                | 0.037                          | 0.040                            | 0.657   | 1.21E-04<br>1.21E-04 | 5.70E-05                       | 1.02E-04                         | 54                        | 54                                  | 190                                   | 0.550 |
|    | 10                | $c_t$            | 0.5504               | c <sub>a</sub>    | 0.9559               | 0.956                | 0.955                          | 0.932                            | 0.037                | 0.037                          | 0.030                            | 0.007   | 9.09E-05             | 4.94E-05                       | 6.73E-05                         | 42                        | 42                                  | 174                                   | 0.551 |
|    | 10                | $c_t$            | 0.5503               | c <sub>a</sub>    | 0.9339               | 0.955                | 0.955                          | 0.927                            | 0.028                | 0.028                          | 0.028                            | 0.990   | 9.09E-05<br>9.17E-05 | 5.03E-05                       | 7.51E-05                         | 50                        | 50                                  | 174                                   | 0.550 |
|    | 2                 | $c_t$            | 0.5541               | $c \neq c_a, c_t$ | 0.1873               | 0.956                | 0.954                          | 0.908                            | 0.028                | 0.028                          | 0.031                            | 1.223   | 9.17E-05<br>9.27E-05 | 3.86E-05                       | 5.20E-05                         | 41                        | 41                                  | 239                                   | 0.554 |
|    |                   | $c_t$            |                      | $c \neq c_a, c_t$ |                      | 0.960                | 0.959                          | 0.909                            |                      |                                |                                  |   |                      | 5.15E-05                       | 8.39E-05                         | 72                        | 72                                  | 197                                   | 0.551 |
|    | 3                 | $c_t$            | 0.5506<br>0.5503     | $c \neq c_a, c_t$ | 0.1676<br>0.5114     | 0.962                | 0.939                          | 0.922                            | 0.034<br>0.021       | 0.033<br>0.021                 | 0.038<br>0.017                   | 0.883<br>1.260  | 1.09E-04<br>6.84E-05 | 4.93E-05                       | 6.48E-05                         | 29                        | 29                                  | 122                                   |       |
|    | 5                 | $c_t$            | 0.5512               | $c \neq c_a, c_t$ | 0.3114               | 0.941                | 0.940                          | 0.767                            | 0.021                | 0.021                          | 0.017                            | 1.100   | 1.06E-04             | 4.93E-05<br>5.78E-05           | 7.37E-05                         | 58                        | 58                                  | 163                                   | 0.550 |
| 8  | 6                 | $c_t$            | 0.5512               | $c \neq c_a, c_t$ | 0.2998               | 0.961                | 0.939                          | 0.698                            | 0.033                | 0.032                          | 0.029                            | 1.100   | 8.76E-05             | 4.79E-05                       | 6.14E-05                         | 44                        | 44                                  | 215                                   | 0.506 |
|    | 7                 | $c_t$            | 0.5509               | $c \neq c_a, c_t$ | 0.1801               | 0.944                | 0.959                          | 0.098                            | 0.027                | 0.026                          | 0.022                            | 0.510   | 1.10E-04             | 4.79E-05<br>5.62E-05           | 1.30E-04                         | 56                        | 56                                  | 213                                   | 0.551 |
|    | 8                 | $c_t$            | 0.5511               | $c_a$             | 0.7428               | 0.962                | 0.950                          | 0.960                            | 0.034                | 0.033                          | 0.065                            | 0.510   | 8.86E-05             | 4.57E-05                       | 1.01E-04                         | 39                        | 39                                  | 224                                   | 0.551 |
|    |                   | $c_t$            |                      | $c_a$             |                      | 0.955                |                                | 0.962                            | 0.027                | 0.027                          | 0.037                            | 0.890   |                      | 7.56E-05                       |                                  | 52                        | 52                                  | 200                                   | 0.552 |
|    | 9                 | $c_t$            | 0.5511               | $c_a$             | 0.975                |                      | 0.958                          |                                  |                      |                                |                                  |   | 1.07E-04             |                                | 1.19E-04                         |                           | -                                   |                                       |       |
|    | 10                | $c_t$            | 0.5537               | $c \neq c_a, c_t$ | 0.8214               | 0.957                | 0.956                          | 0.905                            | 0.030                | 0.030                          | 0.057                            | 0.525   | 9.72E-05             | 5.02E-05                       | 1.15E-04                         | 50                        | 50                                  | 187                                   | 0.554 |
|    | 1                 | XXX              | XXX                  | XXX               | XXX                  | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX   |
|    | 2                 | $c_t$            | 0.5508               | $c \neq c_a, c_t$ | 0.4025               | 0.946                | 0.943                          | 0.919                            | 0.023                | 0.023                          | 0.032                            | 0.698   | 7.36E-05             | 3.80E-05                       | 7.51E-05                         | 32                        | 32                                  | 197                                   | 0.551 |
|    | 3                 | $c_t$            | 0.5512               | $c \neq c_a, c_t$ | 0.3425               | 0.957                | 0.956                          | 0.866                            | 0.029                | 0.029                          | 0.019                            | 1.528   | 9.32E-05             | 3.84E-05                       | 3.63E-05                         | 42                        | 42                                  | 120                                   | 0.297 |
|    | 4                 | XXX              | XXX<br>0.5510        | XXX               | XXX                  | XXX                  | XXX                            | XXX<br>0.000                     | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX   |
| 9  | 5                 | $c_t$            | 0.5518               | $c_t$             | 0.1093               | 0.910                | 0.893                          | 0.892                            | 0.014                | 0.014                          | 0.042                            | 0.334   | 4.65E-05             | 2.88E-05                       | 1.32E-04                         | 21                        | 21                                  | 235                                   | 0.443 |
|    | 6                 | XXX              | XXX                  | XXX               | XXX                  | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX   |
|    | 7                 | $c_t$            | 0.5510               | $c \neq c_a, c_t$ | 0.1876               | 0.952                | 0.947                          | 0.973                            | 0.026                | 0.026                          | 0.069                            | 0.371   | 8.35E-05             | 2.10E-05                       | 6.63E-05                         | 41                        | 41                                  | 201                                   | 0.551 |
|    | 8                 | $c_t$            | 0.5521               | $c_a$             | 0.9567               | 0.924                | 0.918                          | 0.969                            | 0.016                | 0.016                          | 0.064                            | 0.256   | 5.34E-05             | 2.74E-05                       | 1.45E-04                         | 23                        | 23                                  | 247                                   | 0.552 |
|    | 9                 | $c_t$            | 0.5512               | $c_a$             | 0.3441               | 0.947                | 0.943                          | 0.880                            | 0.024                | 0.024                          | 0.024                            | 1.016   | 7.88E-05             | 4.06E-05                       | 6.76E-05                         | 39                        | 39                                  | 237                                   | 0.551 |
|    | 10                | $c_t$            | 0.5505               | $c_t$             | 0.1468               | 0.951                | 0.950                          | 0.783                            | 0.027                | 0.027                          | 0.013                            | 2.073   | 9.05E-05             | 4.68E-05                       | 3.55E-05                         | 43                        | 43                                  | 161                                   | 0.404 |
|    | 1                 | $c_t$            | 0.5506               | $c \neq c_a, c_t$ | 0.3174               | 0.954                | 0.954                          | 0.820                            | 0.028                | 0.028                          | 0.017                            | 1.672   | 8.95E-05             | 4.63E-05                       | 3.92E-05                         | 46                        | 46                                  | 145                                   | 0.517 |
|    | 2                 | $c_t$            | 0.55                 | $c_a$             | 0.8046               | 0.962                | 0.954                          | 0.902                            | 0.036                | 0.035                          | 0.079                            | 0.445   | 1.16E-04             | 5.93E-05                       | 1.63E-04                         | 52                        | 52                                  | 247                                   | 0.550 |
|    | 3                 | $c_t$            | 0.5509               | $c_a$             | 0.4941               | 0.954                | 0.953                          | 0.775                            | 0.029                | 0.029                          | 0.038                            | 0.763   | 9.41E-05             | 4.86E-05                       | 9.26E-05                         | 43                        | 43                                  | 234                                   | 0.551 |
|    | 4                 | $c_t$            | 0.5531               | $c_a$             | 0.1768               | 0.956                | 0.954                          | 0.942                            | 0.029                | 0.029                          | 0.043                            | 0.670   | 9.29E-05             | 5.09E-05                       | 1.04E-04                         | 45                        | 45                                  | 234                                   | 0.553 |
| 10 | 5                 | $c_t$            | 0.5503               | $c_a$             | 0.4116               | 0.960                | 0.952                          | 0.895                            | 0.034                | 0.033                          | 0.074                            | 0.451   | 1.10E-04             | 5.61E-05                       | 1.48E-04                         | 49                        | 49                                  | 245                                   | 0.550 |
|    | 6                 | $c_t$            | 0.5500               | $c_a$             | 0.5706               | 0.964                | 0.964                          | 0.899                            | 0.035                | 0.035                          | 0.025                            | 1.424   | 1.14E-04             | 7.31E-05                       | 7.67E-05                         | 51                        | 51                                  | 160                                   | 0.550 |
|    | 7                 | XXX              | XXX                  | XXX               | XXX                  | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX   |
|    | 8                 | $c_t$            | 0.5503               | $c_a$             | 0.2259               | 0.969                | 0.968                          | 0.964                            | 0.039                | 0.039                          | 0.056                            | 0.703   | 1.27E-04             | 6.03E-05                       | 1.02E-04                         | 66                        | 66                                  | 216                                   | 0.550 |
|    | 9                 | $c_t$            | 0.5501               | $c \neq c_a, c_t$ | 0.3543               | 0.960                | 0.957                          | 0.867                            | 0.035                | 0.035                          | 0.059                            | 0.582   | 1.13E-04             | 5.82E-05                       | 1.18E-04                         | 57                        | 57                                  | 172                                   | 0.550 |
|    | 10                | $c_t$            | 0.5503               | $c \neq c_a, c_t$ | 0.3946               | 0.968                | 0.962                          | 0.960                            | 0.040                | 0.040                          | 0.063                            | 0.628   | 1.30E-04             | 7.10E-05                       | 1.37E-04                         | 67                        | 67                                  | 241                                   | 0.550 |

Table B5.: Combination  $(\rho, \lambda, \rho) = \text{L-N-N}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 23, 64)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (6, 23, 64)$ .

