Intro to

Computer Vision

with Prof. Kosta Derpanis

Image Transformations

translation

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{I}_{2\times 2} & \mathbf{t}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{pmatrix} \mathbf{x}$$

two degrees of freedom


translation
h
x

translation

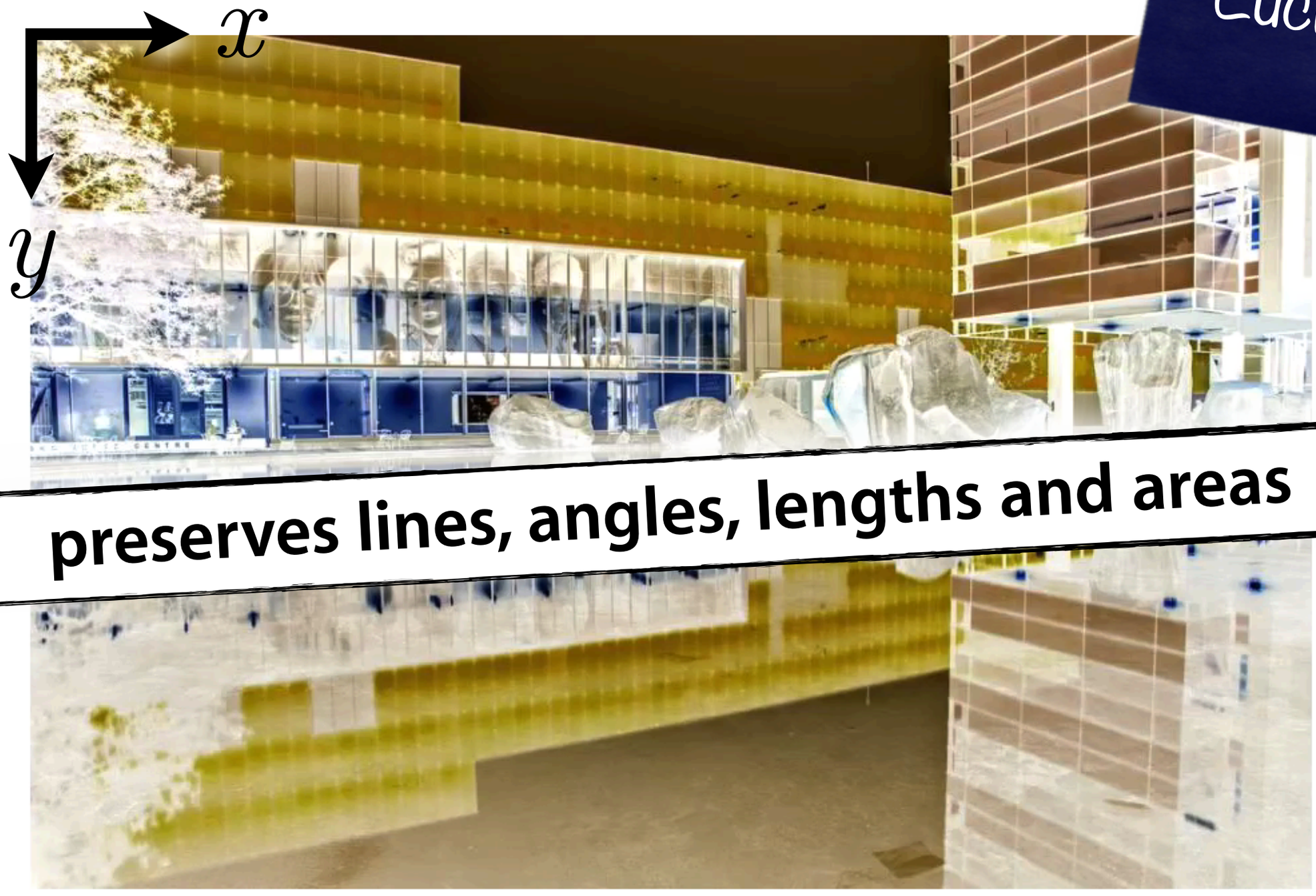
preserves lines, angles, lengths, areas and orientations

Euclidean

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{R}_{2\times 2} & \mathbf{t}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{pmatrix} \mathbf{x}$$

three degrees of freedom



preserves lines, angles, lengths and areas

similarity

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha\cos\theta & -\alpha\sin\theta & t_x \\ \alpha\sin\theta & \alpha\cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha\mathbf{R}_{2\times 2} & \mathbf{t}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{pmatrix} \mathbf{x}$$

four degrees of freedom

similarity

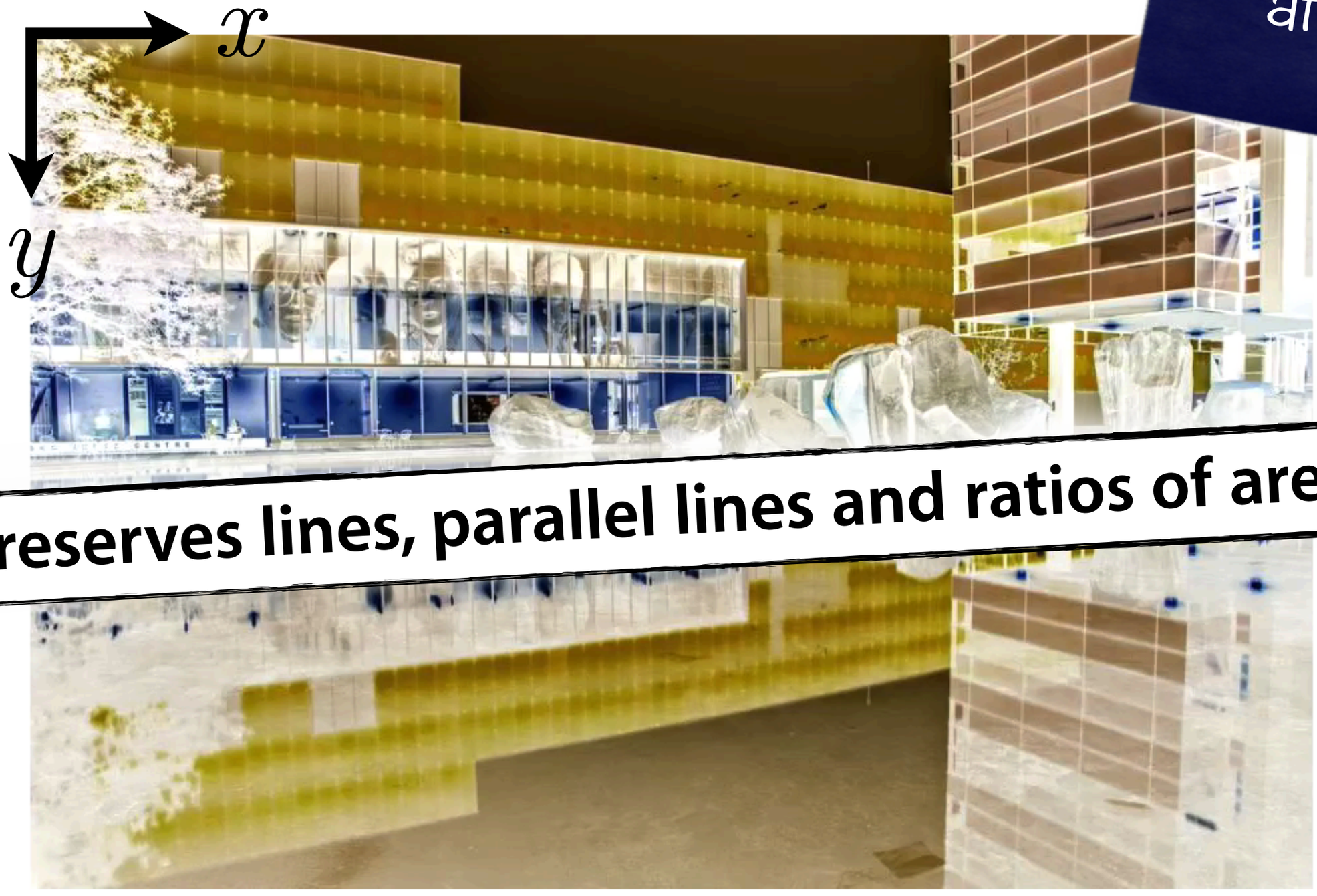
preserves lines, angles and ratios of areas

affine

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

six degrees of freedom

affine



preserves lines, parallel lines and ratios of areas

homography

$$w \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

divide by last component to complete transformation

homography

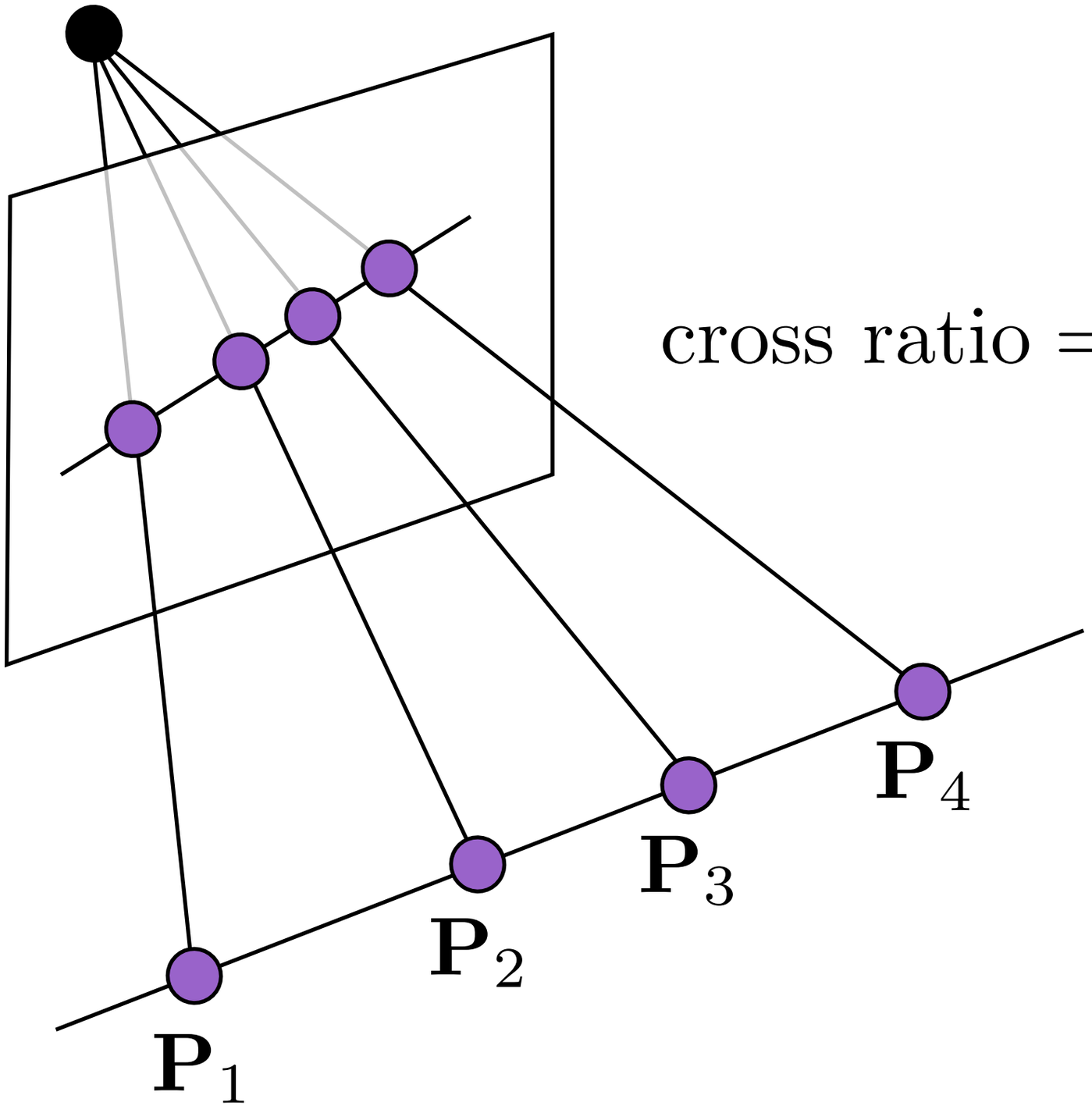
$$w \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

