

Image

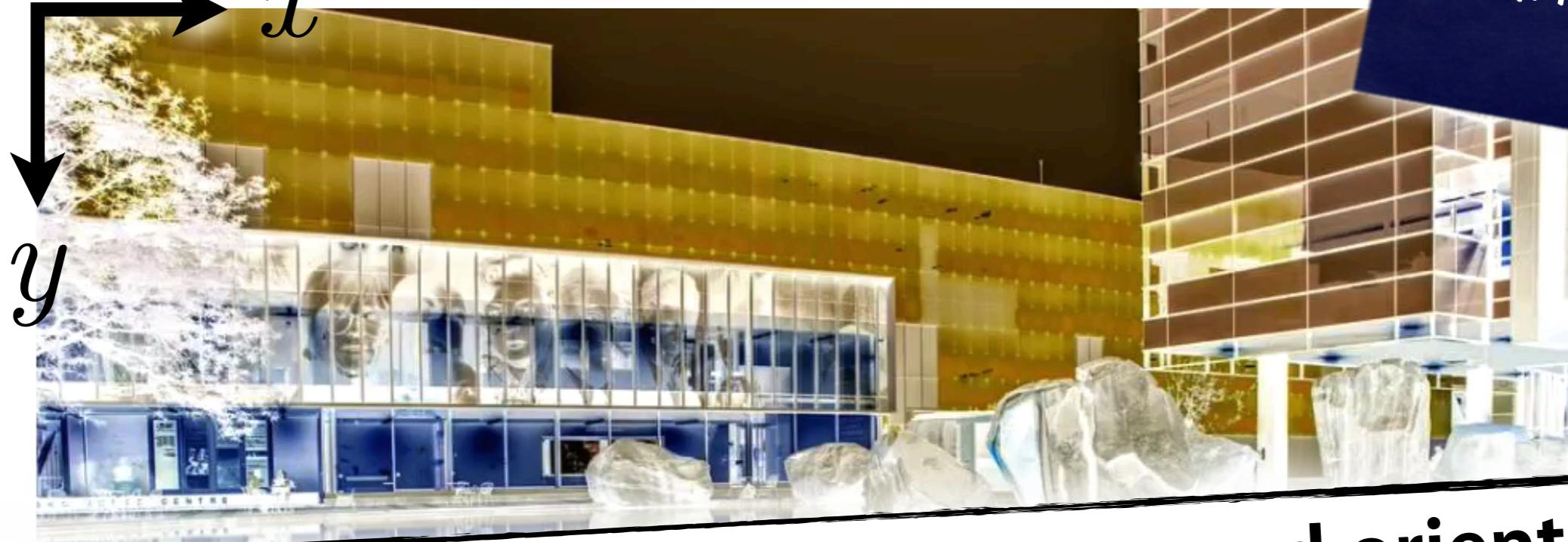
# Transformations

translation

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{I}_{2 \times 2} & \mathbf{t}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & 1 \end{pmatrix} \mathbf{x}$$

**two degrees of freedom**



$x$   
 $y$

translation

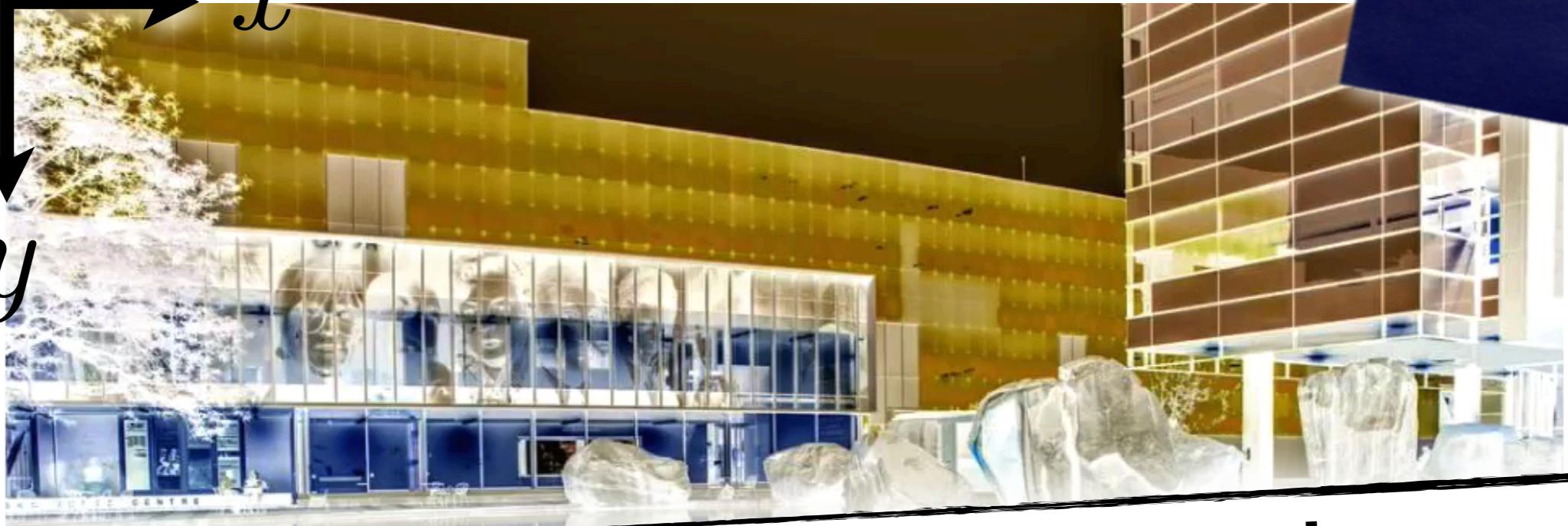
**preserves lines, angles, lengths, areas and orientations**



Euclidean

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & t_x \\ \sin \theta & \cos \theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} \mathbf{R}_{2 \times 2} & \mathbf{t}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & 1 \end{pmatrix} \mathbf{x}$$

**three degrees of freedom**



$x$

$y$

Euclidean

**preserves lines, angles, lengths and areas**



similarity

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \cos \theta & -\alpha \sin \theta & t_x \\ \alpha \sin \theta & \alpha \cos \theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} \alpha R_{2 \times 2} & t_{2 \times 1} \\ 0_{1 \times 2} & 1 \end{pmatrix} \mathbf{x}$$

**four degrees of freedom**



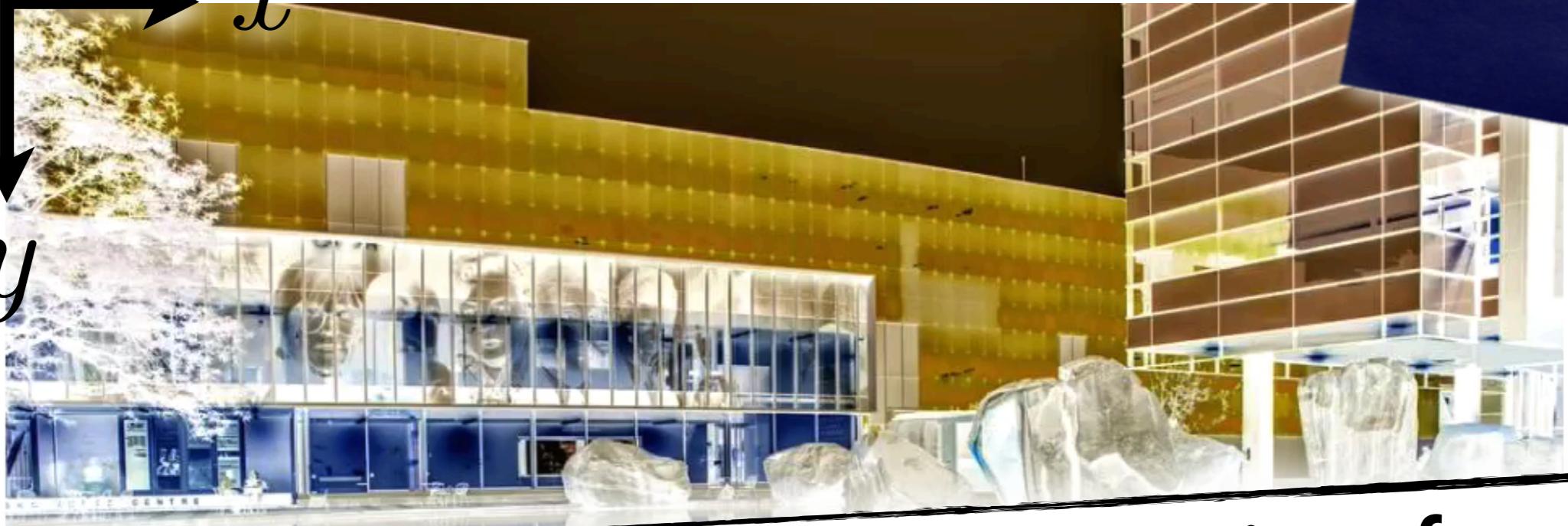
**preserves lines, angles and ratios of areas**



affine

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

**six degrees of freedom**



$x$

$y$

affine

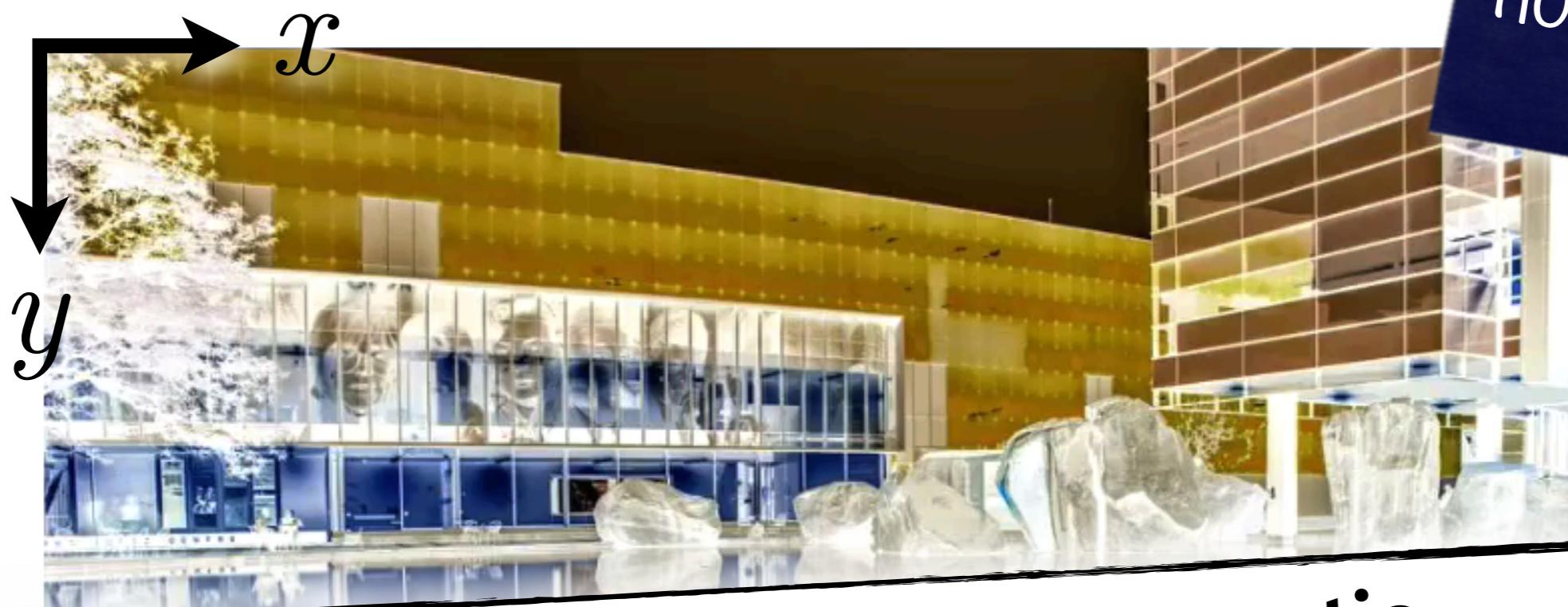
**preserves lines, parallel lines and ratios of areas**