| Γ. | $\mathcal{A}_q^p$ | Step             | o 1 - Step 3       | Step     | 4 - Step 8         |                      | $L_0^{norm}$      |                 |                    | $L_1^i$           | norm                      |  |                       | $L_2^{norm}$                    |                              |                               | $L_{\infty}$                |                              |               |
|----|-------------------|------------------|--------------------|----------|--------------------|----------------------|-------------------|-----------------|--------------------|-------------------|---------------------------|--|-----------------------|---------------------------------|------------------------------|-------------------------------|-----------------------------|------------------------------|---------------|
| q  | p                 | $\tilde{\tau}_c$ | Dominant           | $\tau_c$ | Dominant           | $L_{o,D}^{norm,adv}$ | I norm,adv        | L'norm,clean    | $L_{*}^{norm,adv}$ | L'norm,adv        | L'norm,clean              | $L_{1,\mathcal{H}}^{norm,adv}$           | $L_{a,D}^{norm,adv}$  | Lnorm,adv                       | L'norm,clean                 | $L^{norm,adv}$                | $_{I}$ norm, adv            | $L^{norm,clean}$             | $\mathcal{L}$ |
| 4  |                   |                  | Category<br>0.5504 |          | Category<br>0.5426 | $L_{0,R}$ 0.965      | $L_{0,H} = 0.962$ | $L_{0,H}$ 1.000 | $L_{1,R} = 0.037$  | $L_{1,H} = 0.034$ | $L_{1,\mathcal{H}}$ 0.029 | $L_{1,\mathcal{H}}^{norm,clean}$ $1.175$ | $L_{2,R}$<br>1.19E-04 | $E_{2,\mathcal{H}}$<br>8.70E-06 | $L_{2,\mathcal{H}}$ 1.03E-05 | $L_{\infty,\mathcal{R}}$ $53$ | $L_{\infty,\mathcal{H}}$ 53 | $L_{\infty,\mathcal{H}}$ 217 | 0.008         |
|    | 1 2               | $c_t$            | 0.5519             | $c_t$    | 0.5420             | 0.903                | 0.962             | 1.000           | 0.037              | 0.034             | 0.029                     | 0.700                                    | 8.07E-05              | 1.49E-05                        | 3.29E-05                     | 41                            | 42                          | 227                          | 0.008         |
|    | 3                 | $c_t$            | 0.5519             | $c_t$    | 0.5541             | 0.948                | 0.940             | 1.000           | 0.025              | 0.023             | 0.032                     | 4.619                                    | 9.56E-05              | 7.32E-05                        | 2.26E-05                     | 41                            | 42                          | 56                           | 0.018         |
|    | 4                 | $c_t$            | 0.5501             | $c_t$    | 0.1338             | 0.937                | 0.954             | 0.999           | 0.030              | 0.027             | 0.000                     | 2.734                                    | 1.40E-04              | 6.79E-05                        | 3.58E-05                     | 64                            | 68                          | 127                          | 0.393         |
|    | 5                 | $c_t$            | 0.5501             | $c_t$    | 0.4012             | 0.970                | 0.908             | 1.000           | 0.045              | 0.040             | 0.014                     | 1.247                                    | 9.70E-05              | 4.58E-05                        | 4.31E-05                     | 45                            | 45                          | 140                          | 0.277         |
| 1  | 6                 | $c_t$            | 0.3507 ge          | $c_t$    | 0.4012             | 0.938                | 0.955             | 0.912           | 0.030              | 0.027             | 0.022                     | 2.972                                    | 9.70E-05<br>1.53E-04  | 7.55E-05                        | 4.51E-05<br>4.90E-05         | 66                            | 67                          | 160                          | 0.149         |
|    | 7                 | $c_t$            | 0.5503             | $c_a$    | 0.300              | 0.974                | 0.971             | 1.000           | 0.043              | 0.043             | 0.013                     | 1.149                                    | 1.38E-04              | 6.58E-05                        | 9.18E-05                     | 74                            | 67                          | 259                          | 0.192         |
|    | 8                 | $c_t$            | 0.5506             | $c_t$    | 0.4377             | 0.908                | 0.948             | 1.000           | 0.045              | 0.039             | 0.054                     | 0.678                                    | 1.19E-04              | 5.22E-05                        | 1.20E-04                     | 53                            | 54                          | 263                          | 0.113         |
|    | 9                 | $c_t$            | 0.5505             | $c_t$    | 0.4591             | 0.965                | 0.948             | 1.000           | 0.037              | 0.034             | 0.035                     | 0.078                                    | 1.19E-04<br>1.18E-04  | 4.86E-05                        | 7.98E-05                     | 53                            | 53                          | 242                          | 0.133         |
|    | 10                | $c_t$            | 0.5504             | $c_t$    | 0.4551             | 0.959                | 0.953             | 1.000           | 0.037              | 0.033             | 0.033                     | 0.942                                    | 9.70E-05              | 7.00E-06                        | 1.07E-05                     | 48                            | 49                          | 198                          | 0.000         |
| -  | 1                 | $c_t$            | 0.5504             | $c_t$    | 0.4961             | 0.939                | 0.933             | 1.000           | 0.030              | 0.027             | 0.031                     | 0.570                                    | 5.67E-05              | 2.70E-05                        | 7.22E-05                     | 26                            | 30                          | 208                          | 0.054         |
|    | 2                 | -                | 0.5519             | _        | 0.4773             | 0.928                | 0.922             | 1.000           | 0.017              | 0.016             | 0.027                     | 0.672                                    | 5.53E-05              | 2.76E-05<br>2.36E-05            | 4.68E-05                     | 25                            | 25                          | 215                          | 0.034         |
|    | 3                 | $c_t$            | 0.5502             | $c_t$    | 0.4867             | 0.940                | 0.935             | 1.000           | 0.017              | 0.010             | 0.025                     | 0.072                                    | 6.80E-05              | 3.23E-05                        | 5.75E-05                     | 30                            | 31                          | 153                          | 0.064         |
|    | 4                 | $c_t$            | 0.5524             | $c_t$    | 0.3933             | 0.945                | 0.939             | 1.000           | 0.021              | 0.013             | 0.020                     | 1.673                                    | 7.25E-05              | 3.44E-05                        | 3.36E-05                     | 32                            | 30                          | 125                          | 0.004         |
|    | 5                 | $c_t$            | 0.5524             | $c_t$    | 0.407              | 0.958                | 0.952             | 1.000           | 0.022              | 0.021             | 0.012                     | 1.026                                    | 9.55E-05              | 3.92E-05                        | 6.20E-05                     | 43                            | 46                          | 207                          | 0.144         |
| 2  | 6                 | $c_t$            | 0.5511             | $c_t$    | 0.4754             | 0.952                | 0.932             | 1.000           | 0.029              | 0.024             | 0.020                     | 0.733                                    | 9.55E-05<br>8.67E-05  | 3.56E-05                        | 7.98E-05                     | 39                            | 41                          | 256                          | 0.144         |
|    | 7                 | $c_t$            | 0.5516             | $c_t$    | 0.3993             | 0.946                | 0.943             | 0.997           | 0.024              | 0.024             | 0.008                     | 2.941                                    | 7.85E-05              | 3.76E-05                        | 2.23E-05                     | 34                            | 36                          | 98                           | 0.070         |
|    | 8                 | $c_t$            | 0.5505             | $c_t$    | 0.4494             | 0.951                | 0.946             | 1.000           | 0.024              | 0.022             | 0.009                     | 2.550                                    | 8.17E-05              | 3.88E-05                        | 2.63E-05                     | 39                            | 37                          | 114                          | 0.101         |
|    | 9                 | $c_t$            | 0.5504             | $c_t$    | 0.4868             | 0.943                | 0.938             | 1.000           | 0.023              | 0.023             | 0.003                     | 1.842                                    | 7.26E-05              | 3.11E-05                        | 2.69E-05                     | 33                            | 32                          | 107                          | 0.064         |
|    | 10                | $c_t$            | 0.5511             | $c_t$    | 0.4595             | 0.939                | 0.932             | 1.000           | 0.021              | 0.019             | 0.026                     | 0.759                                    | 6.92E-05              | 3.48E-05                        | 7.04E-05                     | 33                            | 31                          | 180                          | 0.092         |
|    | 1                 | $c_t$            | 0.5507             | $c_t$    | 0.4363             | 0.961                | 0.957             | 1.000           | 0.034              | 0.031             | 0.019                     | 1.621                                    | 1.11E-04              | 4.92E-05                        | 4.43E-05                     | 56                            | 54                          | 135                          | 0.114         |
|    | 2                 | $c_t$            | 0.5504             | $c_a$    | 0.3227             | 0.954                | 0.951             | 0.999           | 0.034              | 0.031             | 0.013                     | 2.350                                    | 9.26E-05              | 7.96E-05                        | 7.61E-05                     | 47                            | 40                          | 166                          | 0.280         |
|    | 3                 | $c_t$            | 0.5500             | $c_a$    | 0.543              | 0.962                | 0.957             | 1.000           | 0.032              | 0.029             | 0.017                     | 1.748                                    | 1.04E-04              | 7.40E-06                        | 6.85E-06                     | 47                            | 54                          | 227                          | 0.007         |
| l  | 4                 | $c_t$            | 0.5503             | $c_{a}$  | 0.5484             | 0.943                | 0.934             | 1.000           | 0.025              | 0.023             | 0.019                     | 1.163                                    | 8.09E-05              | 4.04E-05                        | 6.27E-05                     | 37                            | 37                          | 260                          | 0.344         |
|    | 5                 | $c_t$            | 0.5502             | $c_a$    | 0.3046             | 0.961                | 0.956             | 0.999           | 0.037              | 0.034             | 0.015                     | 2.221                                    | 1.21E-04              | 8.22E-05                        | 5.07E-05                     | 62                            | 58                          | 86                           | 0.373         |
| 3  | 6                 | $c_t$            | 0.5509             | $c_t$    | 0.2442             | 0.950                | 0.945             | 1.000           | 0.026              | 0.024             | 0.018                     | 1.350                                    | 8.44E-05              | 5.59E-05                        | 7.80E-05                     | 42                            | 36                          | 137                          | 0.307         |
|    | 7                 | Ct.              | 0.5518             | Ct       | 0.4378             | 0.937                | 0.938             | 0.997           | 0.022              | 0.021             | 0.010                     | 2.081                                    | 7.37E-05              | 3.43E-05                        | 3.61E-05                     | 32                            | 33                          | 192                          | 0.114         |
|    | 8                 | $c_t$            | 0.5503             | $c_t$    | 0.3679             | 0.952                | 0.946             | 1.000           | 0.027              | 0.025             | 0.022                     | 1.107                                    | 9.00E-05              | 4.41E-05                        | 6.76E-05                     | 40                            | 38                          | 196                          | 0.182         |
|    | 9                 | $c_t$            | 0.5500             | $c_t$    | 0.4231             | 0.964                | 0.959             | 1.000           | 0.034              | 0.031             | 0.029                     | 1.071                                    | 1.11E-04              | 5.27E-05                        | 7.97E-05                     | 50                            | 51                          | 289                          | 0.127         |
|    | 10                | $c_t$            | 0.5505             | $c_t$    | 0.2699             | 0.971                | 0.965             | 1.000           | 0.047              | 0.043             | 0.021                     | 2.066                                    | 1.52E-04              | 9.48E-05                        | 6.20E-05                     | 70                            | 66                          | 180                          | 0.281         |
|    | 1                 | XXX              | XXX                | XXX      | XXX                | XXX                  | XXX               | xxx             | XXX                | XXX               | XXX                       | XXX                                      | XXX                   | xxx                             | XXX                          | XXX                           | XXX                         | XXX                          | xxx           |
|    | 2                 | $c_t$            | 0.5503             | $c_t$    | 0.3197             | 0.951                | 0.960             | 0.952           | 0.032              | 0.032             | 0.010                     | 3.326                                    | 1.08E-04              | 6.14E-05                        | 3.12E-05                     | 53                            | 57                          | 89                           | 0.231         |
|    | 3                 | $c_t$            | 0.5501             | $c_t$    | 0.427              | 0.944                | 0.940             | 0.997           | 0.023              | 0.022             | 0.019                     | 1.171                                    | 7.67E-05              | 3.82E-05                        | 7.20E-05                     | 35                            | 36                          | 182                          | 0.123         |
| İ  | 4                 | $c_t$            | 0.5501             | $c_t$    | 0.1936             | 0.939                | 0.951             | 0.902           | 0.028              | 0.028             | 0.009                     | 3.024                                    | 9.56E-05              | 6.95E-05                        | 4.60E-05                     | 49                            | 45                          | 73                           | 0.357         |
| 4  | 5                 | $c_t$            | 0.5507             | $c_t$    | 0.401              | 0.963                | 0.959             | 1.000           | 0.034              | 0.031             | 0.007                     | 4.309                                    | 1.11E-04              | 4.77E-05                        | 2.48E-05                     | 50                            | 52                          | 144                          | 0.150         |
| 4  | 6                 | $c_t$            | 0.5503             | $c_t$    | 0.4518             | 0.970                | 0.964             | 1.000           | 0.044              | 0.040             | 0.137                     | 0.296                                    | 1.44E-04              | 6.89E-05                        | 2.65E-04                     | 71                            | 69                          | 298                          | 0.098         |
|    | 7                 | $c_t$            | 0.5513             | $c_t$    | 0.4601             | 0.957                | 0.955             | 0.999           | 0.032              | 0.030             | 0.015                     | 2.016                                    | 1.05E-04              | 5.40E-05                        | 4.22E-05                     | 51                            | 50                          | 96                           | 0.091         |
|    | 8                 | $c_t$            | 0.5504             | $c_t$    | 0.3751             | 0.947                | 0.940             | 1.000           | 0.024              | 0.022             | 0.038                     | 0.571                                    | 7.70E-05              | 3.79E-05                        | 1.14E-04                     | 40                            | 38                          | 288                          | 0.175         |
|    | 9                 | $c_t$            | 0.5512             | $c_t$    | 0.3204             | 0.943                | 0.945             | 0.966           | 0.026              | 0.025             | 0.013                     | 1.974                                    | 8.53E-05              | 4.33E-05                        | 5.04E-05                     | 42                            | 40                          | 141                          | 0.231         |
| L  | 10                | $c_t$            | 0.5500             | $c_t$    | 0.4939             | 0.952                | 0.955             | 0.978           | 0.033              | 0.032             | 0.011                     | 2.937                                    | 1.09E-04              | 4.17E-05                        | 2.97E-05                     | 52                            | 50                          | 174                          | 0.056         |
|    | 1                 | $c_t$            | 0.551              | $c_t$    | 0.4661             | 0.960                | 0.956             | 1.000           | 0.031              | 0.029             | 0.028                     | 1.032                                    | 1.01E-04              | 3.63E-05                        | 5.54E-05                     | 46                            | 46                          | 199                          | 0.085         |
|    | 2                 | XXX              | XXX                | XXX      | XXX                | XXX                  | XXX               | XXX             | XXX                | XXX               | XXX                       | XXX                                      | XXX                   | XXX                             | XXX                          | XXX                           | XXX                         | XXX                          | XXX           |
|    | 3                 | $c_t$            | 0.5506             | $c_t$    | 0.4257             | 0.958                | 0.956             | 0.996           | 0.032              | 0.030             | 0.034                     | 0.866                                    | 1.04E-04              | 5.31E-05                        | 8.37E-05                     | 51                            | 48                          | 186                          | 0.125         |
|    | 4                 | $c_t$            | 0.5510             | $c_t$    | 0.3254             | 0.959                | 0.955             | 1.000           | 0.030              | 0.028             | 0.025                     | 1.121                                    | 9.83E-05              | 4.94E-05                        | 5.66E-05                     | 49                            | 48                          | 144                          | 0.226         |
| 5  | 5                 | $c_t$            | 0.5503             | $c_t$    | 0.4866             | 0.956                | 0.949             | 1.000           | 0.029              | 0.026             | 0.054                     | 0.488                                    | 9.33E-05              | 3.12E-05                        | 8.85E-05                     | 42                            | 42                          | 258                          | 0.064         |
| 0  | 6                 | $c_t$            | 0.5505             | $c_t$    | 0.3936             | 0.969                | 0.965             | 1.000           | 0.039              | 0.036             | 0.041                     | 0.867                                    | 1.26E-04              | 6.32E-05                        | 1.00E-04                     | 58                            | 54                          | 172                          | 0.157         |
|    | 7                 | $c_t$            | 0.5501             | $c_t$    | 0.4877             | 0.949                | 0.944             | 1.000           | 0.025              | 0.023             | 0.022                     | 1.050                                    | 8.13E-05              | 3.10E-05                        | 3.56E-05                     | 38                            | 39                          | 107                          | 0.062         |
|    | 8                 | $c_t$            | 0.5501             | $c_t$    | 0.3773             | 0.970                | 0.966             | 1.000           | 0.041              | 0.037             | 0.031                     | 1.186                                    | 1.32E-04              | 6.26E-05                        | 6.83E-05                     | 58                            | 60                          | 153                          | 0.173         |
|    | 9                 | $c_t$            | 0.5500             | $c_t$    | 0.4334             | 0.970                | 0.967             | 1.000           | 0.042              | 0.038             | 0.053                     | 0.710                                    | 1.34E-04              | 6.74E-05                        | 1.23E-04                     | 59                            | 57                          | 239                          | 0.117         |
|    | 10                | $c_t$            | 0.5518             | $c_t$    | 0.461              | 0.952                | 0.947             | 1.000           | 0.026              | 0.024             | 0.031                     | 0.771                                    | 8.50E-05              | 3.37E-05                        | 5.78E-05                     | 40                            | 41                          | 231                          | 0.091         |

