

UNIVERSITY OF SYDNEY

Manual: Making measurements

VERSION 1.0

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1 Introduction

I have written the code to simulate taking interferometric differential polarimetric data from MCFOST models given a set of stellar parameters.

1.1 List of required files

- star.py
- physics.py
- run_MCFOST.py
- write_MCFOST_parafle.py
- Default_parameter class file

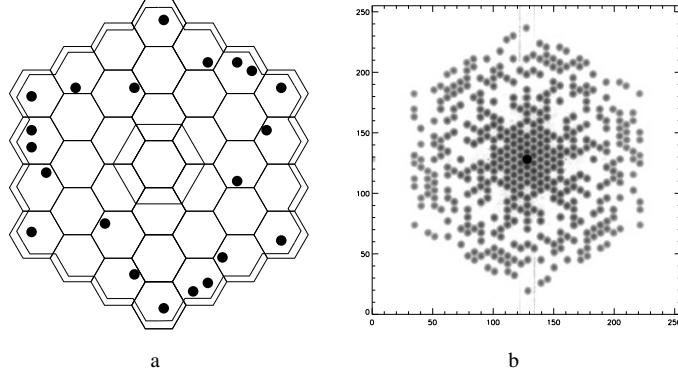


Figure 2.2: 21 hole Golay mask for the Keck telescope (a) and the corresponding coverage of the power spectrum (b). (Tuthill et al., 2000)

2 An introduction to differential polarimetry

2.1 Interferometry and non-redundant aperture masking

Overcoming the limitations of atmospheric turbulence has been a perpetual challenge for astronomers. According to Rayleigh's Criterion, the angular resolution of a telescope with diameter D at wavelength λ should be λ/D , so a 10 m telescope, like the Keck telescope in Hawaii should have a resolution of 10 mas in visible light ($\lambda = 500$ nm). However, atmospheric distortion limits the resolution of all ground based telescopes to around 1 arcsecond (Labeyrie et al., 2006), the same as the fundamental resolution limit of a 10 cm telescope. This would not allow us to resolve even one of the stars with the largest apparent diameter, α Orionis (Betelgeuse), which has an angular size of ~ 50 mas, let alone give us the opportunity to probe its structure.

Stellar interferometry is a technique that has been used since the 1800s and has been a necessary device in the advancement of imaging technologies. It was first suggested by Fizeau (1868), who noted that the size of a source would affect the amount of smearing of the interference fringes it produced. Independent of Fizeau, Michelson invented the stellar interferometer and used it to determine the diameters of Jupiter's four largest moons (Michelson, 1891). This success led to the construction of the Michelson stellar interferometer at the Mount Wilson Observatory in California which was used to determine the diameter of α Orionis (Michelson and Pease, 1921).

The physics principles behind interferometry can be traced back to Young's double slit experiment (Young, 1802). When coherent light passes through an aperture of two slits, a fringe pattern is observed (Figure 2.1). This pattern arises from the optical path difference (OPD) that the light can take to reach the screen. The fringe contrast is called the *visibility* (V) and is defined in terms of I_{\max} and I_{\min} , the maximum and minimum intensities in the fringe pattern.

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

In stellar interferometry the point source is replaced by a star and the pair of slits are replaced by two telescopes of separation B . A pair of such telescopes forms a baseline, and will have the equivalent resolution of a telescopes with diameter B . Further, it is the square of the visibility (called the power) which is recorded for a set of fringes, an observable that, with careful calibration, can be made somewhat robust to noise imposed by atmospheric seeing.

Rather than requiring a number of telescopes to produce a set of baselines, interferometry can be achieved with a single telescope using a technique called non-redundant aperture masking (Readhead et al., 1988; Tuthill et al., 2000). In non-redundant aperture masking a metal mask, such as the one in Figure 2a is placed over the pupil of the telescope. The mask is constructed with a number of carefully placed holes. Each pair of holes forms a baseline, and corresponds to a single sample of the power spectrum (Figure 2b). The mask is non-redundant if the vector separation of each pair of holes is unique. This ensures that the Fourier component of any baseline can be uniquely identified. If there was a redundancy

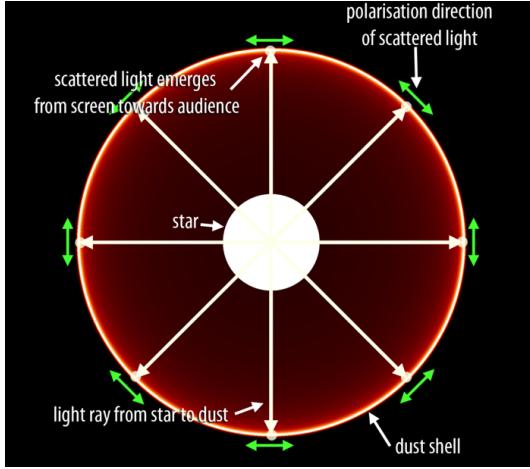


Figure 2.3: Diagram showing how light scattered off a dust shell is linearly polarised. ([Norris et al., 2012](#))

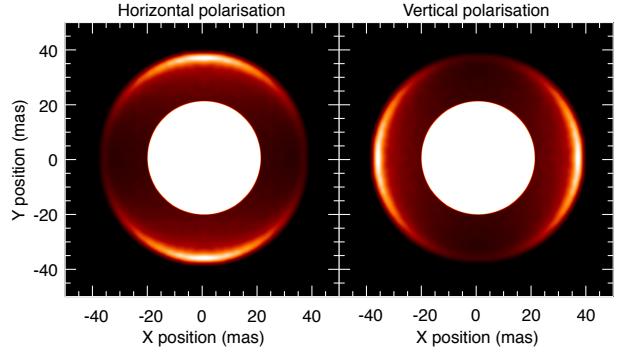


Figure 2.4: Modelled images of W Hya as seen through horizontally and vertically polarised filters. ([Norris et al., 2012](#))

R in a given baseline, then the corresponding power will add as a random walk with R steps, and the power would be more subject to atmospheric noise and hence, more difficult to calibrate accurately ([Tuthill et al., 2010](#)).

2.2 Polarisation of light and Stokes formalism

Starlight is unpolarised, in that it is actually composed of light in many different and rapidly varying polarisation states. However, when this light is scattered off the dust shell it becomes partially linearly polarised (Figure 2.3). The simplest geometry of a star with a thin, spherical circumstellar shell imaged through a horizontally polarised filter will look larger vertically (Figure 2.4).

There are different formalisms to describe the polarised states but we will be using Stokes formalism. All forms of polarisation can be expressed using four Stokes parameters: I , Q , U and V . Suppose that we had filters that allowed through linearly polarised light in the horizontal (H), vertical (V), $+45^\circ$ (A) and -45° (B) directions and we also had filters that allowed through right- (R) and left- (L) circularly polarised light. Then the Stokes parameters are the following:

$$I = H + V = A + B \quad Q = H - V \quad U = A - B \quad V = R - L$$

Conversely, the different polarisation states can be represented in terms of the Stokes vector: $[I \quad Q \quad U \quad V]^T$

Horizontal \mathcal{P} -state (H)	$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	\mathcal{P} -state at $+45^\circ$ (A)	$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$
Vertical \mathcal{P} -state (V)	$\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$	\mathcal{P} -state at -45° (V)	$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$
		\mathcal{R} - state (R)	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$
		\mathcal{L} - state (L)	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

2.3 Differential polarimetry and the VAMPIRES instrument

The goal of differential polarimetry is to image polarised structures, which would otherwise be invisible due to the presence of bright, but unpolarised, sources. It has successfully been used to image dust around AGB stars (E.g. [Ireland et al. \(2005\)](#); [Norris et al. \(2012\)](#); [Kervella et al. \(2016\)](#)).

Traditionally, calibration is done using a reference star, but atmospheric and instrumental conditions change quickly with time. In differential polarimetry, calibration is done by comparing simultaneous observations through two orthogonal polarisation states either by taking the difference or the ratio. The advantage is that errors can be eliminated to obtain highly accurate observations.

The Visible Aperture-Masking Polarimetric Interferometer for Resolving Exoplanetary Signatures (VAMPIRES) is a new instrument, which combines interferometry with differential polarimetry ([Norris et al., 2015](#)). It operates in the visible/IR region ($0.6 \mu\text{m}$ to $1.0 \mu\text{m}$), which is a shorter wavelength than other aperture masking instruments, exploiting the fact that light is more polarised at these wavelengths due to mechanisms involved in Rayleigh and Mie scattering. Although designed to image the inner most regions of protoplanetary disks, it has the capacity to image other polarised structures such as the dust shells around stars.

Differential polarimetry allows for observations to be made of faint polarised structures despite close proximity to very bright sources which would otherwise make them impossible to discern. This is combined with non-redundant aperture

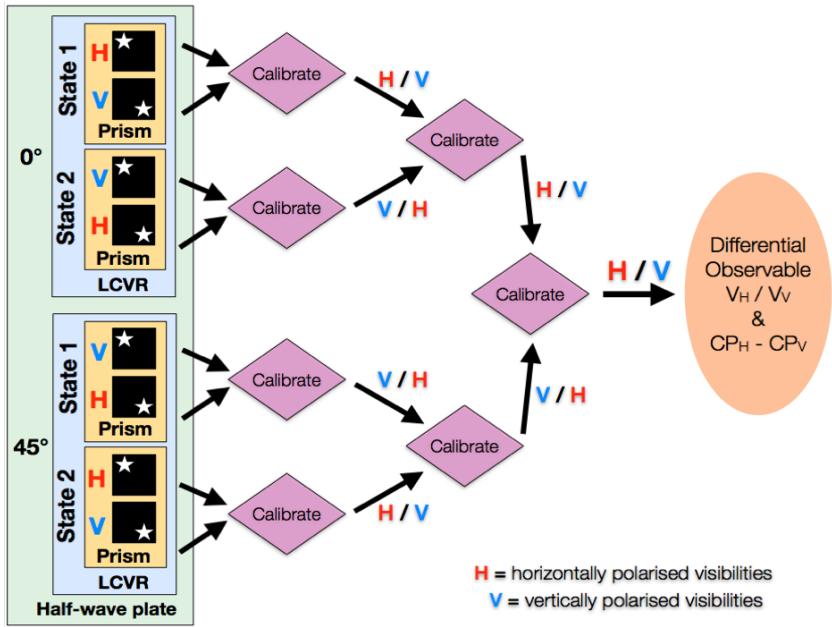


Figure 2.5: Diagrammatic representation of the 3-tier calibration in the VAMPIRES instrument (some details in text, for full details refer to [Norris et al. \(2015\)](#)). Obtained after calibration are the differential observables for the visibilities and closure phases although only the visibilities were used in this project. ([Norris et al., 2015](#))

masking to attain the necessary resolution that cannot be achieved with differential polarimetry alone. VAMPIRES has a resolution of ~ 10 mas while other recent imaging polarimeters: Spectro-Polarimetric High-Contrast Exoplanet Research (SPHERE) ([Beuzit et al., 2008](#)) and the Gemini Planet Imager (GPI) ([Macintosh et al., 2008](#)) have resolutions of ~ 100 mas and ~ 200 mas respectively ([Martinez et al., 2010](#)).

VAMPIRES has a 3-tier calibration system (Figure 2.5). The first tier uses a Wollaston prism, which splits a light beam into two orthogonal polarisations, and allows them to be measured simultaneously. The second tier uses a Liquid-Crystal Variable Retarded (LCVR), which allows for fast switching between the two channels in the Wollaston prism (~ 10 ms), and the third tier uses a half-wave plate, which rotates the polarisations by 45 degrees.

