The concrete delivery problem

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- Constraints:
  - Concrete is produced at several homogeneous production sites located some distance away from the customers.
  - Trucks start at a central depot, travel to a production plant, unload their cargo at one of the construction sites.
- Objective: maximize the number of satisfied customers, weighted by their demand
- Instance:
  - To facilitate a future comparison of CDP implementations, large number of problem instances are published.
- Solution:
  - Integer Programming models
    - first ILP: combination of CVRPTW with Split Deliveries, and the Parallel Machine Scheduling Problem (PMSP) with Time Windows and Maximum Time Lags
      - A truck can only be used once for every customer, and whenever it is used, its delivery must be succeeded by another delivery
    - second ILP: allows a single vehicle to visit customers more than once by representing each customer by a of customer nodes
    - Constraint programming model.
  - Heuristic
    - Steepest Descent and best fit
    - Fix-and-optimize heuristic

First ILP

- The concrete trucks start their trip at a central (source) depot and travel between production sites and customers.
- At the end of the day, the trucks return to an end (sink) depot (which may not be the same as the starting depot).
- directed, weighted graph G(V,A) $V = \{0\} \cup C \cup \{n+1\}$:
  - 0: source
  - n+1: sink
- Arc costs:
  - $c_{0,i} = \min_{p \in P} t_{0,p} + t_{p,i}, \forall i \in C.$
  - $c_{i,j} = \min_{p \in P} t_{i,p} + t_{p,j}, \forall i, j \in C.$
  - $c_{i,n+1} = t_{i,n+1}$
  - $c_{0,n+1} = 0.$
- Parameters:
  - p_k: Time required to empty truck $k \in K$
  - γ: Maximum time lag between consecutive deliveries.
- Variables:
  - x_{ijk} is a binary variable indicating whether vehicle k travels from i to j
  - C_k^i record the time that vehicle k finishes its delivery to customer i.
  - y_i denotes whether customer i is serviced.
  - z_{kl}^i is equal to one if truck k delivers its payload immediately before truck l to customer i, zero otherwise.
  - k_0 represents a dummy truck
  - $K_0 = K \cup \{k_0\}$
- Model
  - $max \sum_{i \in C} q_i y_i$
  - Ensures that sufficient concrete is delivered at construction site
    - $\sum_{k \in K} \sum_{j \in \delta^+(i)} q_k x_{ijk} \geq q_i y_i, i \in C$
  - Determine the shape of a feasible tour: a tour starts at the source depot, visits a number of pickup and delivery points and finally returns to the sink depot.
    - $\sum_{j \in \delta^+(0)} x_{0jk} = \sum_{i \in \delta^-(n+1)} x_{i,n+1,k} = 1, k \in K$
    - $\sum_{j \in \delta^+(i)} x_{ijk} = \sum_{j \in \delta^-(i)} x_{j,i,k}, c \in C, k \in K$
    - The dummy truck must be scheduled
      - $\sum_{l \in K_0} z_{k_0 l}^i = 1, i \in V.$
    - Flow preservation constraints, each delivery has exactly one successor and one predecessor
      - $\sum_{l \in K_0 \setminus \{k\}} z_{kl}^i = \sum_{l \in K_0 \setminus \{k\}} z_{lk}^i, k \neq l$
  - Between two consecutive visits, starting, processing and travel times have to be taken into account
    - $C_k^i + t_{ij} - M(1 - x_{ijk}) \leq C_k^j - p_k, i, j \in V, k \in K$
    - Enforce that deliveries do not overlap in time:
      - $C_k^i - M(1 - z_{kl}^i) \leq C_l^i - p_l$
    - Enforce a maximum time lag between consecutive deliveries
      - $C_l^i - p_l \leq C_k^i + \gamma + M(1 - z_{kl}^i)$
  - $a_i + p_k \leq C_k^i \leq b_i, i \in V$
  - $\sum_{l \in K_0 \setminus \{k\}} z_{kl}^i = \sum_{j \in N} x_{ijk}, \forall k \in K, i \, inC$