| $\mathcal{A}_q^p$ | - 1 | Step             | 1 - Step 3       | Step     | 4 - Step 8 |                      | $L_0^{norm}$        |                     |                | $L_1^r$                        | norm                             |                                  |            | $L_2^{norm}$                   |                      |                | $L_{\infty}$             |                                       |       |
|-------------------|-----|------------------|------------------|----------|------------|----------------------|---------------------|---------------------|----------------|--------------------------------|----------------------------------|----------------------------------|------------|--------------------------------|----------------------|----------------|--------------------------|---------------------------------------|-------|
|                   |     |                  | Dominant         | Î        | Dominant   | r norm.adv           | r norm,adv          | 7 norm,clean        | r norm,adv     |                                |                                  | $L_{1,\mathcal{H}}^{norm,adv}$   | I.norm,adv | r norm.adv                     | 7 norm,clean         | r norm,adv     | r norm,adv               | r norm.clean                          |       |
|                   | p   | $\tilde{\tau}_c$ | Category         | $\tau_c$ | Category   | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}$ | $L_{0,\mathcal{H}}$ | $L_{1,R}$      | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $L_{2,R}$  | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{H}}$  | $L_{\infty,R}$ | $L_{\infty,\mathcal{H}}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | L     |
|                   | 1   | $c_t$            | 0.5532           | $c_t$    | 0.4042     | 0.944                | 0.935               | 1.000               | 0.022          | 0.020                          | 0.059                            | 0.342                            | 7.12E-05   | 3.59E-05                       | 1.31E-04             | 35             | 34                       | 198                                   | 0.149 |
|                   | 2   | $c_t$            | 0.5500           | $c_t$    | 0.4867     | 0.971                | 0.968               | 1.000               | 0.043          | 0.039                          | 0.032                            | 1.240                            | 1.38E-04   | 5.45E-05                       | 5.97E-05             | 62             | 65                       | 161                                   | 0.063 |
|                   | 3   | $c_t$            | 0.5501           | $c_t$    | 0.4259     | 0.965                | 0.962               | 1.000               | 0.037          | 0.034                          | 0.051                            | 0.662                            | 1.21E-04   | 5.71E-05                       | 1.10E-04             | 56             | 56                       | 228                                   | 0.124 |
|                   | 4   | $c_t$            | 0.5508           | $c_t$    | 0.4067     | 0.953                | 0.947               | 1.000               | 0.027          | 0.025                          | 0.031                            | 0.799                            | 8.93E-05   | 4.22E-05                       | 7.21E-05             | 40             | 41                       | 194                                   | 0.144 |
| 6                 | 5   | $c_t$            | 0.5509           | $c_t$    | 0.4832     | 0.947                | 0.941               | 1.000               | 0.024          | 0.022                          | 0.022                            | 0.982                            | 7.81E-05   | 3.21E-05                       | 6.65E-05             | 36             | 37                       | 300                                   | 0.068 |
|                   | 6   | $c_t$            | 0.5510           | $c_t$    | 0.4046     | 0.956                | 0.952               | 1.000               | 0.029          | 0.026                          | 0.026                            | 1.013                            | 9.27E-05   | 3.88E-05                       | 5.52E-05             | 46             | 44                       | 139                                   | 0.146 |
|                   | 7   | $c_t$            | 0.5502           | $c_t$    | 0.5412     | 0.964                | 0.960               | 1.000               | 0.034          | 0.032                          | 0.038                            | 0.833                            | 1.11E-04   | 1.75E-05                       | 2.95E-05             | 51             | 53                       | 196                                   | 0.009 |
|                   | 8   | $c_t$            | 0.5508           | $c_t$    | 0.3759     | 0.960                | 0.951               | 1.000               | 0.034          | 0.031                          | 0.036                            | 0.885                            | 1.13E-04   | 5.64E-05                       | 1.12E-04             | 54             | 51                       | 320                                   | 0.175 |
|                   | 9   | $c_t$            | 0.5501           | $c_t$    | 0.5504     | 0.939                | 0.932               | 1.000               | 0.021          | 0.019                          | 0.057                            | 0.339                            | 6.85E-05   | 7.93E-06                       | 2.98E-05             | 32             | 33                       | 214                                   | 0.000 |
|                   | 10  | $c_t$            | 0.5505           | $c_t$    | 0.4484     | 0.964                | 0.959               | 0.997               | 0.037          | 0.034                          | 0.062                            | 0.549                            | 1.21E-04   | 5.73E-05                       | 1.49E-04             | 57             | 54                       | 284                                   | 0.102 |
|                   | 1   | $c_t$            | 0.5501           | $c_t$    | 0.4305     | 0.957                | 0.947               | 1.000               | 0.032          | 0.029                          | 0.076                            | 0.376                            | 1.03E-04   | 4.84E-05                       | 1.62E-04             | 48             | 47                       | 275                                   | 0.120 |
|                   | 2   | $c_t$            | 0.5519           | $c_t$    | 0.5418     | 0.965                | 0.960               | 1.000               | 0.036          | 0.032                          | 0.037                            | 0.887                            | 1.15E-04   | 1.70E-05                       | 2.92E-05             | 56             | 57                       | 236                                   | 0.010 |
|                   | 3   | $c_t$            | 0.5503           | $c_t$    | 0.3892     | 0.961                | 0.956               | 1.000               | 0.032          | 0.029                          | 0.034                            | 0.844                            | 1.03E-04   | 5.18E-05                       | 8.90E-05             | 47             | 49                       | 238                                   | 0.161 |
|                   | 4   | $c_t$            | 0.5504           | $c_t$    | 0.3653     | 0.949                | 0.942               | 1.000               | 0.024          | 0.022                          | 0.024                            | 0.919                            | 7.89E-05   | 4.47E-05                       | 8.05E-05             | 35             | 37                       | 270                                   | 0.185 |
|                   | 5   | $c_t$            | 0.5511           | $c_t$    | 0.3502     | 0.962                | 0.958               | 1.000               | 0.033          | 0.030                          | 0.025                            | 1.209                            | 1.06E-04   | 4.12E-05                       | 5.22E-05             | 48             | 49                       | 167                                   | 0.201 |
|                   | 6   | $c_t$            | 0.5519           | $c_t$    | 0.3524     | 0.962                | 0.956               | 1.000               | 0.033          | 0.030                          | 0.041                            | 0.736                            | 1.07E-04   | 5.07E-05                       | 9.03E-05             | 52             | 47                       | 238                                   | 0.200 |
|                   | 7   | $c_t$            | 0.5501           | $c_t$    | 0.3548     | 0.956                | 0.953               | 0.995               | 0.030          | 0.027                          | 0.028                            | 0.984                            | 9.59E-05   | 4.13E-05                       | 5.24E-05             | 48             | 48                       | 212                                   | 0.195 |
|                   | 8   | $c_t$            | 0.5507           | $c_t$    | 0.445      | 0.964                | 0.961               | 1.000               | 0.036          | 0.033                          | 0.029                            | 1.121                            | 1.15E-04   | 5.25E-05                       | 5.68E-05             | 54             | 52                       | 105                                   | 0.106 |
|                   | 9   | $c_t$            | 0.5501           | $c_t$    | 0.4401     | 0.968                | 0.964               | 1.000               | 0.040          | 0.037                          | 0.062                            | 0.602                            | 1.31E-04   | 5.70E-05                       | 1.19E-04             | 60             | 63                       | 237                                   | 0.110 |
|                   | 10  | $c_t$            | 0.5504           | $c_t$    | 0.3963     | 0.952                | 0.948               | 1.000               | 0.027          | 0.025                          | 0.025                            | 0.993                            | 8.66E-05   | 4.33E-05                       | 6.33E-05             | 40             | 39                       | 210                                   | 0.154 |
|                   | 1   | $c_t$            | 0.5501           | $c_t$    | 0.4962     | 0.953                | 0.949               | 1.000               | 0.027          | 0.025                          | 0.023                            | 1.077                            | 8.70E-05   | 4.38E-05                       | 6.49E-05             | 37             | 37                       | 173                                   | 0.054 |
|                   | 2   | $c_t$            | 0.5515           | $c_t$    | 0.4302     | 0.957                | 0.953               | 1.000               | 0.021          | 0.025                          | 0.023                            | 1.224                            | 9.43E-05   | 3.60E-05                       | 4.98E-05             | 39             | 41                       | 241                                   | 0.034 |
|                   | 3   |                  | 0.5517           |          | 0.4012     | 0.961                | 0.958               | 1.000               | 0.023          | 0.021                          | 0.022                            | 0.923                            | 1.10E-04   | 4.80E-05                       | 7.89E-05             | 51             | 54                       | 234                                   | 0.044 |
|                   | 4   | $c_t$            | 0.5501           | $c_t$    | 0.3382     | 0.946                | 0.940               | 1.000               | 0.034          | 0.031                          | 0.034                            | 2.009                            | 7.46E-05   | 4.94E-05                       | 4.16E-05             | 33             | 32                       | 103                                   | 0.044 |
|                   | 5   | $c_t$            | 0.5510           | $c_t$    | 0.3362     | 0.940                | 0.940               | 1.000               | 0.023          | 0.021                          | 0.010                            | 1.424                            | 1.08E-04   | 5.47E-05                       | 5.77E-05             | 59             | 58                       | 148                                   | 0.212 |
|                   | 6   | $c_t$            | 0.5505           | $c_t$    | 0.3214     | 0.943                | 0.944               | 0.992               | 0.033          | 0.031                          | 0.022                            | 1.424                            | 8.48E-05   | 4.38E-05                       | 5.09E-05             | 39             | 39                       | 222                                   | 0.230 |
|                   | 7   | $c_t$            | 0.5505           | $c_t$    | 0.3895     | 0.945                | 0.944               | 1.000               | 0.020          | 0.025                          | 0.016                            | 0.445                            | 1.08E-04   | 4.38E-05<br>5.10E-05           | 1.47E-04             | 58             | 59<br>59                 | 257                                   | 0.161 |
| _                 | _   | $c_t$            |                  | $c_t$    |            |                      |                     |                     |                |                                |                                  |                                  |            |                                |                      |                |                          |                                       |       |
|                   | 8   | $c_t$            | 0.5533           | $c_t$    | 0.4905     | 0.955                | 0.951               | 1.000               | 0.028          | 0.026                          | 0.046                            | 0.568                            | 9.25E-05   | 4.40E-05                       | 9.44E-05             | 45             | 45                       | 191                                   | 0.063 |
|                   | 9   | $c_t$            | 0.5520<br>0.5520 | $c_t$    | 0.2184     | 0.962<br>0.952       | 0.959<br>0.948      | 1.000<br>1.000      | 0.035<br>0.027 | 0.032<br>0.024                 | 0.024<br>0.058                   | 1.350<br>0.420                   | 1.12E-04   | 7.31E-05<br>4.10E-05           | 8.04E-05<br>1.27E-04 | 54<br>43       | 53<br>38                 | 131<br>227                            | 0.334 |
|                   | 10  | $c_t$            |                  | $c_t$    | 0.5021     |                      |                     |                     |                |                                |                                  |                                  | 8.60E-05   |                                |                      |                |                          |                                       |       |
|                   | 1   | XXX              | XXX              | XXX      | XXX        | XXX                  | XXX                 | XXX                 | XXX            | XXX                            | XXX                              | XXX                              | XXX        | XXX                            | XXX                  | XXX            | XXX                      | XXX                                   | XXX   |
|                   | 2   | $c_t$            | 0.5507           | $c_t$    | 0.4716     | 0.943                | 0.937               | 1.000               | 0.022          | 0.020                          | 0.031                            | 0.653                            | 7.08E-05   | 3.36E-05                       | 7.31E-05             | 35             | 33                       | 183                                   | 0.079 |
|                   | 3   | $c_t$            | 0.5500           | $c_t$    | 0.4752     | 0.955                | 0.952               | 1.000               | 0.028          | 0.026                          | 0.014                            | 1.871                            | 9.13E-05   | 3.46E-05                       | 2.81E-05             | 44             | 41                       | 160                                   | 0.075 |
|                   | 4   | XXX              | XXX              | XXX      | XXX        | XXX                  | XXX                 | XXX                 | XXX            | XXX                            | XXX                              | XXX                              | XXX        | XXX                            | XXX                  | XXX            | XXX                      | XXX                                   | XXX   |
| 9 🗀               | 5   | XXX              | XXX              | XXX      | XXX        | XXX                  | XXX                 | XXX                 | XXX            | XXX                            | XXX                              | XXX                              | XXX        | XXX                            | XXX                  | XXX            | XXX                      | XXX                                   | XXX   |
| _                 | 6   | XXX              | XXX              | XXX      | XXX        | XXX                  | XXX                 | XXX                 | XXX            | XXX                            | XXX                              | XXX                              | XXX        | XXX                            | XXX                  | XXX            | XXX                      | XXX                                   | XXX   |
|                   | 7   | XXX              | XXX              | XXX      | XXX        | XXX                  | XXX                 | XXX                 | XXX            | XXX                            | XXX                              | XXX                              | XXX        | XXX                            | XXX                  | XXX            | XXX                      | XXX                                   | XXX   |
|                   | 8   | $c_t$            | 0.5508           | $c_t$    | 0.4884     | 0.942                | 0.933               | 1.000               | 0.022          | 0.020                          | 0.064                            | 0.307                            | 7.00E-05   | 3.31E-05                       | 1.55E-04             | 34             | 32                       | 302                                   | 0.062 |
|                   | 9   | $c_t$            | 0.5509           | $c_t$    | 0.3998     | 0.944                | 0.939               | 1.000               | 0.023          | 0.021                          | 0.022                            | 0.958                            | 7.49E-05   | 3.56E-05                       | 6.29E-05             | 34             | 36                       | 277                                   | 0.151 |
|                   | 10  | $c_t$            | 0.5512           | $c_t$    | 0.4205     | 0.947                | 0.944               | 1.000               | 0.026          | 0.023                          | 0.011                            | 2.093                            | 8.44E-05   | 3.98E-05                       | 2.98E-05             | 43             | 44                       | 218                                   | 0.131 |
|                   | 1   | $c_t$            | 0.5512           | $c_t$    | 0.3319     | 0.954                | 0.950               | 1.000               | 0.027          | 0.025                          | 0.013                            | 1.915                            | 8.76E-05   | 4.17E-05                       | 3.18E-05             | 41             | 42                       | 120                                   | 0.219 |
| _                 | 2   | $c_t$            | 0.5509           | $c_t$    | 0.4614     | 0.960                | 0.954               | 0.999               | 0.034          | 0.031                          | 0.079                            | 0.397                            | 1.12E-04   | 5.29E-05                       | 1.75E-04             | 57             | 65                       | 305                                   | 0.090 |
|                   | 3   | $c_t$            | 0.5503           | $c_t$    | 0.2933     | 0.955                | 0.954               | 0.987               | 0.031          | 0.029                          | 0.039                            | 0.739                            | 9.99E-05   | 4.78E-05                       | 1.00E-04             | 45             | 46                       | 255                                   | 0.257 |
|                   | 4   | $c_t$            | 0.5500           | $c_t$    | 0.4241     | 0.956                | 0.951               | 1.000               | 0.028          | 0.026                          | 0.037                            | 0.696                            | 9.10E-05   | 4.58E-05                       | 9.02E-05             | 42             | 41                       | 212                                   | 0.126 |
|                   | 5   | $c_t$            | 0.5510           | $c_t$    | 0.4442     | 0.963                | 0.956               | 0.989               | 0.038          | 0.035                          | 0.075                            | 0.463                            | 1.22E-04   | 5.81E-05                       | 1.60E-04             | 54             | 60                       | 252                                   | 0.107 |
|                   | 6   | $c_t$            | 0.5506           | $c_t$    | 0.357      | 0.965                | 0.962               | 1.000               | 0.035          | 0.032                          | 0.014                            | 2.313                            | 1.14E-04   | 6.76E-05                       | 4.41E-05             | 52             | 50                       | 127                                   | 0.194 |
|                   | 7   | $c_t$            | 0.5515           | $c_t$    | 0.4162     | 0.957                | 0.949               | 1.000               | 0.030          | 0.028                          | 0.055                            | 0.501                            | 9.88E-05   | 4.67E-05                       | 1.31E-04             | 45             | 49                       | 227                                   | 0.135 |
|                   | 8   | $c_t$            | 0.5509           | $c_t$    | 0.4562     | 0.970                | 0.967               | 1.000               | 0.042          | 0.038                          | 0.051                            | 0.751                            | 1.35E-04   | 5.88E-05                       | 9.62E-05             | 65             | 63                       | 184                                   | 0.095 |
|                   | 9   | $c_t$            | 0.5504           | $c_t$    | 0.4244     | 0.959                | 0.958               | 0.980               | 0.035          | 0.033                          | 0.061                            | 0.538                            | 1.15E-04   | 5.50E-05                       | 1.31E-04             | 56             | 59                       | 221                                   | 0.126 |
|                   | 10  | $c_t$            | 0.5502           | $c_t$    | 0.344      | 0.968                | 0.961               | 1.000               | 0.041          | 0.038                          | 0.059                            | 0.637                            | 1.34E-04   | 6.72E-05                       | 1.36E-04             | 58             | 59                       | 243                                   | 0.206 |