Calibration between two channels is accomplished by taking the ratio of the visibilities obtained from the two channels of the Wollaston prism ($V_{\text{Ch } 1}$ and $V_{\text{Ch } 2}$). This is repeated with the polarisations rotated 90° by the LCVR ($V'_{\text{Ch } 1}$ and $V'_{\text{Ch } 2}$) and then calibrated against each other:

$$\left(\frac{V_{\text{Ch } 1}}{V_{\text{Ch } 2}} / \frac{V'_{\text{Ch } 1}}{V'_{\text{Ch } 2}} \right)^{\frac{1}{2}} = \left(\frac{V_{\text{Horiz}}}{V_{\text{Vert}}} / \frac{V'_{\text{Horiz}}}{V'_{\text{Vert}}} \right)^{\frac{1}{2}} = \frac{V_{\text{Horiz}}}{V_{\text{Vert}}}$$

This is further repeated for the HWP, and for all other states as shown in Figure 2.5. Calibration is done to remove instrumental noise that arises in different parts of the telescope, for example, non-common path error and imperfections of the LCVR. For full details, refer to [Norris et al. \(2015\)](#).

3 MCFOST

3.1 Making models: MCFOST

Stellar models were produced using the software MCFOST (Pinte et al. 2006, 2009). MCFOST runs a Monte Carlo radiative code used for modelling circumstellar environments. We produced 3 dimension envelope models using a spherical geometry. Scattering was calculated using Mie theory i.e. spherical grains. The inner and outer radius of the dust shell could be controlled and had a hard edge. It was assumed that both radiative and local thermal equilibrium were reached but the gas and dust were not in hydrostatic equilibrium. The star was modelled at a blackbody corresponding to the specified effective temperature. For a star centred at the origin, the image had vertical, point and axial symmetry. To create asymmetries, the star was offset from the origin while the dust remains centred. In this case there were no assumed symmetries.

MCFOST reads in properties of a star from a parameter file and observables are made using ray tracing methods. MCFOST outputted polarised images corresponding to the Stokes parameters I, Q, U and V. These images are produced using ray tracing methods. Note that only the Stokes I, Q and U images are required in this project. A typical set of images are shown in Figure 3.6 and the corresponding MCFOST parameters are given in Table. 1.

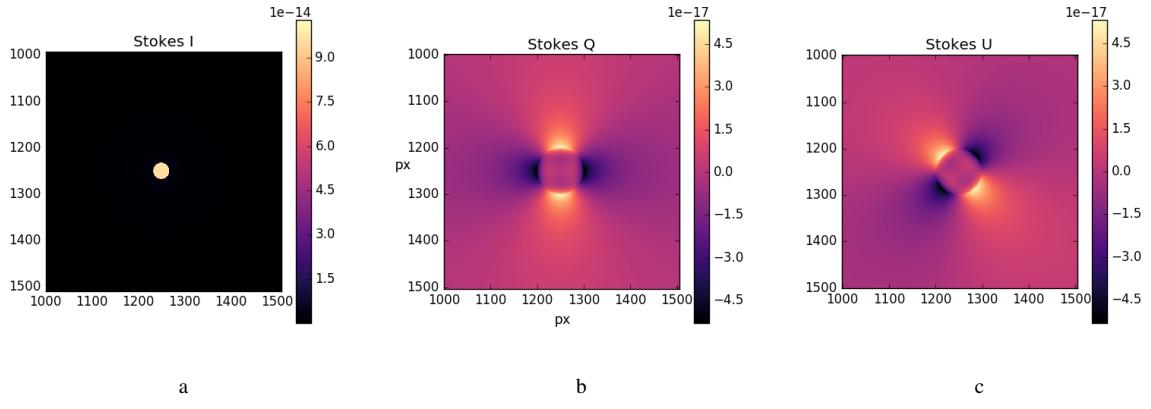


Figure 3.6: Example output of the Stokes I, Q, U images from MCFOST. The original map size was 2501×2501 px but has been cropped to 1000:1500 in both axes. The inner radius of the dust shell is clear in the Stokes Q and U images, as a large amount of light is scattered here. The dust shell extends beyond the depicted region.

Property	Value
stellar radius (R_{\odot})	150
distance (pc)	100
temperature (K)	3000
mass (M_{\odot})	1.0
dust mass (M_{\odot})	3.5E-6
inner radius of dust shell (AU)	2
outer radius of dust shell (AU)	300
grid (px)	2501
map size (AU)	150

Table 1: Some of the MCFOST parameter values used for the production of Figure. 3.6

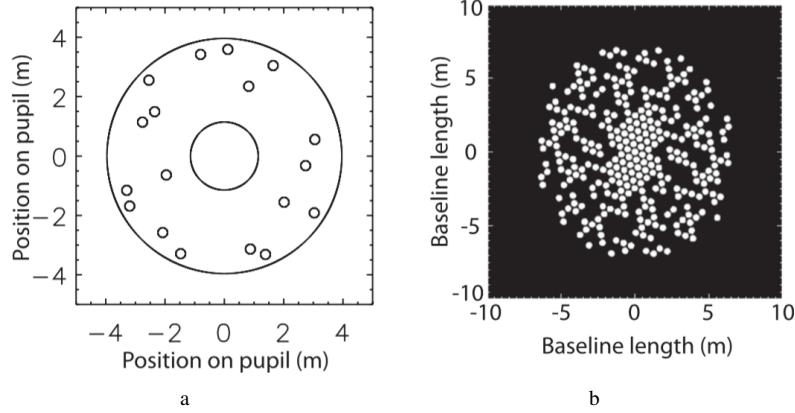


Figure 4.7: (a) 18 hole nudged mask of VAMPIRES (b) corresponding coverage of the power spectrum. ([Norris et al., 2015](#))

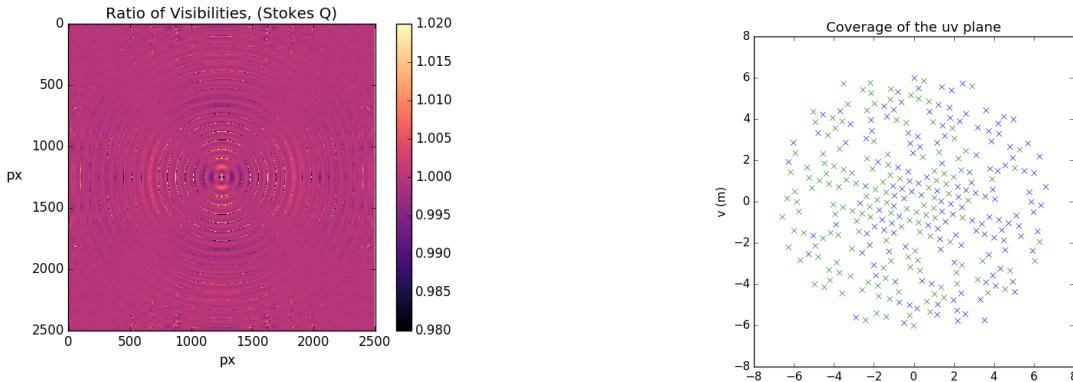


Figure 4.8: PVR_Q corresponding to the images produced in Figure 3.6

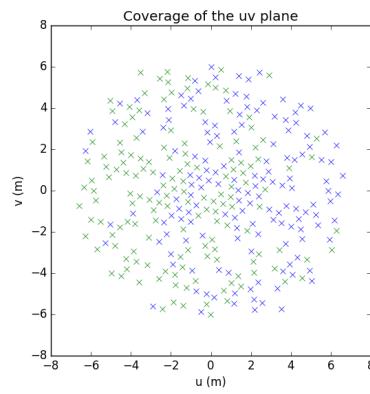


Figure 4.9: Coverage of the uv plane by an 18 hole aperture mask generating 153 baselines. Note that the coverage and PVR is inversion-symmetric as a pair of holes forms two baselines corresponding to the direction the vector of separation points but is counted as one since only one piece of information is recovered. This u,v coverage corresponds to the VAMPIRES 18 hole nudged mask (Figure 4.7)

4 Making measurements

4.0.1 Polarised visibility ratios (PVR)

Images of the horizontally and vertically polarised light (H and V) and images of light polarised in the $+45^\circ$ and -45° (A and B) can be obtained from the I, Q and U images as follows:

$$H = I + Q \quad V = I - Q \quad A = I + U \quad B = I - U$$

For each of these, we obtain the power spectrum (PS) by taking the absolute value of the square of the two dimensional Fourier transform.

$$PS(u, v) = |FFT2(\vec{x})|^2$$

These are then re-centered so that the 0 baseline (brightest point) is at the center and then normalised by dividing through by the maximum value. The two following polarised visibility ratios (PVR) corresponding to Figure 3.6 were produced.

$$PVR_Q = \frac{\text{normalise}(\text{center}(PS_H))}{\text{normalise}(\text{center}(PS_V))} \quad PVR_U = \frac{\text{normalise}(\text{center}(PS_A))}{\text{normalise}(\text{center}(PS_B))}$$

4.0.2 Sampling the PVRs

To produce data from the model that corresponds to that obtained from the telescope, the PVRs need to be sampled at points to achieve the same coverage as the aperture mask in the uv-plane; in this case, the VAMPIRES 18 hole nudge

mask (Figure 4.7). Note that the units of these co-ordinates are baseline length divided by observation wavelength (750 nm). In order to correctly sample the PVRs we multiplied the co-ordinates by the wavelength so that co-ordinates are given in meters, then determined the scale of the PVRs in meters.

The original images (Figure 3.6) had a field of view of 150 AU and were 2501 pixels across. Given that the distance to the image was 100 pc, 1 pixel corresponded to ≈ 0.600 mas. By the Nyquist criterion, the highest frequency in the PVRs will be $\theta \approx 1.200$ mas $\approx 5.815 \times 10^{-9}$ rad. It can be derived from the double slit experiment that $\theta = \frac{\lambda}{B}$ where λ is the wavelength of observation and B is the baseline length. Hence, the maximum baseline length the PVR can represent is $B = \frac{\lambda}{\theta} = \frac{750 \text{ nm}}{5.815 \times 10^{-9} \text{ rad}} \approx 129.0$ m. This maximum baseline is the radius of the largest circle that can be drawn inside the PVRs images. The sampling was then done using the `scipy interpolate` module with a 2d cubic interpolation. The sampling of the visibility ratios are shown in Figure 4.10.