eight degrees of freedom

homography

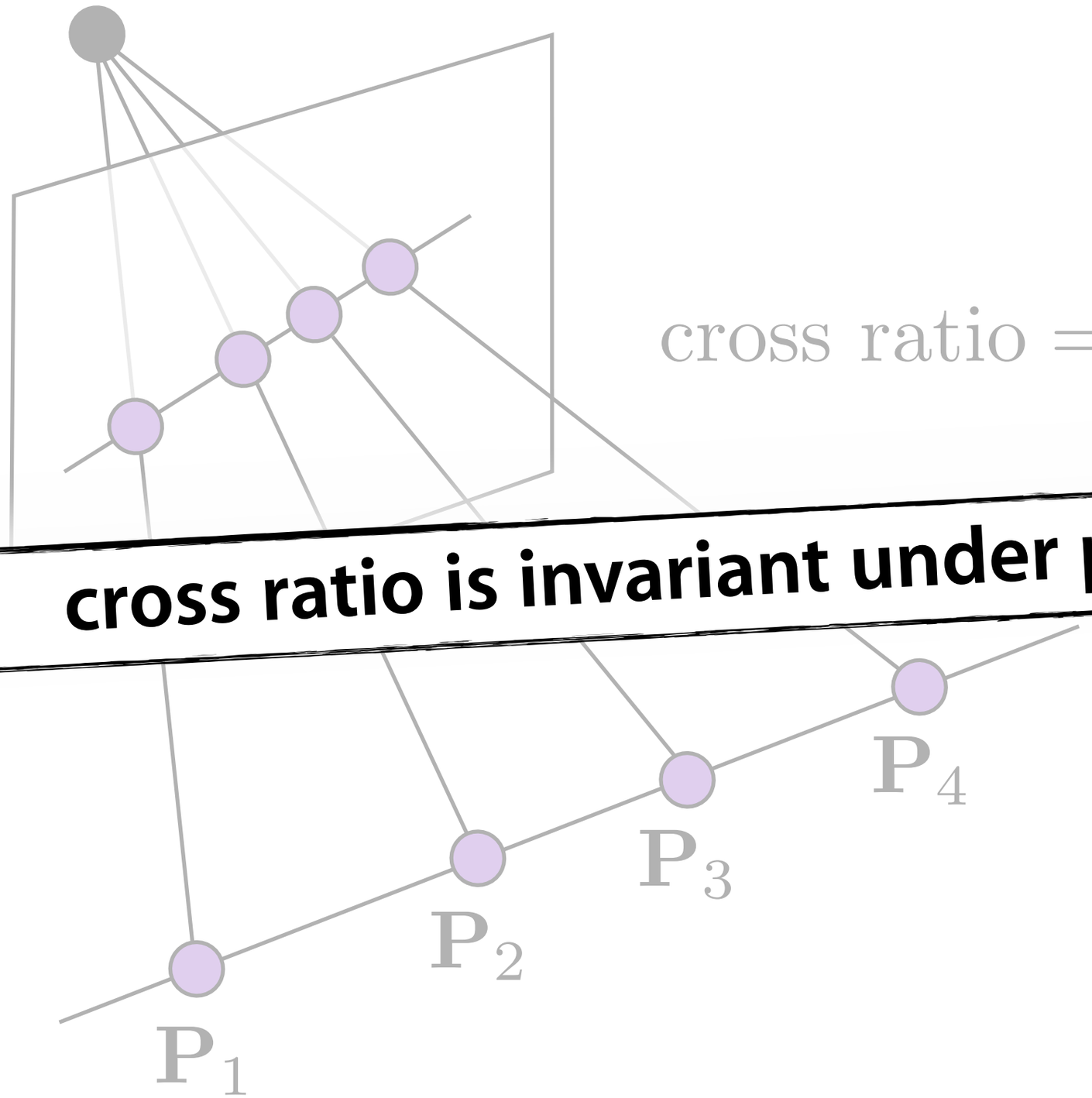
preserves lines and cross ratio

cross ratio



$$\text{cross ratio} = \frac{\|\mathbf{P}_3 - \mathbf{P}_1\|\|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\|\|\mathbf{P}_4 - \mathbf{P}_1\|}$$

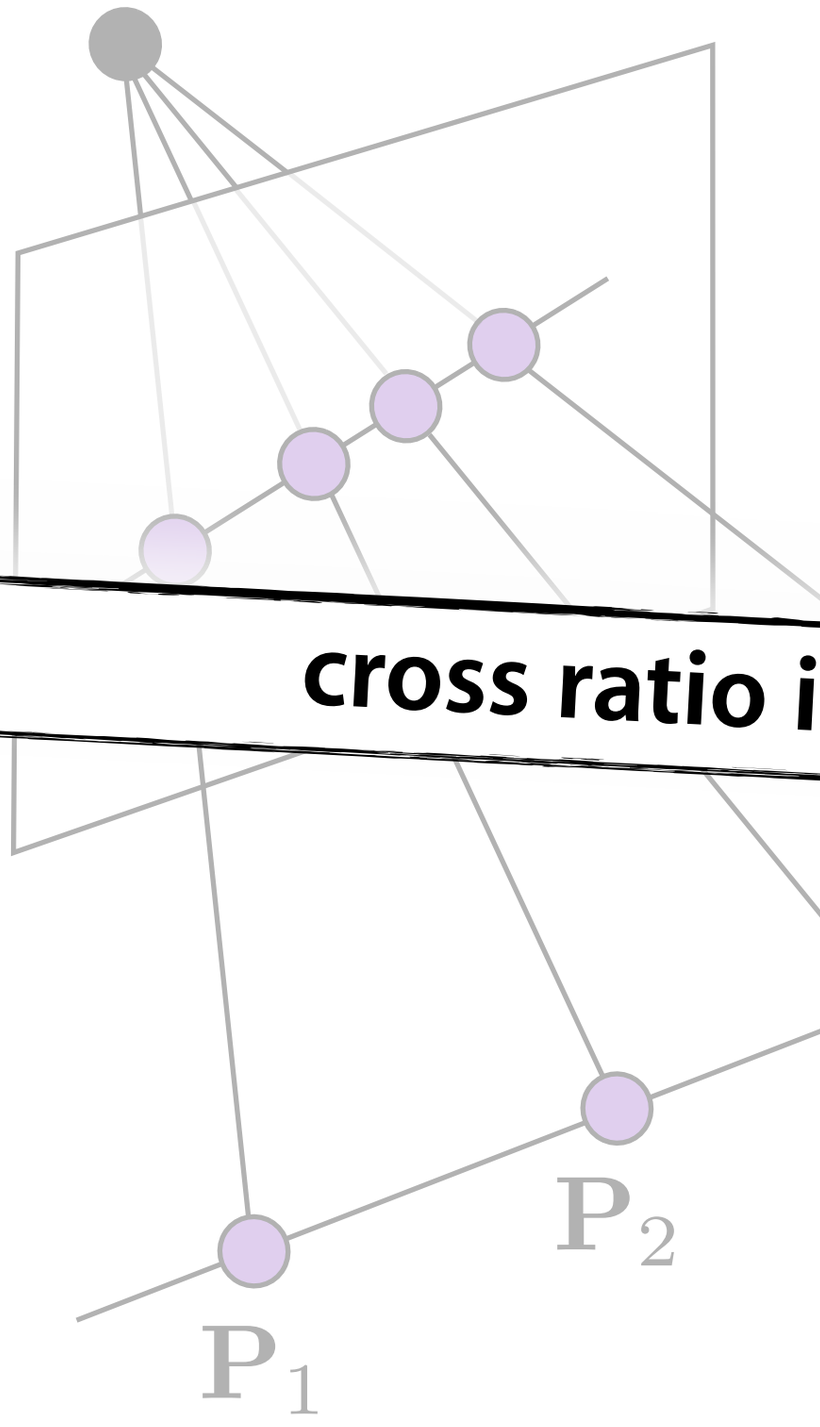
cross ratio



$$\text{cross ratio} = \frac{\|\mathbf{P}_3 - \mathbf{P}_1\|\|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\|\|\mathbf{P}_4 - \mathbf{P}_1\|}$$