Table B6.: Combination  $(\rho, \lambda, \rho) = \text{N-L-N}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 0, 4)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (89, 0, 4)$ .

| $\mathcal{A}_q^p$ | Step 1 - Step 3   Step 4 - Step 8 |                    |                              |                    |                      | $L_0^{norm}$                   |                                  |                      | $L^{i}$                        | norm                             |   |                      | $L_2^{norm}$                   |                                  |                           | $L_{\infty}$                        |                                       |               |
|-------------------|-----------------------------------|--------------------|------------------------------|--------------------|----------------------|--------------------------------|----------------------------------|----------------------|--------------------------------|----------------------------------|---|----------------------|--------------------------------|----------------------------------|---------------------------|-------------------------------------|---------------------------------------|---------------|
| q p               | $\tilde{\tau}_c$                  | Dominant           | $\tau_c$                     | Dominant           | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | $\mathcal{L}$ |
| 1                 | $c_t$                             | Category<br>0.5504 | $c_a$                        | Category<br>0.6887 | 0.965                | 0.964                          | 0.901                            | 0.037                | 0.037                          | 0.031                            | $\frac{L_{1,H}}{1.176}$   | 1.19E-04             | 9.44E-06                       | 1.26E-05                         | 53                        | 53                                  | 230                                   | 0.550         |
| 2                 | $c_t$                             | 0.5519             | $c \neq c_a, c_t$            | 0.3168             | 0.948                | 0.943                          | 0.859                            | 0.025                | 0.037                          | 0.034                            | 0.714   | 8.07E-05             | 1.62E-05                       | 3.88E-05                         | 41                        | 41                                  | 246                                   | 0.439         |
| 3                 | $c_t$                             | 0.5511             | $c_{\tau}$ $c_{a}$ , $c_{t}$ | 0.5694             | 0.957                | 0.957                          | 0.645                            | 0.030                | 0.030                          | 0.012                            | 2.460   | 9.56E-05             | 7.99E-05                       | 5.25E-05                         | 41                        | 41                                  | 135                                   | 0.518         |
| 4                 | $c_t$                             | 0.5501             | $c_a$                        | 0.5705             | 0.970                | 0.969                          | 0.675                            | 0.043                | 0.043                          | 0.025                            | 1.742   | 1.40E-04             | 7.39E-05                       | 6.85E-05                         | 64                        | 64                                  | 194                                   | 0.527         |
| , 5               | $c_t$                             | 0.5501             | $c_a$                        | 0.701              | 0.958                | 0.957                          | 0.737                            | 0.030                | 0.030                          | 0.025                            | 1.196   | 9.70E-05             | 5.00E-05                       | 5.70E-05                         | 45                        | 45                                  | 217                                   | 0.531         |
| 1 6               | $c_t$                             | 0.3507 ge          | $c_a$                        | 0.9933             | 0.974                | 0.974                          | 0.376                            | 0.048                | 0.048                          | 0.019                            | 2.441   | 1.53E-04             | 8.23E-05                       | 8.22E-05                         | 66                        | 66                                  | 195                                   | 0.351         |
| 7                 | $c_t$                             | 0.5503             | $c_a$                        | 0.4633             | 0.968                | 0.965                          | 0.664                            | 0.043                | 0.042                          | 0.038                            | 1.108   | 1.38E-04             | 7.12E-05                       | 1.23E-04                         | 74                        | 74                                  | 255                                   | 0.523         |
| 8                 | $c_t$                             | 0.5506             | $c_a$                        | 0.5062             | 0.957                | 0.946                          | 0.576                            | 0.036                | 0.035                          | 0.045                            | 0.793   | 1.19E-04             | 5.58E-05                       | 1.36E-04                         | 53                        | 53                                  | 255                                   | 0.551         |
| 9                 | $c_t$                             | 0.5505             | $c_a$                        | 0.9991             | 0.965                | 0.963                          | 0.761                            | 0.037                | 0.036                          | 0.037                            | 0.992   | 1.18E-04             | 5.28E-05                       | 9.61E-05                         | 53                        | 53                                  | 255                                   | 0.550         |
| 10                |                                   | 0.5504             | $c \neq c_a, c_t$            | 0.4151             | 0.959                | 0.957                          | 0.928                            | 0.030                | 0.030                          | 0.036                            | 0.825   | 9.70E-05             | 7.63E-06                       | 1.47E-05                         | 48                        | 48                                  | 223                                   | 0.489         |
| 1                 | $c_t$                             | 0.5504             | $c \neq c_a, c_t$            | 0.8666             | 0.928                | 0.927                          | 0.712                            | 0.017                | 0.017                          | 0.034                            | 0.516   | 5.67E-05             | 2.93E-05                       | 9.12E-05                         | 26                        | 26                                  | 236                                   | 0.550         |
| 2                 | $c_t$                             | 0.5519             | $c_a$                        | 0.7673             | 0.929                | 0.929                          | 0.737                            | 0.017                | 0.017                          | 0.026                            | 0.664   | 5.53E-05             | 2.56E-05                       | 5.48E-05                         | 25                        | 25                                  | 243                                   | 0.552         |
| 3                 | $c_t$                             | 0.5502             | $c \neq c_a, c_t$            | 0.139              | 0.940                | 0.940                          | 0.746                            | 0.021                | 0.021                          | 0.035                            | 0.593   | 6.80E-05             | 3.51E-05                       | 7.93E-05                         | 30                        | 30                                  | 176                                   | 0.414         |
| 4                 | $c_t$                             | 0.5524             | $c_a$                        | 0.8271             | 0.945                | 0.945                          | 0.696                            | 0.022                | 0.022                          | 0.017                            | 1.330   | 7.25E-05             | 3.75E-05                       | 4.55E-05                         | 32                        | 32                                  | 177                                   | 0.552         |
| 2 5               | $c_t$                             | 0.5508             | $c_a$                        | 0.2806             | 0.958                | 0.957                          | 0.552                            | 0.029                | 0.029                          | 0.028                            | 1.046   | 9.55E-05             | 4.27E-05                       | 7.21E-05                         | 43                        | 43                                  | 244                                   | 0.551         |
| 2 6               | $c_t$                             | 0.5511             | $c_a$                        | 0.3694             | 0.952                | 0.949                          | 0.747                            | 0.027                | 0.027                          | 0.036                            | 0.745   | 8.67E-05             | 3.87E-05                       | 9.05E-05                         | 39                        | 39                                  | 255                                   | 0.458         |
| 7                 | $c_t$                             | 0.5516             | $c_a$                        | 0.2765             | 0.946                | 0.945                          | 0.619                            | 0.024                | 0.024                          | 0.012                            | 2.050   | 7.85E-05             | 4.06E-05                       | 3.50E-05                         | 34                        | 34                                  | 185                                   | 0.441         |
| 8                 | $c_t$                             | 0.5505             | $c_a$                        | 0.1529             | 0.951                | 0.951                          | 0.648                            | 0.025                | 0.025                          | 0.013                            | 1.968   | 8.17E-05             | 4.23E-05                       | 3.99E-05                         | 39                        | 39                                  | 177                                   | 0.424         |
| 9                 | $c_t$                             | 0.5504             | $c \neq c_a, c_t$            | 0.1515             | 0.943                | 0.942                          | 0.674                            | 0.022                | 0.022                          | 0.016                            | 1.390   | 7.26E-05             | 3.38E-05                       | 4.02E-05                         | 33                        | 33                                  | 184                                   | 0.492         |
| 10                | $c_t$                             | 0.5511             | $c_a$                        | 0.9992             | 0.939                | 0.935                          | 0.714                            | 0.021                | 0.021                          | 0.032                            | 0.665   | 6.92E-05             | 3.79E-05                       | 9.54E-05                         | 33                        | 33                                  | 235                                   | 0.551         |
| 1                 | $c_t$                             | 0.5507             | $c_a$                        | 0.7145             | 0.961                | 0.957                          | 0.684                            | 0.034                | 0.034                          | 0.028                            | 1.223   | 1.11E-04             | 5.32E-05                       | 7.42E-05                         | 56                        | 56                                  | 209                                   | 0.535         |
| 2                 | $c_t$                             | 0.5504             | $c_a$                        | 0.9216             | 0.954                | 0.951                          | 0.585                            | 0.029                | 0.028                          | 0.021                            | 1.322   | 9.26E-05             | 8.63E-05                       | 1.61E-04                         | 47                        | 47                                  | 255                                   | 0.543         |
| 3                 | $c_t$                             | 0.5500             | $c_a$                        | 0.4874             | 0.962                | 0.961                          | 0.893                            | 0.032                | 0.032                          | 0.020                            | 1.560   | 1.04E-04             | 8.07E-06                       | 9.05E-06                         | 47                        | 47                                  | 232                                   | 0.392         |
| 4                 | $c_t$                             | 0.5503             | $c_a$                        | 0.9828             | 0.943                | 0.936                          | 0.683                            | 0.025                | 0.024                          | 0.025                            | 0.966   | 8.09E-05             | 4.42E-05                       | 9.10E-05                         | 37                        | 37                                  | 254                                   | 0.547         |
| 3 5               | $c_t$                             | 0.5502             | $c_a$                        | 0.9276             | 0.961                | 0.954                          | 0.616                            | 0.037                | 0.036                          | 0.029                            | 1.269   | 1.21E-04             | 8.90E-05                       | 1.10E-04                         | 62                        | 62                                  | 178                                   | 0.545         |
| 6                 | $c_t$                             | 0.5509             | $c_a$                        | 0.6108             | 0.950                | 0.947                          | 0.577                            | 0.026                | 0.026                          | 0.029                            | 0.883   | 8.44E-05             | 6.08E-05                       | 1.67E-04                         | 42                        | 42                                  | 245                                   | 0.551         |
| 7                 | $c_t$                             | 0.5518             | $c_a$                        | 0.921              | 0.937                | 0.935                          | 0.553                            | 0.022                | 0.022                          | 0.012                            | 1.913   | 7.37E-05             | 3.67E-05                       | 4.59E-05                         | 32                        | 32                                  | 194                                   | 0.549         |
| 8                 | $c_t$                             | 0.5503             | $c_a$                        | 0.9859             | 0.952                | 0.949                          | 0.700                            | 0.027                | 0.027                          | 0.026                            | 1.038   | 9.00E-05             | 4.83E-05                       | 9.49E-05                         | 40                        | 40                                  | 255                                   | 0.549         |
| 9                 | $c_t$                             | 0.5500             | $c_a$                        | 0.4988             | 0.964                | 0.963                          | 0.732                            | 0.034                | 0.034                          | 0.035                            | 0.991   | 1.11E-04             | 5.74E-05                       | 1.03E-04                         | 50                        | 50                                  | 255                                   | 0.550         |
| 10                | $c_t$                             | 0.5505             | $c_a$                        | 0.8508             | 0.971                | 0.965                          | 0.691                            | 0.047                | 0.046                          | 0.033                            | 1.399   | 1.52E-04             | 1.03E-04                       | 1.13E-04                         | 70                        | 70                                  | 220                                   | 0.551         |
| 1                 | XXX                               | XXX                | XXX                          | XXX                | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX           |
| 2                 | $c_t$                             | 0.5503             | $c_t$                        | 0.1618             | 0.951                | 0.950                          | 0.418                            | 0.032                | 0.032                          | 0.016                            | 2.032   | 1.08E-04             | 6.44E-05                       | 6.00E-05                         | 53                        | 53                                  | 190                                   | 0.389         |
| 3                 | $c_t$                             | 0.5501             | $c_a$                        | 0.2935             | 0.944                | 0.942                          | 0.406                            | 0.023                | 0.023                          | 0.023                            | 1.008   | 7.67E-05             | 4.12E-05                       | 9.56E-05                         | 35                        | 35                                  | 204                                   | 0.457         |
| 4                 | $c_t$                             | 0.5501             | $c_a$                        | 0.9624             | 0.939                | 0.937                          | 0.258                            | 0.028                | 0.028                          | 0.017                            | 1.618   | 9.56E-05             | 7.18E-05                       | 9.35E-05                         | 49                        | 49                                  | 145                                   | 0.550         |
| 4 5               | $c_t$                             | 0.5507             | $c_a$                        | 0.1927             | 0.963                | 0.962                          | 0.432                            | 0.034                | 0.034                          | 0.009                            | 3.806   | 1.11E-04             | 5.20E-05                       | 3.74E-05                         | 50                        | 50                                  | 216                                   | 0.512         |
| 6                 | $c_t$                             | 0.5503             | $c_a$                        | 0.9777             | 0.970                | 0.965                          | 0.758                            | 0.044                | 0.044                          | 0.146                            | 0.301   | 1.44E-04             | 7.48E-05                       | 2.94E-04                         | 71                        | 71                                  | 255                                   | 0.550         |
| 7                 | $c_t$                             | 0.5513             | $c_a$                        | 0.5677             | 0.957                | 0.954                          | 0.612                            | 0.032                | 0.032                          | 0.022                            | 1.476   | 1.05E-04             | 5.84E-05                       | 7.06E-05                         | 51                        | 51                                  | 160                                   | 0.535         |
| 8                 | $c_t$                             | 0.5504             | $c_a$                        | 0.4599             | 0.947                | 0.944                          | 0.635                            | 0.024                | 0.024                          | 0.043                            | 0.552   | 7.70E-05             | 4.12E-05                       | 1.34E-04                         | 40                        | 40                                  | 255                                   | 0.538         |
| 9                 | $c_t$                             | 0.5512             | $c_a$                        | 0.8991             | 0.943                | 0.942                          | 0.347                            | 0.026                | 0.026                          | 0.017                            | 1.549   | 8.53E-05             | 4.59E-05                       | 7.09E-05                         | 42                        | 42                                  | 173                                   | 0.551         |
| 10                |                                   | 0.55               | $c_a$                        | 0.4666             | 0.952                | 0.950                          | 0.355                            | 0.033                | 0.033                          | 0.013                            | 2.473   | 1.09E-04             | 4.39E-05                       | 3.83E-05                         | 52                        | 52                                  | 224                                   | 0.550         |
| 1                 | $c_t$                             | 0.551              | $c_a$                        | 0.7041             | 0.960                | 0.959                          | 0.755                            | 0.031                | 0.031                          | 0.031                            | 0.992   | 1.01E-04             | 3.96E-05                       | 6.51E-05                         | 46                        | 46                                  | 236                                   | 0.551         |
| 2                 | XXX                               | XXX                | XXX                          | XXX                | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | XXX           |
| 3                 | $c_t$                             | 0.5506             | $c_a$                        | 0.9752             | 0.958                | 0.955                          | 0.708                            | 0.032                | 0.032                          | 0.041                            | 0.776   | 1.04E-04             | 5.71E-05                       | 1.04E-04                         | 51                        | 51                                  | 255                                   | 0.551         |
| 4                 | $c_t$                             | 0.551              | $c_a$                        | 0.3894             | 0.959                | 0.958                          | 0.740                            | 0.030                | 0.030                          | 0.032                            | 0.932   | 9.83E-05             | 5.39E-05                       | 7.92E-05                         | 49                        | 49                                  | 253                                   | 0.551         |
| 5 5               | $c_t$                             | 0.5503             | $c_a$                        | 0.4587             | 0.956                | 0.953                          | 0.795                            | 0.029                | 0.029                          | 0.058                            | 0.490   | 9.33E-05             | 3.40E-05                       | 9.96E-05                         | 42                        | 42                                  | 253                                   | 0.550         |
| 6                 | $c_t$                             | 0.5505             | $c_a$                        | 0.9062             | 0.969                | 0.968                          | 0.671                            | 0.039                | 0.039                          | 0.043                            | 0.911   | 1.26E-04             | 6.89E-05                       | 1.13E-04                         | 58                        | 58                                  | 185                                   | 0.551         |
| 7                 | $c_t$                             | 0.5501             | $c \neq c_a, c_t$            | 0.0851             | 0.949                | 0.948                          | 0.778                            | 0.025                | 0.025                          | 0.028                            | 0.903   | 8.13E-05             | 3.39E-05                       | 4.89E-05                         | 38                        | 38                                  | 127                                   | 0.550         |
| 8 9               | $c_t$                             | 0.5501             | $c_a$                        | 0.7727             | 0.970                | 0.970                          | 0.742                            | 0.041                | 0.041                          | 0.036                            | 1.119   | 1.32E-04             | 6.83E-05                       | 8.20E-05                         | 58<br>59                  | 58<br>59                            | 175                                   | 0.550         |
|                   | $c_t$                             | 0.5500<br>0.5518   | $c_a$                        | 0.1352<br>0.8296   | 0.970<br>0.952       | 0.969<br>0.951                 | 0.753<br>0.782                   | 0.042                | 0.041                          | 0.067                            | 0.615<br>0.777  | 1.34E-04             | 7.34E-05                       | 1.59E-04<br>6.67E-05             | 40                        | 40                                  | 231<br>237                            | 0.550         |
| 10                | $c_t$                             | 0.5518             | $c_a$                        | 0.8290             | 0.952                | 0.951                          | 0.782                            | 0.020                | 0.020                          | 0.034                            | 0.777   | 8.50E-05             | 3.67E-05                       | 0.07E-05                         | 40                        | 40                                  | 231                                   | 0.552         |