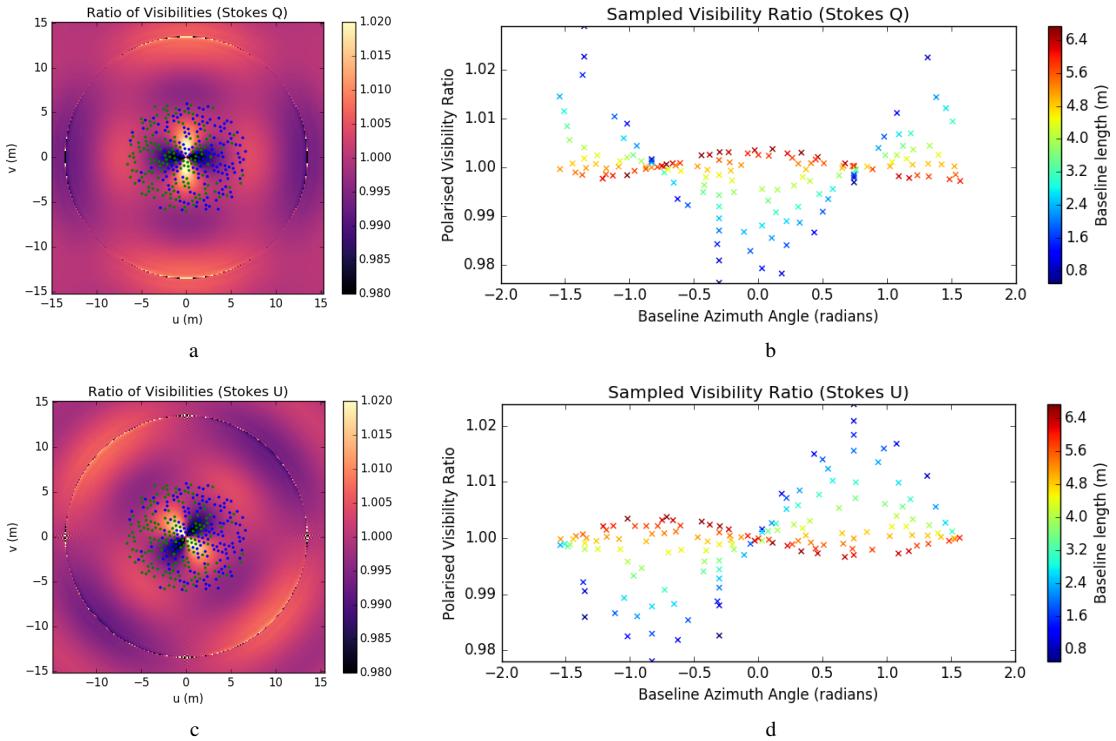


Figure 4.10: Artificial data taken from a model produced by MCFOST (Figure 3.6). Each cross in the sampled visibility ratio corresponds to a point in the ratio of visibilities image.

4.1 Reduced χ^2 Error

The χ^2 error is defined as:

$$\chi^2_{\text{reduced}} = \frac{1}{N-p} \sum_{i=0}^N \frac{(o_i - m_i)^2}{e_i^2} \quad (1)$$

where N is the number of data points, p is the number of free parameters, o_i is a data point in the observation data, m_i is a data point from the model (Figure 4.10) and e_i is the error in o_i . Note that $N - p$ is the degree of freedom.

5 The star class

5.1 Initialising a star

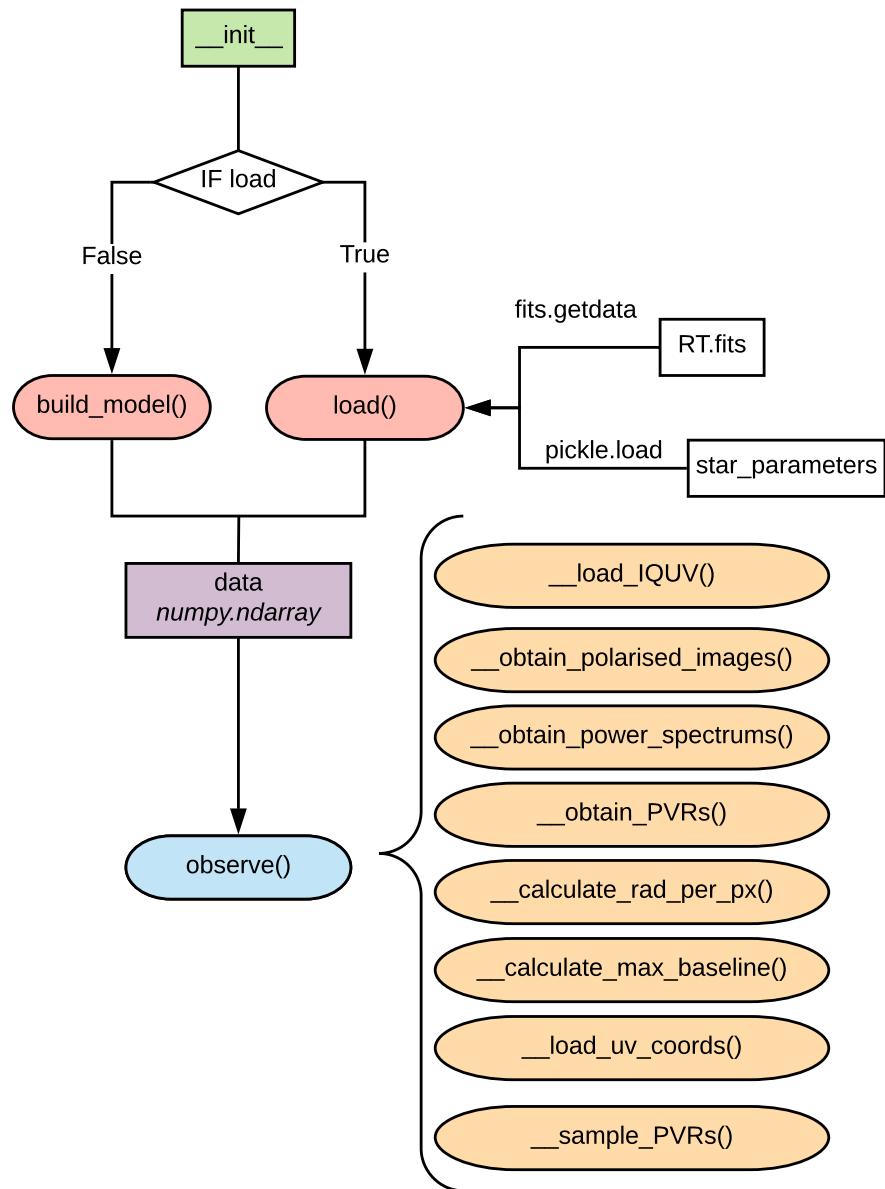


Figure 5.11: Flow chart showing the initialisation of the star class.

5.2 Building an MCFOST model

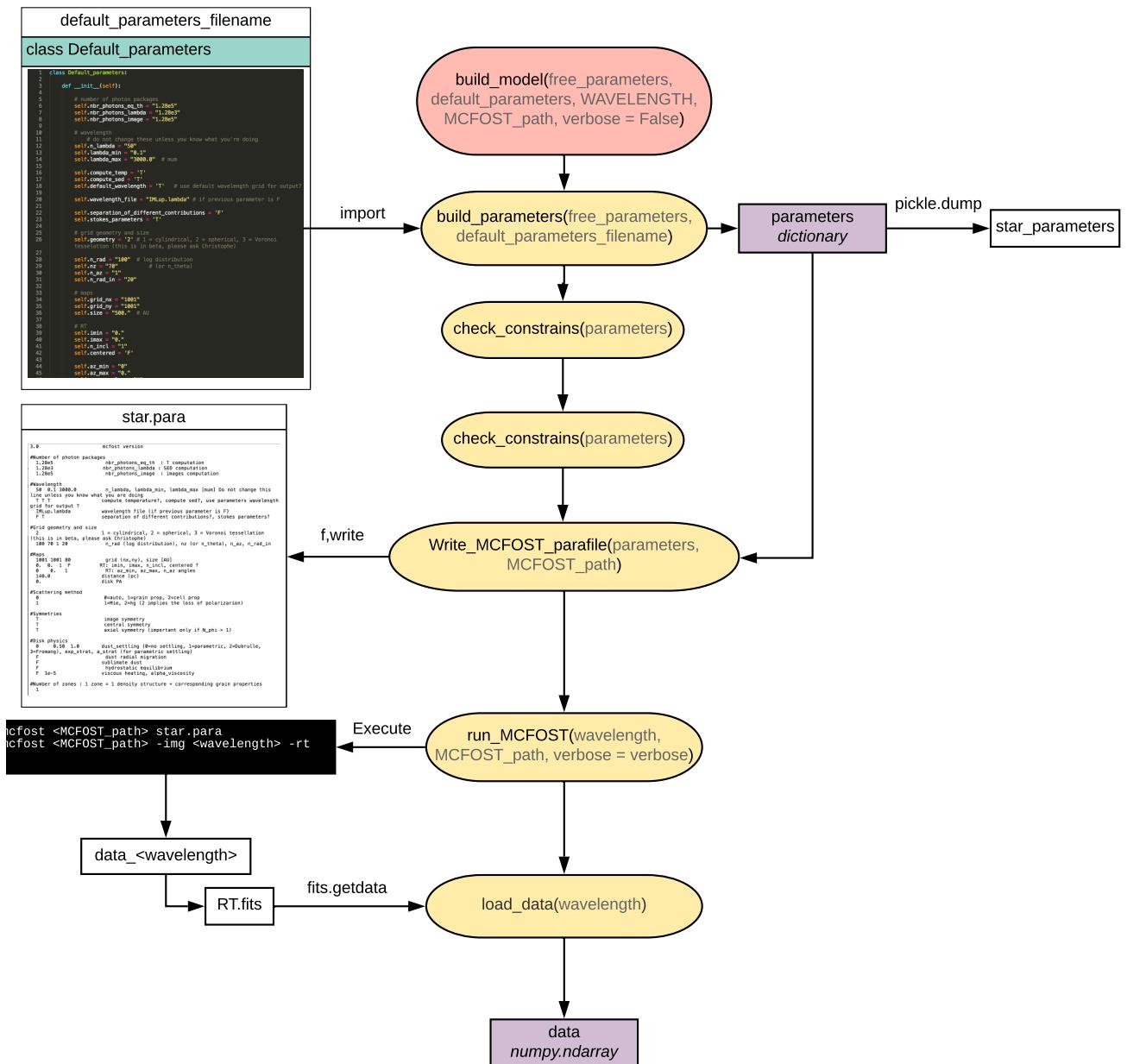


Figure 5.12: Flow chart showing how MCFOST is called to build a model.

5.3 Create a simple star

5.3.1 Arguments

To create a star, you call the function as follows:

```
Star(free_parameters, default_parameters, WAVELENGTH, MCFOST_path, obs_data)
```

The following is a description of each argument:

Argument	Description	Type
free_parameters	A dictionary of free parameters. The name of these parameters must match the naming of these parameters in the default_parameters.	dictionary
default_parameters	The name of the default_parameters file. The default parameter file must create an object with the class name Default_parameters which has attributes which correspond to the MCFOST parameters. An example default_parameters file is included in Appendix A	String
WAVELENGTH	The wavelength of observation in microns.	float
MCFOST_path	Path specifying the location where the parameter file will be built and where the data folders will be created	string
obs_data	An object containing observation data. See Section 5.3.2	An object

Figure 5.13

5.3.2 Observation data

Observation data must be stored in an object and passed into the star class. The observation data *must* have the following attributes:

Attribute	Description	Type
u_coords, v_coords	The u-v coordinates of each baseline (in instrument coordinates)	np.ndarray
blengths	The baseline lengths (in meters)	np.ndarray
bazims	The baseline azimuths (in instrument coordinates)	np.ndarray

Figure 5.14

The attributes above contain information about the mask used during the observation. They are necessary for simulate observations of the model star because they dictate where the polarised visibility ratio is sampled.

If the model star is to be compared with observation data, it is useful to pass the star the following observation data:

Attribute	Description	Type
vhvv, vhvu	The polarised differential visibilities in Stokes Q and U	np.ndarray
vhvverr, vhvuerr	The errors on the polarised differential visibilities in Stokes Q and U	np.ndarray

Figure 5.15

The observation data is necessary for some methods (e.g `calculate_reduced_chi2error()`). Any additional attributes may be helpful, but are not necessary.