cross ratio is invariant under perspective projection

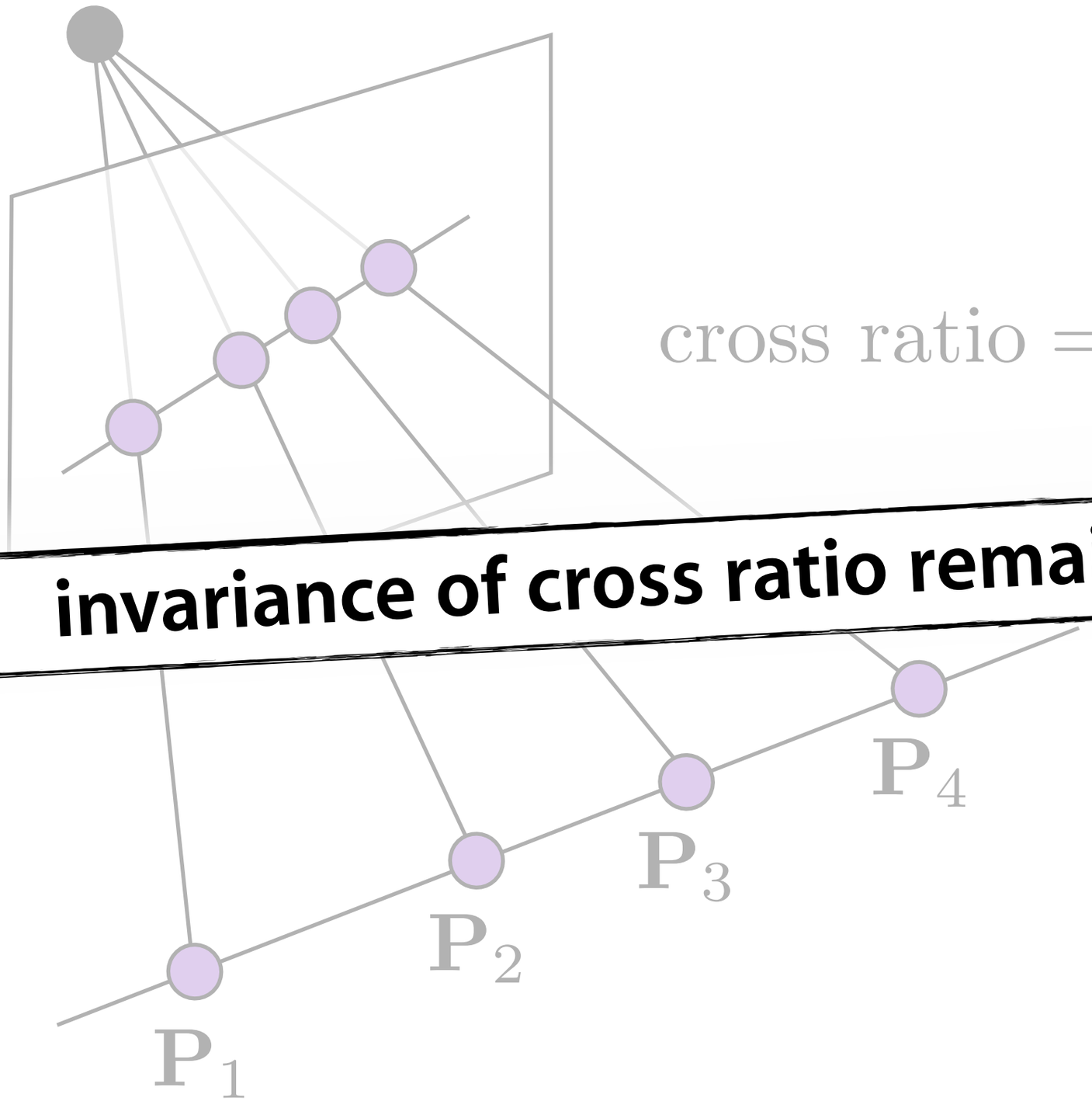
cross ratio



$$\text{cross ratio} = \frac{\|\mathbf{P}_3 - \mathbf{P}_1\|\|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\|\|\mathbf{P}_4 - \mathbf{P}_1\|}$$

cross ratio is invariant under homography

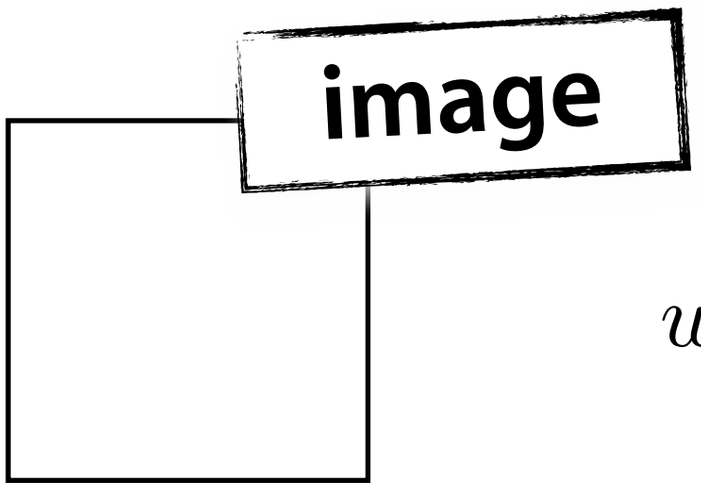
cross ratio



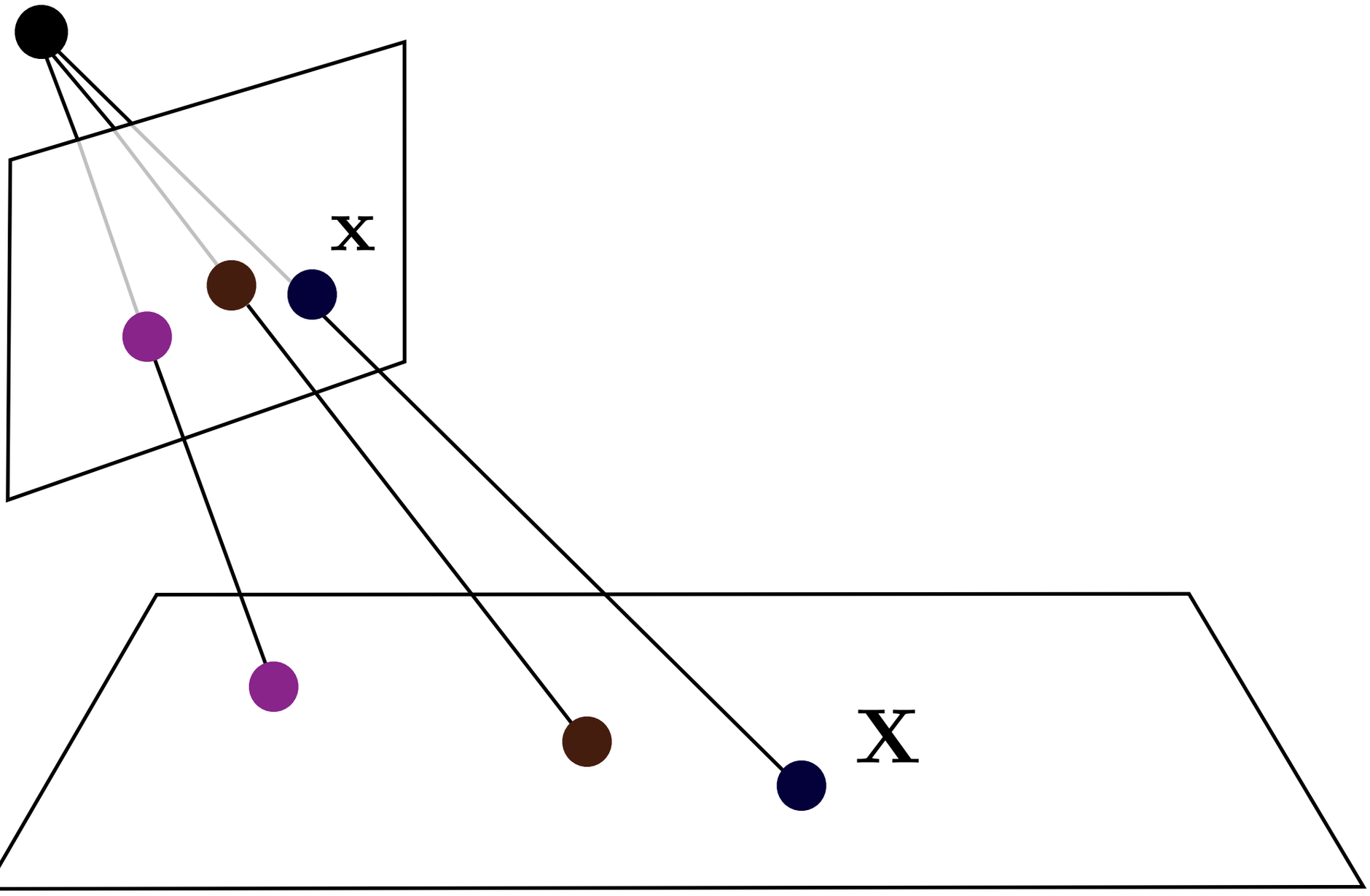
$$\text{cross ratio} = \frac{\|\mathbf{P}_3 - \mathbf{P}_1\|\|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\|\|\mathbf{P}_4 - \mathbf{P}_1\|}$$

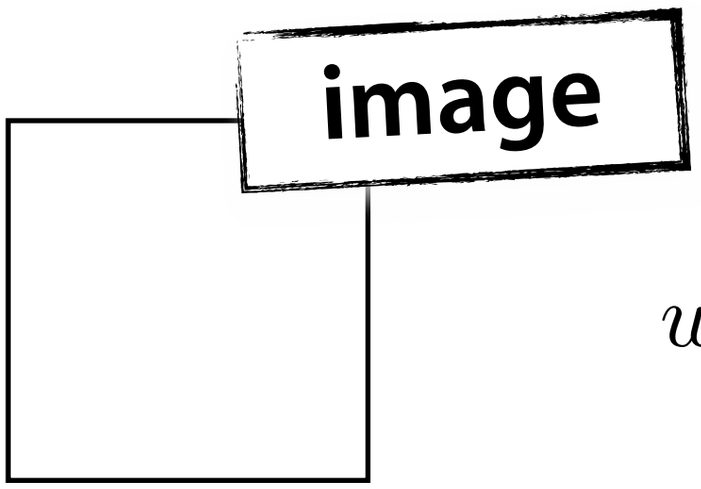
invariance of cross ratio remains under permutation