|    | $l_a^p$   | Step             | 1 - Step 3 | Step 4            | - Step 8 |       | $L_0^{norm}$        |                             |              | L:    | norm                | -2  |           |                     |                     |                |                          |                                       |       |
|----|---|------------------|------------|-------------------|----------|-------|---------------------|-----------------------------|--------------|-------|---------------------|---|-----------|---------------------|---------------------|----------------|--------------------------|---------------------------------------|-------|
|    | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ |                  |            |                   |          |       | r norm,adv          | $L_{\infty}$ $t_{norm,adv}$ | + norm.clean |       |                     |   |           |                     |                     |                |                          |                                       |       |
| q  | p   | $\tilde{\tau}_c$ | Category   | $	au_c$           | Category |       | $L_{0,\mathcal{H}}$ | $L_{0,\mathcal{H}}$         |              |       | $L_{1,\mathcal{H}}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}$ | $L_{2,\mathcal{H}}$ | $L_{2,\mathcal{H}}$ | $L_{\infty,R}$ | $L_{\infty,\mathcal{H}}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | L     |
|    | 1   | $c_t$            | 0.5532     | $c_a$             | 0.5155   | 0.944 | 0.941               | 0.743                       | 0.022        | 0.022 | 0.069               | 0.319   | 7.12E-05  | 3.90E-05            | 1.64E-04            | 35             | 35                       | 254                                   | 0.553 |
|    | 2   | $c_t$            | 0.55       | $c_a$             | 0.2994   | 0.971 | 0.971               | 0.762                       | 0.043        | 0.043 | 0.036               | 1.181   | 1.38E-04  | 5.94E-05            | 7.20E-05            | 62             | 62                       | 213                                   | 0.550 |
|    | 3   | $c_t$            | 0.5501     | $c_a$             | 0.3357   | 0.965 | 0.964               | 0.746                       | 0.037        | 0.037 | 0.055               | 0.671   | 1.21E-04  | 6.22E-05            | 1.31E-04            | 56             | 56                       | 255                                   | 0.550 |
|    | 4   | $c_t$            | 0.5508     | $c_a$             | 0.4556   | 0.953 | 0.950               | 0.718                       | 0.027        | 0.027 | 0.036               | 0.754   | 8.93E-05  | 4.61E-05            | 9.46E-05            | 40             | 40                       | 219                                   | 0.551 |
| 6  | 5   | $c_t$            | 0.5509     | $c \neq c_a, c_t$ | 0.2022   | 0.947 | 0.945               | 0.739                       | 0.024        | 0.024 | 0.027               | 0.892   | 7.81E-05  | 3.50E-05            | 8.86E-05            | 36             | 36                       | 255                                   | 0.551 |
| "  | 6   | $c_t$            | 0.5510     | $c_a$             | 0.4884   | 0.956 | 0.955               | 0.728                       | 0.029        | 0.029 | 0.031               | 0.938   | 9.27E-05  | 4.23E-05            | 7.23E-05            | 46             | 46                       | 204                                   | 0.551 |
|    | 7   | $c_t$            | 0.5502     | $c \neq c_a, c_t$ | 0.3266   | 0.964 | 0.963               | 0.901                       | 0.034        | 0.034 | 0.045               | 0.768   | 1.11E-04  | 1.90E-05            | 3.53E-05            | 51             | 51                       | 237                                   | 0.550 |
|    | 8   | $c_t$            | 0.5508     | $c_a$             | 0.727    | 0.960 | 0.951               | 0.709                       | 0.034        | 0.034 | 0.045               | 0.754   | 1.13E-04  | 6.13E-05            | 1.47E-04            | 54             | 54                       | 255                                   | 0.551 |
|    | 9   | $c_t$            | 0.5501     | $c_a$             | 0.1327   | 0.939 | 0.936               | 0.929                       | 0.021        | 0.021 | 0.060               | 0.348   | 6.85E-05  | 8.61E-06            | 3.27E-05            | 32             | 32                       | 235                                   | 0.550 |
|    | 10  | $c_t$            | 0.5505     | $c_a$             | 0.7708   | 0.964 | 0.961               | 0.711                       | 0.037        | 0.037 | 0.069               | 0.532   | 1.21E-04  | 6.20E-05            | 1.80E-04            | 57             | 57                       | 255                                   | 0.551 |
|    | 1   | $c_t$            | 0.5501     | $c_a$             | 0.891    | 0.957 | 0.948               | 0.755                       | 0.032        | 0.031 | 0.081               | 0.381   | 1.03E-04  | 5.25E-05            | 1.84E-04            | 48             | 48                       | 255                                   | 0.550 |
|    | 2   | $c_t$            | 0.5519     | $c_a$             | 0.7635   | 0.965 | 0.964               | 0.910                       | 0.036        | 0.035 | 0.043               | 0.816   | 1.15E-04  | 1.85E-05            | 3.43E-05            | 56             | 56                       | 245                                   | 0.552 |
|    | 3   | $c_t$            | 0.5503     | $c_a$             | 0.8846   | 0.961 | 0.960               | 0.746                       | 0.032        | 0.032 | 0.046               | 0.688   | 1.03E-04  | 5.64E-05            | 1.29E-04            | 47             | 47                       | 255                                   | 0.550 |
|    | 4   | $c_t$            | 0.5504     | $c \neq c_a, c_t$ | 0.0571   | 0.949 | 0.948               | 0.691                       | 0.024        | 0.024 | 0.033               | 0.731   | 7.89E-05  | 4.88E-05            | 1.11E-04            | 35             | 35                       | 255                                   | 0.550 |
| 7  | 5   | $c_t$            | 0.5511     | $c_a$             | 0.3862   | 0.962 | 0.962               | 0.748                       | 0.033        | 0.033 | 0.028               | 1.161   | 1.06E-04  | 4.49E-05            | 6.55E-05            | 48             | 48                       | 255                                   | 0.551 |
| 1  | 6   | $c_t$            | 0.5519     | $c_a$             | 0.9881   | 0.962 | 0.959               | 0.747                       | 0.033        | 0.033 | 0.048               | 0.690   | 1.07E-04  | 5.53E-05            | 1.13E-04            | 52             | 52                       | 255                                   | 0.552 |
|    | 7   | $c_t$            | 0.5501     | $c \neq c_a, c_t$ | 0.1151   | 0.956 | 0.955               | 0.712                       | 0.030        | 0.029 | 0.029               | 1.006   | 9.59E-05  | 4.48E-05            | 6.12E-05            | 48             | 48                       | 255                                   | 0.550 |
|    | 8   | $c_t$            | 0.5507     | $c \neq c_a, c_t$ | 0.0809   | 0.964 | 0.964               | 0.731                       | 0.036        | 0.036 | 0.038               | 0.930   | 1.15E-04  | 5.73E-05            | 8.71E-05            | 54             | 54                       | 181                                   | 0.526 |
|    | 9   | $c_t$            | 0.5501     | $c_a$             | 0.9831   | 0.968 | 0.966               | 0.745                       | 0.040        | 0.040 | 0.062               | 0.646   | 1.31E-04  | 6.18E-05            | 1.28E-04            | 60             | 60                       | 246                                   | 0.550 |
|    | 10  | $c_t$            | 0.5504     | $c_a$             | 0.9954   | 0.952 | 0.952               | 0.733                       | 0.027        | 0.027 | 0.030               | 0.900   | 8.66E-05  | 4.71E-05            | 8.07E-05            | 40             | 40                       | 199                                   | 0.550 |
|    | 1   | $c_t$            | 0.5501     | $c \neq c_a, c_t$ | 0.1989   | 0.953 | 0.953               | 0.725                       | 0.027        | 0.027 | 0.031               | 0.864   | 8.70E-05  | 4.77E-05            | 8.70E-05            | 37             | 37                       | 197                                   | 0.550 |
|    | 2   | $c_t$            | 0.5515     | $c_a$             | 0.1491   | 0.957 | 0.956               | 0.765                       | 0.029        | 0.029 | 0.025               | 1.178   | 9.43E-05  | 3.92E-05            | 6.20E-05            | 39             | 39                       | 255                                   | 0.552 |
|    | 3   | $c_t$            | 0.5517     | $c_a$             | 0.3067   | 0.961 | 0.959               | 0.727                       | 0.034        | 0.034 | 0.038               | 0.891   | 1.10E-04  | 5.21E-05            | 9.69E-05            | 51             | 51                       | 255                                   | 0.552 |
|    | 4   | $c_t$            | 0.5501     | $c \neq c_a, c_t$ | 0.2328   | 0.946 | 0.946               | 0.615                       | 0.023        | 0.023 | 0.017               | 1.356   | 7.46E-05  | 5.38E-05            | 7.34E-05            | 33             | 33                       | 133                                   | 0.550 |
|    | 5   | $c_t$            | 0.5510     | $c \neq c_a, c_t$ | 0.067    | 0.961 | 0.960               | 0.708                       | 0.033        | 0.033 | 0.030               | 1.098   | 1.08E-04  | 5.92E-05            | 8.46E-05            | 59             | 59                       | 207                                   | 0.526 |
| 8  | 6   | $c_t$            | 0.5505     | $c_a$             | 0.1924   | 0.943 | 0.939               | 0.560                       | 0.026        | 0.026 | 0.022               | 1.143   | 8.48E-05  | 4.63E-05            | 7.18E-05            | 39             | 39                       | 245                                   | 0.551 |
|    | 7   | $c_t$            | 0.5505     | $c_a$             | 0.8727   | 0.961 | 0.956               | 0.754                       | 0.033        | 0.033 | 0.072               | 0.458   | 1.08E-04  | 5.55E-05            | 1.65E-04            | 58             | 58                       | 255                                   | 0.551 |
|    | 8   | $c_t$            | 0.5533     | $c_a$             | 0.1861   | 0.955 | 0.953               | 0.751                       | 0.028        | 0.028 | 0.053               | 0.534   | 9.25E-05  | 4.78E-05            | 1.19E-04            | 45             | 45                       | 253                                   | 0.553 |
|    | 9   | $c_t$            | 0.5520     | $c_a$             | 0.6761   | 0.962 | 0.960               | 0.671                       | 0.035        | 0.034 | 0.038               | 0.906   | 1.12E-04  | 7.94E-05            | 1.37E-04            | 54             | 54                       | 210                                   | 0.552 |
|    | 10  | $c_t$            | 0.5520     | $c_a$             | 0.7294   | 0.952 | 0.951               | 0.709                       | 0.027        | 0.026 | 0.062               | 0.425   | 8.60E-05  | 4.44E-05            | 1.44E-04            | 43             | 43                       | 241                                   | 0.552 |
|    | 1   | XXX              | XXX        | XXX               | XXX      | XXX   | XXX                 | XXX                         | xxx          | XXX   | xxx                 | XXX   | XXX       | XXX                 | XXX                 | XXX            | XXX                      | XXX                                   | XXX   |
|    | 2   | $c_t$            | 0.5507     | $c_a$             | 0.162    | 0.943 | 0.941               | 0.728                       | 0.022        | 0.022 | 0.034               | 0.645   | 7.08E-05  | 3.65E-05            | 8.94E-05            | 35             | 35                       | 216                                   | 0.551 |
|    | 3   | $c_t$            | 0.5500     | $c_a$             | 0.7399   | 0.955 | 0.955               | 0.740                       | 0.028        | 0.028 | 0.019               | 1.468   | 9.13E-05  | 3.77E-05            | 4.14E-05            | 44             | 44                       | 147                                   | 0.499 |
|    | 4   | XXX              | XXX        | xxx               | XXX      | xxx   | XXX                 | XXX                         | XXX          | XXX   | XXX                 | XXX   | XXX       | XXX                 | XXX                 | xxx            | xxx                      | XXX                                   | XXX   |
|    | 5   | XXX              | XXX        | xxx               | XXX      | XXX   | XXX                 | XXX                         | XXX          | XXX   | xxx                 | XXX   | XXX       | XXX                 | XXX                 | XXX            | XXX                      | XXX                                   | XXX   |
| 9  | 6   | XXX              | XXX        | xxx               | XXX      | xxx   | XXX                 | XXX                         | XXX          | XXX   | xxx                 | XXX   | xxx       | XXX                 | XXX                 | xxx            | XXX                      | XXX                                   | XXX   |
|    | 7   | XXX              | XXX        | XXX               | XXX      | XXX   | XXX                 | XXX                         | XXX          | XXX   | XXX                 | XXX   | XXX       | XXX                 | XXX                 | xxx            | XXX                      | XXX                                   | XXX   |
|    | 8   | $c_t$            | 0.5508     | $c_a$             | 0.8219   | 0.942 | 0.937               | 0.756                       | 0.022        | 0.021 | 0.067               | 0.319   | 7.00E-05  | 3.61E-05            | 1.79E-04            | 34             | 34                       | 255                                   | 0.494 |
|    | 9   | $c_t$            | 0.5509     | $c_a$             | 0.7332   | 0.944 | 0.942               | 0.702                       | 0.023        | 0.023 | 0.024               | 0.945   | 7.49E-05  | 3.86E-05            | 7.95E-05            | 34             | 34                       | 255                                   | 0.551 |
|    | 10  | $c_t$            | 0.5512     | $c \neq c_a, c_t$ | 0.1304   | 0.947 | 0.946               | 0.645                       | 0.026        | 0.025 | 0.014               | 1.885   | 8.44E-05  | 4.36E-05            | 4.08E-05            | 43             | 43                       | 235                                   | 0.551 |
|    | 1   | $c_t$            | 0.5512     | $c \neq c_a, c_t$ | 0.3562   | 0.954 | 0.953               | 0.661                       | 0.027        | 0.027 | 0.017               | 1.616   | 8.76E-05  | 4.53E-05            | 4.48E-05            | 41             | 41                       | 184                                   | 0.551 |
|    | 2   | $c_t$            | 0.5509     | $c_a$             | 0.6846   | 0.960 | 0.954               | 0.705                       | 0.034        | 0.034 | 0.086               | 0.395   | 1.12E-04  | 5.71E-05            | 2.03E-04            | 57             | 57                       | 255                                   | 0.551 |
|    | 3   | $c_t$            | 0.5503     | $c_a$             | 0.0874   | 0.955 | 0.955               | 0.619                       | 0.031        | 0.031 | 0.041               | 0.751   | 9.99E-05  | 5.17E-05            | 1.15E-04            | 45             | 45                       | 255                                   | 0.550 |
|    | 4   | $c_t$            | 0.5500     | $c \neq c_a, c_t$ | 0.2809   | 0.956 | 0.954               | 0.741                       | 0.028        | 0.028 | 0.044               | 0.640   | 9.10E-05  | 4.98E-05            | 1.22E-04            | 42             | 42                       | 242                                   | 0.550 |
|    | 5   | $c_t$            | 0.5510     | $c_a$             | 0.3481   | 0.963 | 0.957               | 0.696                       | 0.038        | 0.037 | 0.081               | 0.458   | 1.22E-04  | 6.26E-05            | 1.86E-04            | 54             | 54                       | 255                                   | 0.551 |
| 10 | 6   | $c_t$            | 0.5506     | $c \neq c_a, c_t$ | 0.6706   | 0.965 | 0.965               | 0.695                       | 0.035        | 0.035 | 0.024               | 1.450   | 1.14E-04  | 7.37E-05            | 8.50E-05            | 52             | 52                       | 204                                   | 0.536 |
|    | 7   | $c_t$            | 0.5515     | $c \neq c_a, c_t$ | 0.1164   | 0.957 | 0.951               | 0.728                       | 0.030        | 0.030 | 0.064               | 0.465   | 9.88E-05  | 5.07E-05            | 1.75E-04            | 45             | 45                       | 255                                   | 0.552 |
|    | 8   | $c_t$            | 0.5509     | $c \neq c_a, c_t$ | 0.4144   | 0.970 | 0.970               | 0.753                       | 0.042        | 0.042 | 0.056               | 0.747   | 1.35E-04  | 6.41E-05            | 1.17E-04            | 65             | 65                       | 226                                   | 0.551 |
|    | 9   | $c_t$            | 0.5504     | $c_a$             | 0.1488   | 0.959 | 0.957               | 0.675                       | 0.035        | 0.035 | 0.065               | 0.539   | 1.15E-04  | 5.92E-05            | 1.47E-04            | 56             | 56                       | 249                                   | 0.550 |
|    | 10  | $c_t$            | 0.5502     | $c \neq c_a, c_t$ | 0.5014   | 0.968 | 0.964               | 0.750                       | 0.041        | 0.041 | 0.067               | 0.605   | 1.34E-04  | 7.31E-05            | 1.66E-04            | 58             | 58                       | 255                                   | 0.550 |
|    |   | t                |            | 1 / -a, -t        | 0.00     |       |                     | *****                       |              |       |                     |   |           |                     |                     |                |                          |                                       | 7.000 |