5.3.3 Demo star

The following is code to create a simple star:

```

1 import numpy as np
2 from star import *
3 from scipy import io
4
5 class VAMPIRES_data:
6
7     def __init__(self, filename, cube_info_filename):
8         ''' Reads in the polarised-differential calibrated VAMPIRES data produced by the
9             IDL pipeline: param filename: The full filename of the data file. E.g.
10            diffdata_vega_.....idlvar '''
11
12         data_object = io.readsav(filename, python_dict=False, verbose=False)
13         self.vhvv = data_object.vhvv
14         self.vhvverr = data_object.vhvverr
15         self.vhvvu = data_object.vhvvu
16         self.vhvvuerr = data_object.vhvvuerr
17         self.blengths = data_object.blengths
18         self.bazims = data_object.bazims
19         self.u_coords = data_object.u_coords
20         self.v_coords = data_object.v_coords
21
22 VAMP_data = "diffdata_RLeo_03_20170313_750-50_18holeNudged_0_0.idlvar"
23 VAMP_data_info = "cubeinfoMar2017.idlvar"
24 obs_data = VAMPIRES_data(VAMP_data, VAMP_data_info)
25
26 free_params = {"Rin": 3, "Rout": 5, "dust_mass": 1e-7, "radius": 100, "size": 80}
27 default_params = "default_parameters_star"
28
29 WAVELENGTH = 0.75          # microns
30
31 # location where the parameter file will be built and where the data folders will be created
32 MCFOST_path = ""
33
34 # Create a star
35 s = Star(free_params, default_params, WAVELENGTH, MCFOST_path, obs_data, load = False)

```

5.4 Optional arguments

These are the optional arguments you can use when creating your star:

Argument	Description	possible values
load	Will load in the last RT.fits file produced by MCFOST at the location specified by MCFOST_path. The parameters it loads will be those passed to the star on creation, and not necessarily the parameters used to produce the RT.fits file	True, False, default: False
verbose	Will either display or suppress all the MCFOST calculations shown when running MCFOST	True, False, default: False

5.5 Attributes

After initialisation the star will have the following attributes:

Attribute	Description	Type
WAVELENGTH	The wavelength of observation in microns	float
MCFOST_path	Path specifying the location where the parameter file will be built and where the data folders will be created	string
free_parameters	A dictionary carrying all the free parameters that will overwrite the default parameters. The keys are the parameter names (must match the parameter names in default_parameters) and the values are strings	dictionary
default_parameters	The name of the python file defining a Default_parameter class.	string
n_free_parameters	The number of free parameters. i.e. the number of keys in free_parameters	int

parameters	A Default_parameters object which is defined in the default_parameters python file. The attributes of the object correspond to the MCOST parameters	Default_parameters object
obs_data	An object which holds all the observational data. See Section 5.3.2	object
resolution	The resolution of the image in radians. Tells you the smallest thing in the IQUV images that we can detect	numpy.float64
resolution_mask_soft	The resolution of the mask in mas for object with a soft edge. $R = \lambda/B$	numpy.float64
resolution_mask_hard	The resolution of the mask in mas for an object with a hard edge. $R = 1.22\lambda/B$	numpy.float64
max_baseline	The maximum baseline that can be represented in the PVRs	numpy.float64
angular_size_image_rad	Gives the angular size of the image in radians	numpy.float64
rad_per_px	The angular size of a pixel in radians	numpy.float64
mas_per_px	The angular size of a pixel in mas	numpy.float64
distance_m	Distance to the star in meters	float
distance_AU	Distance to the star in AU	float
radius_m	Radius of the star in meters	float
diameter_mas	Diameter of the star in mas	numpy.float64
rin_m	The inner radius of the dust shell in meters	float
din_mas	The inner diameter of the dust shell in mas	numpy.float64
rout_m	The outer radius of the dust shell in meters	float
dout_mas	The outer diameter of the dust shell in mas	numpy.float64
I, Q, U, V	Images produced by MCFOST corresponding to the polarised images in Stokes I, Q, U and V	numpy.ndarray
Hp, Vp, Ap, Bp	Polarised images defined by: $Hp = I + Q$, $Vp = I - Q$, $Ap = I + U$, $Bp = I - U$	numpy.ndarray
Hps, Vps, Aps, Bps	The power spectrums of Hps, Vps, Aps and Bps where the power spectrum of \vec{x} is given by $FFT2(\vec{x} ^2)$. The power spectrums have been recentered so that the maximum value is at the center of the image, and normalised by dividing through the image by the maximum value	numpy.ndarray
PVR_Q, PVR_U	The polarised visibility ratio. $PVR_Q = Hps / Vps$, $PVR_U = Aps / Vps$	numpy.ndarray
U_vals, V_vals	The grid spacings of the PVR images in terms of baseline length in meters. The edges of the images correspond to $\pm max_baseline$ and the center is at 0 meters	numpy.ndarray
u_meters, v_meters	Lists storing the u, v co-ordinates of the baselines in meters. The co-ordinates of baseline _i is stored at index i in both lists	list
u_neg_meters, v_neg_meters	The same as u_meters and v_meters but each value is multiplied by -1	list
data_Q, data_U	Stores the polarised visibility ratios obtained from sampling PVR_Q and PVR_U. The order of the samples are in the same order as the baselines i.e. sample _i corresponds to baseline _i	numpy.ndarray
data	The data from the fits file produced by MCFOST unpacked using <code>fits.getdata</code>	numpy.ndarray
*reduced_chi2err_Q	The reduced χ^2 error of data_Q compared with obs_data.vhvv	numpy.float64
*reduced_chi2err_U	The reduced χ^2 error of data_U compared with obs_data.vhvvu	numpy.float64
*reduced_chi2err	The average of reduced_chi2err_Q and reduced_chi2err_U	numpy.float64
†data_I	Values from sampling Ips the same way PVR_Q and PVR_U are sampled to obtain data_Q and data_U	numpy.ndarray

Figure 5.16: Attributes of the Star class.

* Attribute not created during the initialisation process. Requires `calculate_reduced_chi2err()` to be called first.

† Attribute not created during the initialisation process. It is only calculated in `sanity_check_star()`

5.6 Methods

5.6.1 Callable methods

Method	Description	Optional arguments and requirements
observe()	Simulates the observation of the star. It produces interferometric data: the polarised differential visibilities in Stokes Q and U. Executed upon initialisation.	None
display_IQUV()	Displays the MCFOST images corresponding to the polarised images in Stokes I, Q U and V.	None
display_power_spectrums()	Displays the power spectrums of H, V, A and B	None
display_PVRs()	Displays the polarised visibility ratios	None
display_data()	Provides an easy way to plot the simulated and observed data. More details in Section 5.6.6	Some options require observation data outlined in Table 5.15
print_parameters()	Prints the MCFOST parameters and their values in alphabetical order	None
calculate_reduced_chi2error()	Calculated the reduced χ^2 error (defined by Equation 1. Returns the reduced χ^2 error in both Stokes Q and U, as well as overall.)	Observational data specified in Table 5.15
checksum()	For monochromatic coherent radiation, $I^2 = Q^2 + U^2 + V^2$. This method returns $I^2 - Q^2 - U^2 - V^2$. It is expected that this result is close to 0.	None
sanity_check_star()	A basic check looking at a star with no shell. See Section 6.1	None
sanity_check_shell()	A basic check looking at a just resolved star with a shell twice the size of the star. See section 6.2	None

5.6.2 Hidden methods

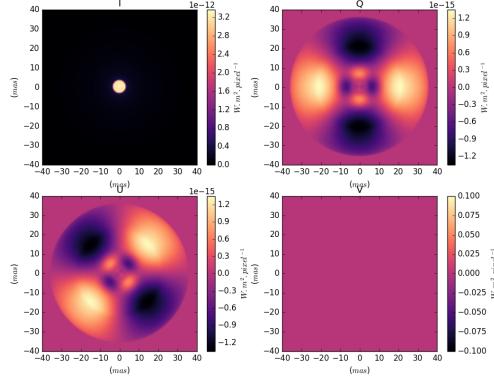
Method	Description
__load_IQUV()	Unloads the fits file into the IQUV images
__obtain_polarised_images()	Obtains the polarised images (Hp, Vp, Ap, Bp) from the IQUV images
__obtain_power_spectrums()	Obtains the power spectrums (Hps, Vps, Aps, Bps) from the Hp, Vp, Ap, Bp images
__obtain_PVRs()	Obtains the polarised visibility ratios in Stokes Q and U (PVR_Q, PVR_U) from the power spectrums Hps, Vps, Aps, Bps
__calculate_rad_per_px()	Calculates the angular size of a pixel in radians
__calculate_max_baseline()	Determines the maximum baseline that the PVR_Q and PVR_U images can represent and is used to determine where to sample the PVRs
__load_uv_coords()	Determines the uv co-ordinates where we should sample the PVRs in units of meters
__sample_PVR()	A function called by __sample_PVRs(). This function samples a single PVR using the interpolate module with a 2d quintic interpolation.
__sample_PVRs()	Samples PVR_Q and PVR_U. The samples are stored in data_Q and data_U
__display_sim_data()	Plots the simulated data of either Stokes Q or Stokes U on a specified ax (plot)
__display_observed_data()	Plots the observed data of either Stokes Q or Stokes U on a specified ax (plot)
__sampled_power_spectrum()	This samples the power spectrum of I (Ips) the same way sample_PVR() samples PVR_Q and PVR_U
__display_sampled_power_spectrum_I()	Plots Ips with the sample points overlaid on the image and data_I as a function of baseline length.

5.6.3 `display_IQUV(scale = "auto", interpolation = None)`

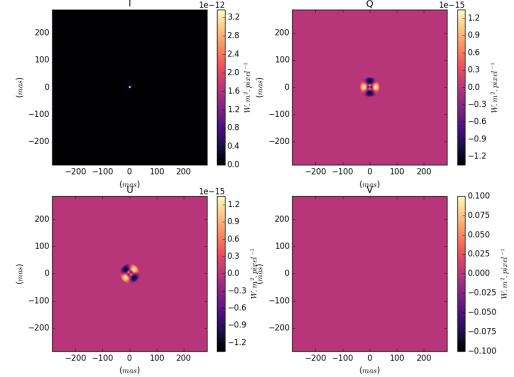
By default, `scale = "auto"` automatically zooms into the images to show the outer radius of the shell with a 10% margin. To turn this off set `scale = "off"`. Note: This does not work if the outer edge of the envelope extends beyond the figure.

By default, `interpolation = None` and is passed as an argument into `imshow()`. To turn interpolation off, set `interpolation = "nearest"`.