transformations are nested

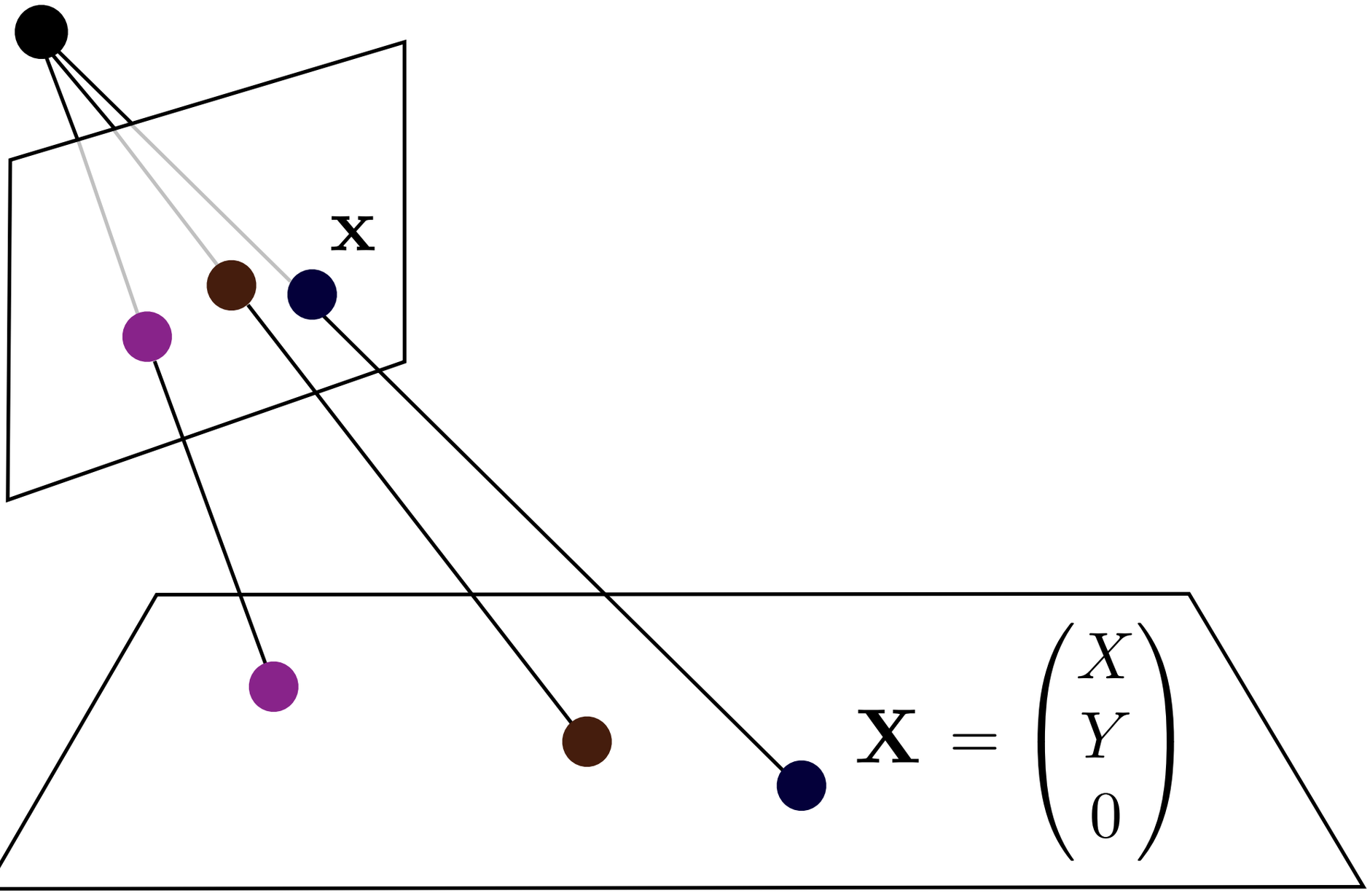


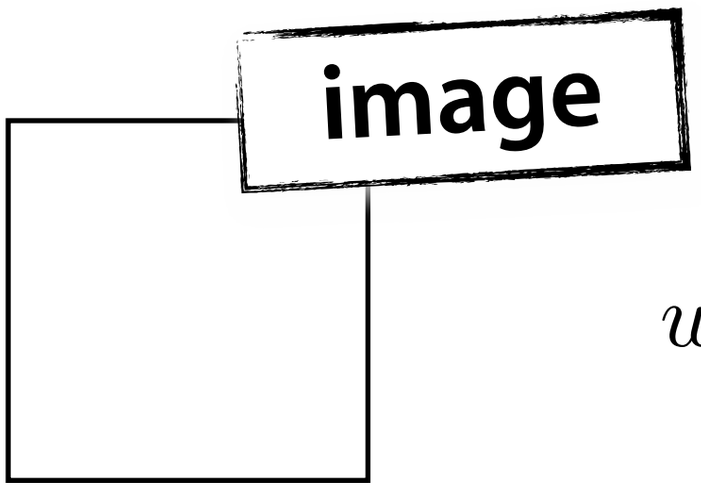
$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \mathbf{M}_{\text{int}} \ \mathbf{M}_{\text{ext}} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



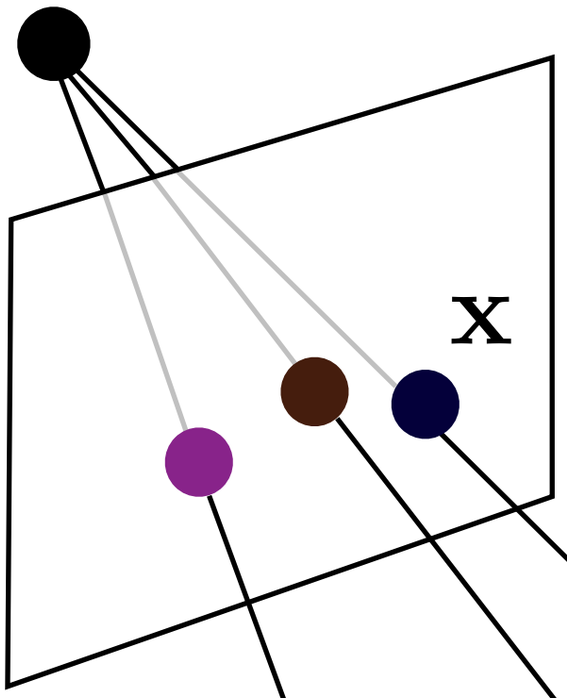


$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



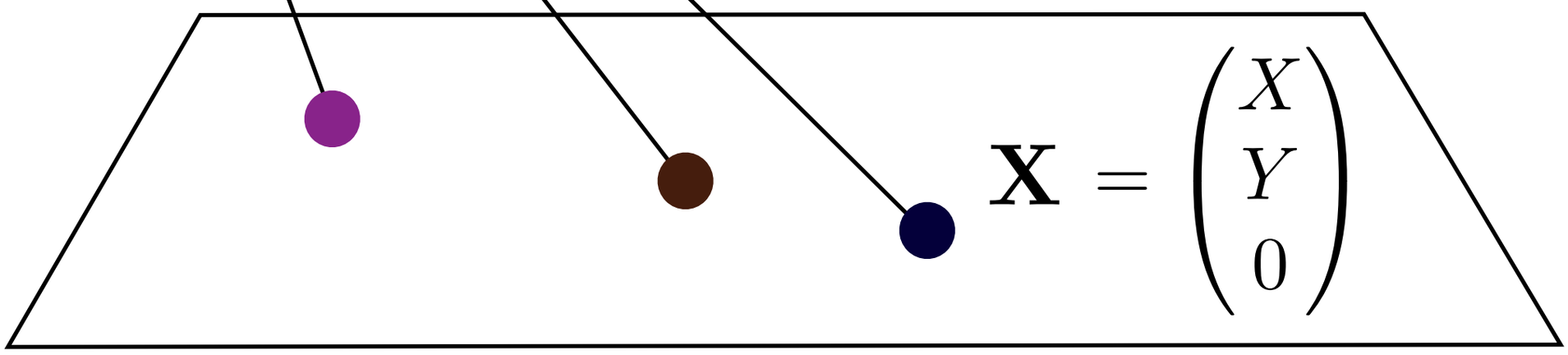


$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix} \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix}$$

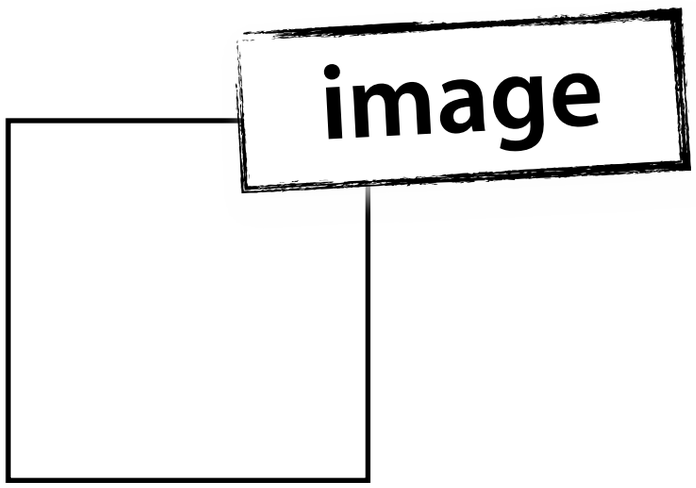


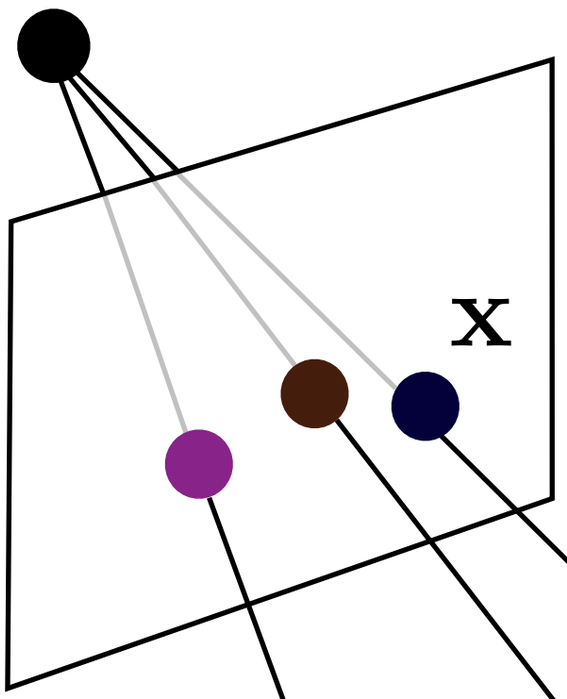
$$= \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

