Alistair Pattison Fall 2023 Comps

Topic: Two great theorems from algebraic number theory

Your topic is to study and present the material necessary to prove two of the famous theorems of algebraic number theory: the finiteness of the class group and Dirichlet's unit theorem.

Your main reference is Daniel Marcus' book "Number Fields", Chapters 1-3 and 5. Some consultation of Appendix B on Galois theory may be necessary. You can find a pdf copy of this book through the college's subscription to SpringerLink (https://link.springer.com/book/10.1007/978-3-319-90233-3). There is also a hard copy available in the library, but it is the first edition, which is identical but in a harder-to-read font.

You indicated an interest in computational number theory in your comps survey. Though it is not required for the project, one way to enhance your paper is to learn enough either SAGE or MAGMA to do computations in algebraic number fields, and use this to provide solutions to some of the exercises in Marcus' text.

There are two components to the final product of this project:

(1) One product of your project is a paper. Your paper should rigorously summarize what you've learned and should be oriented around presenting key results. In addition to definitions, theorems, and proofs, it should contain useful examples and exercises that you have worked out, perhaps organized via appendices. It is certainly not necessary to work through all examples or exercises. Be careful to reformulate the ideas and proofs in your own words to avoid plagiarism. It can also be helpful to come up with your own examples or explanatory metaphors, which you can use both as an aid while you are reading and in writing your paper.

(2) The other product of your project is a 25-minute talk. The intended audience is your fellow math majors. Your talk should not cover everything that you've learned; select pieces to give a coherent narrative that builds towards a key result. Be sure to include motivating examples and figures, in addition to definitions and theorems. Convey the key ideas of proofs or arguments, if possible, but focus on communicating big concepts rather than precise details.