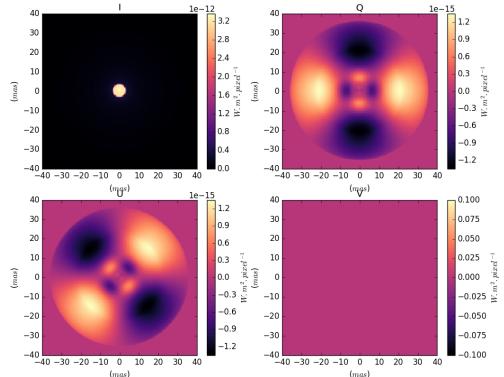
```
display_IQUV()
plot.show()
```



```
display_IQUV(scale = "off")
plot.show()
```



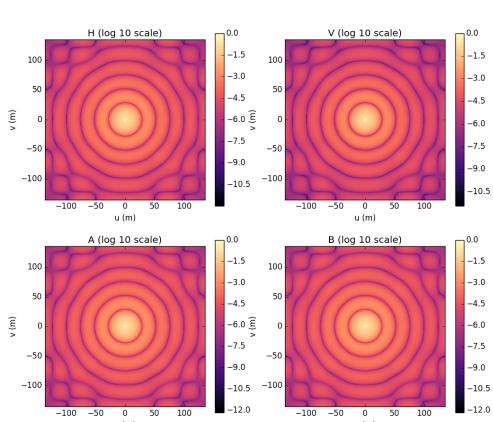
```
display_IQUV(interpolation = "nearest")
plot.show()
```



5.6.4 `display_power_spectrums()`

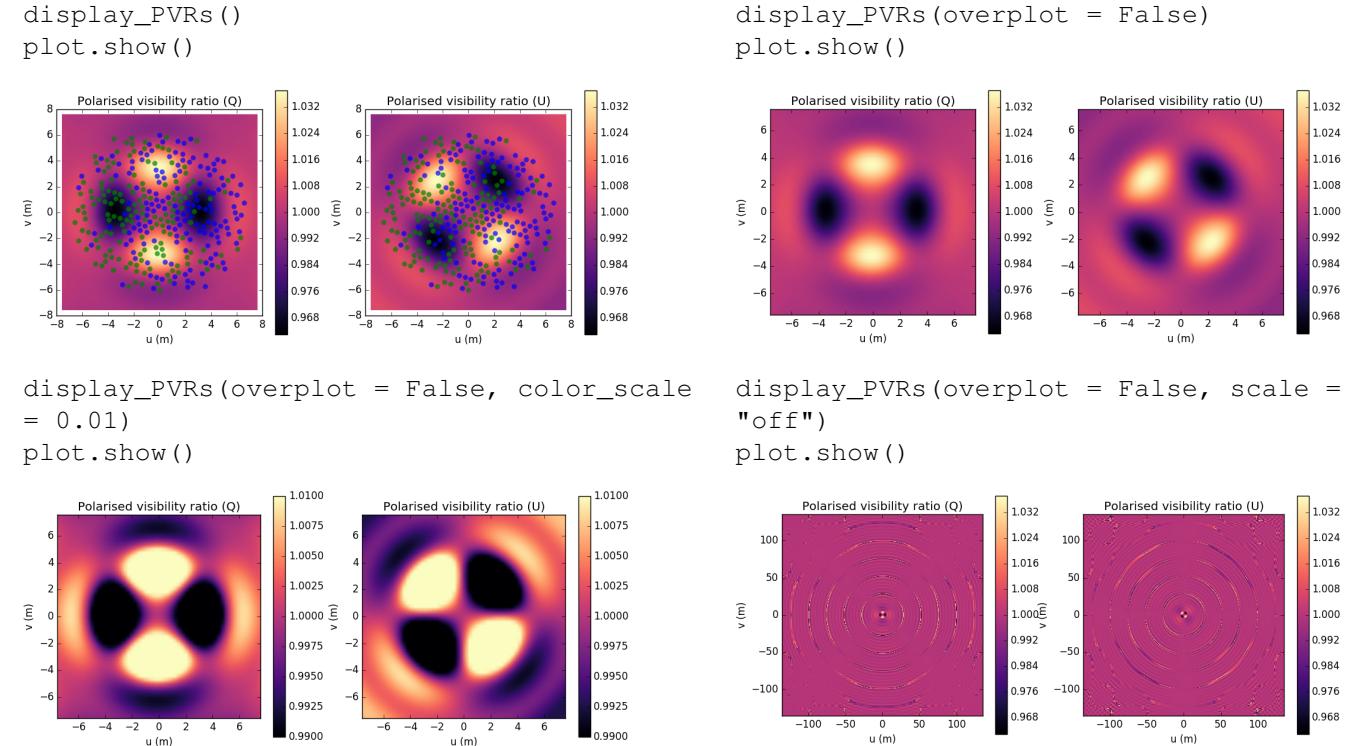
Displays the power spectrums (Hps, Vps, Aps, Bps) on a log 10 scale.

```
display_power_spectrums()
plot.show()
```



5.6.5 `display_PVRs(overplot = True, scale = "auto", interpolation = None, color_scale = "auto")`

Displays the polarised visibility ratios (PVR_Q, PVR_U). It automatically plots the points where the PVRs are sampled. To turn this off, set `overplot = False`. The images automatically scale to display the region of the PVRs where they are sampled. To view the entire PVR set `scale = "off"`. The images automatically adjust the colour scale so that the maximum and minimum colour values correspond to the maximum value of the sampled points, with the colourbar centered at 1. To manually adjust the colour bar so that it ranges from $1 - \alpha$ to $1 + \alpha$, set `color_scale = α` .



5.6.6 `display_data(option = 6, ylims = "auto", epsilon = 0.001)`

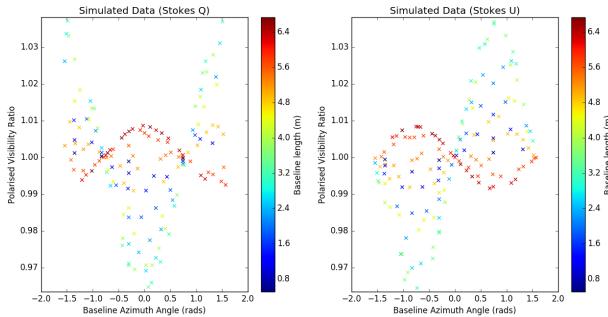
For these images, we have subtracted out the bias from the observation data using the following lines:

```
1 self.vhvv -= np.mean(self.vhvv) - 1
2 self.vhvuu -= np.mean(self.vhvuu) - 1
```

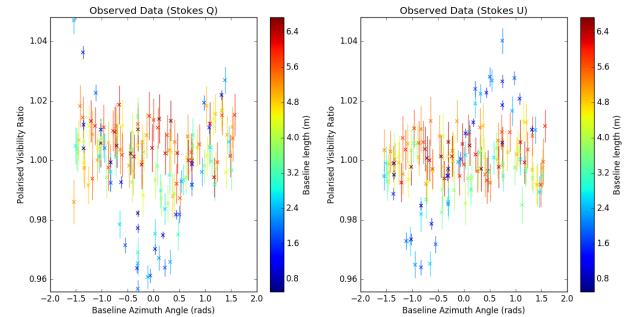
The `display_data()` function has 6 options that displays various combinations of the simulated and observed data:

- **Option 1:** simulated data
- **Option 2:** observed data
- **Option 3:** simulated and observed data in Stokes Q
- **Option 4:** simulated and observed data in Stokes U
- **Option 5:** simulated and observed data in Stokes Q and U (separate plots)
- **Option 6 (default):** simulated and observed data in Stokes Q and U (Q's and U's on the same plot)

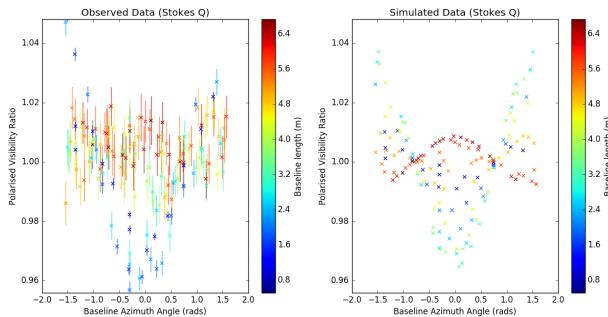
```
display_data(option = 1)
plot.show()
```



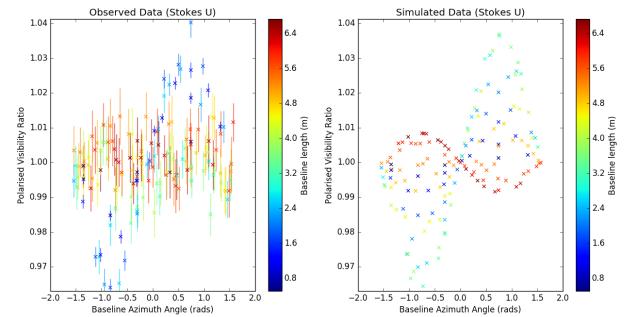
```
display_data(option = 2)
plot.show()
```



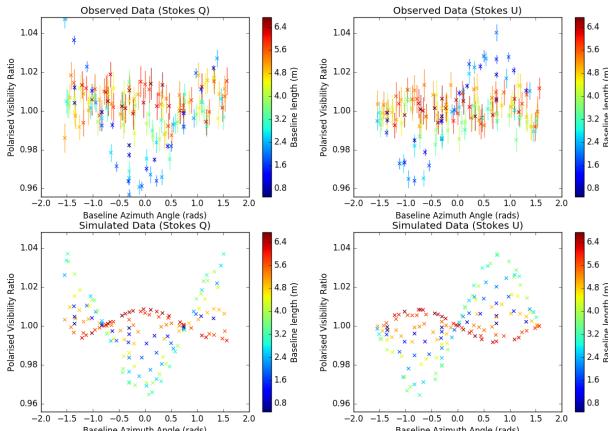
```
display_data(option = 3)
plot.show()
```



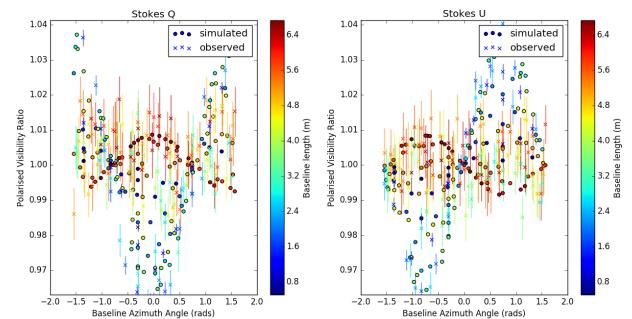
```
display_data(option = 4)
plot.show()
```



```
display_data(option = 5)
plot.show()
```

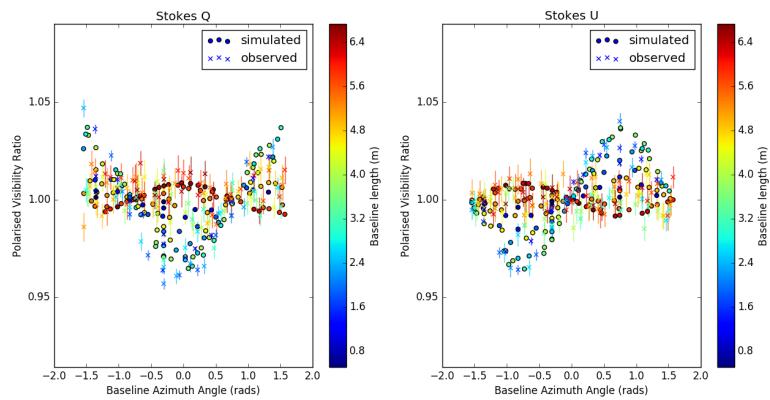


```
display_data()
plot.show()
```



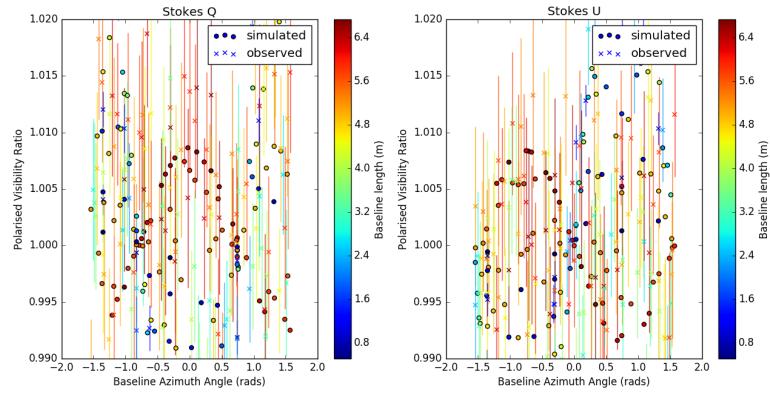
The y limits are automatically adjusted so that the maximum y value is $\max + \epsilon$ and the minimum y value is $\min - \epsilon$ where \max and \min are the maximum and minimum values of the data points being displayed. ϵ can be changed by specifying `epsilon`.

```
display_data(epsilon = 0.05)
plot.show()
```