homography

$$w \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

**eight degrees of freedom**



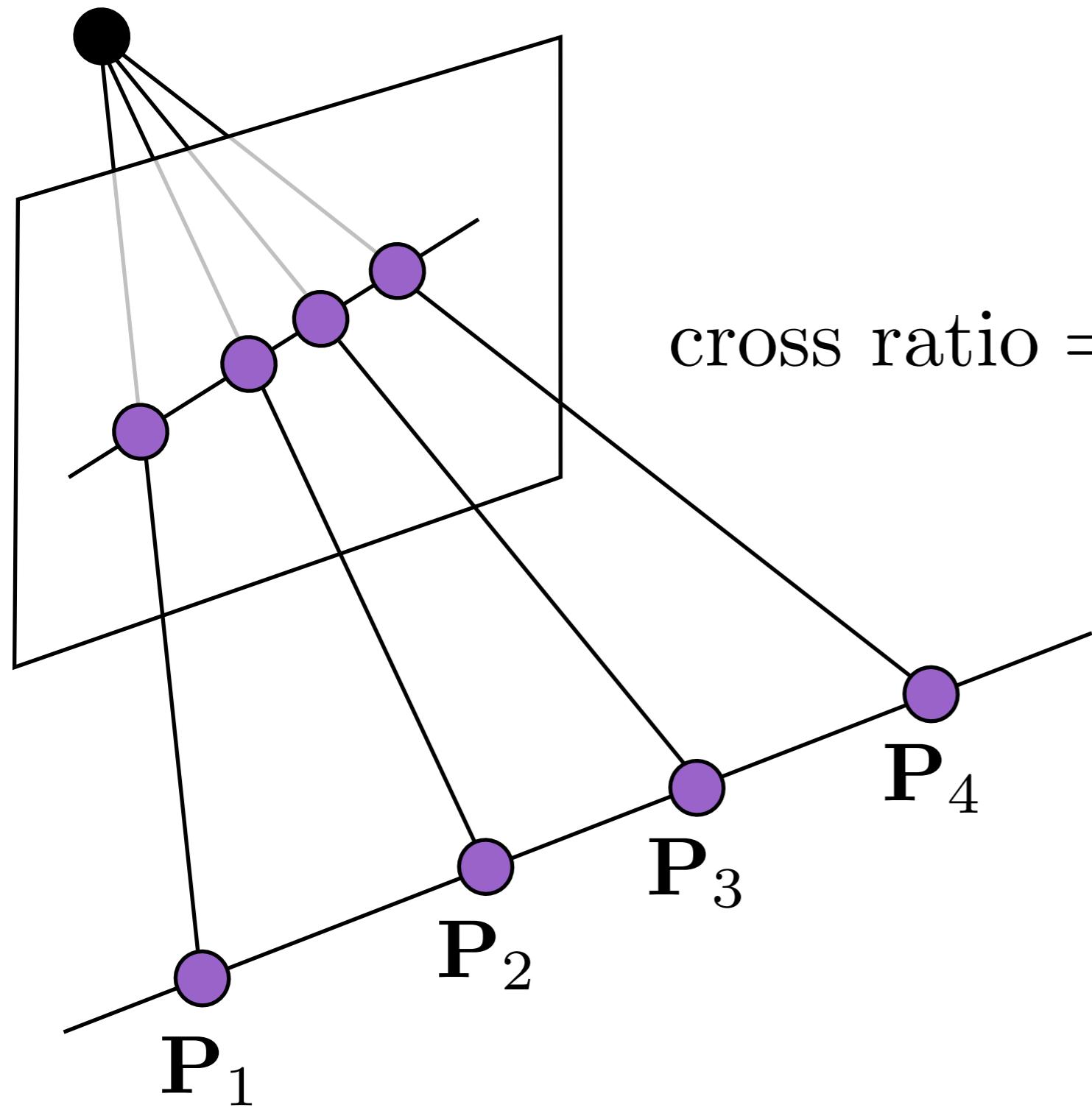
$x$   
 $y$

homography

preserves lines and cross ratio

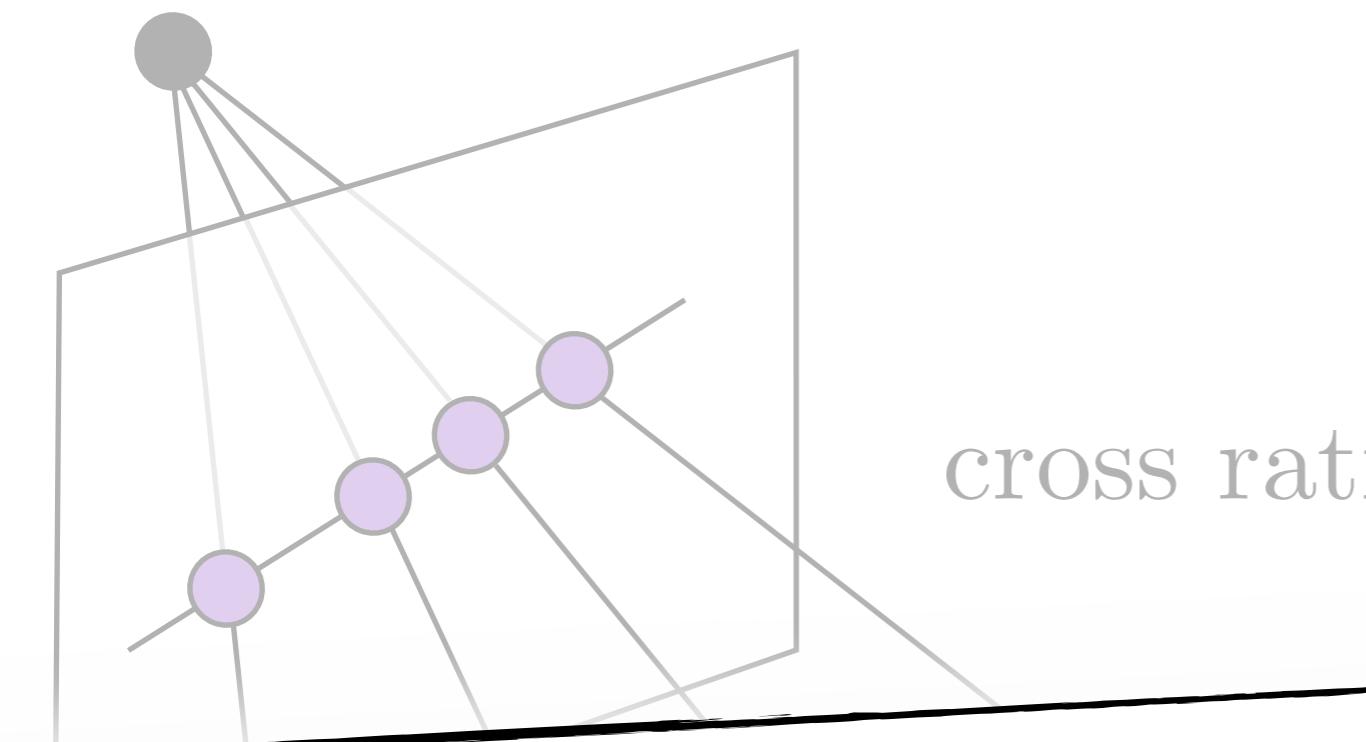


*cross ratio*



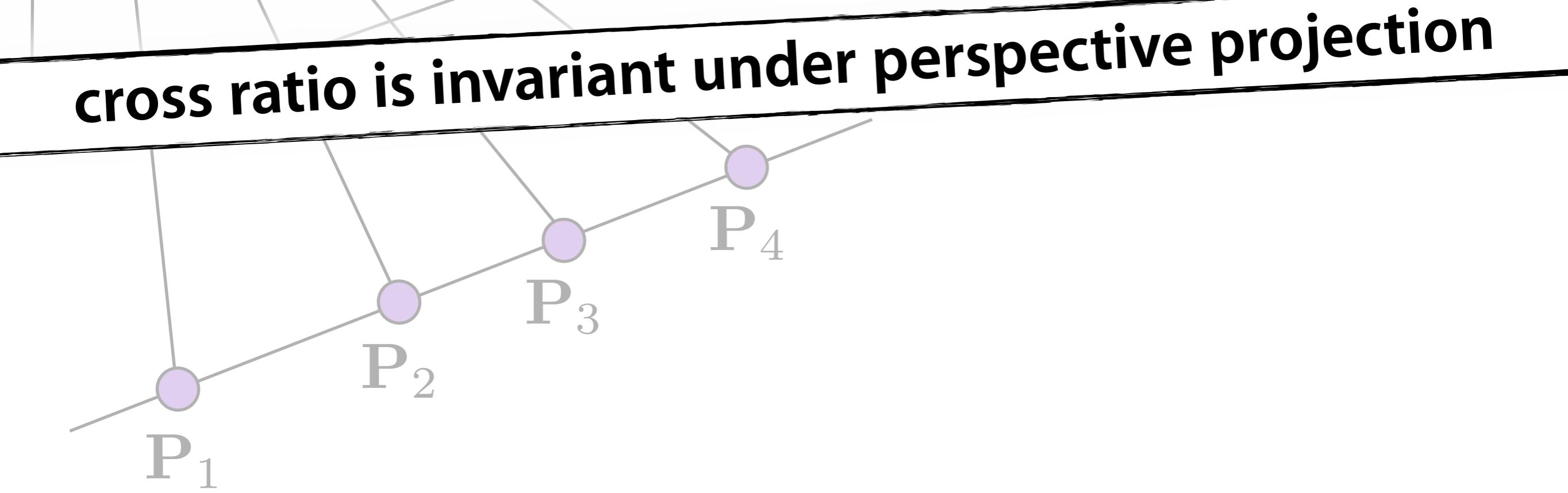
$$\text{cross ratio} = \frac{\|P_3 - P_1\| \|P_4 - P_2\|}{\|P_3 - P_2\| \|P_4 - P_1\|}$$