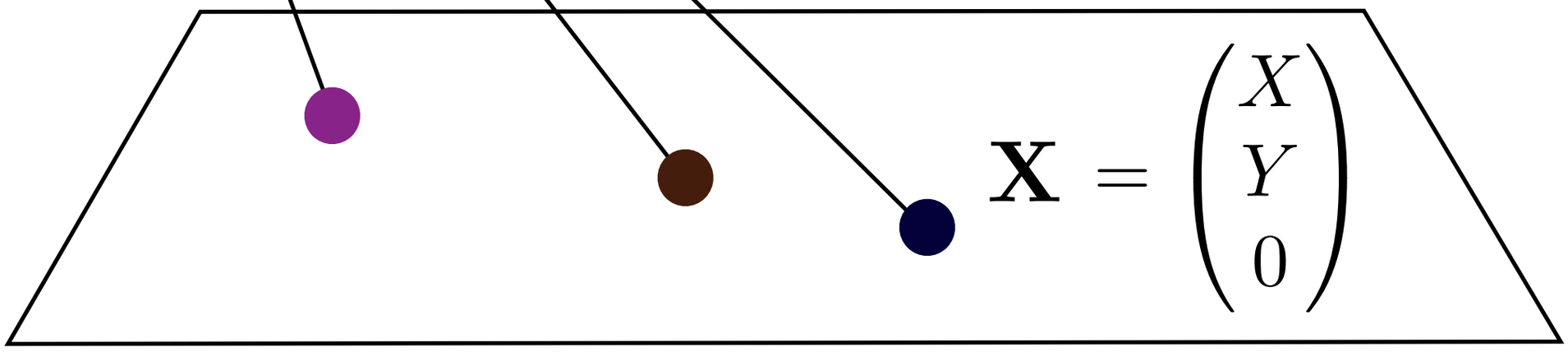


$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

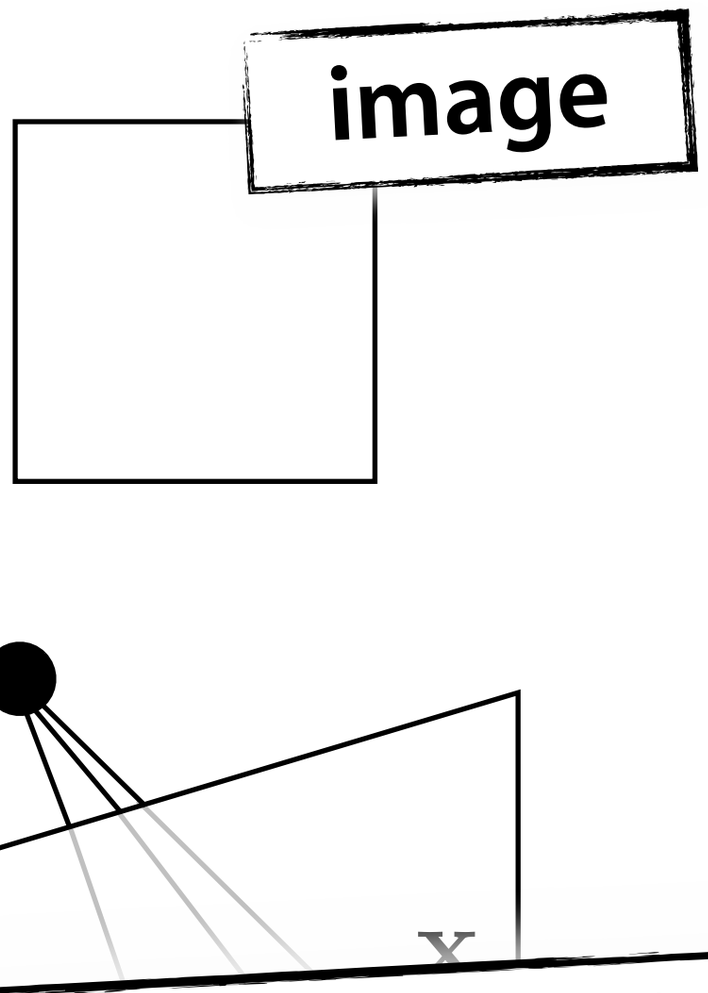




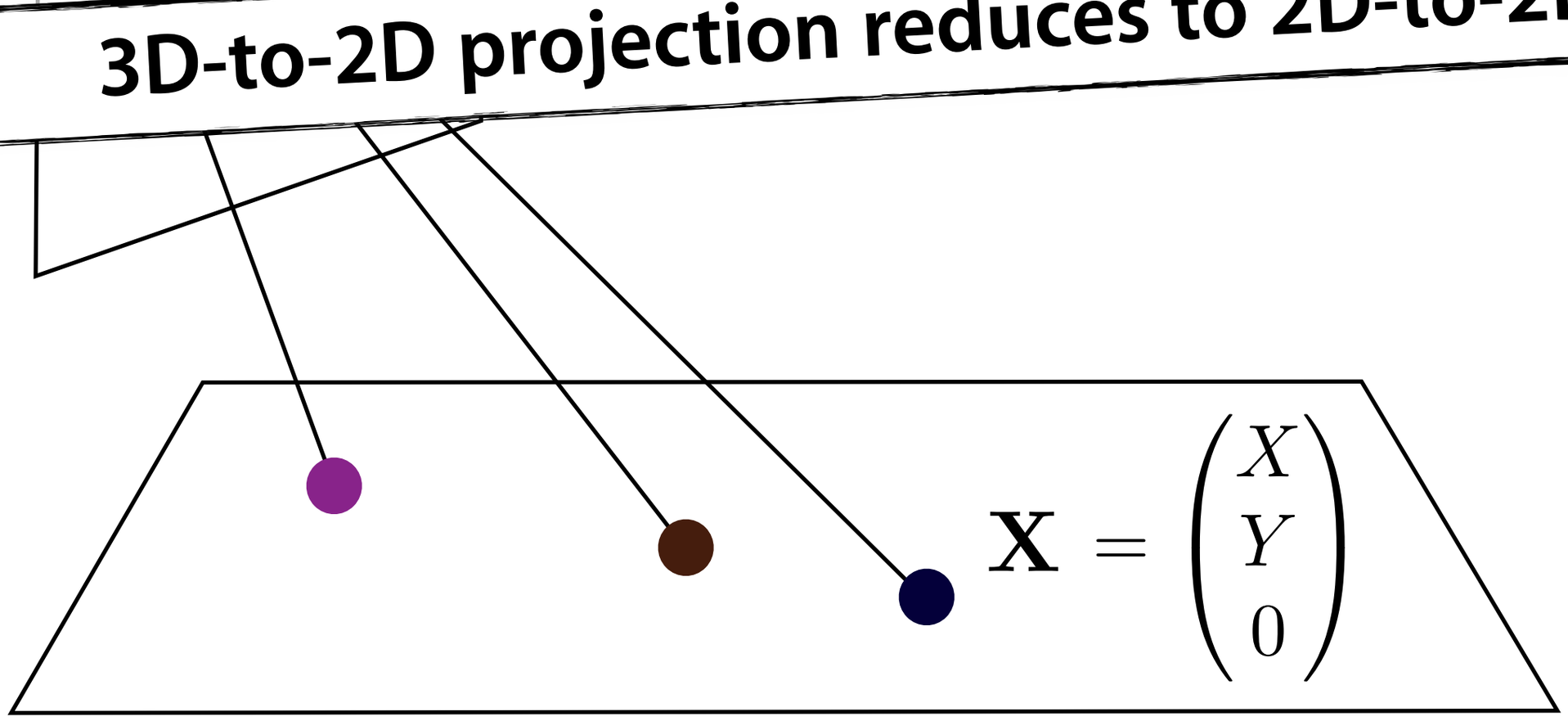
$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$



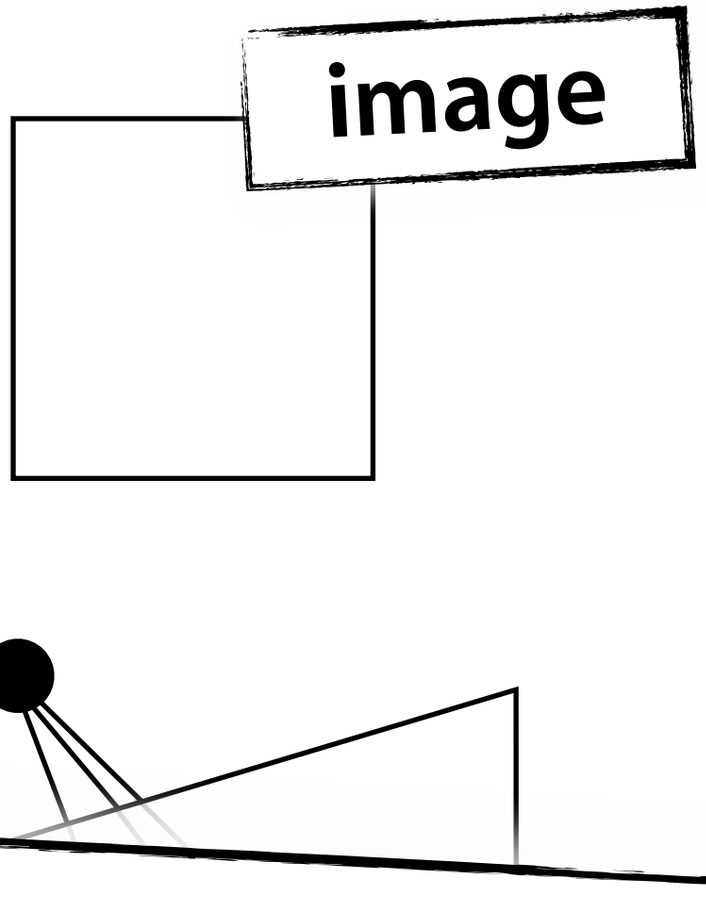
$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$



$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$



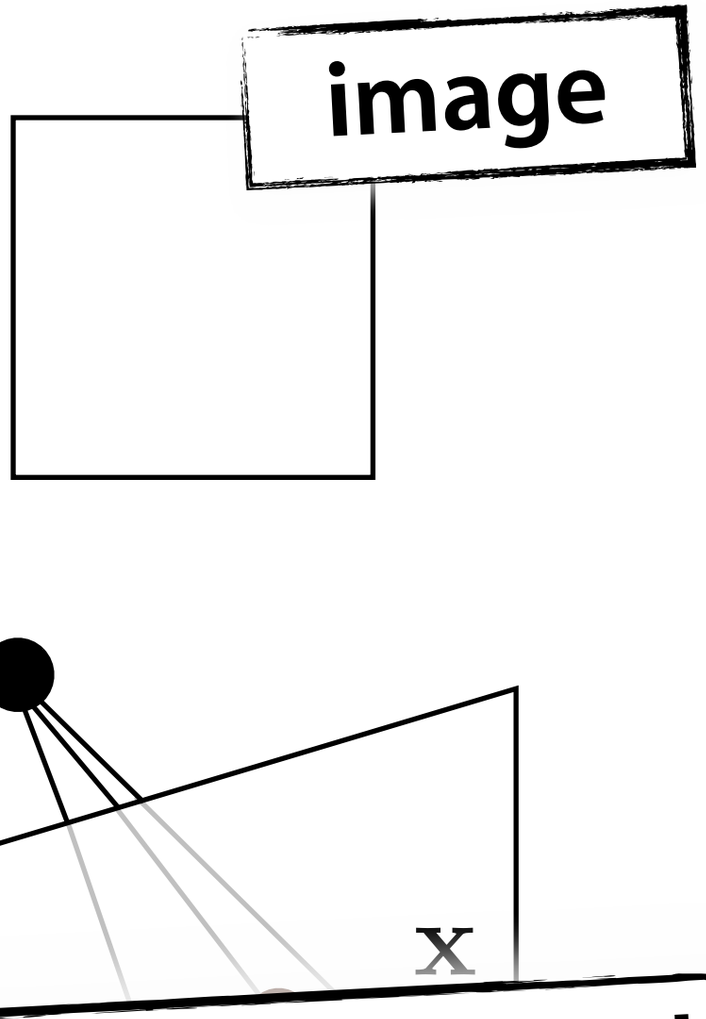
3D-to-2D projection reduces to 2D-to-2D mapping



$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

transformation is called a homography

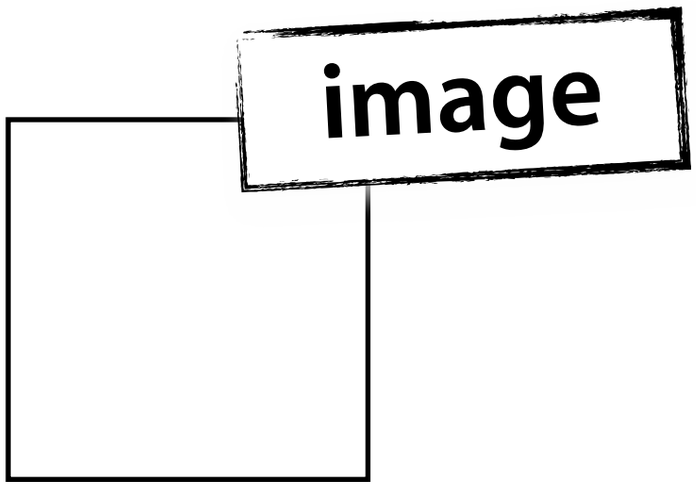
$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$