Table B7.: Combination  $(\rho, \lambda, \rho) = \text{N-N-L}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 22, 71)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (1, 21, 71)$ .

| $\mathcal{A}_q^p$                            | Step 1 - Step 3 Step 4 - Step 8 |                    |                              | - Step 8           |                      | $L_0^{norm}$                   |                                  |                      | $L_1^r$                        | norm                             |   |                      | $L_2^{norm}$                   |                                  |                           | $L_{\infty}$                        |                                       |               |
|--|---------------------------------|--------------------|------------------------------|--------------------|----------------------|--------------------------------|----------------------------------|----------------------|--------------------------------|----------------------------------|---|----------------------|--------------------------------|----------------------------------|---------------------------|-------------------------------------|---------------------------------------|---------------|
| $q \mid p$                                   | $\tilde{	au}_c$                 | Dominant           | $\tau_c$                     | Dominant           | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,clean}$ | $\mathcal{L}$ |
| 1  | $c_t$                           | Category<br>0.5504 | $c_t$                        | Category<br>0.5504 | 0.965                | 0.964                          | 0.901                            | 0.037                | 0.037                          | 0.031                            | $\frac{L_{1,H}}{1.176}$   | 1.19E-04             | 9.44E-06                       | 1.26E-05                         | 53                        | 53                                  | 230                                   | 0.000         |
| 2  | $c_t$                           | 0.5519             | $c_t$                        | 0.5519             | 0.948                | 0.943                          | 0.859                            | 0.037                | 0.037                          | 0.034                            | 0.714   | 8.07E-05             | 1.62E-05                       | 3.88E-05                         | 41                        | 41                                  | 246                                   | 0.000         |
| 3  | $c_t$                           | 0.5511             | $c_t$                        | 0.7392             | 0.946                | 0.943                          | 0.645                            | 0.025                | 0.025                          | 0.034                            | 2.460   | 9.56E-05             | 7.99E-05                       | 5.25E-05                         | 41                        | 41                                  | 135                                   | 0.551         |
| 4  | $c_t$                           | 0.5501             |                              | 0.7569             | 0.970                | 0.969                          | 0.675                            | 0.030                | 0.043                          | 0.012                            | 1.742   | 1.40E-04             | 7.39E-05                       | 6.85E-05                         | 64                        | 64                                  | 194                                   | 0.538         |
| 1 5  | $c_t$                           | 0.5501             | $c_a$                        | 0.1774             | 0.958                | 0.957                          | 0.737                            | 0.043                | 0.045                          | 0.025                            | 1.196   | 9.70E-05             | 5.00E-05                       | 5.70E-05                         | 45                        | 45                                  | 217                                   | 0.379         |
| 1 6  | $c_t$                           | 0.3507 ge          | $c_a$                        | 0.1774             | 0.933                | 0.974                          | 0.737                            | 0.030                | 0.030                          | 0.025                            | 2.441   | 1.53E-04             | 8.23E-05                       | 8.22E-05                         | 66                        | 66                                  | 195                                   | 0.379         |
| 7  | $c_t$                           | 0.5503             | $c_a$                        | 0.549              | 0.968                | 0.965                          | 0.664                            | 0.043                | 0.048                          | 0.019                            | 1.108   | 1.38E-04             | 7.12E-05                       | 1.23E-04                         | 74                        | 74                                  | 255                                   | 0.405         |
| 8  | $c_t$                           | 0.5506             | $c_a$                        | 0.5506             | 0.957                | 0.946                          | 0.576                            | 0.046                | 0.042                          | 0.045                            | 0.793   | 1.19E-04             | 5.58E-05                       | 1.36E-04                         | 53                        | 53                                  | 255                                   | 0.000         |
| 9  | $c_t$                           | 0.5505             |                              | 0.5505             | 0.965                | 0.963                          | 0.761                            | 0.037                | 0.036                          | 0.037                            | 0.992   | 1.13E-04<br>1.18E-04 | 5.28E-05                       | 9.61E-05                         | 53                        | 53                                  | 255                                   | 0.000         |
| 10   | $c_t$                           | 0.5504             | $c_t$                        | 0.5504             | 0.959                | 0.957                          | 0.928                            | 0.030                | 0.030                          | 0.036                            | 0.825   | 9.70E-05             | 7.63E-06                       | 1.47E-05                         | 48                        | 48                                  | 223                                   | 0.000         |
| 1  | _                               | 0.5504             |                              | 0.3774             | 0.938                | 0.927                          | 0.712                            | 0.030                | 0.030                          | 0.034                            | 0.525   | 5.67E-05             | 2.93E-05                       | 9.12E-05                         | 26                        | 26                                  | 236                                   | 0.173         |
| 2  | $c_t$                           | 0.5519             | $c_t$                        | 0.5519             | 0.929                | 0.921                          | 0.712                            | 0.017                | 0.017                          | 0.034                            | 0.664   | 5.53E-05             | 2.56E-05                       | 5.48E-05                         | 25                        | 25                                  | 243                                   | 0.000         |
| 3  | $c_t$                           | 0.5513             | $c_t$                        | 0.4317             | 0.940                | 0.929                          | 0.746                            | 0.017                | 0.017                          | 0.020                            | 0.593   | 6.80E-05             | 3.51E-05                       | 7.93E-05                         | 30                        | 30                                  | 176                                   | 0.000         |
| 4  | $c_t$                           | 0.5524             | $c_t$                        | 0.4317             | 0.945                | 0.940                          | 0.696                            | 0.021                | 0.021                          | 0.033                            | 1.330   | 7.25E-05             | 3.75E-05                       | 4.55E-05                         | 32                        | 32                                  | 177                                   | 0.118         |
| 5  | $c_t$                           | 0.5524             | $c_t$                        | 0.2512             | 0.958                | 0.943                          | 0.552                            | 0.022                | 0.022                          | 0.017                            | 1.046   | 9.55E-05             | 4.27E-05                       | 7.21E-05                         | 43                        | 43                                  | 244                                   | 0.000         |
| $\begin{vmatrix} 2 & 3 \\ 6 & \end{vmatrix}$ | $c_t$                           | 0.5508             | $c_t$                        | 0.5511             | 0.952                | 0.949                          | 0.332                            | 0.029                | 0.029                          | 0.028                            | 0.745   | 9.55E-05<br>8.67E-05 | 3.87E-05                       | 9.05E-05                         | 39                        | 39                                  | 255                                   | 0.000         |
| 7  | $c_t$                           | 0.5511             | $c_t$                        | 0.3311             | 0.932                | 0.949                          | 0.619                            | 0.024                | 0.024                          | 0.030                            | 2.050   | 7.85E-05             | 4.06E-05                       | 3.50E-05                         | 34                        | 34                                  | 185                                   | 0.000         |
| 8  | $c_t$                           | 0.5505             | $c_t$                        | 0.3197             | 0.940                | 0.945                          | 0.648                            | 0.024                | 0.024                          | 0.012                            | 1.968   | 8.17E-05             | 4.00E-05<br>4.23E-05           | 3.99E-05                         | 39                        | 39                                  | 177                                   | 0.232         |
| 9  |                                 | 0.5504             | $c_t$                        | 0.5504             | 0.943                | 0.942                          | 0.674                            | 0.023                | 0.023                          | 0.013                            | 1.390   | 7.26E-05             | 3.38E-05                       | 4.02E-05                         | 33                        | 33                                  | 184                                   | 0.000         |
| 10   | $c_t$                           | 0.5504             |                              | 0.4823             | 0.939                | 0.942                          | 0.074                            | 0.022                | 0.022                          | 0.010                            | 0.665   | 6.92E-05             | 3.79E-05                       | 9.54E-05                         | 33                        | 33                                  | 235                                   | 0.332         |
| 10   | $c_t$                           | 0.5511             | $c_a$                        | 0.4023             | 0.961                | 0.957                          | 0.684                            | 0.021                | 0.021                          | 0.032                            | 1.223   | 1.11E-04             | 5.79E-05<br>5.32E-05           | 7.42E-05                         | 56                        | 56                                  | 209                                   | 0.332         |
| 2  | $c_t$                           | 0.5504             | $c_t$                        | 0.9695             | 0.954                | 0.951                          | 0.585                            | 0.034                | 0.034                          | 0.023                            | 1.322   | 9.26E-05             | 8.63E-05                       | 1.61E-04                         | 47                        | 47                                  | 255                                   | 0.550         |
| 3  | $c_t$                           | 0.5504             | $c_a$                        | 0.9095             | 0.962                | 0.961                          | 0.893                            | 0.029                | 0.028                          | 0.021                            | 1.560   | 1.04E-04             | 8.07E-06                       | 9.05E-06                         | 47                        | 47                                  | 232                                   | 0.000         |
| 4  | $c_t$                           | 0.5503             | $c_t$                        | 0.9465             | 0.962                | 0.936                          | 0.683                            | 0.032                | 0.032                          | 0.020                            | 0.966   | 8.09E-05             | 4.42E-05                       | 9.03E-00<br>9.10E-05             | 37                        | 37                                  | 254                                   | 0.542         |
| 5  | $c_t$                           | 0.5503             | $c_a$                        | 0.9403             | 0.943                | 0.954                          | 0.616                            | 0.023                | 0.024                          | 0.029                            | 1.269   | 1.21E-04             | 8.90E-05                       | 1.10E-04                         | 62                        | 62                                  | 178                                   | 0.542         |
| 3 6  | $c_t$                           | 0.5502             | $c_a$                        | 0.2329             | 0.950                | 0.947                          | 0.577                            | 0.026                | 0.036                          | 0.029                            | 0.883   | 8.44E-05             | 6.08E-05                       | 1.67E-04                         | 42                        | 42                                  | 245                                   | 0.551         |
| 7  | $c_t$                           | 0.5518             | $c_a$ $c_t$                  | 0.2646             | 0.937                | 0.935                          | 0.553                            | 0.020                | 0.020                          | 0.023                            | 1.913   | 7.37E-05             | 3.67E-05                       | 4.59E-05                         | 32                        | 32                                  | 194                                   | 0.331         |
| 8  | $c_t$                           | 0.5503             | $c_t$                        | 0.2540             | 0.952                | 0.949                          | 0.700                            | 0.022                | 0.022                          | 0.012                            | 1.038   | 9.00E-05             | 4.83E-05                       | 9.49E-05                         | 40                        | 40                                  | 255                                   | 0.518         |
| 9  | $c_t$                           | 0.5500             | $c_a$                        | 0.3313             | 0.964                | 0.943                          | 0.732                            | 0.021                | 0.027                          | 0.025                            | 0.991   | 1.11E-04             | 5.74E-05                       | 1.03E-04                         | 50                        | 50                                  | 255                                   | 0.358         |
| 10   | $c_t$                           | 0.5505             | $c_a$                        | 0.7996             | 0.971                | 0.965                          | 0.691                            | 0.034                | 0.034                          | 0.033                            | 1.399   | 1.52E-04             | 1.03E-04                       | 1.13E-04                         | 70                        | 70                                  | 220                                   | 0.551         |
| 1  | XXX                             | XXX                | XXX                          | XXX                | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | 0.000         |
| 2  | $c_t$                           | 0.5563             | $c_t$                        | 0.5563             | 0.953                | 0.951                          | 0.890                            | 0.028                | 0.028                          | 0.047                            | 0.584   | 8.95E-05             | 1.80E-05                       | 3.71E-05                         | 42                        | 42                                  | 255                                   | 0.000         |
| 3  | $c_t$                           | 0.5506             | $c_a$                        | 0.1774             | 0.958                | 0.955                          | 0.708                            | 0.020                | 0.020                          | 0.041                            | 0.776   | 1.04E-04             | 5.71E-05                       | 1.04E-04                         | 51                        | 51                                  | 255                                   | 0.493         |
| 4  | $c_t$                           | 0.5510             | $c \neq c_a, c_t$            | 0.1485             | 0.959                | 0.958                          | 0.740                            | 0.030                | 0.032                          | 0.032                            | 0.932   | 9.83E-05             | 5.39E-05                       | 7.92E-05                         | 49                        | 49                                  | 253                                   | 0.490         |
| 5  | $c_t$                           | 0.5503             | $c_{\tau}$ $c_{a}$ , $c_{t}$ | 0.5503             | 0.956                | 0.953                          | 0.795                            | 0.029                | 0.029                          | 0.058                            | 0.490   | 9.33E-05             | 3.40E-05                       | 9.96E-05                         | 42                        | 42                                  | 253                                   | 0.000         |
| 4 6  | $c_t$                           | 0.5505             | $c_t$                        | 0.5505             | 0.969                | 0.968                          | 0.671                            | 0.039                | 0.039                          | 0.043                            | 0.911   | 1.26E-04             | 6.89E-05                       | 1.13E-04                         | 58                        | 58                                  | 185                                   | 0.000         |