*cross ratio*

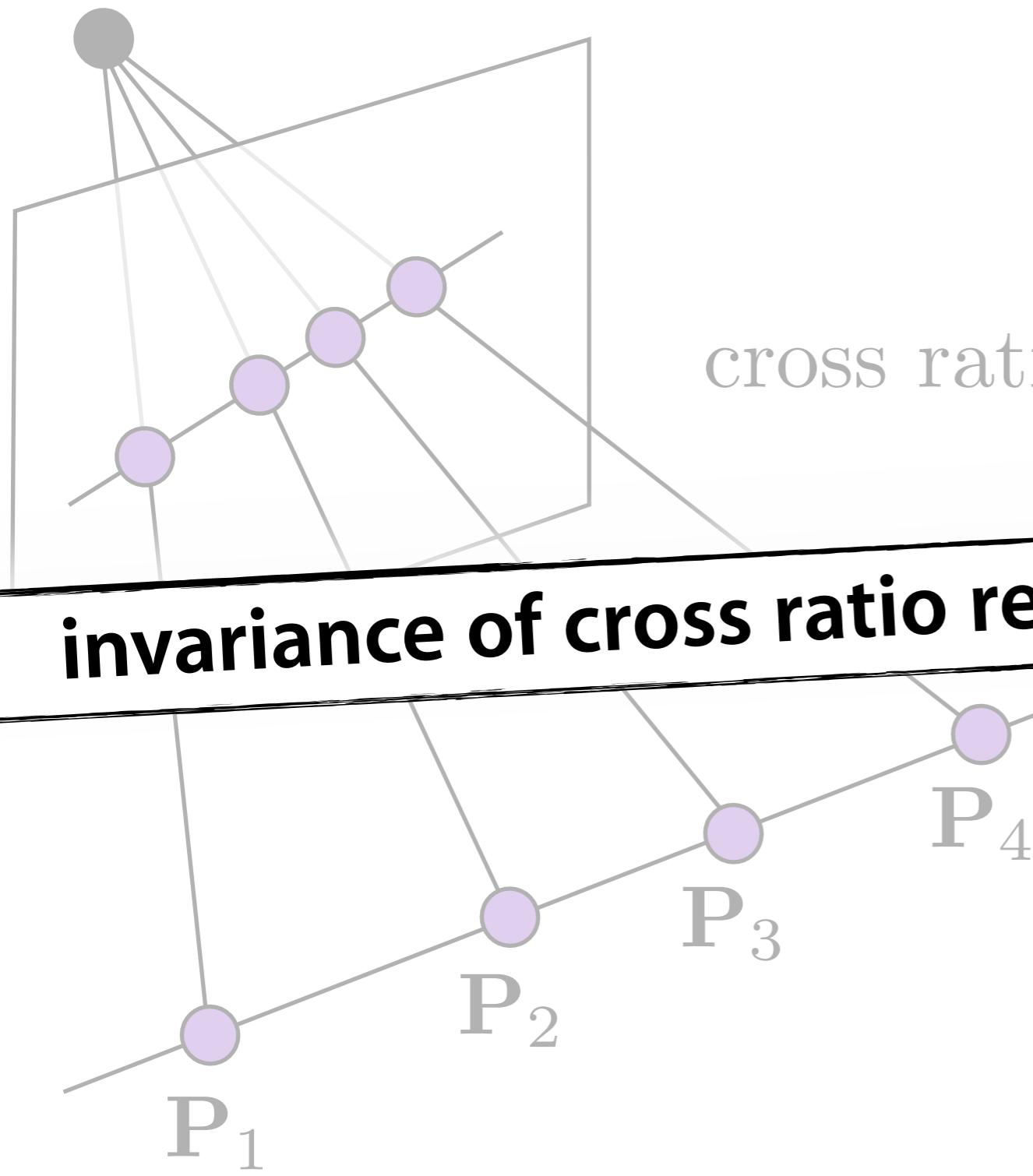


$$\text{cross ratio} = \frac{\|P_3 - P_1\| \|P_4 - P_2\|}{\|P_3 - P_2\| \|P_4 - P_1\|}$$

**cross ratio is invariant under perspective projection**



*cross ratio*



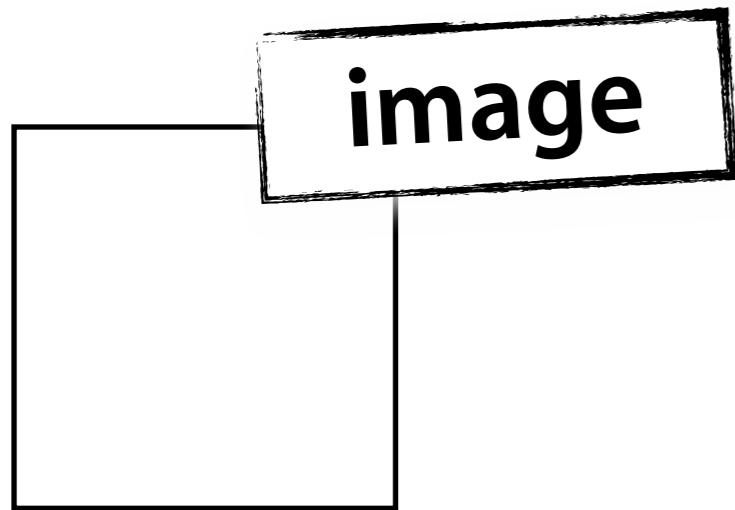
**image**

$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix} \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix}$$

$$= \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

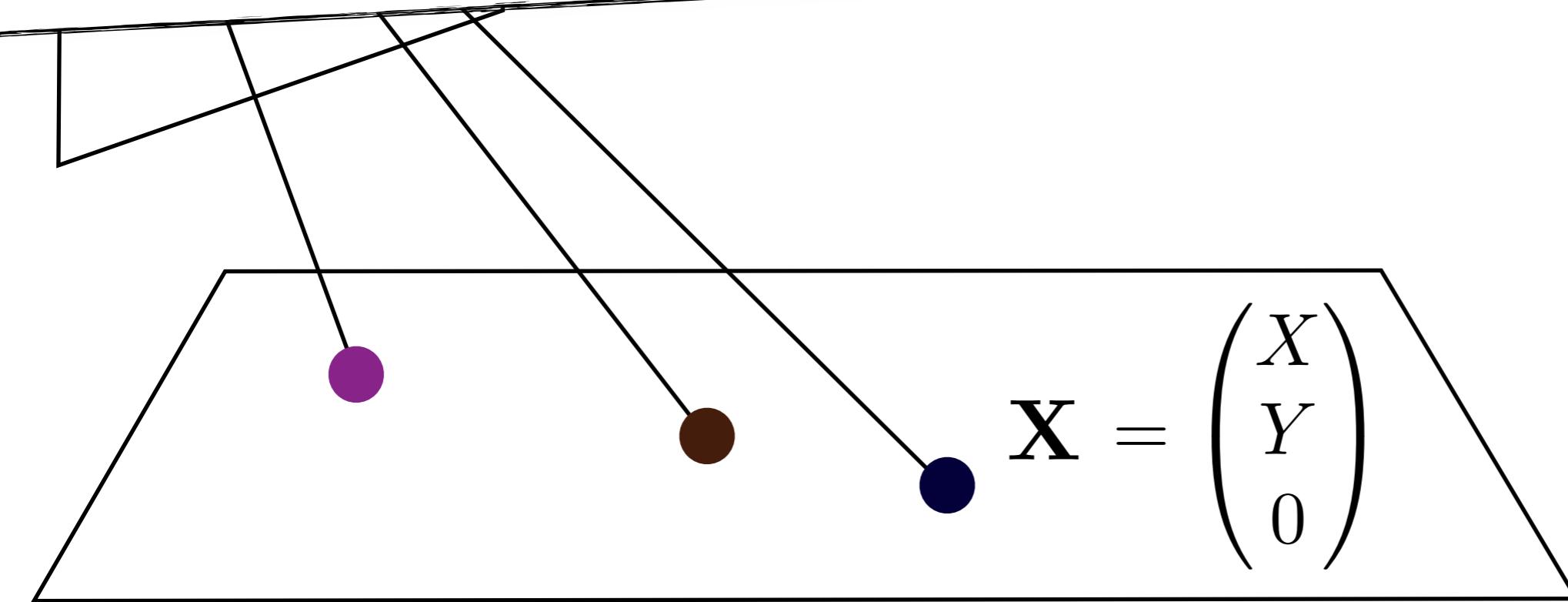
$$= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$



$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

**3D-to-2D projection reduces to 2D-to-2D mapping**



$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

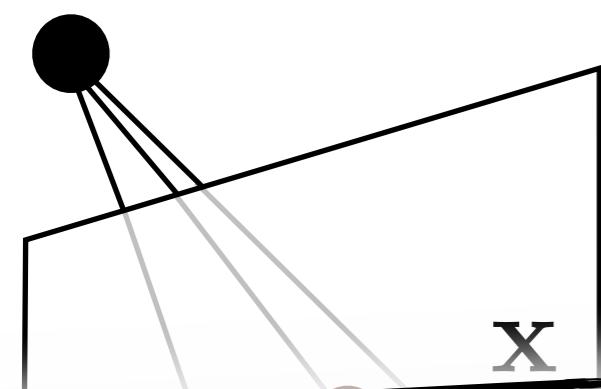
image

$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

transformation is called a **homography**

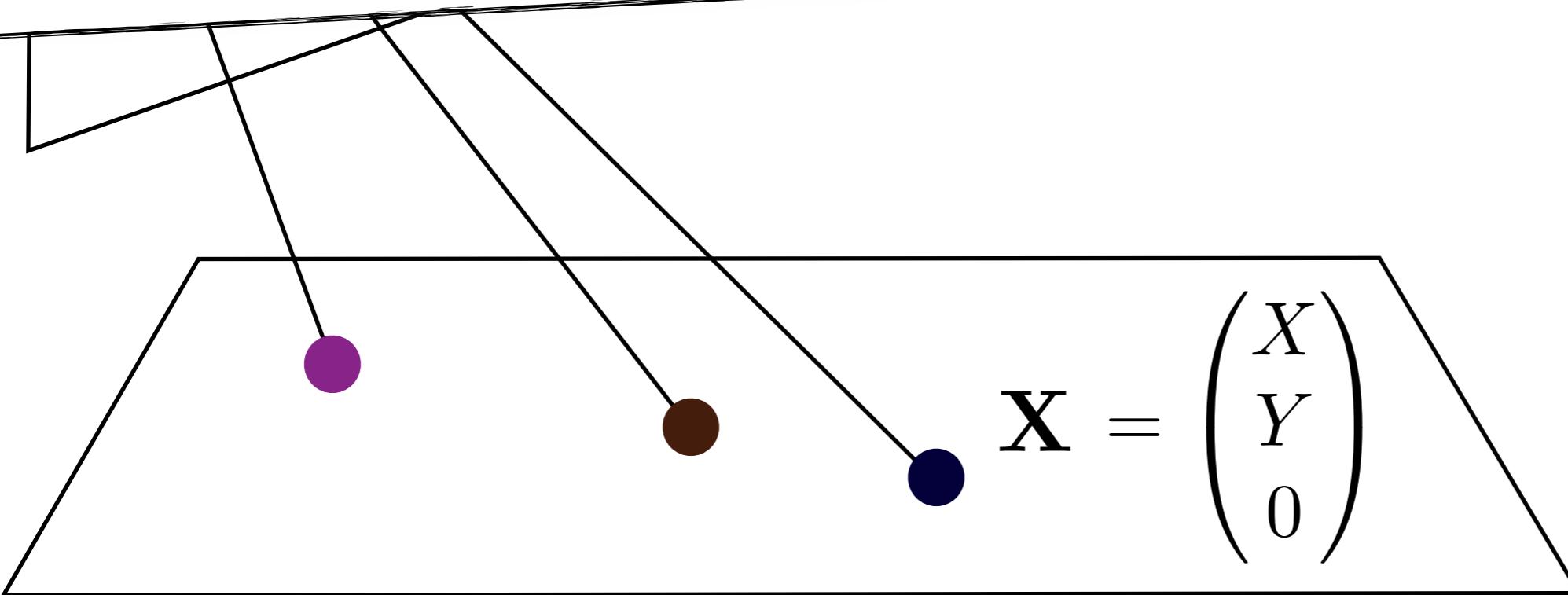
$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