$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

homography is an invertible transformation

$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

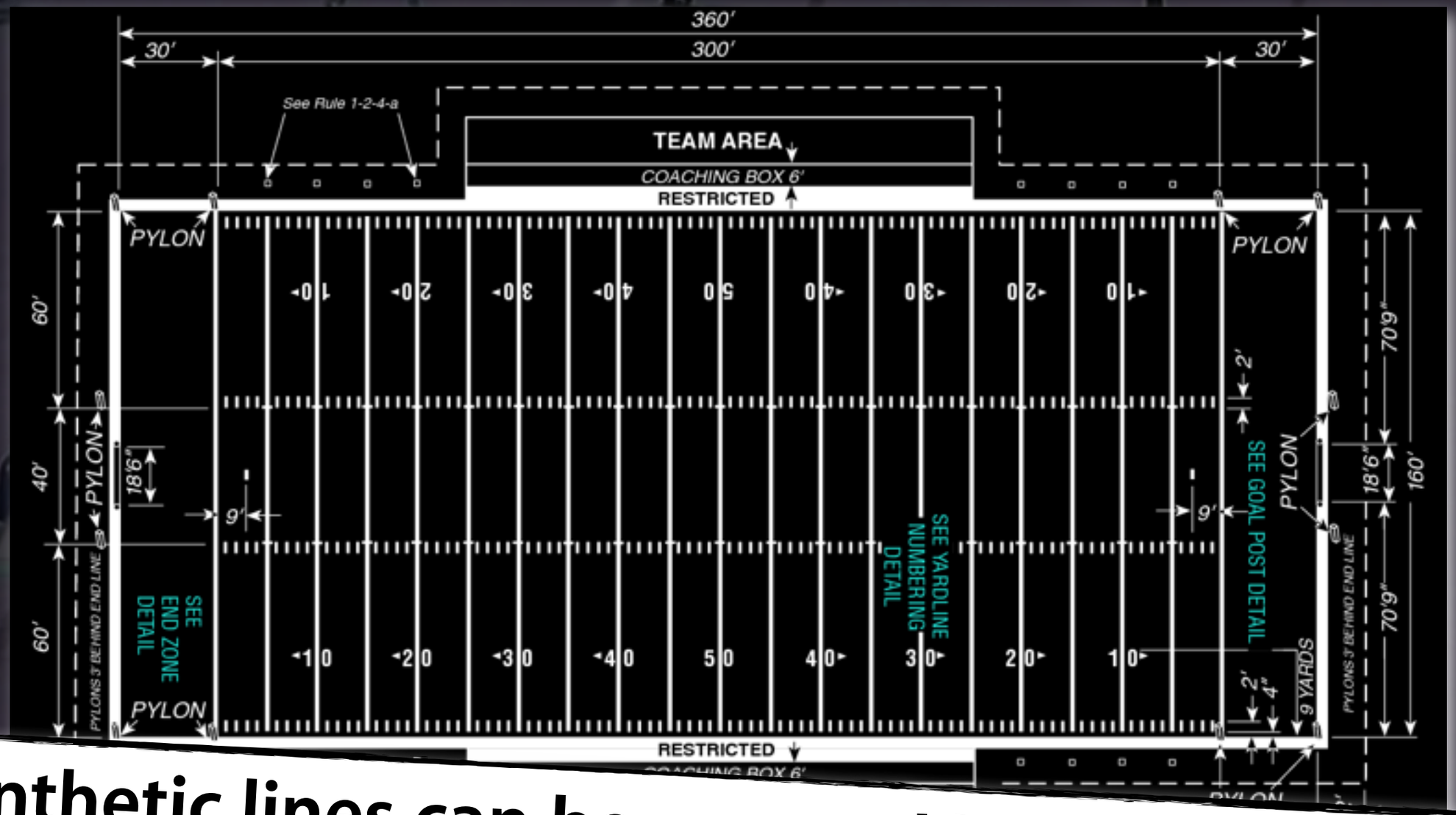


$$w \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

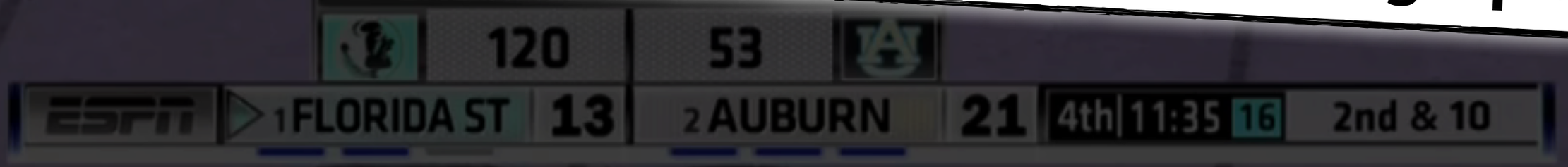
homography defined up to a scalar multiple

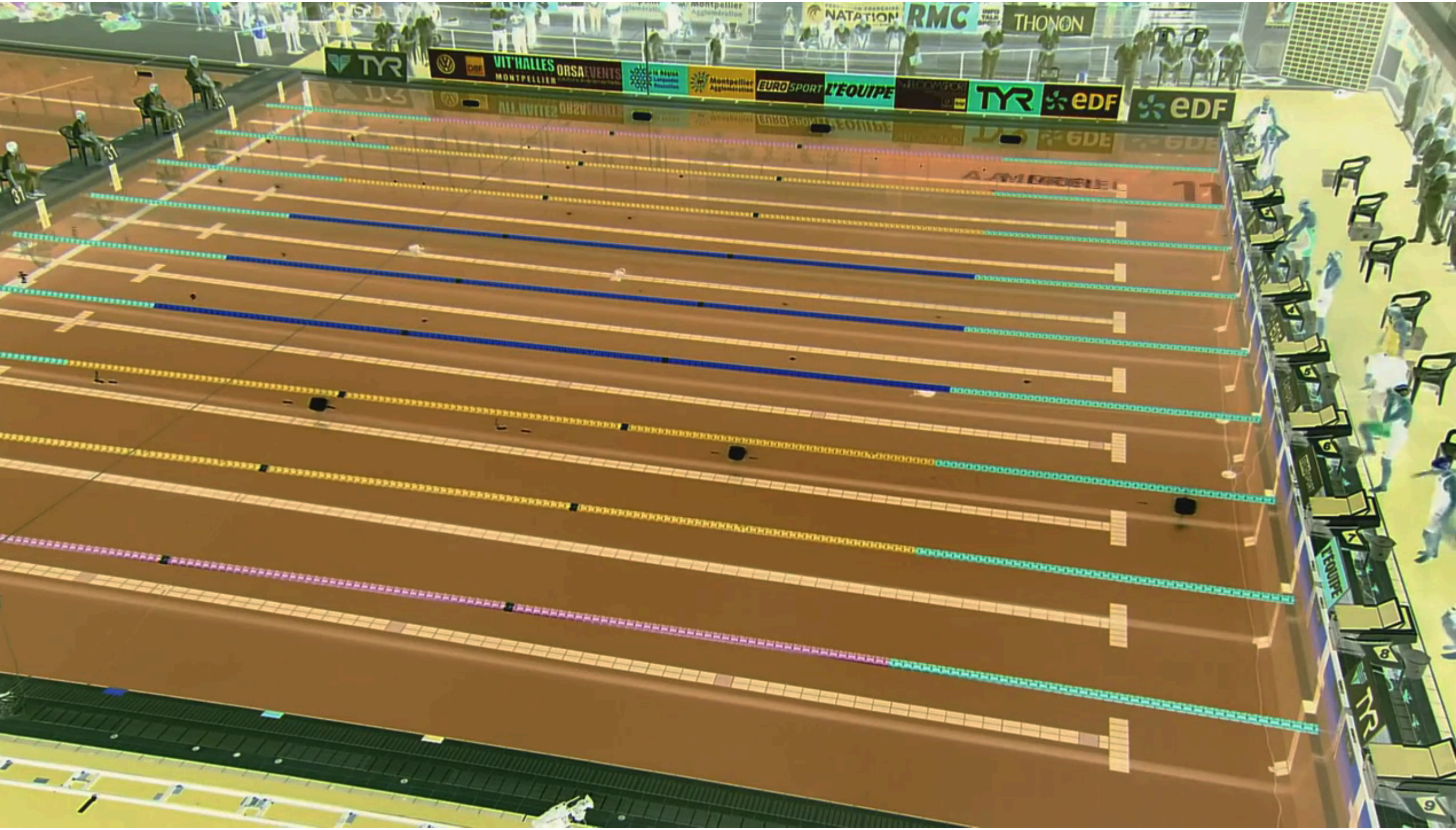
$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