Alternatively, you can specify the values of the ylims by passing ylims a list with the minimum and maximum y values.

```
display_data(ylims = [0.99, 1.02])
plot.show()
```



6 Sanity checks

6.1 Sanity check 1: Star

6.1.1 Worked example

Here you can see a working example of the first sanity check. The default parameters were specified in appendix A. The free parameters used are:

```
free_params = {"Rin": 3, "Rout": 5, "dust_mass": 1e-20, "radius": 500, "size":  
    → 80}
```

and the sanity check was called with:

```
s.sanity_check_star()
```

and the following images were produced:

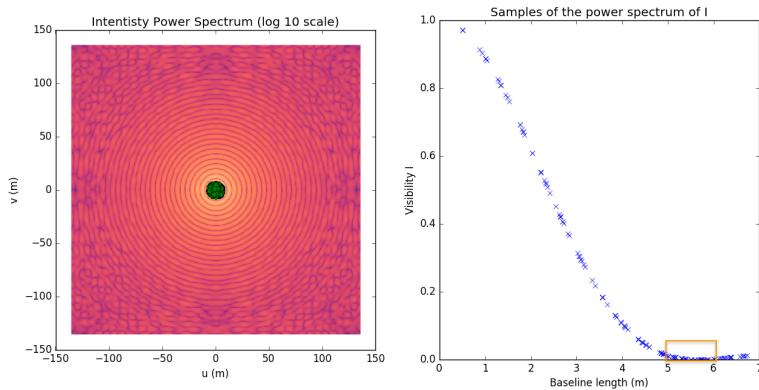


Figure 6.17: A figure produced during the star sanity check.

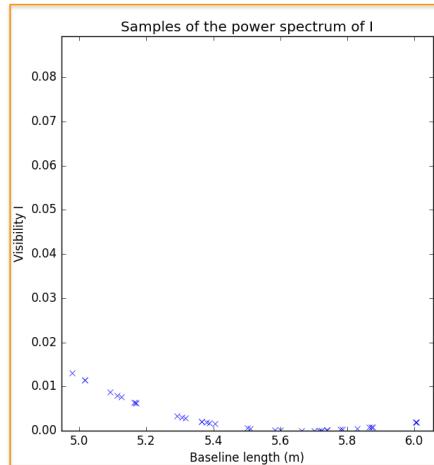


Figure 6.18: Figure 6.17 zoomed in on the orange rectangle. The visibility goes to 0 between 5.4 and 5.8 meters.

As can be seen in Figure 6.18, the visibility goes to 0 between 5.4 and 5.8 meters. Enter these values into the terminal where prompted.

```

--- SANITY CHECK (star) ---

Distance to the star (pc):          140.0
Radius of the star (R_sun):        500

Distance to the star (m):           4.3199e+18
Radius of the star (m):            3.48e+11

Radius (diameter) of the star (mas): 16.616      (33.232)

Look for the baseline length where the visibility goes to 0.
Give an UNDERESTIMATE for this baseline length (in meters): 5.4
Give an OVERESTIMATE for this baseline length (in meters): 5.8

Wavelength of observation (microns): 0.75
The baseline at which the visibility is zero corresponds to a resolution range (mas):
32.54 - 34.95
The diameter of the star should lie in this resolution range.

```

Figure 6.19: Terminal output for the sanity check. Enter the range of baselines where the visibility goes to 0 as shown in Figure 6.18. The terminal should estimate a range for the diameter of the star which should match the actual diameter of the star.

The terminal should then give you an estimate for the diameter of the star. In Figure 6.19 this range is 32.54 - 34.95 mas. We can see that the diameter of our star is 33.232 which does indeed lie in the provided range.

6.1.2 Optional Arguments

```
sanity_check_star(self, star_edge = "hard", shell = False, prec = 5)
```

star_edge	The determines how resolution (R) is calculated for a given wavelength (λ) and baseline (B). If the edge is hard, $R = 1.22\lambda/B$, if the edge is soft, $R = \lambda/B$.
shell	If shell = True, the terminal output will display the inner and outer radius of the dust shell in AU, meter and mas.
prec	Changes the precision of the values that are displayed in the terminal.

6.1.3 Trouble shooting: too much dust (the predicted diameter of the star is wrong)

If you try to run this sanity check on a star that has a substantial dust shell, it won't work!

Significantly reduce the dust mass!

The following demonstrates what happens when you run this sanity check with too much dust.

Performing a sanity check with the following parameters:

```
free_params = {"Rin": 3, "Rout": 5, "dust_mass": 1e-6, "radius": 500, "size":
→ 80}
```

The following figure is produced:

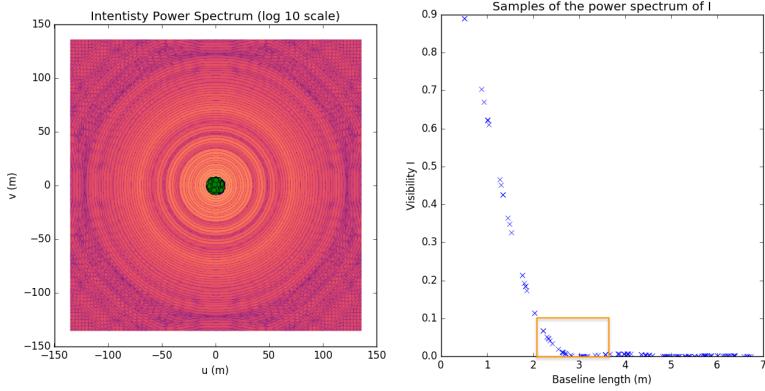


Figure 6.20: An example of a figure produced during a star sanity check of a star with a dust shell.

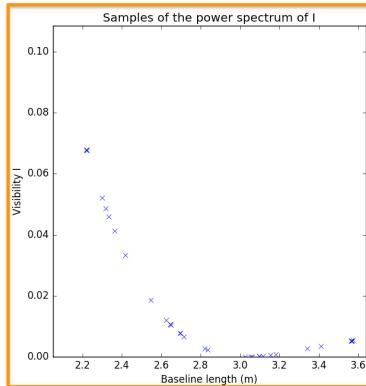


Figure 6.21: Figure 6.20 zoomed in on the orange rectangle.

Reading off Figure 6.21, the baseline at which the visibility of I goes to 0 is between **2.8** and **3.2** meters. Feeding this as input into the terminal produces the following output:

```
--- SANITY CHECK (star) ---

Distance to the star (pc):          140.0
Radius of the star (R_sun):         500

Distance to the star (m):           4.3199e+18
Radius of the star (m):            3.48e+11

Radius (diameter) of the star (mas): 16.616      (33.232)

Look for the baseline length where the visibility goes to 0.
Give an UNDERESTIMATE for this baseline length (in meters): 2.8
Give an OVERESTIMATE for this baseline length (in meters): 3.2

Wavelength of observation (microns): 0.75
The baseline at which the visibility is zero corresponds to a resolution range (mas):
58.979 - 67.404
The diameter of the star should lie in this resolution range.
```

Figure 6.22: Terminal output after providing an underestimate of **2.8** and an overestimate of **3.2** meters of where the visibility of I goes to 0. This corresponds to a resolution range of **58.979 - 67.404 mas** which is inconsistent with the diameter of the star (**33.232 mas**).

6.1.4 Trouble shooting: unresolved star (visibility doesn't reach zero)

Check the resolution (mas) of the mask with:

```
print(s.resolution_mask_soft)
```

If your star is smaller than the resolution of the mask than this sanity check will not work!

Make your star bigger!

Here is an example of an unresolved star:

```
free_params = {"Rin": 3, "Rout": 5, "dust_mass": 1e-20, "radius": 300, "size":  
    → 80}
```

The size of the star is mas is displayed in the terminal output.

```
--- SANITY CHECK (star) ---  
  
Distance to the star (pc): 140.0  
Radius of the star (R_sun): 300  
  
Distance to the star (m): 4.3199e+18  
Radius of the star (m): 2.088e+11  
  
Radius (diameter) of the star (mas): 9.9696 (19.939)  
  
Look for the baseline length where the visibility goes to 0.  
Give an UNDERESTIMATE for this baseline length (in meters):
```

Figure 6.23: Terminal output for a star that is unresolved. The diameter of the star is 19.939 mas and the resolution of the mask is 28 mas.

The figures produced by an unresolved star might look like this:

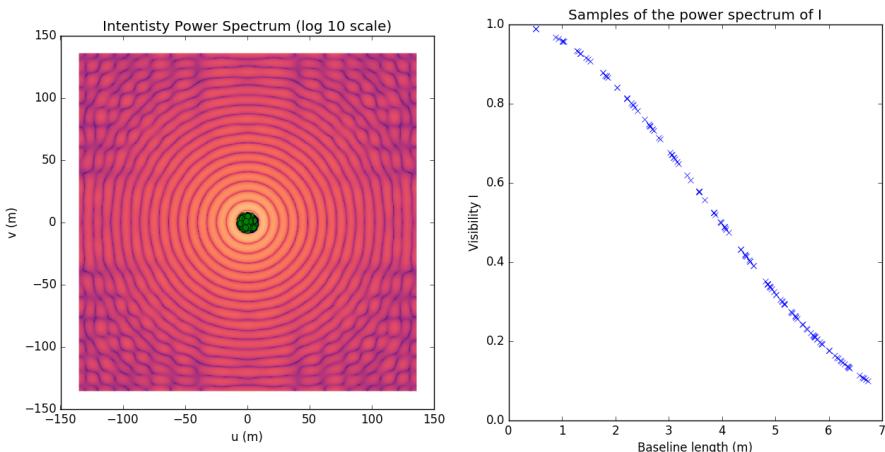


Figure 6.24: Figures from an unresolved star. The visibility never reaches zero.

The visibility of I for an unresolved will never reach 0. If it appears to reach the x-axis, check the scale on the y-axis, because the figure might automatically scale.