| 7  | $c_t$                           | 0.5501             | $c_t$                        | 0.5501             | 0.949                | 0.948                          | 0.778                            | 0.025                | 0.025                          | 0.028                            | 0.903   | 8.13E-05             | 3.39E-05                       | 4.89E-05                         | 38                        | 38                                  | 127                                   | 0.000         |
| 8  | $c_t$                           | 0.5501             | $c_t$                        | 0.1792             | 0.970                | 0.970                          | 0.742                            | 0.041                | 0.041                          | 0.036                            | 1.119   | 1.32E-04             | 6.83E-05                       | 8.20E-05                         | 58                        | 58                                  | 175                                   | 0.371         |
| 9  | $c_t$                           | 0.5500             | $c_t$                        | 0.1991             | 0.970                | 0.969                          | 0.753                            | 0.042                | 0.041                          | 0.067                            | 0.615   | 1.34E-04             | 7.34E-05                       | 1.59E-04                         | 59                        | 59                                  | 231                                   | 0.351         |
| 10   | $c_t$                           | 0.5518             | $c_t$                        | 0.4151             | 0.952                | 0.951                          | 0.782                            | 0.026                | 0.026                          | 0.034                            | 0.777   | 8.50E-05             | 3.67E-05                       | 6.67E-05                         | 40                        | 40                                  | 237                                   | 0.137         |
| 1  | $c_t$                           | 0.5516             | $c_t$                        | 0.2099             | 0.939                | 0.938                          | 0.705                            | 0.021                | 0.021                          | 0.024                            | 0.871   | 6.74E-05             | 3.70E-05                       | 7.48E-05                         | 32                        | 32                                  | 238                                   | 0.342         |
| 2  | XXX                             | XXX                | XXX                          | XXX                | XXX                  | XXX                            | XXX                              | XXX                  | XXX                            | XXX                              | XXX   | XXX                  | XXX                            | XXX                              | XXX                       | XXX                                 | XXX                                   | 0.000         |
| 3  | $c_t$                           | 0.5501             | $c_a$                        | 0.2025             | 0.944                | 0.942                          | 0.406                            | 0.023                | 0.023                          | 0.023                            | 1.008   | 7.67E-05             | 4.12E-05                       | 9.56E-05                         | 35                        | 35                                  | 204                                   | 0.406         |
| 4  | $c_t$                           | 0.5501             | $c_a$                        | 0.9795             | 0.939                | 0.937                          | 0.258                            | 0.028                | 0.028                          | 0.017                            | 1.618   | 9.56E-05             | 7.18E-05                       | 9.35E-05                         | 49                        | 49                                  | 145                                   | 0.550         |
| 5  | $c_t$                           | 0.5507             | $c_t$                        | 0.2642             | 0.963                | 0.962                          | 0.432                            | 0.034                | 0.034                          | 0.009                            | 3.806   | 1.11E-04             | 5.20E-05                       | 3.74E-05                         | 50                        | 50                                  | 216                                   | 0.286         |
| 5 6  | $c_t$                           | 0.5503             | $c_t$                        | 0.2798             | 0.970                | 0.965                          | 0.758                            | 0.044                | 0.044                          | 0.146                            | 0.301   | 1.44E-04             | 7.48E-05                       | 2.94E-04                         | 71                        | 71                                  | 255                                   | 0.271         |
| 7  | $c_t$                           | 0.5513             | $c \neq c_a, c_t$            | 0.1083             | 0.957                | 0.954                          | 0.612                            | 0.032                | 0.032                          | 0.022                            | 1.476   | 1.05E-04             | 5.84E-05                       | 7.06E-05                         | 51                        | 51                                  | 160                                   | 0.450         |
| 8  | $c_t$                           | 0.5504             | $c_a$                        | 0.1182             | 0.947                | 0.944                          | 0.635                            | 0.024                | 0.024                          | 0.043                            | 0.552   | 7.70E-05             | 4.12E-05                       | 1.34E-04                         | 40                        | 40                                  | 255                                   | 0.490         |
| 9  | $c_t$                           | 0.5512             | $c \neq c_a, c_t$            | 0.2135             | 0.943                | 0.942                          | 0.347                            | 0.026                | 0.026                          | 0.017                            | 1.549   | 8.53E-05             | 4.59E-05                       | 7.09E-05                         | 42                        | 42                                  | 173                                   | 0.551         |
| 10   | $c_t$                           | 0.5500             | $c_t$                        | 0.55               | 0.952                | 0.950                          | 0.355                            | 0.033                | 0.033                          | 0.013                            | 2.473   | 1.09E-04             | 4.39E-05                       | 3.83E-05                         | 52                        | 52                                  | 224                                   | 0.000         |
|  |                                 |                    |                              |                    |                      |                                |                                  |                      |                                |                                  |   |                      |                                |                                  |                           |                                     |                                       |               |

| The first content of the content                   | A        | p  | Step 1 - Step 3 Step 4 - Step 8 |        |                   | - Step 8 |            | $L_0^{norm}$        |                     |            | L!                  | norm                |                                |            | $L_2^{norm}$        |                     | $L_{\infty}$             |                          |                          |               |
|--|----------|----|---------------------------------|--------|-------------------|----------|------------|---------------------|---------------------|------------|---------------------|---------------------|--------------------------------|------------|---------------------|---------------------|--------------------------|--------------------------|--------------------------|---------------|
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          | 4  | † î                             |        |                   |          | r norm,adv |                     | τ norm,clean        | r norm,adv | r norm,adv          | r norm,clean        | $L_{1 \mathcal{H}}^{norm,adv}$ | r norm,adv |                     | r norm,clean        | r norm,adv               |                          | r norm,clean             | -             |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | q        | p  | $\tau_c$                        |        | $	au_c$           |          | $L_{0,R}$  | $L_{0,\mathcal{H}}$ | $L_{0,\mathcal{H}}$ | $L_{1,R}$  | $L_{1,\mathcal{H}}$ | $L_{1,\mathcal{H}}$ |                                | $L_{2,R}$  | $L_{2,\mathcal{H}}$ | $L_{2,\mathcal{H}}$ | $L_{\infty,\mathcal{R}}$ | $L_{\infty,\mathcal{H}}$ | $L_{\infty,\mathcal{H}}$ | $\mathcal{L}$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | $c_t$                           |        | $c_t$             |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.419         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | [        |    | $c_t$                           |        | $c_t$             |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $\begin{array}{c c c c c c c c c c c c c c c c c c c $   | [        | 3  | $c_t$                           |        | $c_t$             |          |            |                     |                     |            |                     |                     |                                |            |                     |                     | 56                       |                          |                          | 0.387         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          | 4  | $c_t$                           |        | $c_t$             |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.213         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 6        | 5  | $c_t$                           | 0.5509 | $c_t$             | 0.4426   | 0.947      | 0.945               | 0.739               | 0.024      | 0.024               | 0.027               | 0.892                          | 7.81E-05   | 3.50E-05            | 8.86E-05            | 36                       | 36                       | 255                      | 0.108         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | ا ۲      | 6  | $c_t$                           | 0.5510 | $c_a$             | 0.2329   | 0.956      | 0.955               | 0.728               | 0.029      | 0.029               | 0.031               | 0.938                          | 9.27E-05   | 4.23E-05            | 7.23E-05            | 46                       | 46                       | 204                      | 0.318         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          | 7  | $c_t$                           | 0.5502 | $c_t$             | 0.5502   | 0.964      | 0.963               | 0.901               | 0.034      | 0.034               | 0.045               | 0.768                          | 1.11E-04   | 1.90E-05            | 3.53E-05            | 51                       | 51                       | 237                      | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          | 8  | $c_t$                           | 0.5508 | $c_a$             | 0.2071   | 0.960      | 0.951               | 0.709               | 0.034      | 0.034               | 0.045               | 0.754                          | 1.13E-04   | 6.13E-05            | 1.47E-04            | 54                       | 54                       | 255                      | 0.344         |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |          | 9  | $c_t$                           | 0.5501 | $c_t$             | 0.5501   | 0.939      | 0.936               | 0.929               | 0.021      | 0.021               | 0.060               | 0.348                          | 6.85E-05   | 8.61E-06            | 3.27E-05            | 32                       | 32                       | 235                      | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          | 10 | $c_t$                           | 0.5505 |                   | 0.3134   | 0.964      | 0.961               | 0.711               | 0.037      | 0.037               | 0.069               | 0.532                          | 1.21E-04   | 6.20E-05            | 1.80E-04            | 57                       | 57                       | 255                      | 0.237         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          | 1  | $c_t$                           | 0.5501 | $c_t$             | 0.1007   | 0.957      | 0.948               | 0.755               | 0.032      | 0.031               | 0.081               | 0.381                          | 1.03E-04   | 5.25E-05            | 1.84E-04            | 48                       | 48                       | 255                      | 0.449         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | li       | 2  | _                               | 0.5519 |                   | 0.5519   | 0.965      | 0.964               | 0.910               | 0.036      | 0.035               | 0.043               | 0.816                          | 1.15E-04   | 1.85E-05            | 3.43E-05            | 56                       | 56                       | 245                      | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | l        | 3  | -                               | 0.5503 |                   | 0.135    | 0.961      | 0.960               | 0.746               | 0.032      | 0.032               | 0.046               | 0.688                          | 1.03E-04   | 5.64E-05            | 1.29E-04            | 47                       | 47                       | 255                      | 0.415         |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.515         |
| $ \begin{vmatrix} f & c_{c} & 0.5519 & c_{c} & 0.5519 & c_{c} & 0.5519 & c_{c} & 0.5501 & 0.956 & 0.955 & 0.955 & 0.712 & 0.033 & 0.033 & 0.048 & 0.090 & 1.07E-04 & 55.2E-05 & 1.13E-04 & 52 & 52 & 255 & 255 & 0.0550 & 0.0550 & 0.0550 & 0.958 & 0.956 & 0.0712 & 0.030 & 0.029 & 0.069 & 0.038 & 0.330 & 1.15E-04 & 57.2E-05 & 8.71E-05 & 54 & 848 & 255 & 20.0550 & 0.0550 & 0.0500 & 0.098 & 0.966 & 0.745 & 0.040 & 0.040 & 0.040 & 0.040 & 0.040 & 1.15E-04 & 618E-05 & 54E-05 & 54 & 848 & 255 & 0.05501 & 0.0550 & 0.$ | _        |    | -                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 7        |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.446         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 1 1      |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.125         |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | l 1      |    | -                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.433         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          | -                        |                          | 0.194         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.550         |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.330         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 8        |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.499         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.333         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.177         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     | -                        | -                        |                          | 0.238         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          | _  | -                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          |               |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.237         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.291         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.216         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 9        |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.204         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.346         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |          |    | -                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.195         |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |          |    | _                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          | _                        |                          | 0.350         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $\sqcup$ |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.342         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | -                               |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     | 1                        |                          |                          | 0.449         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |          |    | _                               |        | -                 |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.306         |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | [        |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.521         |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  | [        |    | $c_t$                           |        | $c_t$             |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.375         |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 10       |    | $c_t$                           |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.276         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 10       |    | $c_t$                           |        | $c \neq c_a, c_t$ |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.551         |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | [        |    | $c_t$                           |        | $c_t$             |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.316         |
|  | [        |    | $c_t$                           |        | $c_t$             |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          |                          | 0.000         |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  | [        |    |                                 |        |                   |          |            |                     |                     |            |                     |                     |                                |            |                     |                     |                          |                          | -                        | 0.360         |
|  |          | 10 | $c_t$                           | 0.5502 | $c \neq c_a, c_t$ | 0.1397   | 0.968      | 0.964               | 0.750               | 0.041      | 0.041               | 0.067               | 0.605                          | 1.34E-04   | 7.31E-05            | 1.66E-04            | 58                       | 58                       | 255                      | 0.522         |