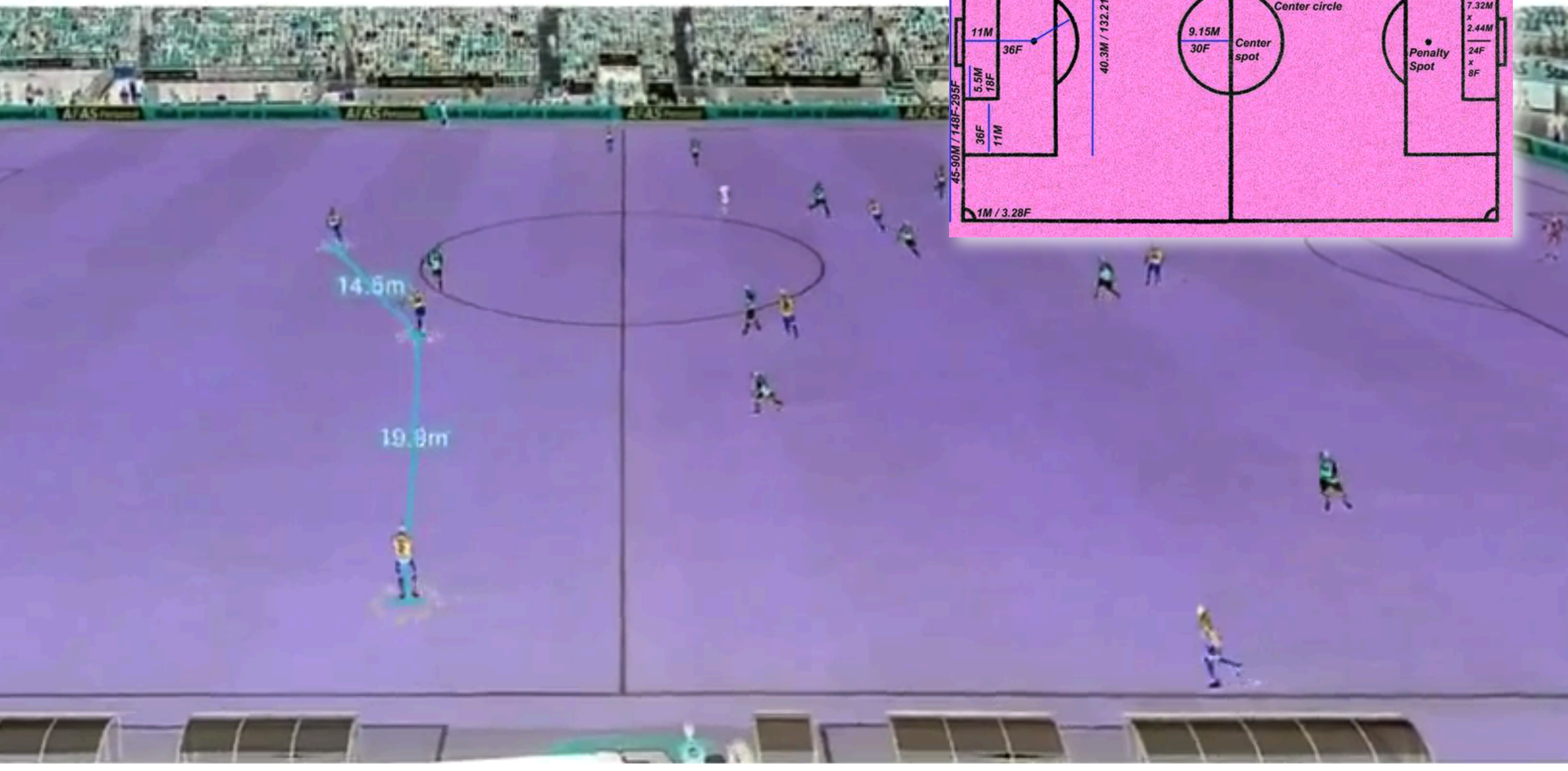

16
2ND & 10
2ND HALF TOTAL YARDS
120
53
ESPN
1 FLORIDA ST
13
2 AUBURN
21
4th
11:35
16
2nd & 10



synthetic lines can be mapped by a homography




TYR
VIT'HALLES
MONTPELLIER
ORSA EVENTS
Montpellier Agglomération
EUROSPORT
L'ÉQUIPE
NATATION
RMC
THONON
TYR
EDF
EDF


14.6m
19.9m

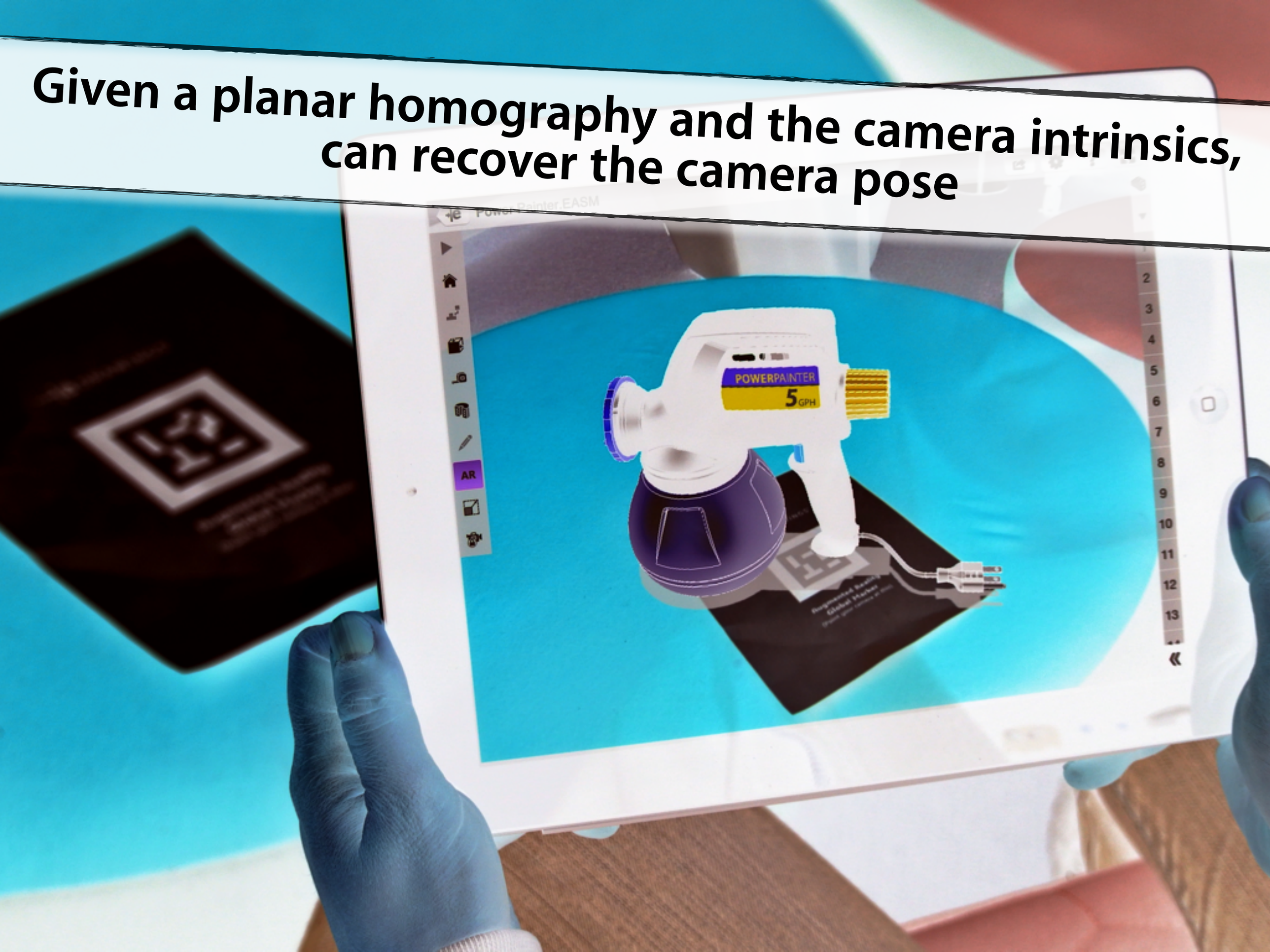
90-120M / 295-394F
16.5M / 54F
5.5M / 18F
11M
36F
5.5M
18F
36F
11M
45-90M / 148F-295F
1M / 3.28F
40.3M / 132.21F
Half-way line
Center circle
9.15M
30F
Center spot
Penalty area
18-yard box
Penalty Spot
Goal
7.32M
x
2.44M
24F
x
8F


14.6m
19.9m
Welkom
in het AFAS

Why do the players appear stretched?


Search, Compare & Book All Major Airlines
HOME
HOME
-20
-30

Given a planar homography and the camera intrinsics, can recover the camera pose



Behind the Scenes

the creators project

Box by Bot and Dolly

Creators Project

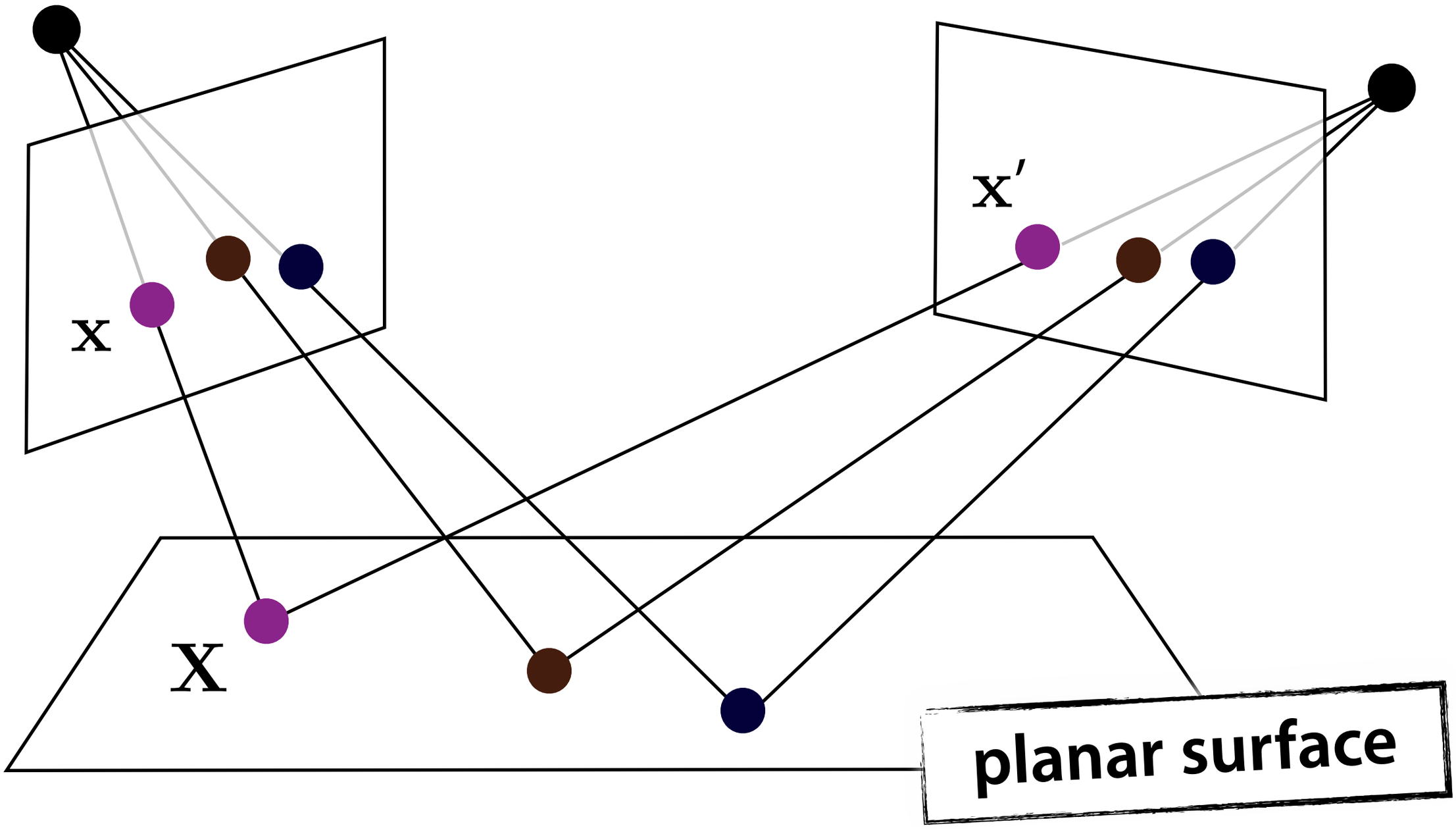

x
x′
x
planar surface

image 1

$$w \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{H}_1 \mathbf{H}_2^{-1} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