image



$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

homography is an invertible transformation



$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

image

$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

homography defined up to a scalar multiple

$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

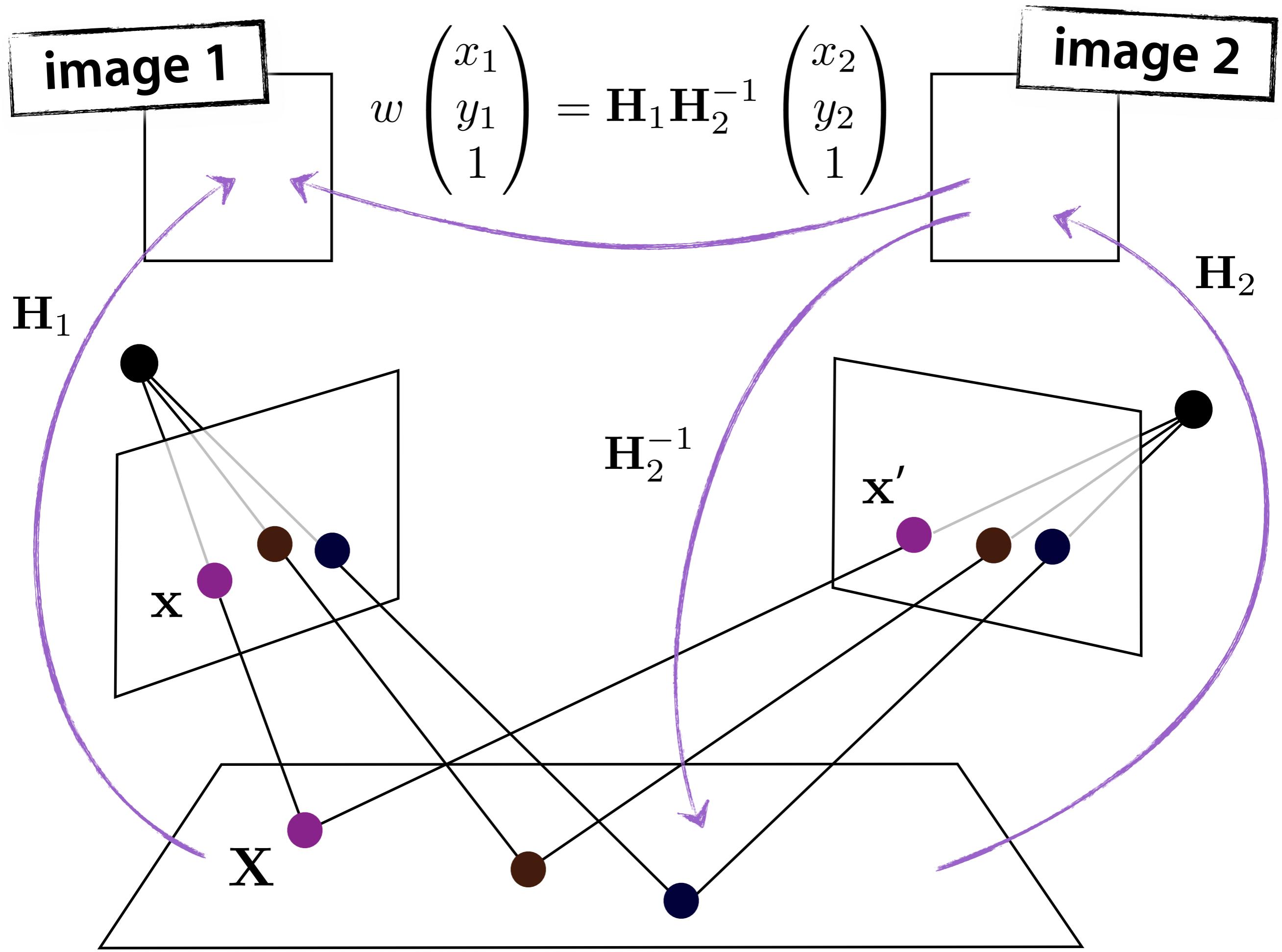
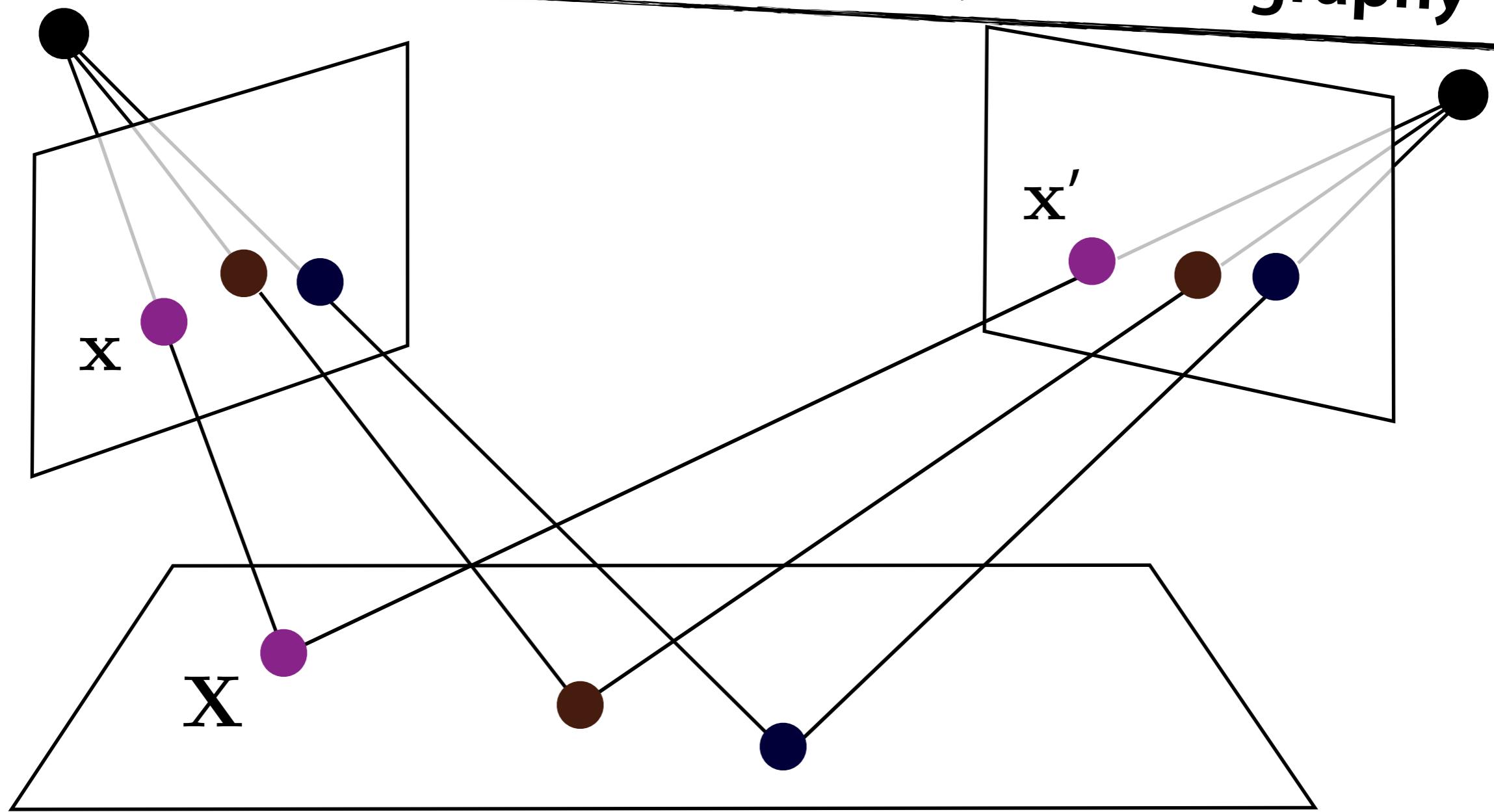