- $x_{ijk} \in \{0,1\}$ $C_k^i \in Z$, $y_i \in \{0,1\}$

Second ILP

- The concrete trucks start their trip at a central (source) depot and travel between production sites and customers.
- At the end of the day, the trucks return to an end (sink) depot (which may not be the same as the starting depot).
- $\forall$ customer $i \in C$, $C^i = \{1, \dots, n(i)\}$, $n(i) = \lceil \frac{q_i}{\min_{k\in K}(q_k)} \rceil$
- Time window $[a_u, b_u] = [a_i, b_i]\ i \in C, u \in C^i$
- $D = \cup_{i\in C} C^i$
- directed, weighted graph G(V,A) $V = \{0\} \cup D \cup \{n+1\}$: 0: source, n+1: sink
- delivery node c^i_h has a directed edge to a delivery node c^i_j if $h < j, i \in C, h, j \in C_i$.
- directed edge from c^i_u to $c^j_v, i \neq j$, except if c^j_v needs to be scheduled earlier than c^i_u
- Arc costs:
  - $c_{0,c^i_j} = \min_{p\in P} t_{0,p} + t_{p,i}, \forall c^i_j \in D.$
  - $c_{c^iu,c^jv} = \min_{p\in P} t_{i,p} + t_{p,j}, \forall c^i u, c^j v \in D, c^i u \neq c^j v$
  - $c_{c^ij,n+1} = t_{i,n+1}$
  - $c_{0,n+1} = 0.$

- Parameters:
  - p_k: Time required to empty truck $k \in K$
  - γ: Maximum time lag between consecutive deliveries.
- Variables:
  - x_{ijk} is a binary variable indicating whether vehicle k travels from i to j
  - c^i records the time that delivery $i \in D$ is completed.
  - y_i denotes whether customer i is serviced.
- Model
  - $max \sum_{i\in C} q_i y_i$
  - Determine the shape of a feasible tour: a tour starts at the source depot, visits a number of pickup and delivery points and finally returns to the sink depot.
    - $\sum_{j\in\delta^+(0)} x_{0jk} = \sum_{i\in\delta^-(n+1)} x_{i,n+1,k} = 1, k \in K$
    - $\sum_{j\in\delta^+(i)} x_{ijk} = \sum_{j\in\delta^-(i)} x_{j,i,k}, c \in C, k \in K$
  - $S(i, \alpha)$ = $\sum_{k\in K} \sum_{j\in\delta^+(i)} \alpha x_{ijk}, i \in D$
    - Number of times a delivery can be made
      - $S(i, 1) \leq 1, \forall i \in D$
    - Ordering of deliveries
      - $S(j + 1, 1) \leq S(j, 1)$,$\forall i \in C, j \in \{1, \dots, n(i) - 1\}$
    - Sum of capacities of the vehicles performing the deliveries for customer $i \in C$ should cover the customer's demand
      - $\sum_{j\in C^i} S(j, q_k) \geq q_i y_i, i \in C$
  - $c^i - M(1 - x_{ijk}) \leq c^j - p_k - c_{ij}, i \neq 0$
  - $c^i - M(1 - x_{ijk}) \leq c^j - c_{ij}, \forall (0, j) \in A$
  - $c^i - S(i, p_k) \geq a_i$
  - Enforce a maximum time lag between consecutive deliveries
    - $c^{j+1} - S(j + 1, p_k) - c^j \leq \gamma, i \in C$,j \in {1,...,n(i)-1}$
  - Enforce that deliveries do not overlap in time:

- $c^{j+1} \geq c^j + S(j, p_k)$,i \in C, $j \in \{1, \ldots, n(i) - 1\}$
- $a_i + p_k \leq C_k^i \leq b_i$, $i \in V$

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- $x_{ijk} \in \{0,1\}$ $C^{i}_k \in Z$, $y_i \in \{0,1\}$
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