6.2 Sanity check 2: Shell

6.2.1 Worked example

Here you can see a working example of the second sanity check. The default parameters for the star were as specified in appendix A. A star was created without modifying the default parameters and at a wavelength of 0.75 microns. The *shell* sanity check was called.

```
free_params = {}
```

and the sanity check was called with:

```
s.sanity_check_star()
```

A window with a figure should pop up. This can be closed and ignored for now. The terminal output should look like Figure 6.25

```

--- SANITY CHECK (shell) ---

The maximum baseline for this mask is (meters) 6.7378
The resolution of this mask is (mas): 28.011
Distance to the star (pc): 140.0

A star that is just resolved has radius (solar radii): 421.45
A shell that is twice the size of the star has radius (AU): 3.9216

--- Make a star that is just resolved with a shell twice the size of the star ---
Radius of the star (R_sun): 200.0
Inner radius of dust shell (AU): 3.0
Outer radius of dust shell (AU): 300.

Distance to the star (m): 4.3199e+18
Radius of the star (m): 1.392e+11
Inner radius of dust shell (m): 4.4879e+11
Outer radius of dust shell (m): 4.4879e+13

Radius (diameter) of the star (mas): 6.6464 (13.293)
Inner radius (diameter) of the dust shell (mas): 21.429 (42.857)
Outer radius (diameter) of the dust shell (mas): 2142.9 (4285.7)

You should see distinct sinusoidal shapes. If you do not, you may need to zoom in.
Look at the blue-green sinusoid where the amplitude is largest. The colour tells you baseline.
What is an UNDERESTIMATE for the baseline length of this sinusoid? █

```

Figure 6.25: Terminal output after running `sanity_check_shell()`. The box indicates the lines that tell: the radius of a star (in solar radii) that is just resolved at the given distance with the given mask, and the corresponding radius of a dust shell (in AU) twice the radius of the star.

The terminal will be waiting for input, but for now, it is not important, so you can terminate the program.

You will see a box Figure 6.25. The box indicates the lines that tell: the radius of a star (in solar radii) that is just resolved at the given distance with the given mask, and the corresponding radius of a dust shell (in AU) twice the radius of the star. We created a star with these parameters. We wanted to make the dust shell thin, so we made the inner radius (`Rin`) and outer radius (`Rout`) approximately the same. We used the following parameters:

```
free_params = {"Rin": 4, "Rout": 4.1, "dust_mass": 1e-10, "radius": 422,
   → "size": 101}
```

You may need to fiddle with the amount of dust. You need enough dust to obtain a polarised signal, but if you have too much dust you will be unable to 'see through' the dust shell.

The second half of the terminal output should change, as shown if Figure 6.26.

```

--- SANITY CHECK (shell) ---

The maximum baseline for this mask is (meters) 6.7378
The resolution of this mask is (mas): 28.011
Distance to the star (pc): 140.0

A star that is just resolved has radius (solar radii): 421.45
A shell that is twice the size of the star has radius (AU): 3.9216

--- Make a star that is just resolved with a shell twice the size of the star ---
Radius of the star (R_sun): 422
Inner radius of dust shell (AU): 4
Outer radius of dust shell (AU): 4.1

Distance to the star (m): 4.3199e+18
Radius of the star (m): 2.9371e+11
Inner radius of dust shell (m): 5.9839e+11
Outer radius of dust shell (m): 6.1335e+11

Radius (diameter) of the star (mas): 14.024 (28.048)
Inner radius (diameter) of the dust shell (mas): 28.571 (57.143)
Outer radius (diameter) of the dust shell (mas): 29.286 (58.571)

You should see distinct sinusoidal shapes. If you do not, you may need to zoom in.
Look at the blue-green sinusoid where the amplitude is largest. The colour tells you baseline.
What is an UNDERESTIMATE for the baseline length of this sinusoid? █

```

Figure 6.26: Terminal output after changing the free parameters of the star to create a star that is just resolved with a thin shell with radius twice that of the star.

To look at your star, you can add the following line to your code:

```
s.display_IQUV(interpolation = "nearest")
```

```
plot.show()
```

Our star is shown in Figure 6.27.

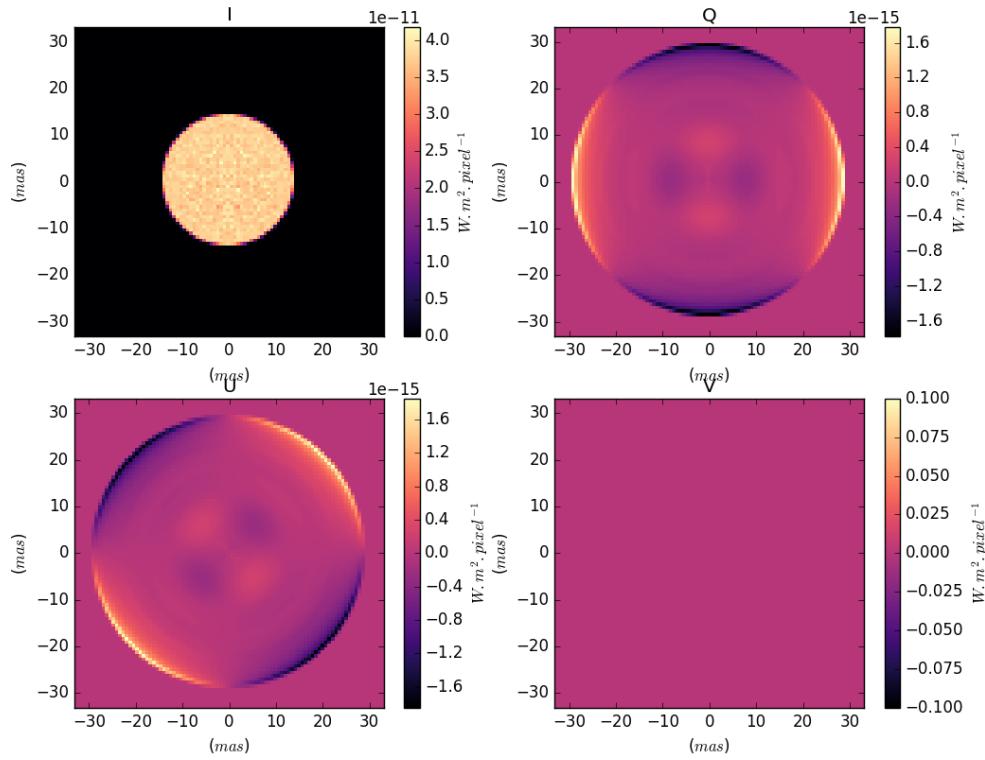


Figure 6.27: Images produced by MCFOST in Stokes polarised intensities.

You should be able to zoom in and check that the radius of the star and the shell are consistent with the input parameters (Figure 6.26).

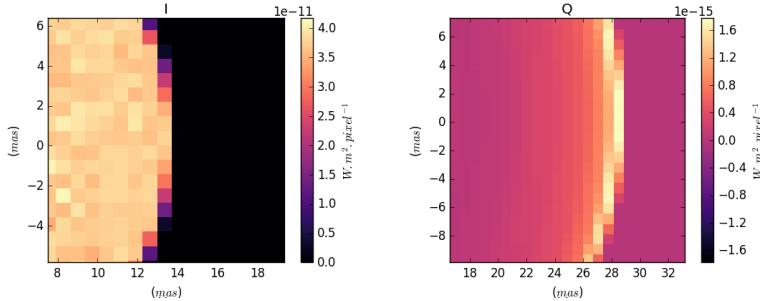


Figure 6.28: I and Q of Figure. 6.27 zoomed in. This is consistent with a stellar radius of 14.0 mas and a shell inner and outer radius of 27.9 mas and 29.1 mas as read off the output shown in Figure 6.26.

A window with another figure should also pop up (Figure 6.29). You may need to zoom in on the data centered around 1.00. Data points much larger or smaller than 1.00 are due to noise and can be ignored for this test.

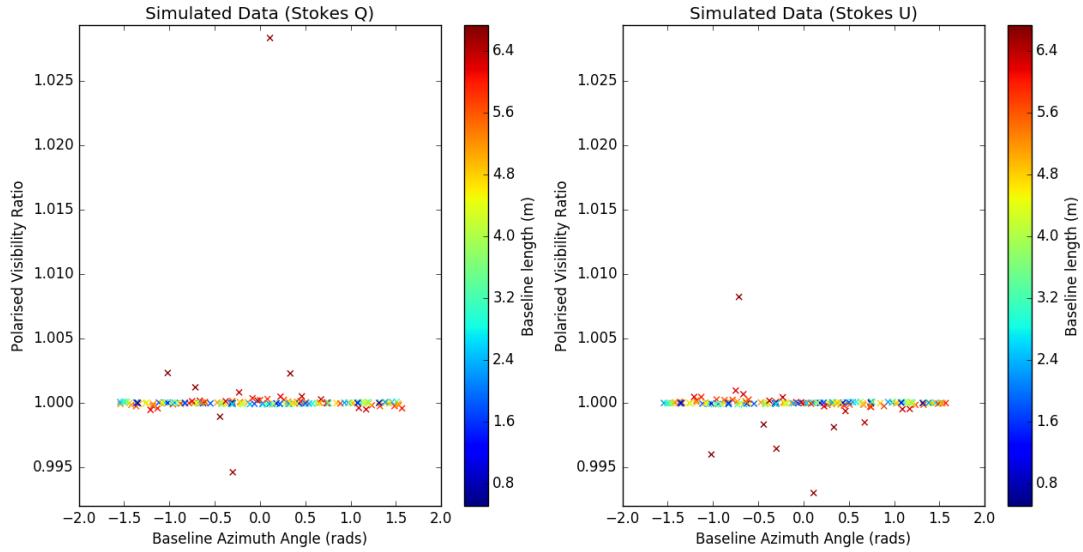


Figure 6.29: Simulated polarimetric differential data produced from the MCFOST images shown in Figure 6.27 by sampling the polarised visibility ratios. The figures automatically scale to show all data points. Noise causes some data points to stray large amounts from 1.00.

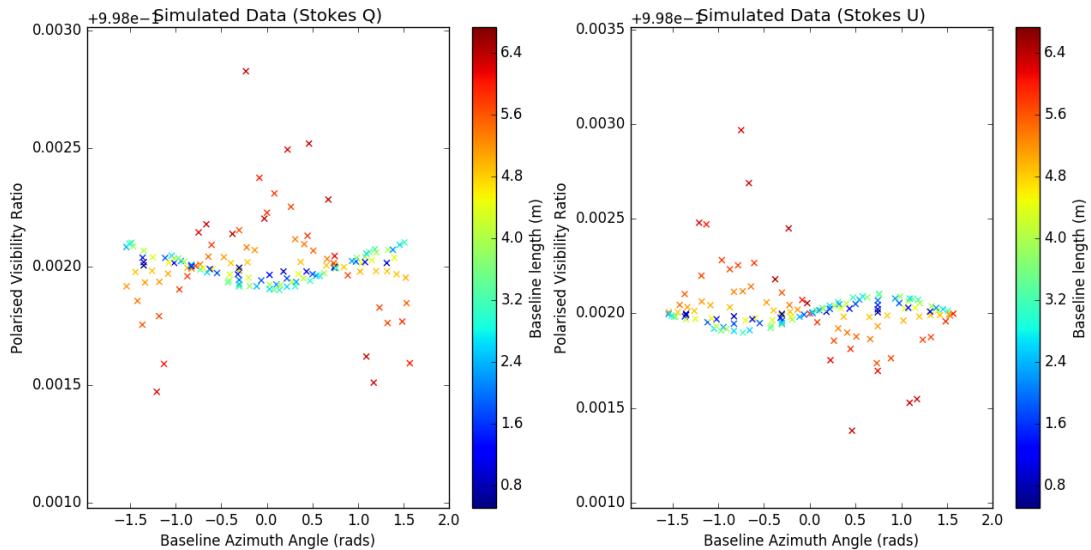


Figure 6.30: The same data as shown in 6.29 but zoomed in around 1.00.

