A Gentle Introduction to Algebraic Number Theory

(and two big theorems)

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Outline

- What is algebraic number theory?

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- Field extensions, number fields, number rings
- The ideal class group
- Two big theorems
  - Ideal class groups are finite
  - Dirichlet's unit theorem

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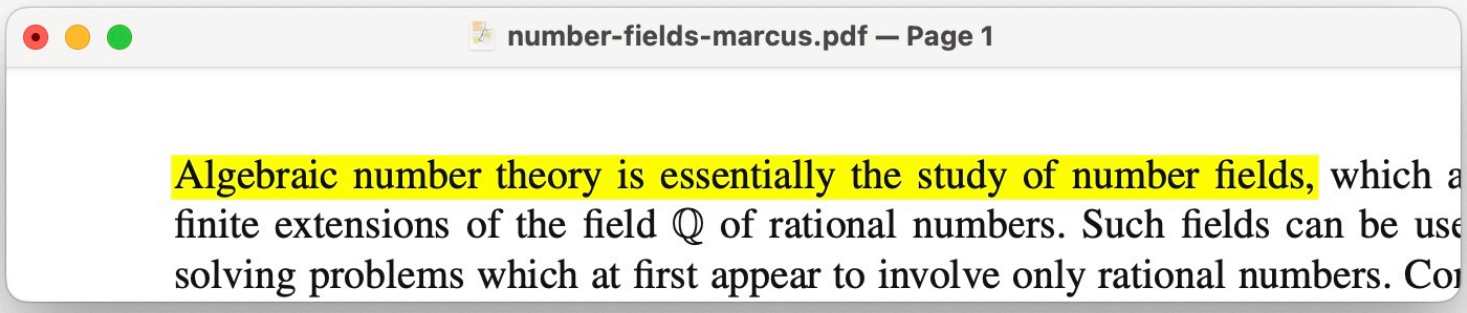

So what is a number field?

$$f(x) = x^2 + 5$$

This is irriducable over $\mathbb{Q}$, but it factors over $\mathbb{C}$:

$$f(x) = (x - i\sqrt{5})(x + i\sqrt{5})$$

But do we really need all of $\mathbb{C}$?

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