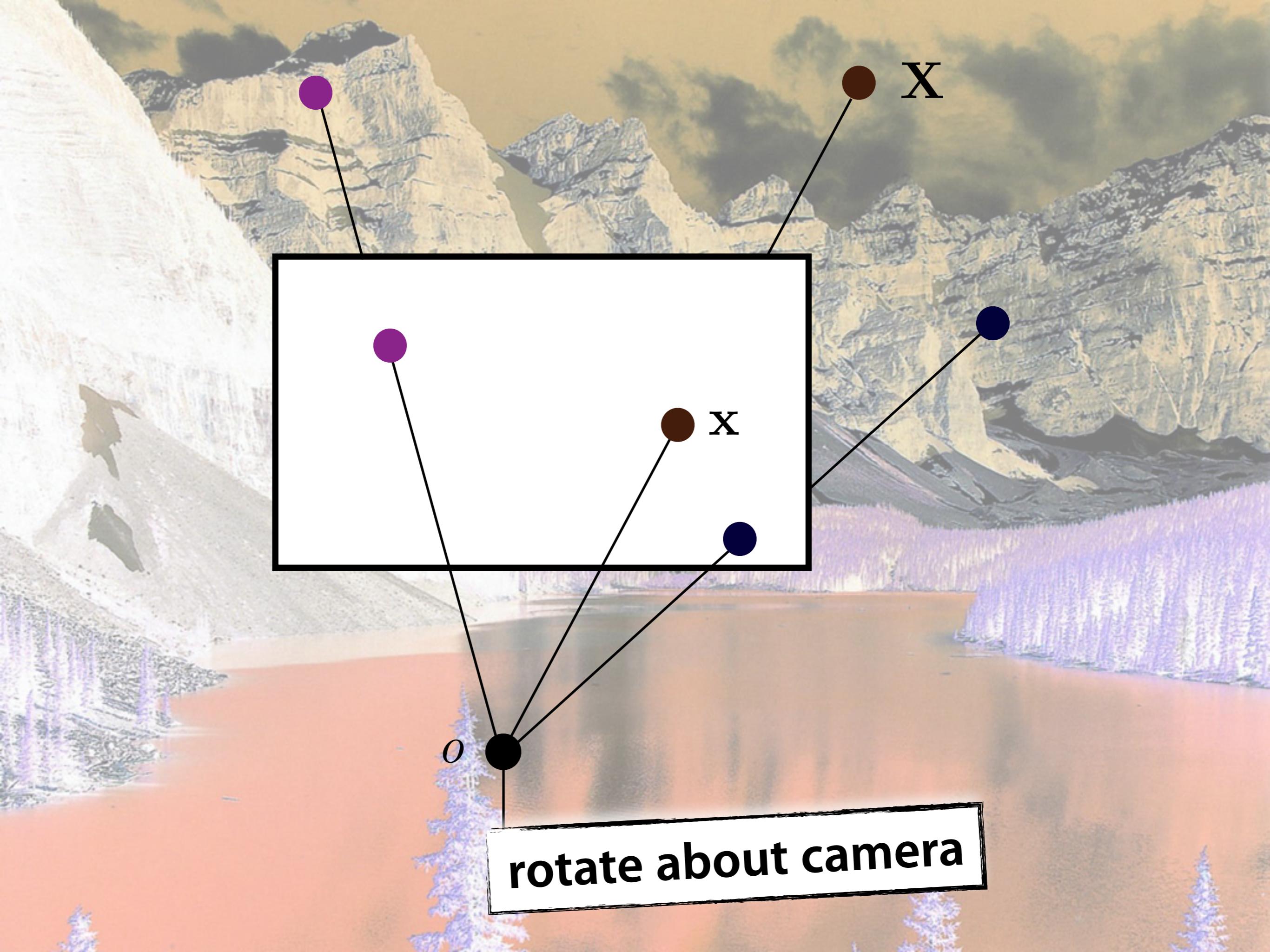
image 1

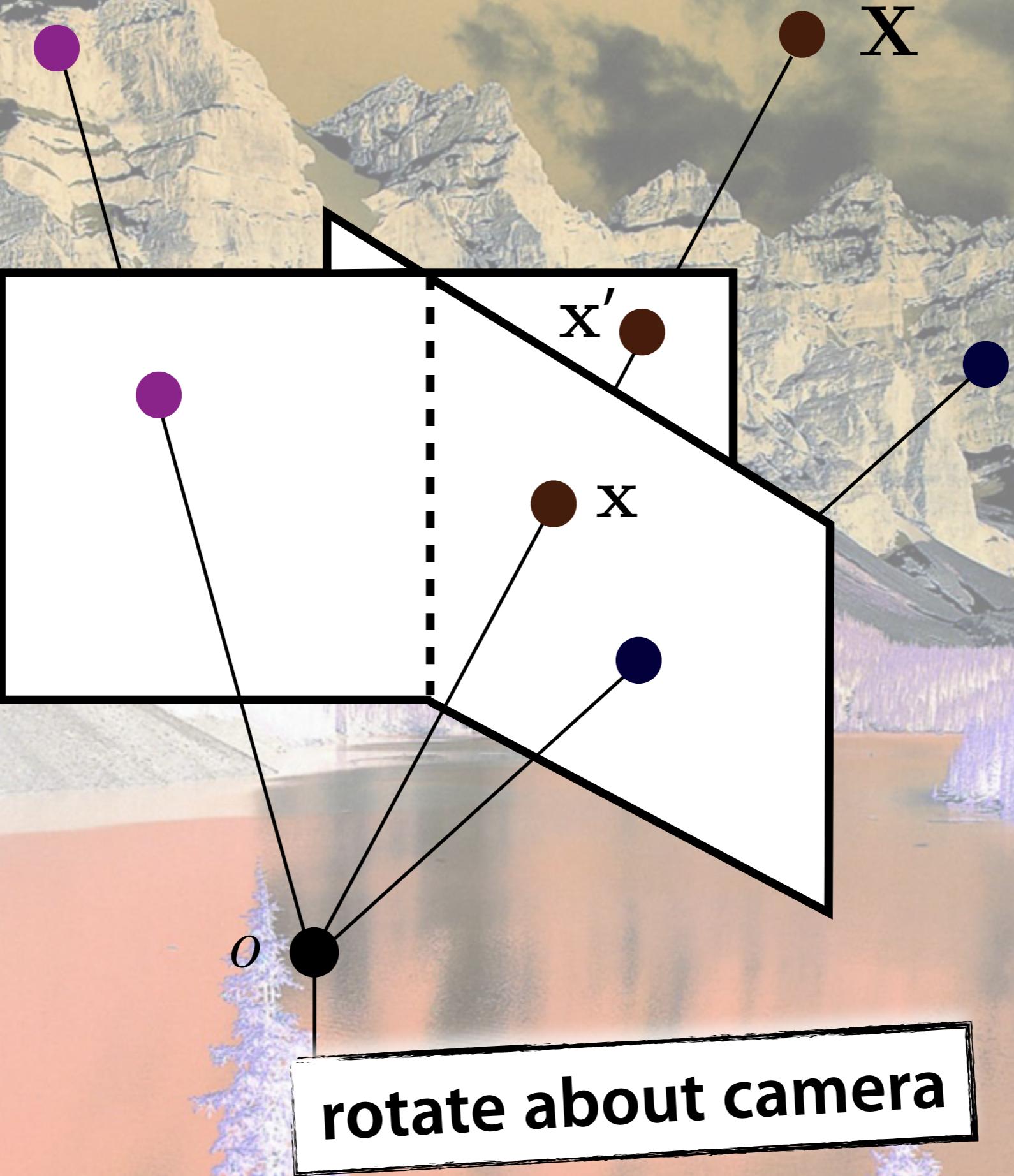
image 2

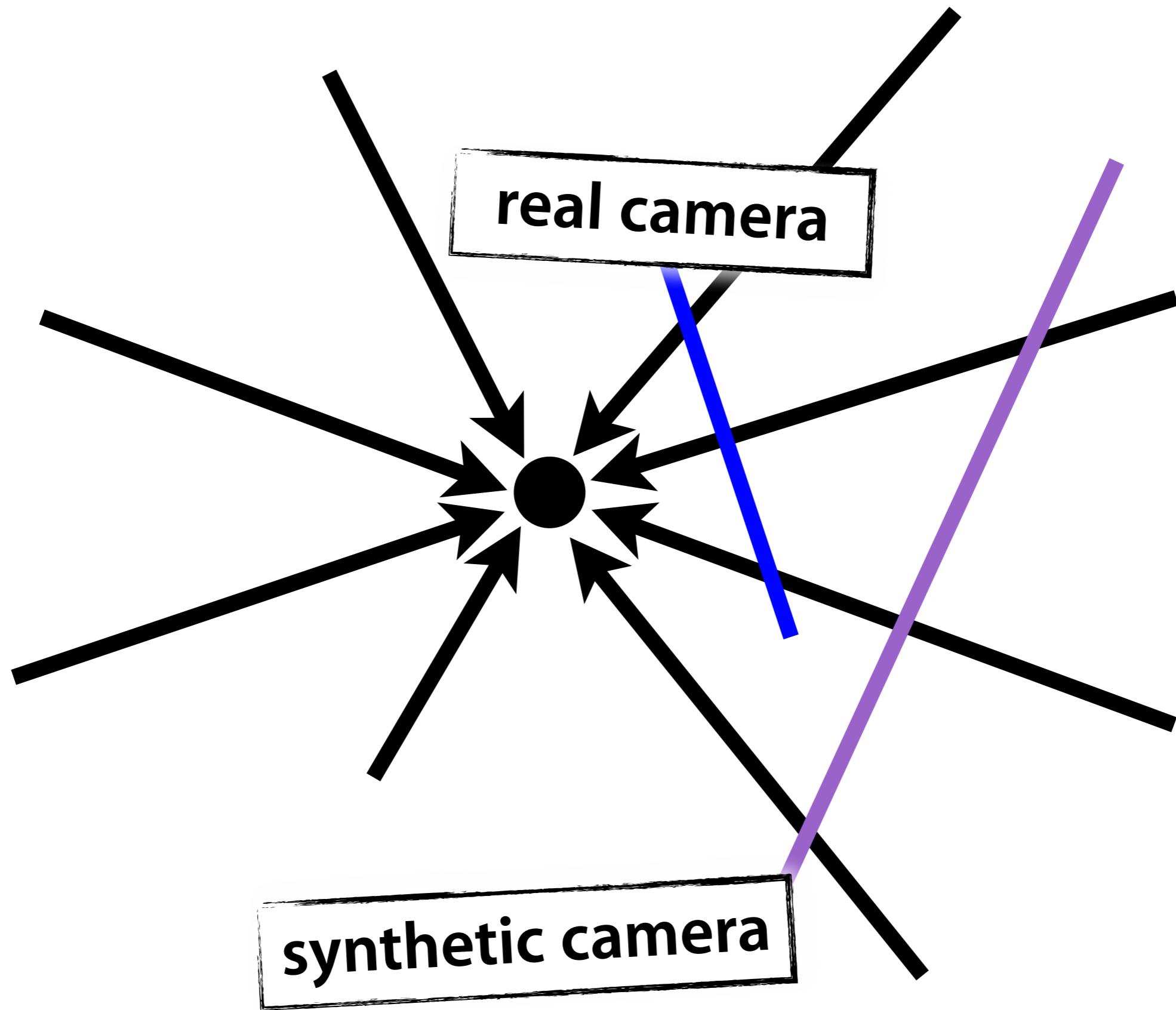
$$w \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = H \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

images of a plane are related by a homography









*rotating  
camera*

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$= \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$= \mathbf{H}_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

*rotating  
camera*

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{H}_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$w_2 \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \mathbf{H}_2 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = w_2 \mathbf{H}_1 \mathbf{H}_2^{-1} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

rotating  
camera

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = w_2 \mathbf{H}_1 \mathbf{H}_2^{-1} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

**images from a rotating camera are related by a homography**

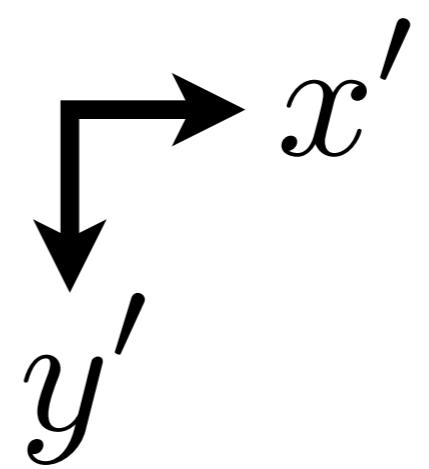
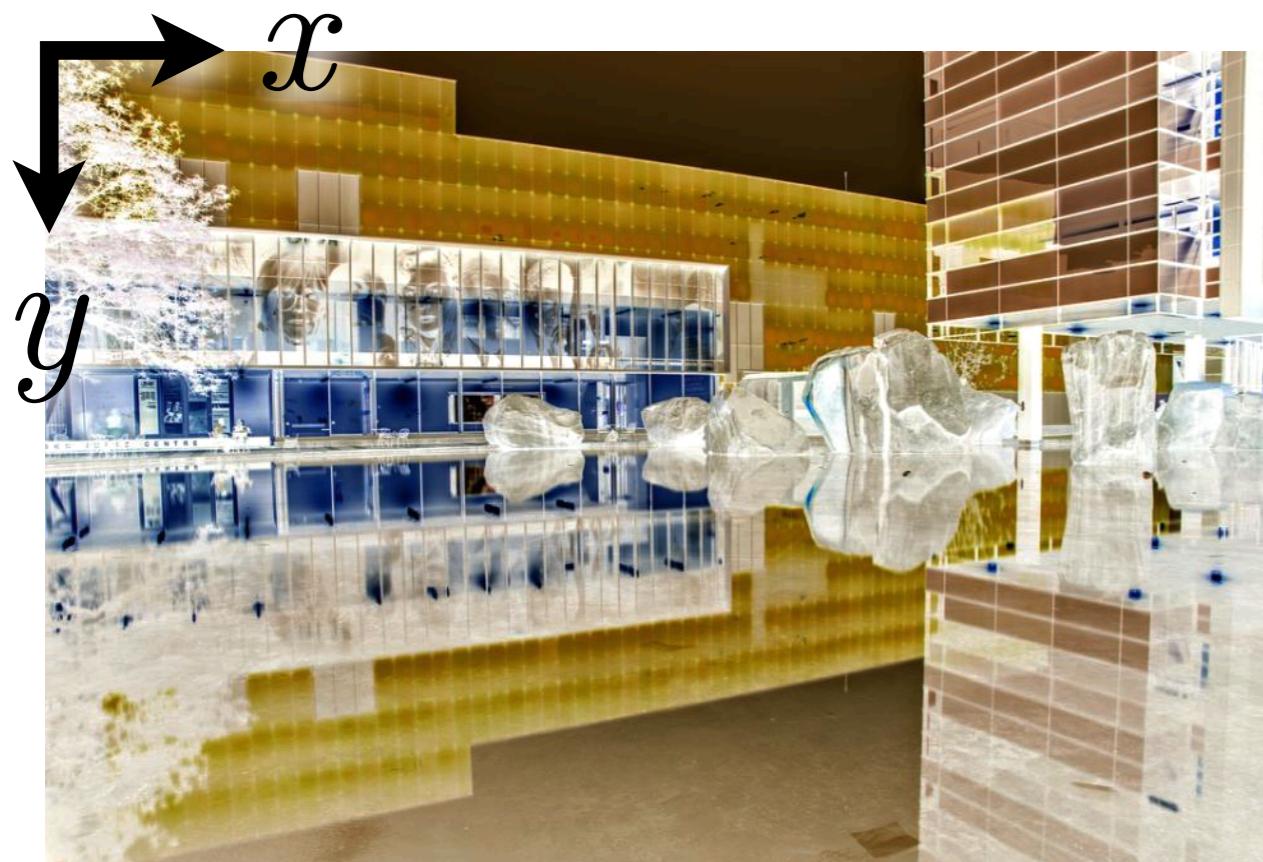
# **FORWARD v INVERSE**

