Exercises for particle kinematics

1) Consider the motion of a continuum body given by the equation

$$x_1 = X_1(1 + \alpha t^3), \quad x_2 = X_2, \quad x_3 = X_3$$

where α is a constant. Determine the displacement, the velocity and the acceleration fields in each of the material and spatial descriptions.

Answer: The displacement fields are: $u(X_i) = [\alpha X_1 t^3,\ 0,\ 0]^T,\ u(x_i) = \left[\frac{\alpha x_1 t^3}{1+\alpha t^3},\ 0, 0\right]^T$ *; the velocity fields are:* $v\ (X_i) = [3X_1 \alpha t^2,\ 0,\ 0]^T$*,* $v(x_i) = \left[\frac{3x_1 \alpha t^2}{1+\alpha t^3},\ 0,\ 0\right]^T$ *; and the acceleration fields are:* $f(X_i) = [6X_1 \alpha t,\ 0,\ 0]^T$*,* $f(x_i) = \left[\frac{6x_1 \alpha t}{1+\alpha t^3},\ 0\ \ 0\right]^T$ *in the material and spatial description, respectively.*

2) For a velocity field $v = [x - z, \quad z(e^t + e^{-t}), \quad 0]^T$, calculate the acceleration at time $t = 2$ in point $(2,1,3)^T$

Answer: The spatial expression for the acceleration field is $f = [x - z,\ \ z(e^t - e^{-t}),\ \ 0]^T$.

3) Solve problem 4.1 on page 42 in the book by Spencer.

4) In a certain region, the spatial velocity components of $v_i = v_i(x_i, t)$ are given as

$$v_1 = -\alpha(x_1^3 + x_1 x_2^2)\exp(-\beta t) \qquad v_2 = -\alpha(x_1^2 x_2 + x_2^3)\exp(-\beta t) \qquad v_3 = 0$$

where $\alpha, \beta \geq 0$ are given constants. Find the components of the spatial acceleration field f_i at point $(1,0,0)$ and time $t = 0$.

Answer: $f = [\alpha\beta + 3\alpha^2,\ 0,\ 0]$

5) A motion of a continuum body given by the equations (grade A-C)

$$v_1 = \frac{3x_1}{1+t} \qquad v_2 = \frac{x_2}{1+t} \qquad v_3 = \frac{5x_3^2}{1+t}$$

Assume that the reference configuration of the continuum body is at $t = 0$, with the consistency condition $X_i = x_i$.

a) Derive the particle path, i.e. the motion x_i
b) Compute the velocity components in terms of the material coordinates and time and the associated accelerations in the material and spatial description

Answers:

(a)

$$x(X_i) = \left[(1+t)^3 X_1, \qquad (1+t)X_2, \qquad \frac{-X_3}{5\ln(1+t)X_3 - 1}\right]^T$$

(b)

$$v(X_i) = \left[3(1+t)^2 X_1, \qquad X_2, \qquad \frac{5X_3^2}{(1+t)(5X_3\ln(1+t)-1)^2}\right]^T$$

$$f(X_i) = \left[6(1+t)X_1, \qquad 0, \qquad \frac{5X_3^2(5X_3\ln(1+t)+10X_3-1)}{(1+t)^2(5X_3\ln(1+t)-1)^3}\right]^T$$

$$f(x_i) = \left[\frac{6x_1}{(1+t)^2}, \qquad 0, \qquad \frac{5x_3^2(10x_3-1)}{(1+t)^2}\right]^T$$