Table B8.: Combination  $(\rho, \lambda, \rho) = \text{N-N-N}$ . One has  $\#\mathcal{S}_{clean}^{VGG-16}(L) = 93$ ,  $(\gamma_{st}, \gamma_{ge}, \theta, \phi)_{\mathcal{R}} = (92, 1, 10, 21)$ , and  $(\Gamma, \Theta, \Phi)_{\mathcal{H}} = (62, 10, 21)$ .

| $\rho, \lambda, \rho$ |     | Step 1-3       | Step 4-8 |                      | $L_0^{norm}$                   |                                  |                      | L                              | norm<br>1              |   |                      | $L_2^{norm}$                   |                                  |                           | $L_{\infty}$                        |                             | $\mathcal{L}$ |
|-----------------------|-----|----------------|----------|----------------------|--------------------------------|----------------------------------|----------------------|--------------------------------|------------------------|---|----------------------|--------------------------------|----------------------------------|---------------------------|-------------------------------------|-----------------------------|---------------|
| $\rho, \lambda, \rho$ |     | $	ilde{	au}_c$ | $\tau_c$ | $L_{0,R}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,adv}$ | $L_{0,\mathcal{H}}^{norm,clean}$ | $L_{1,R}^{norm,adv}$ | $L_{1,\mathcal{H}}^{norm,adv}$ | $L_{1,R}^{norm,clean}$ | $\frac{L_{1,\mathcal{H}}^{norm,adv}}{L_{1,\mathcal{H}}^{norm,clean}}$ | $L_{2,R}^{norm,adv}$ | $L_{2,\mathcal{H}}^{norm,adv}$ | $L_{2,\mathcal{R}}^{norm,clean}$ | $L_{\infty,R}^{norm,adv}$ | $L_{\infty,\mathcal{H}}^{norm,adv}$ | $L_{\infty,R}^{norm,clean}$ |               |
|                       | Avg | 0.5482         | 0.5045   | 0.955                | 0.949                          | 0.996                            | 0.0300               | 0.0274                         | 0.0255                 | 1.7798  | 9.9E-05              | 4.5E-05                        | 5.3E-05                          | 46.60                     | 46.27                               | 125.10                      | 0.0437        |
| LLL                   | Min | 0.2947         | 0.2734   | 0.910                | 0.888                          | 0.903                            | 0.0140               | 0.0120                         | 0.0020                 | 0.2500  | 4.7E-05              | 7.0E-06                        | 5.4E-06                          | 21                        | 22                                  | 18                          | 0.0073        |
|                       | Max | 0.5541         | 0.5434   | 0.974                | 0.970                          | 1.000                            | 0.0470               | 0.0430                         | 0.1140                 | 13.5000   | 1.6E-04              | 9.7E-05                        | 2.0E-04                          | 77                        | 74                                  | 200                         | 0.1168        |
|                       | Avg | 0.5482         | 0.4563   | 0.955                | 0.950                          | 0.853                            | 0.0304               | 0.0279                         | 0.0357                 | 0.9834  | 9.9E-05              | 4.5E-05                        | 7.9E-05                          | 46.60                     | 46.27                               | 196.44                      | 0.4995        |
| LLN                   | Min | 0.2947         | 0.0737   | 0.910                | 0.889                          | 0.350                            | 0.0143               | 0.0129                         | 0.0086                 | 0.2360  | 4.7E-05              | 7.0E-06                        | 7.8E-06                          | 21                        | 22                                  | 113                         | 0.2564        |
|                       | Max | 0.5541         | 0.9997   | 0.974                | 0.970                          | 0.975                            | 0.0475               | 0.0435                         | 0.1295                 | 3.8698  | 1.6E-04              | 9.7E-05                        | 2.3E-04                          | 77                        | 74                                  | 255                         | 0.5541        |
|                       | Avg | 0.5482         | 0.2906   | 0.955                | 0.952                          | 0.853                            | 0.0304               | 0.0302                         | 0.0357                 | 1.0623  | 9.9E-05              | 4.9E-05                        | 7.9E-05                          | 46.60                     | 46.60                               | 196.44                      | 0.3492        |
| LNL                   | Min | 0.2947         | 0.0809   | 0.910                | 0.893                          | 0.350                            | 0.0143               | 0.0139                         | 0.0086                 | 0.2563  | 4.7E-05              | 7.7E-06                        | 7.8E-06                          | 21                        | 21                                  | 113                         | 0.1003        |
|                       | Max | 0.5541         | 0.8624   | 0.974                | 0.974                          | 0.975                            | 0.0475               | 0.0469                         | 0.1295                 | 4.2283  | 1.6E-04              | 1.0E-04                        | 2.3E-04                          | 77                        | 77                                  | 255                         | 0.5511        |
|                       | Avg | 0.5486         | 0.4237   | 0.956                | 0.951                          | 0.996                            | 0.0307               | 0.0282                         | 0.0319                 | 1.2599  | 1.0E-04              | 4.6E-05                        | 7.3E-05                          | 46.87                     | 46.76                               | 196.06                      | 0.4785        |
| NLL                   | Min | 0.3507         | 0.0537   | 0.928                | 0.921                          | 0.902                            | 0.0171               | 0.0157                         | 0.0059                 | 0.2958  | 5.5E-05              | 7.0E-06                        | 6.9E-06                          | 25                        | 25                                  | 56                          | 0.1292        |
|                       | Max | 0.5533         | 0.9978   | 0.974                | 0.971                          | 1.000                            | 0.0476               | 0.0434                         | 0.1366                 | 4.6191  | 1.5E-04              | 9.5E-05                        | 2.7E-04                          | 74                        | 69                                  | 320                         | 0.5532        |
|                       | Avg | 0.5482         | 0.5364   | 0.955                | 0.952                          | 0.853                            | 0.0304               | 0.0302                         | 0.0357                 | 1.0623  | 9.9E-05              | 4.9E-05                        | 7.9E-05                          | 46.60                     | 46.60                               | 196.44                      | 0.5269        |
| LNN                   | Min | 0.2947         | 0.0764   | 0.910                | 0.893                          | 0.350                            | 0.0143               | 0.0139                         | 0.0086                 | 0.2563  | 4.7E-05              | 7.7E-06                        | 7.8E-06                          | 21                        | 21                                  | 113                         | 0.2947        |
|                       | Max | 0.5541         | 0.9995   | 0.974                | 0.974                          | 0.975                            | 0.0475               | 0.0469                         | 0.1295                 | 4.2283  | 1.6E-04              | 1.0E-04                        | 2.3E-04                          | 77                        | 77                                  | 255                         | 0.5541        |
|                       | Avg | 0.5486         | 0.4169   | 0.956                | 0.951                          | 0.996                            | 0.0307               | 0.0282                         | 0.0319                 | 1.2599  | 1.0E-04              | 4.6E-05                        | 7.3E-05                          | 46.87                     | 46.76                               | 196.06                      | 0.1389        |
| NLN                   | Min | 0.3507         | 0.1558   | 0.928                | 0.921                          | 0.902                            | 0.0171               | 0.0157                         | 0.0059                 | 0.2958  | 5.5E-05              | 7.0E-06                        | 6.9E-06                          | 25                        | 25                                  | 56                          | 0.0002        |
|                       | Max | 0.5533         | 0.5504   | 0.974                | 0.971                          | 1.000                            | 0.0476               | 0.0434                         | 0.1366                 | 4.6191  | 1.5E-04              | 9.5E-05                        | 2.7E-04                          | 74                        | 69                                  | 320                         | 0.3953        |
|                       | Avg | 0.5486         | 0.5312   | 0.956                | 0.953                          | 0.693                            | 0.0307               | 0.0306                         | 0.0375                 | 1.0344  | 1.0E-04              | 5.0E-05                        | 9.5E-05                          | 46.87                     | 46.87                               | 224.84                      | 0.5326        |
| NNL                   | Min | 0.3507         | 0.0571   | 0.928                | 0.927                          | 0.258                            | 0.0171               | 0.0171                         | 0.0090                 | 0.3009  | 5.5E-05              | 7.6E-06                        | 9.0E-06                          | 25                        | 25                                  | 127                         | 0.3507        |
|                       | Max | 0.5533         | 0.9992   | 0.974                | 0.974                          | 0.929                            | 0.0476               | 0.0476                         | 0.1456                 | 3.8063  | 1.5E-04              | 1.0E-04                        | 2.9E-04                          | 74                        | 74                                  | 255                         | 0.5533        |
|                       | Avg | 0.5487         | 0.4027   | 0.955                | 0.953                          | 0.697                            | 0.0306               | 0.0304                         | 0.0378                 | 1.0175  | 9.9E-05              | 4.9E-05                        | 9.5E-05                          | 46.60                     | 46.60                               | 225.56                      | 0.2624        |
| NNN                   | Min | 0.3507         | 0.0981   | 0.928                | 0.927                          | 0.258                            | 0.0171               | 0.0171                         | 0.0090                 | 0.3009  | 5.5E-05              | 7.6E-06                        | 9.0E-06                          | 25                        | 25                                  | 127                         | 0.0000        |
|                       | Max | 0.5563         | 0.9795   | 0.974                | 0.974                          | 0.929                            | 0.0476               | 0.0476                         | 0.1456                 | 3.8063  | 1.5E-04              | 1.0E-04                        | 2.9E-04                          | 74                        | 74                                  | 255                         | 0.5520        |

Table B9.: Summary of Tables B2 to B9, presenting the average, maximum, and minimum dominant category label values before and after the application of the noise blowing-up technique  $(\tilde{\tau}_c, \tau)$ , along with  $L_p$  norms (where  $p = 0, 1, 2, \infty$ ) and loss  $\mathcal{L}$  for each combination of  $(\rho, \lambda, \rho)$ . In this summary, the calculations include *good-enough* adversarial images.

| $\rho, \lambda, \rho$ | Number of $\tilde{\mathcal{D}}_{targeted}^{VGG16}(\mathcal{A}_a)$ | Numbe     | er of $\mathcal{D}_{targete}^{hr,VG}$ | $G^{16}(\mathcal{A}_a^{\mathrm{hr}})$ | Average Loss $\mathcal{L}$ |
|-----------------------|---|-----------|---------------------------------------|---------------------------------------|----------------------------|
| ' ' ' '               | turgeteu ( = )  |           |                                       |                                       |                            |
|                       |   | $c = c_t$ | $c \neq c_a, c_t$                     | $c = c_a$                             |                            |
| L-L-L                 | 92  | 92        | 0                                     | 0                                     | 0.0439                     |
| L-L-N                 | 92  | 10        | 25                                    | 57                                    | 0.5019                     |
| L-N-L                 | 92  | 59        | 11                                    | 22                                    | 0.3501                     |
| N-L-L                 | 92  | 16        | 21                                    | 55                                    | 0.4802                     |
| L-N-N                 | 92  | 6         | 23                                    | 63                                    | 0.5295                     |
| N-L-N                 | 92  | 89        | 0                                     | 3                                     | 0.1384                     |
| N-N-L                 | 92  | 1         | 21                                    | 70                                    | 0.5345                     |
| N-N-N                 | 92  | 62        | 10                                    | 20                                    | 0.2615                     |

Table B10.: The table presents the results of a case study conducted on 92 adversarial images obtained with EA<sup>target,C</sup> for C = VGG-16 and  $\tilde{\tau}_t \geq 0.55$  (with notations consistent with Section 3). The technique involves manipulating the adversarial images by extracting noise and applying different combinations  $(\rho, \lambda, \rho)$  in Steps 1, 5, and 7 (see Subsection 3.1).

Table B10 summarizes the main findings from the comparison study for different interpolation techniques. The table includes information on the interpolation methods utilized, Lanczos (L) and Nearest (N), which are shown in *Column 1*. The remaining columns present the following data: *Column 2*: the number of adversarial images used for testing noise blowing-up technique, *Column 3*: the number of images classified in the target category, *Column 4*: the number of images that remained adversarial in the untargeted category, *Column 5*: the number of images classified in the ancestor category after employing the noise blowing-up technique, and *Column 6*: the resulting average loss in target category dominance.

Table B9 indicates that there are no significant differences observed when using different combinations of  $(\rho, \lambda, \rho)$  in relation to  $L_p$  norms (where  $p = 0, 1, 2, \infty$ ). However, Table B10 demonstrates that the combination of L-L-L produces optimal results in terms of both the loss function  $(\mathcal{L})$  and the number of adversarial images remaining in the target category  $(c_t)$  when utilizing the noise blowing-up technique for generating high-resolution adversarial images. Therefore, in our experiments (see Scheme 11), we employ the **L-L-L** combination for  $(\rho, \lambda, \rho)$ .