## **WARP**

**practical issues when warping images**

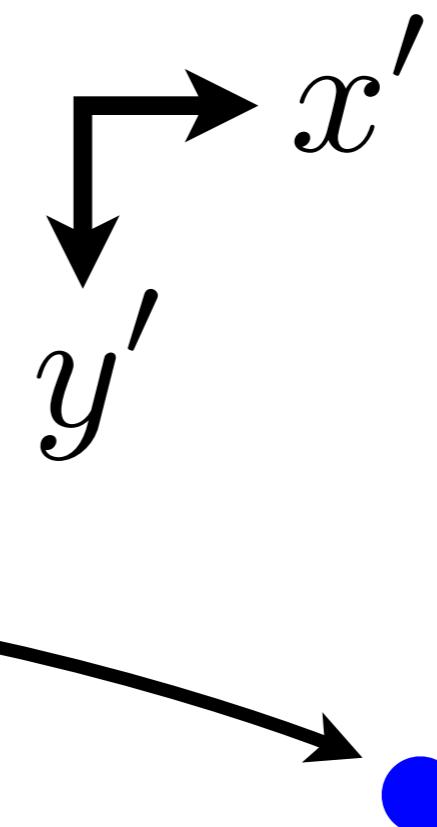
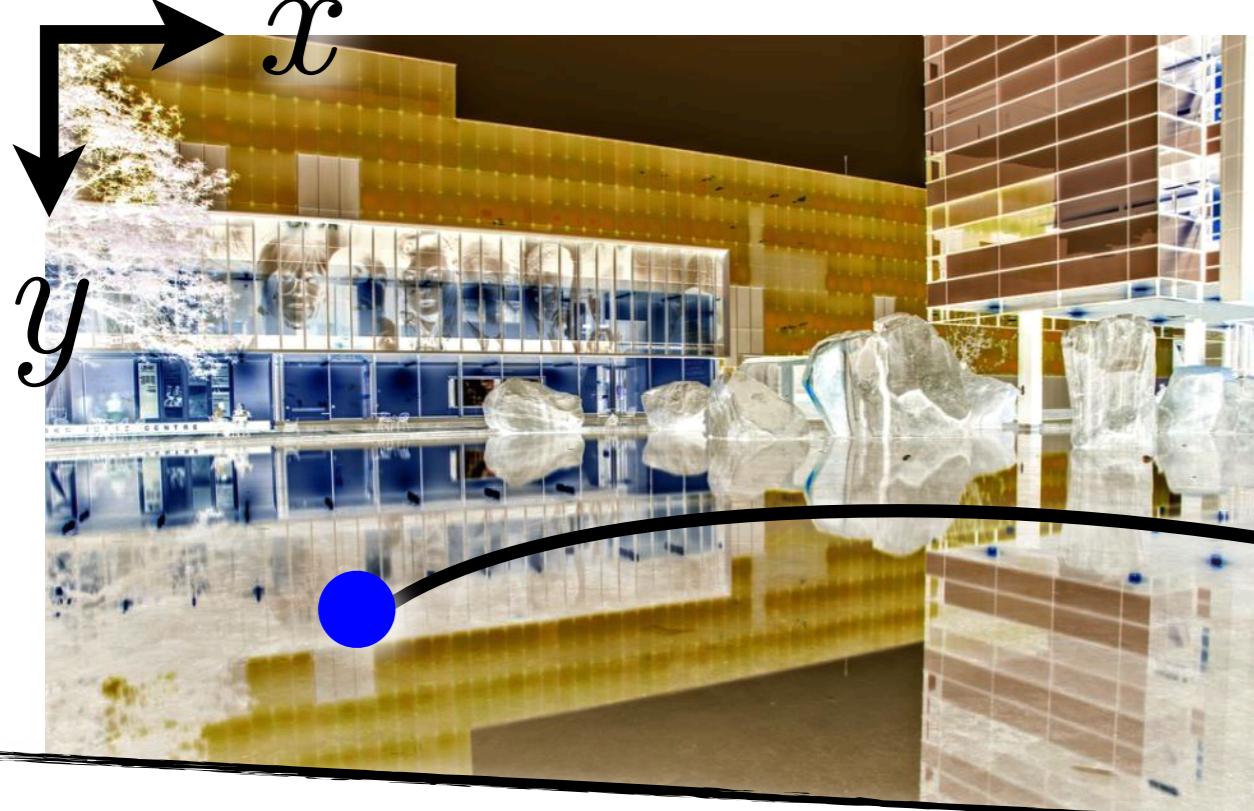
forward  
warp