You will notice that there appear to be two sinusoidal functions. One of these is in red. This corresponds to features of the star since the star we created is just resolved so is picked up by the longest baselines (red). The other sinusoid (in bright blue and green) corresponds to features of the shell. We are interested in the baseline that corresponds to this sinusoid with the largest amplitude. By eye, this sinusoid looks to be light blue to green, which corresponds to a range of baselines of 2.8 to 4.0 meters.

Enter these values into the terminal prompt.

```
You should see distinct sinusoidal shapes. If you do not, you may need to zoom in.  
Look at the blue-green sinusoid where the amplitude is largest. The colour tells you baseline.  
What is an UNDERESTIMATE for the baseline length of this sinusoid? 2.8  
What is an OVERESTIMATE for the baseline length of this sinusoid? 4.0
```

Figure 6.31: Input the range of baselines (2.8 - 4.0 meters) that correspond to the light blue-green sinusoid in Figure 6.30.

Directly after this, the terminal should give you a range telling you what the diameter of the shell should be.

```

Wavelength of observation (microns): 0.75
The diameter of the SHELL should lie between (mas):
47.183 - 67.404

```

Figure 6.32: An estimate for the shell diameter calculated from Figure 6.29. It predicts the diameter of the shell should be between 47 and 67 mas.

We see that an estimate of our shell diameter using Figure 6.29 is between 47 and 67 mas. If we look at the properties of our star in Figure 6.26, our shell had an inner diameter of 56 mas and an outer diameter of 58 mas, which is consistent with our estimation.

6.2.2 Optional Arguments

```
sanity_check_shell(self, star_edge = "hard", dp = 5, prec = 5)
```

star_edge	The determines how resolution (R) is calculated for a given wavelength (λ) and baseline (B). If the edge is hard, $R = 1.22\lambda/B$, if the edge is soft, $R = \lambda/B$.
prec	Changes the precision of the values that are displayed in the terminal.

6.2.3 Trouble Shooting: too much dust

If your star has too much dust, this check won't work!

Reduce the dust mass!

For example, the following free parameters produce the images show in Figure 6.33. You cannot distinguish the star from the shell in these images.

```
free_params = {"Rin": 4, "Rout": 4.1, "dust_mass": 1e-7, "radius": 422, "size":
    → 101}
```

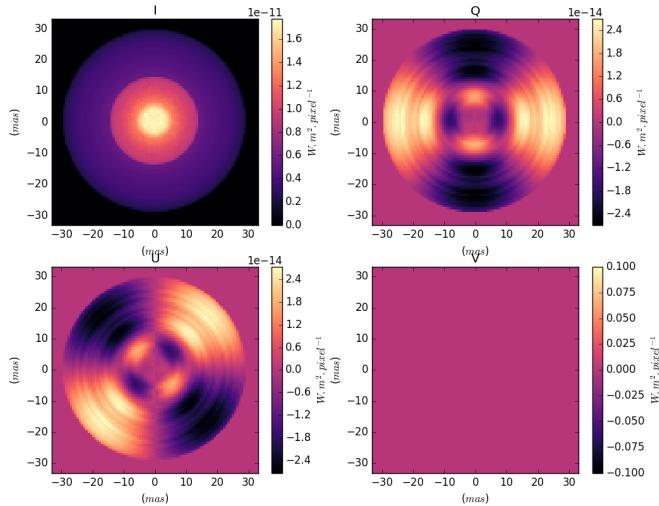


Figure 6.33: A star with too much dust. The shell sanity check will not work.

The following is the simulated data corresponding to these images:

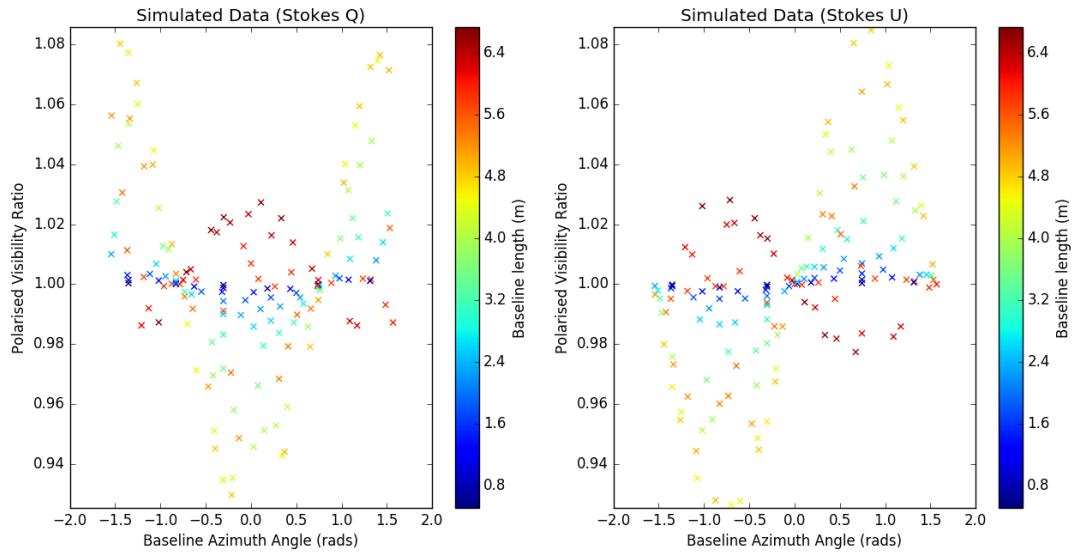


Figure 6.34: Simulated data from the images shown in figure 6.33

You can see that the sinusoid is now yellow-orange colour instead of light blue-green. This corresponds to a baseline range of 4.0-4.4 which estimates the diameter of the shell to be between 42.894 and 47.183 mas which is an *underestimate* for the diameter of the dust shell which is 58 mas.

A Default parameter class

```
1  class Default_parameters:
2
3      def __init__(self):
4
5          # number of photon packages
6          self.nbr_photons_eq_th = "1.28e5"
7          self.nbr_photons_lambda = "1.28e3"
8          self.nbr_photons_image = "1.28e5"
9
10         # wavelength
11             # do not change these unless you know what you're doing
12             self.n_lambda = "50"
13             self.lambda_min = "0.1"
14             self.lambda_max = "3000.0"           # mum
15
16             self.compute_temp = 'T'
17             self.compute_sed = 'T'
18             self.default_wavelength = 'T'        # use default wavelength grid for output?
19
20             self.wavelength_file = "IMLup.lambda" # if previous parameter is F
21
22             self.separation_of_different_contributions = 'F'
23             self.stokes_parameters = 'T'
24
25             # grid geometry and size
26             self.geometry = '2' # 1 = cylindrical, 2 = spherical, 3 = Voronoi tessellation (this
27             # → is in beta, please ask Christophe)
28
29             self.n_rad = "100"           # log distribution
30             self.nz = "70"              # (or n_theta)
31             self.n_az = "1"
32             self.n_rad_in = "20"
33
34             # maps
35             self.grid_nx = "1001"
36             self.grid_ny = "1001"
37             self.size = "500."          # AU
38
39             # RT
40             self.imin = "0."
41             self.imax = "0."
42             self.n_incl = "1"
43             self.centered = 'F'
44
45             self.az_min = "0"
46             self.az_max = "0."
47             self.n_az_angles = "1"
48
49             self.distance = "140.0"       # pc
50             self.disk_PA = "0."
51
52             # Scattering method
53             self.scattering_method = "0"      # 0=auto, 1=grain prop, 2=cell prop
54             self.mie_hg = "1"                # 1=Mie, 2=hg (2 implies the loss
55             # → of polarization)
56
57             # Symmetries
58             self.image_symmetry = 'T'
59             self.central_symmetry = 'T'
60             self.axial_symmetry = 'T'        # (important only if N_phi > 1)
61
62             # Disk physics
63             self.dust_settling = "0"        # (0=no settling, 1=parametric, 2=Dubrulle,
64             # → 3=Fromang)
65             self.exp_strat = "0.50"
66             self.a_strat = "1.0"            # (for parametric settling)
67
68             self.dust_radial_migration = 'F'
69             self.sublimate_dust = 'F'
70             self.hydrostatic_equilibrium = 'F'
71
72             self.viscous_heating = 'F'
73             self.alpha_viscosity = "1e-5"
```

```

72 # Number of zones: 1 zone = 1 density structure + corresponding grain properties
73 self.number_of_zones = "1"
74
75 # Density structure
76 self.zone_type = "3"           # zone type : 1 = disk, 2 = tapered-edge disk, 3 =
77   → envelope, 4 = debris disk, 5 = wall
78
79 self.dust_mass = "1.e-6"
80 self.gas_to_dust_mass_ratio = "100."
81
82 self.scale_height = "10."          # AU, unused for envelope
83 self.reference_radius = "100.0"      # AU, unused for envelope
84 self.vertical_profile_exponent = "2"      # only for debris disk
85
86 self.Rin = "3.0"
87 self.edge = "0.0"
88 self.Rout = "300."
89 self.Rc = "100."                  # AU, Rc is only used for tapered-edge & debris
90   → disks (Rout set to 8*Rc if Rout==0)
91
92 self.flaring_exponent = "1.125"     # unused for envelope
93
94 self.surface_density_exponent = "-0.5"    # (or -gamma for tapered-edge disk or
95   → volume density for envelope), usually < 0
96 self.negative_gamma_exp = "0.0"        # or alpha_in & alpha_out for debris disk
97
98 # Grain properties
99 self.number_of_species = "1"
100
101 self.grain_type = "Mie"            # Mie of DHS
102 self.N_components = "1"
103 self.mixing_rule = "2" # 1 = EMT or 2 = coating
104 self.porosity = "0.0"
105 self.max_fraction = "1.0"
106 self.Vmax = "0.9"                 # for DHS
107
108 self.optical_indicies_file = "Draine_Si_sUV.dat"
109 self.volume_fraction = "1.0"
110
111 self.heating_method = "1"          # 1 = RE + LTE, 2 = RE + NLTE, 3 = NRE
112
113 self.amin = "0.03"                # mum
114 selfamax = "1000.0" # mum
115 self.aexp = "3.5"
116 self.n_grains = "100"             # log distribution
117
118 # Molecular RT settings
119 self.lpop = 'T'
120 self.laccurate_pop = 'T'
121 self.LTE = 'T'
122 self.profile_width = "15."        # km.s^-1
123
124 self.v_turb = "0.2"               # delta
125
126 self.nmol = "1"
127
128 self.molecular_data_filename = "co@xpol.dat"
129 self.level_max = "6"
130
131 self.vmax = "1.0"                 # km.s^-1
132 self.n_speed = "20"
133
134 self.cst_molecule_abundance = 'T'
135 self.abundance = "1.e-6"
136 self.abundance_file = "abundance.fits.gz"
137
138 self.ray_tracing = 'T'
139 self.number_lines_in_RT = "3"
140
141 self.transition_number_1 = "1"
142 self.transition_number_2 = "2"
143 self.transition_number_3 = "3"
144
145 # Star properties
146 self.number_of_stars = "1"
147 self.temp = "4000.0"

```

```
145     self.radius = "200.0"          # solar radius
146     self.mass = "1.0"             # solar mass
147     self.x = "0.0"               # AU
148     self.y = "0.0"               # AU
149     self.z = "0.0"               # AU
150     self.is_blackbody = 'T'
151     self.fUV = "0.1"
152     self.slope_FUV = "2.2"
```

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