$$\mathbf{x}' = \mathbf{T}\mathbf{x}$$

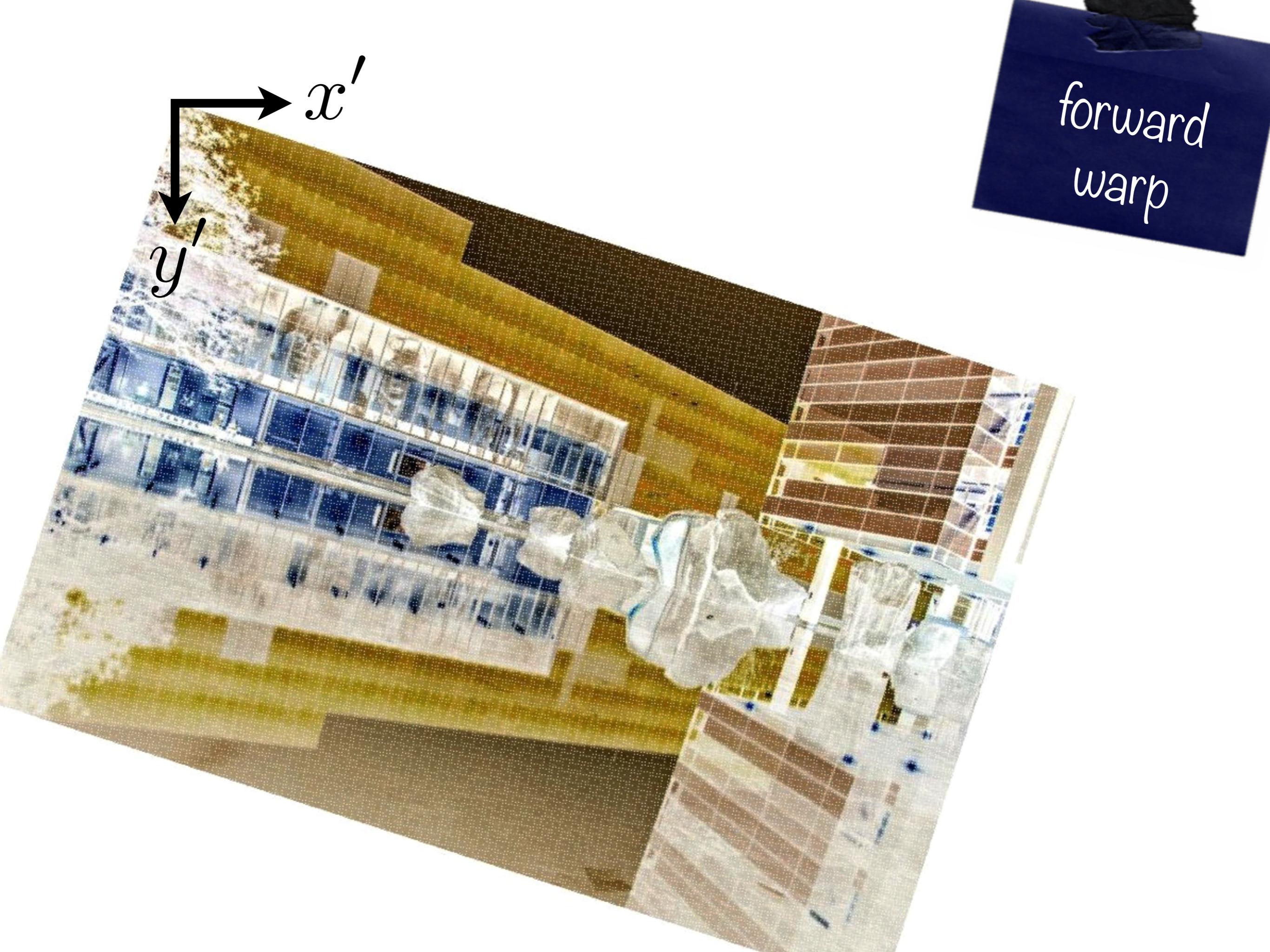


forward  
warp

$$\mathbf{x}' = \mathbf{T}\mathbf{x}$$



copy pixel value from input to mapped output



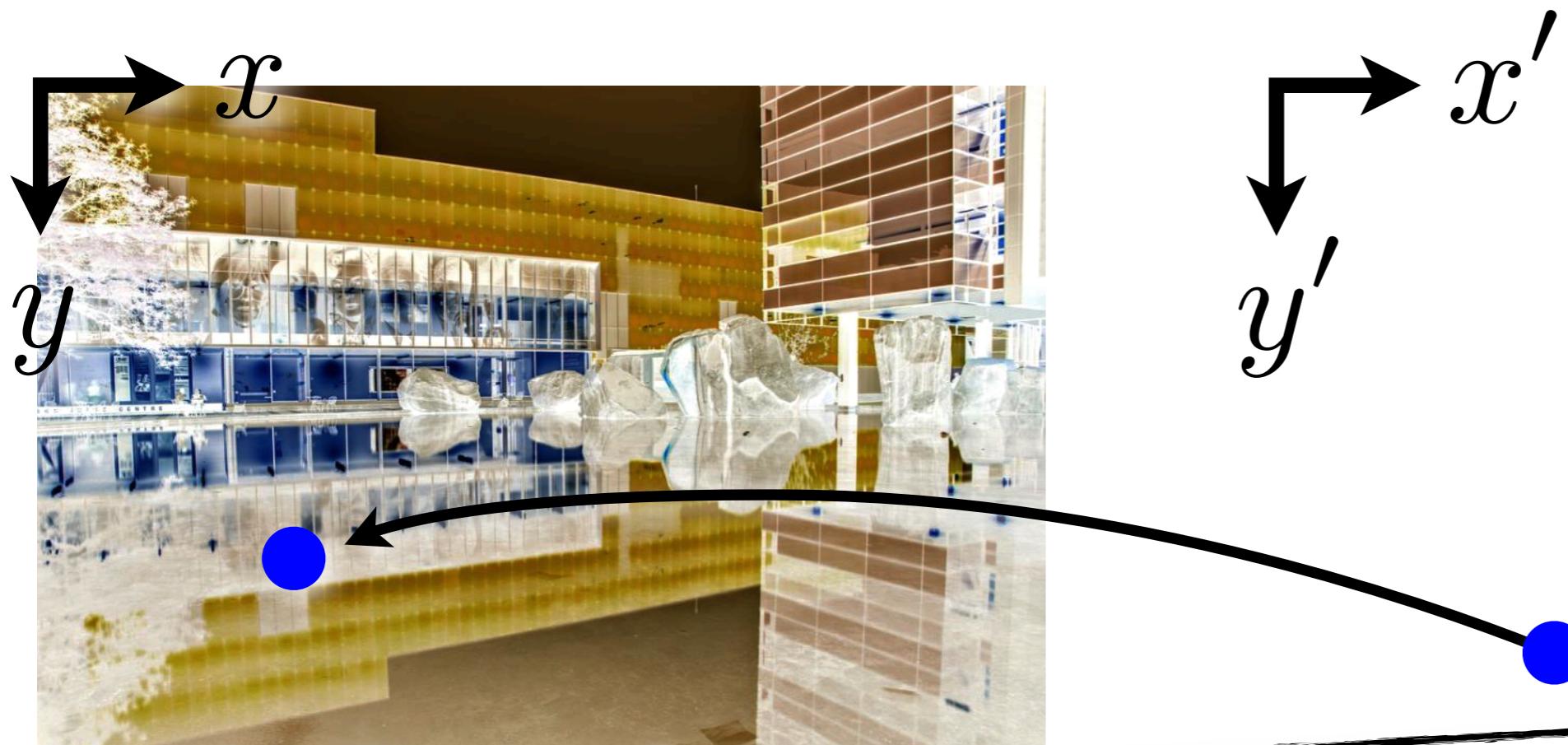
$x'$

$y'$

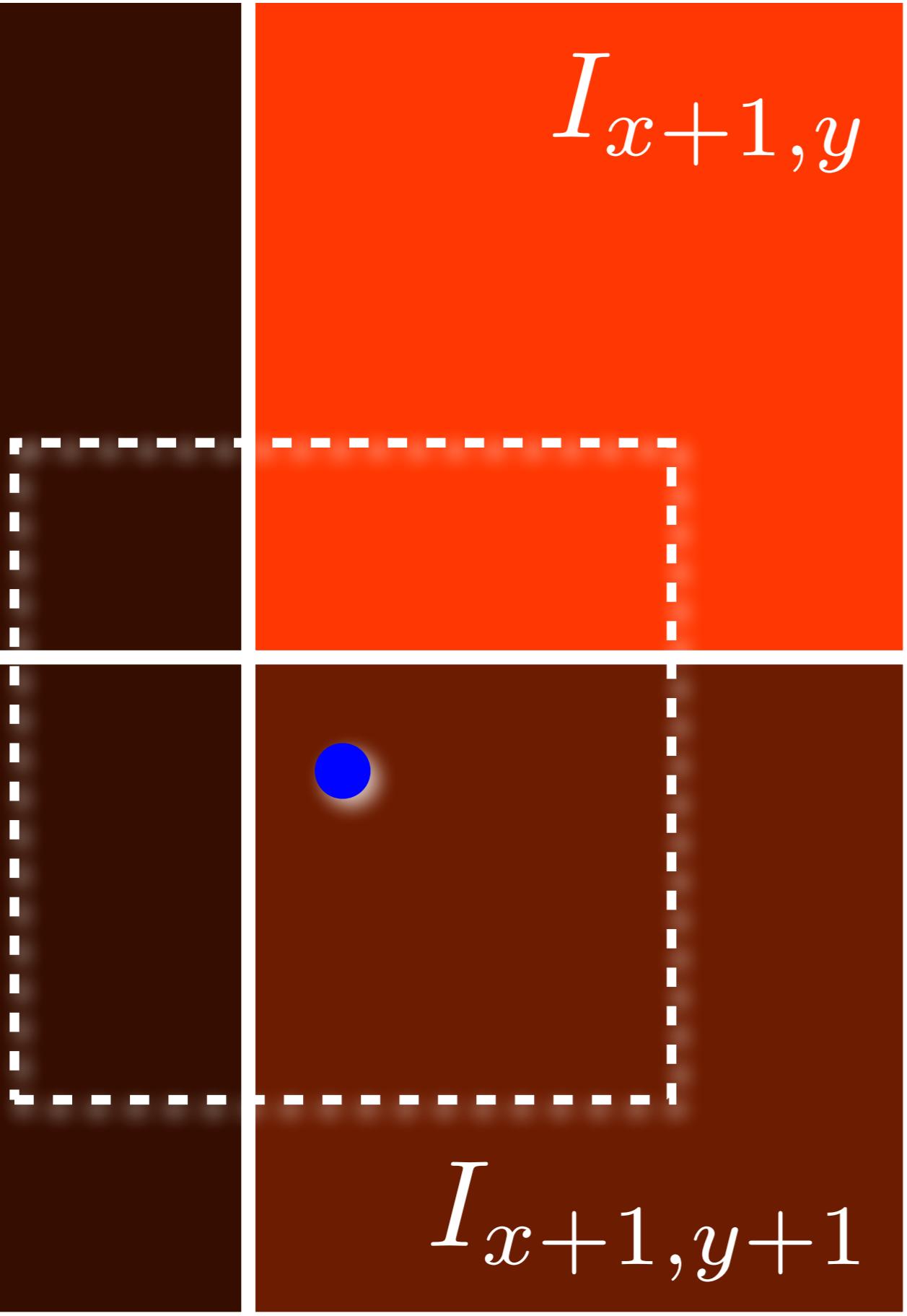
forward  
warp

inverse  
warp

$$\mathbf{x} = \mathbf{T}^{-1} \mathbf{x}'$$



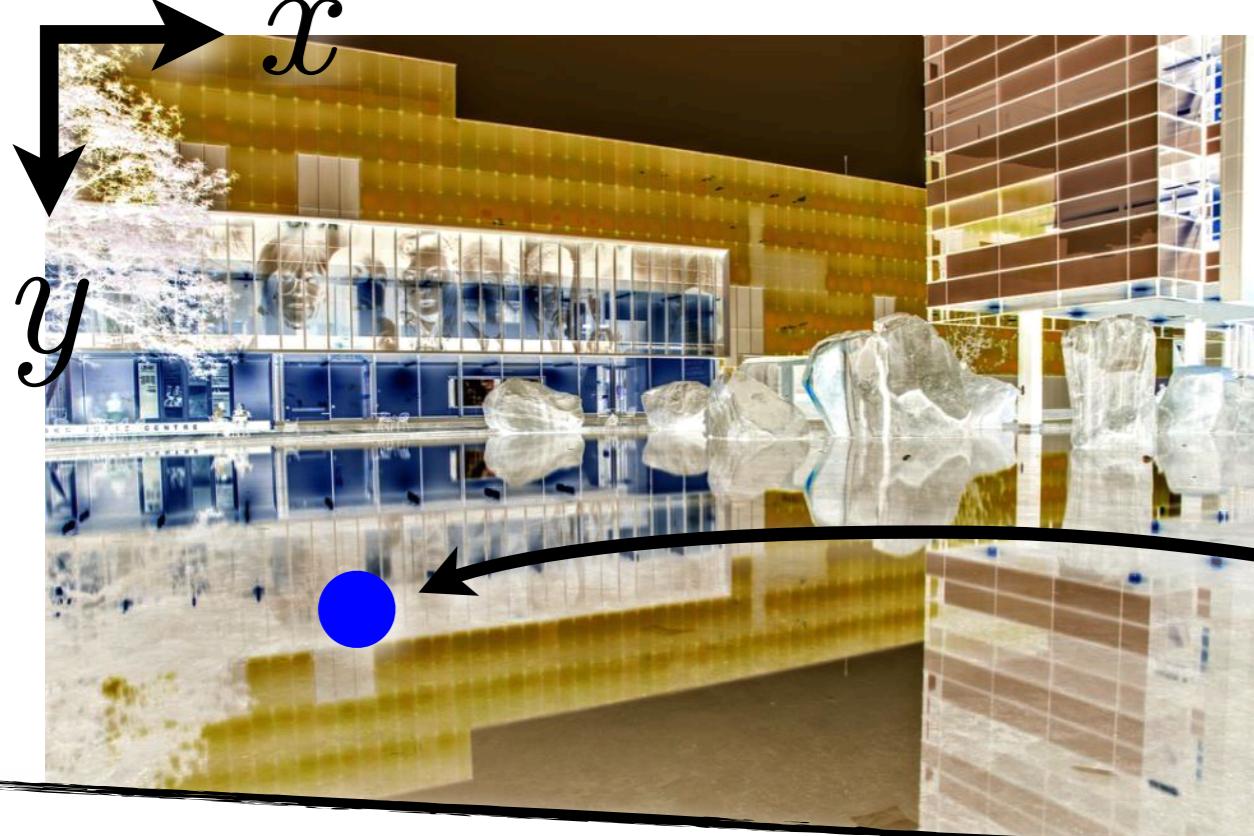
for each output pixel, map to input and “copy” value

 $I_{x+1,y}$  $I_{x+1,y+1}$ 

**Take weighted average of  
colours of closest pixels**

inverse  
warp

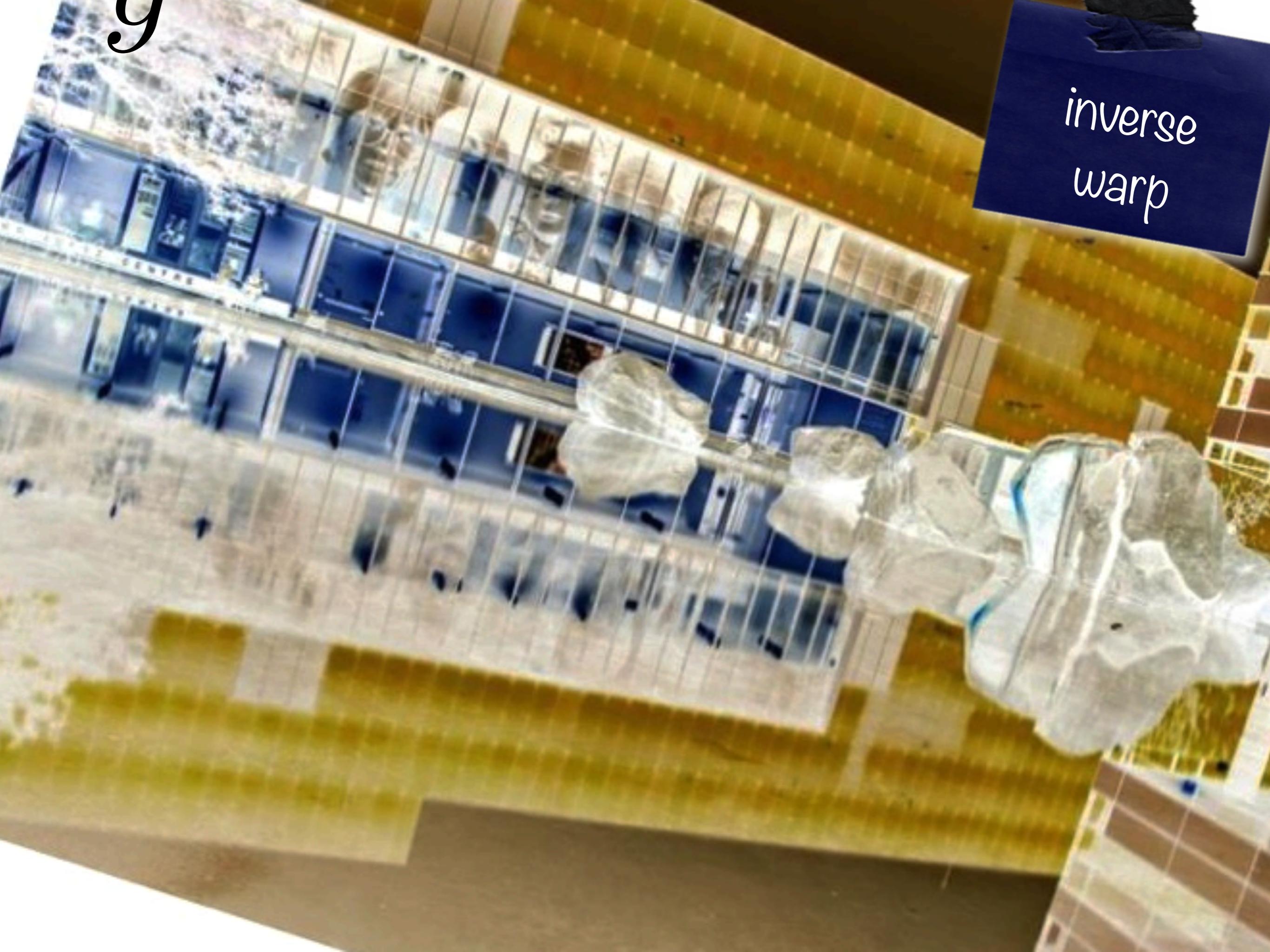
$$\mathbf{x} = \mathbf{T}^{-1} \mathbf{x}'$$



$x'$   
 $y'$

**assumes transformation is invertible**

*inverse  
warp*



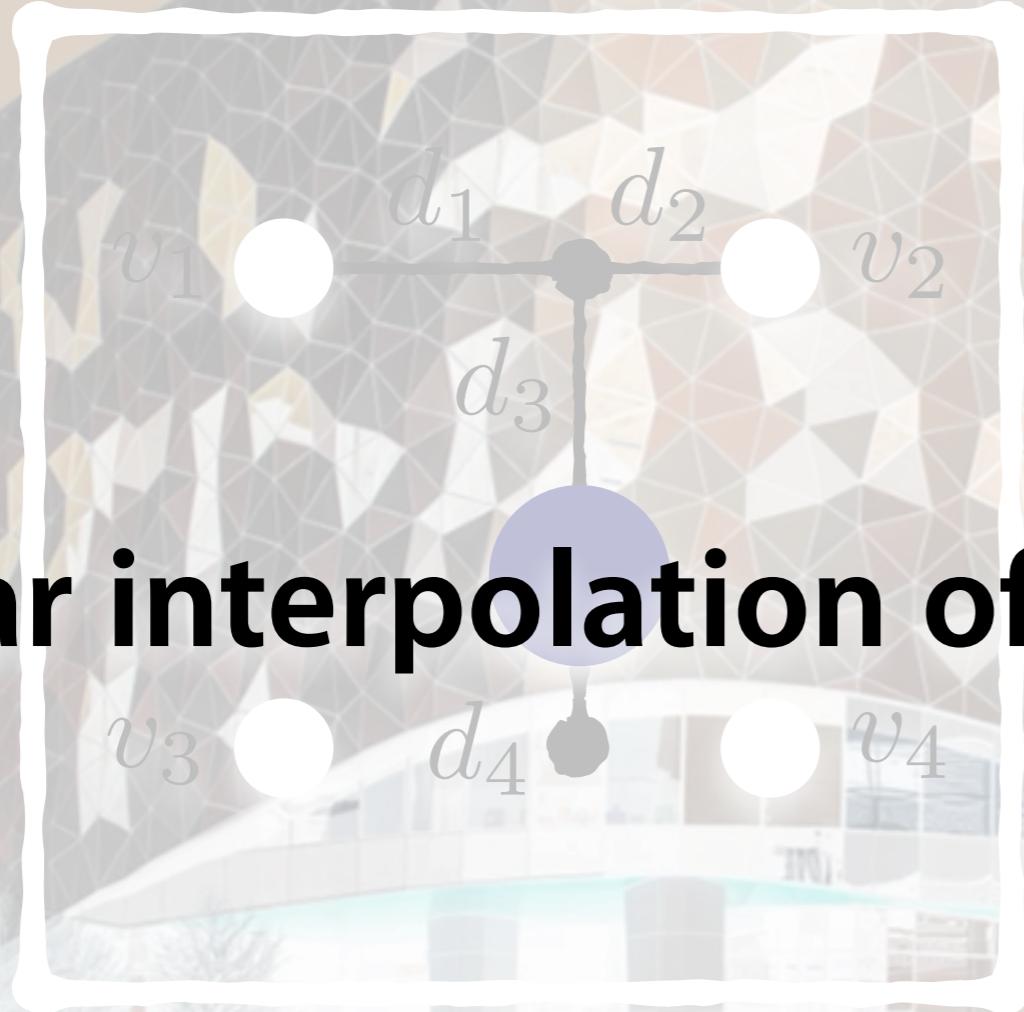


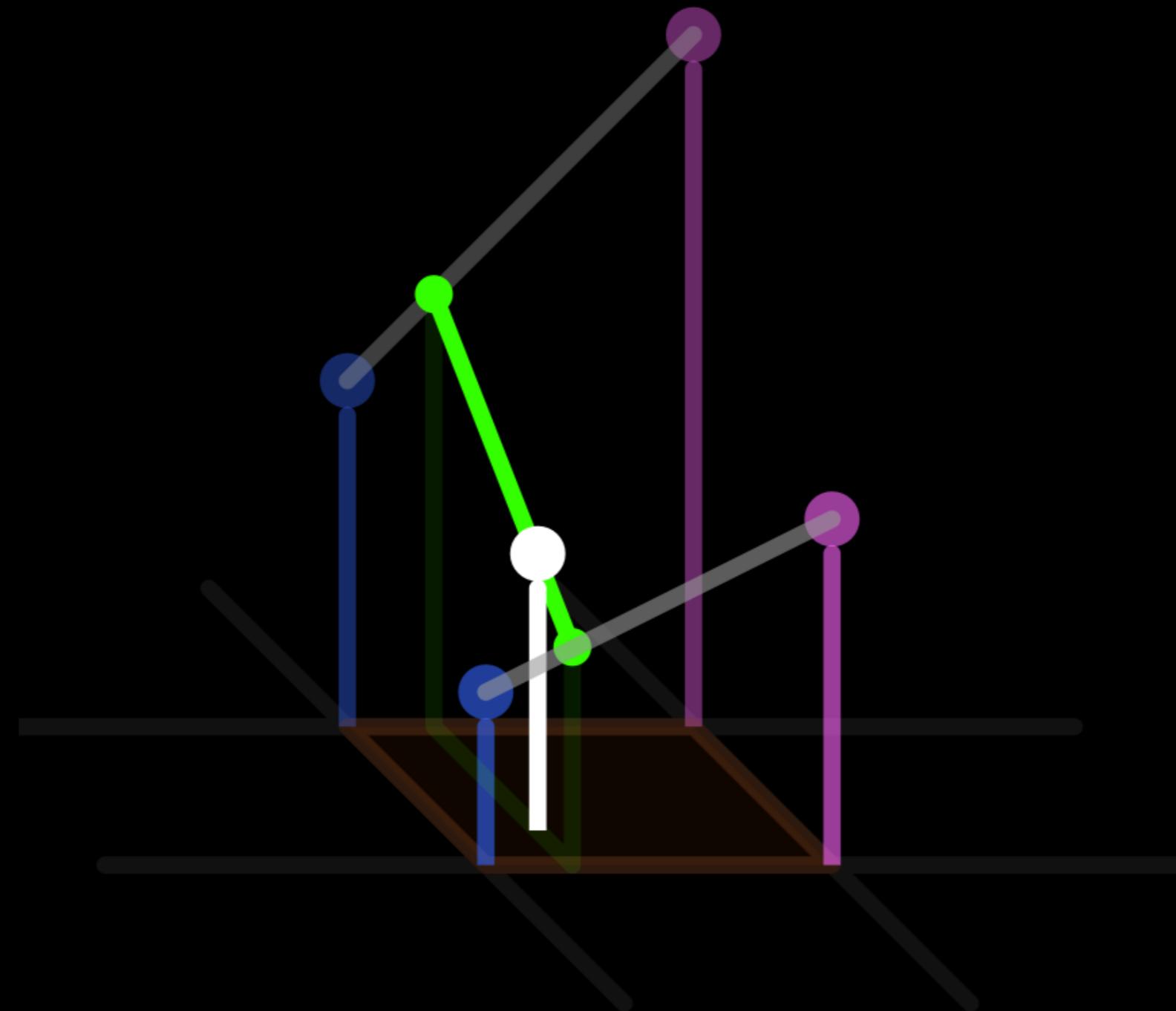
$$q_1 = v_1 d_2 + v_2 d_1$$

$$q_2 = v_3 d_2 + v_4 d_1$$

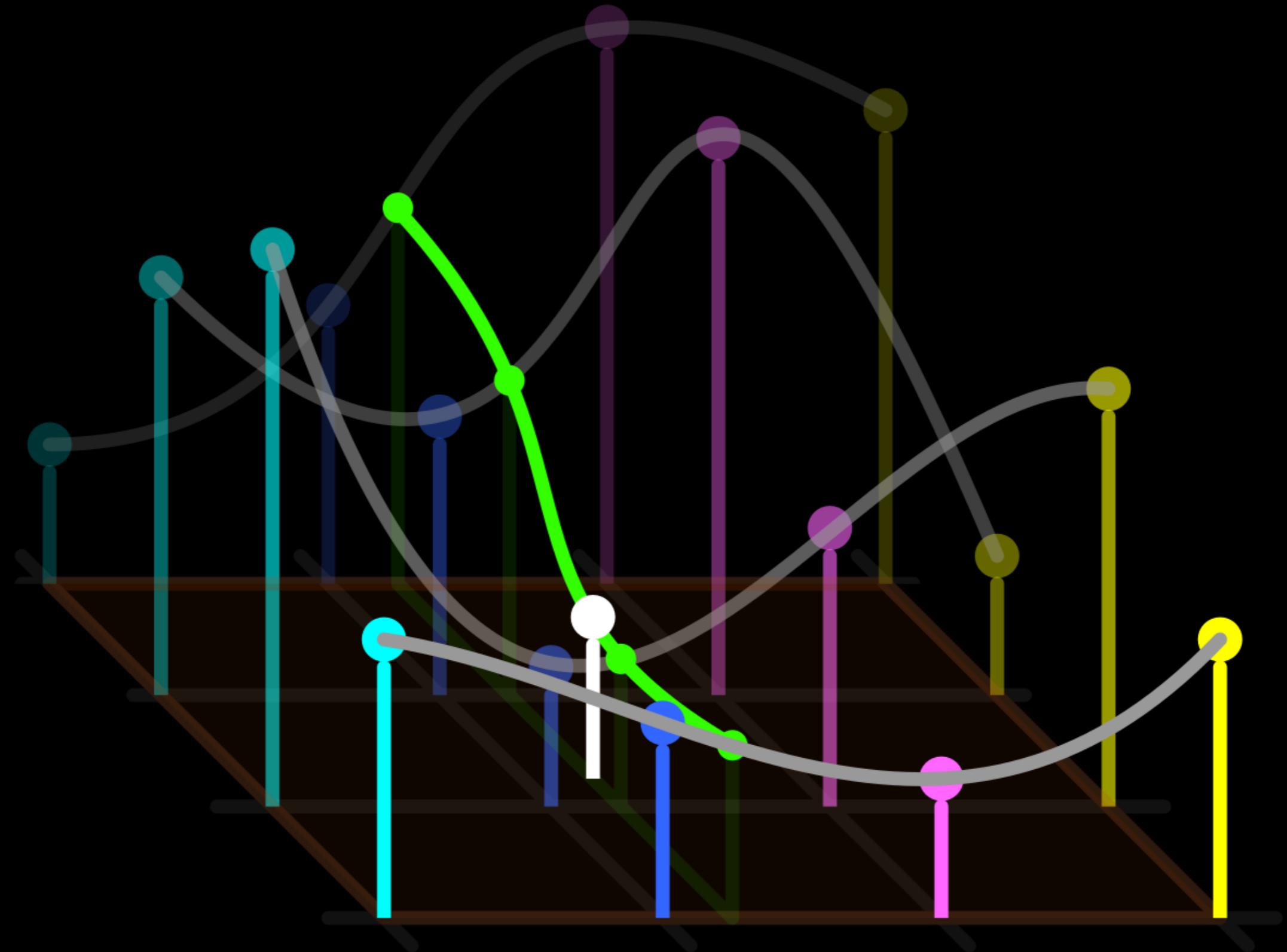
$$q = q_1 d_4 + q_2 d_3$$

# linear interpolation of linear interpolations





bilinear interpolation



bicubic interpolation

planar  
rectification



```
img = imread('ghost-building-original.jpg');

imshow(img)
[X Y] = ginput(4);

X2 = [X(1:2); X(1:2)];
Y2 = [Y(1); Y(1); Y(3) + 50; Y(3) + 50];

tform = maketform('projective', [X Y], [X2 Y2]);
img_out = imtransform(img, tform, 'bicubic');

imshow(img_out)
```