image 2

$\mathbf{H}_1$

$\mathbf{H}_2$

$\mathbf{H}_2^{-1}$

$\mathbf{x}$

$\mathbf{x}'$

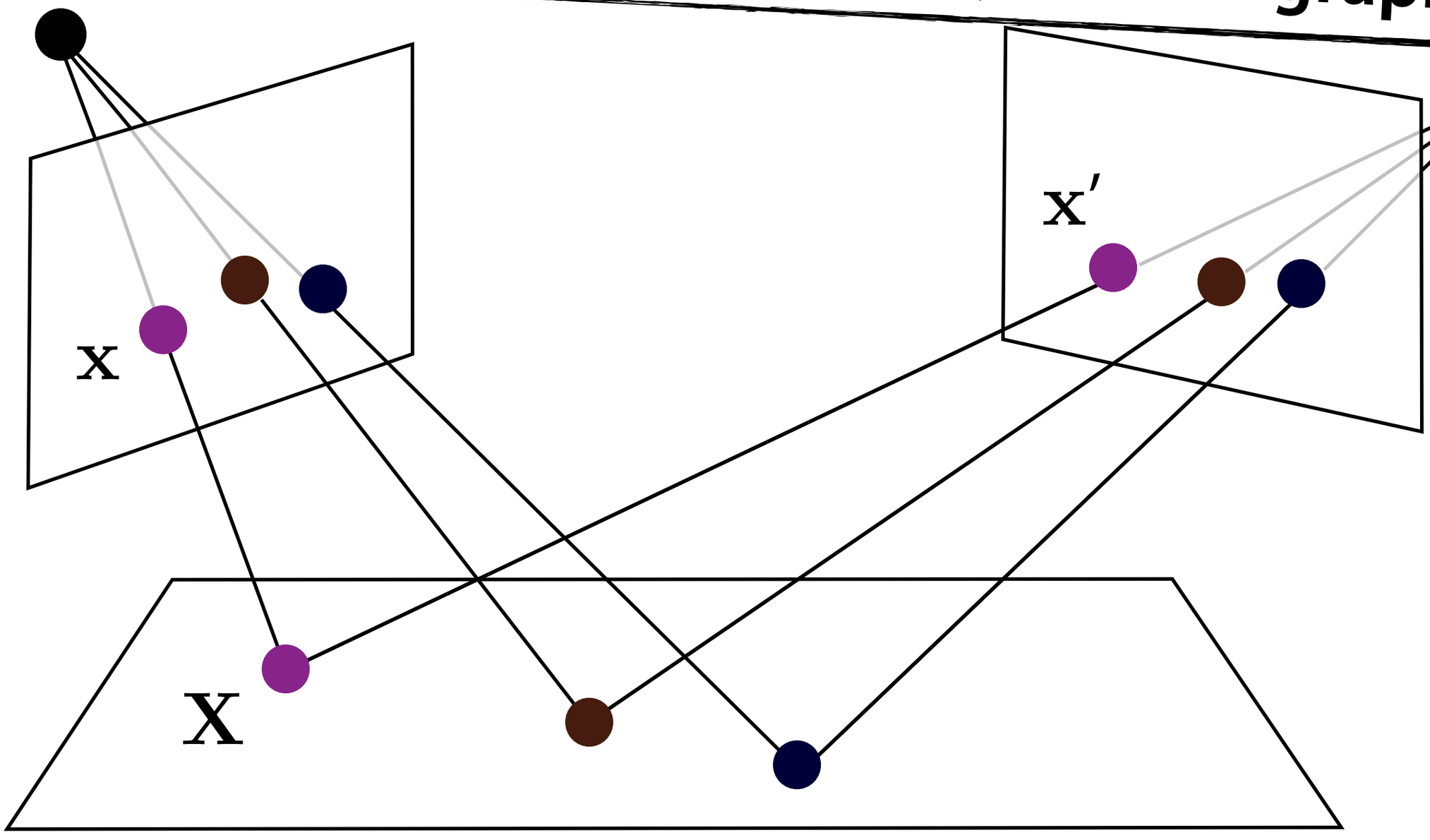
$\mathbf{X}$

image 1

$$w \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

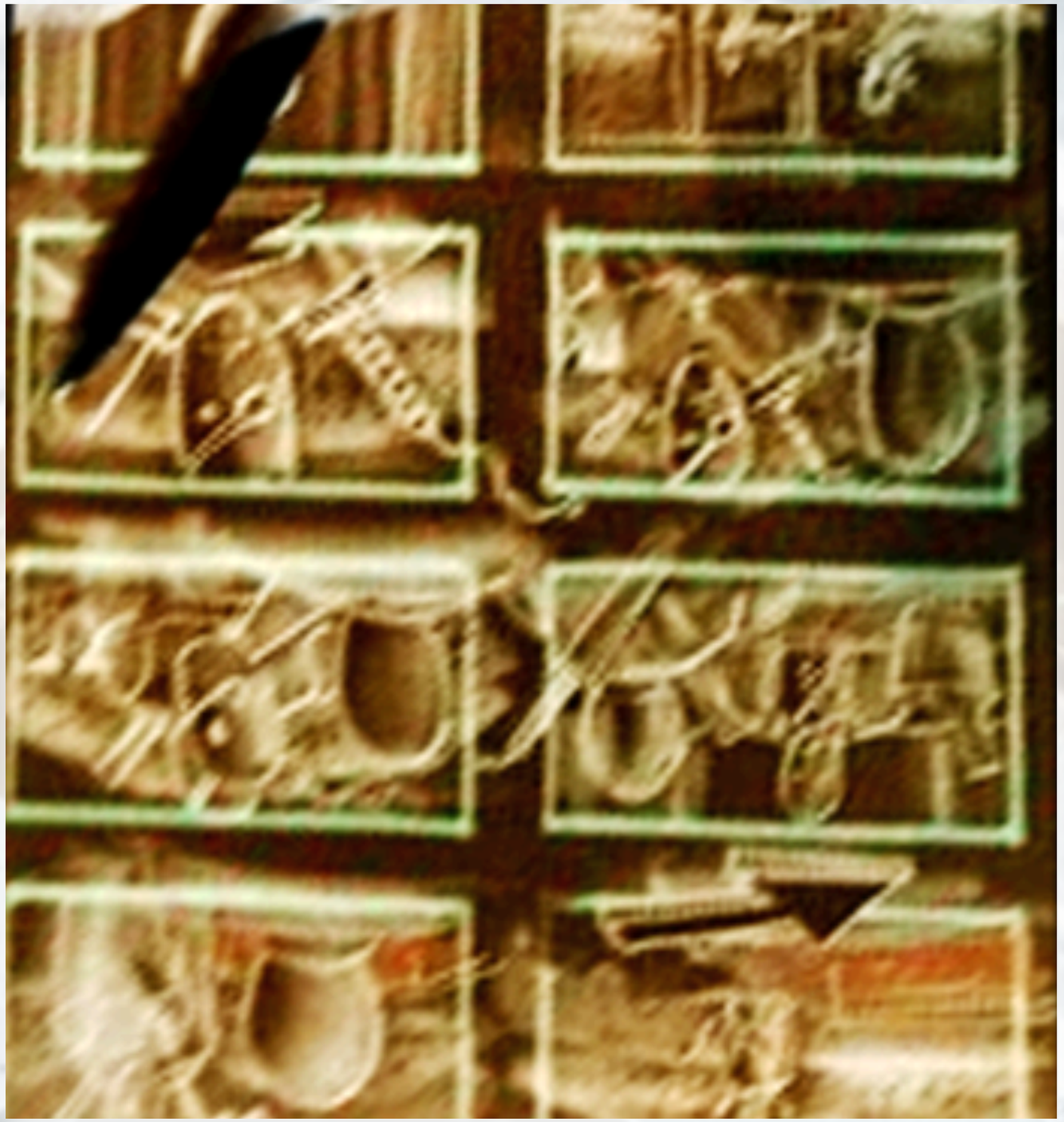
image 2

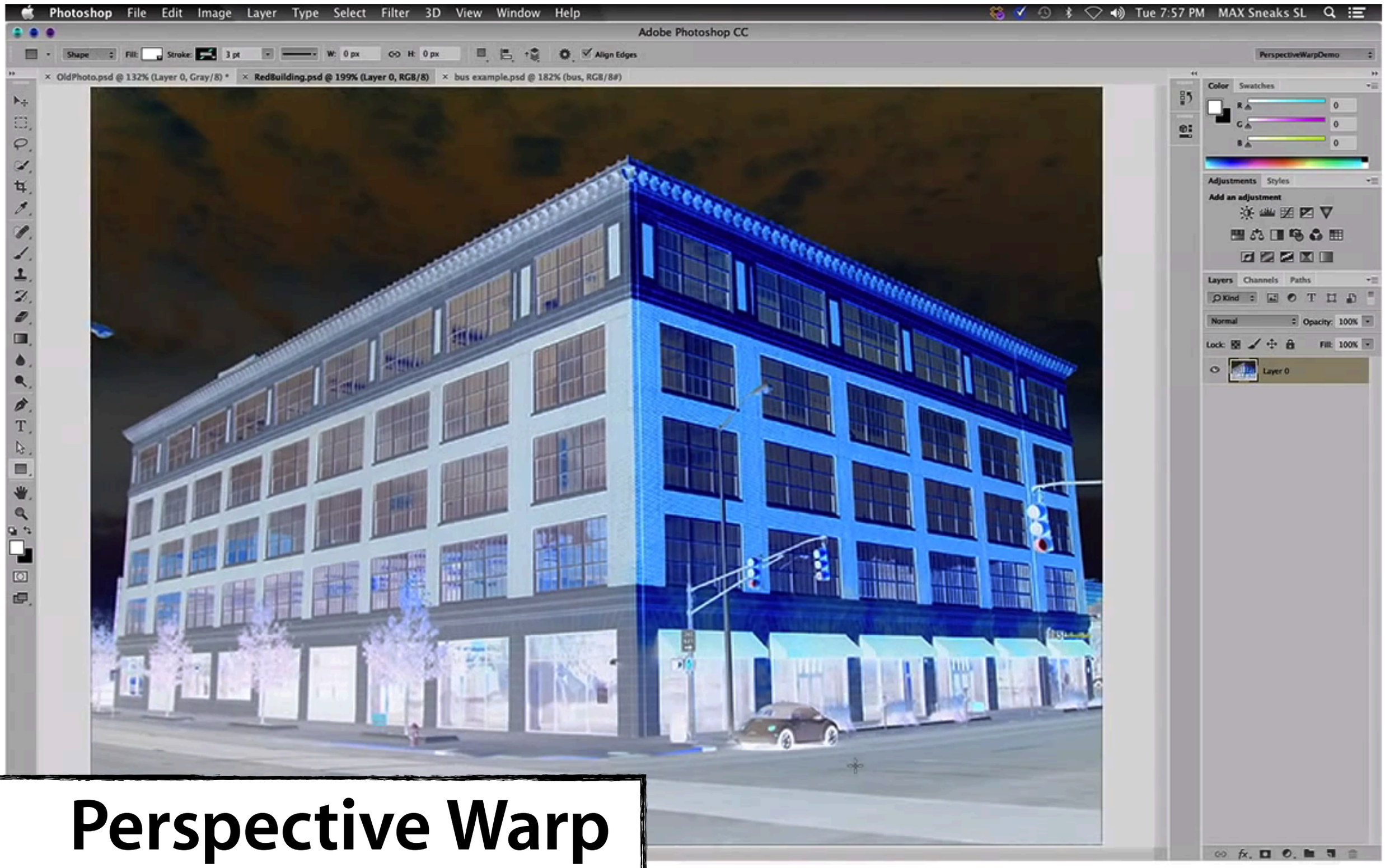
images of a plane are related by a homography



3D Lego Terracotta Army

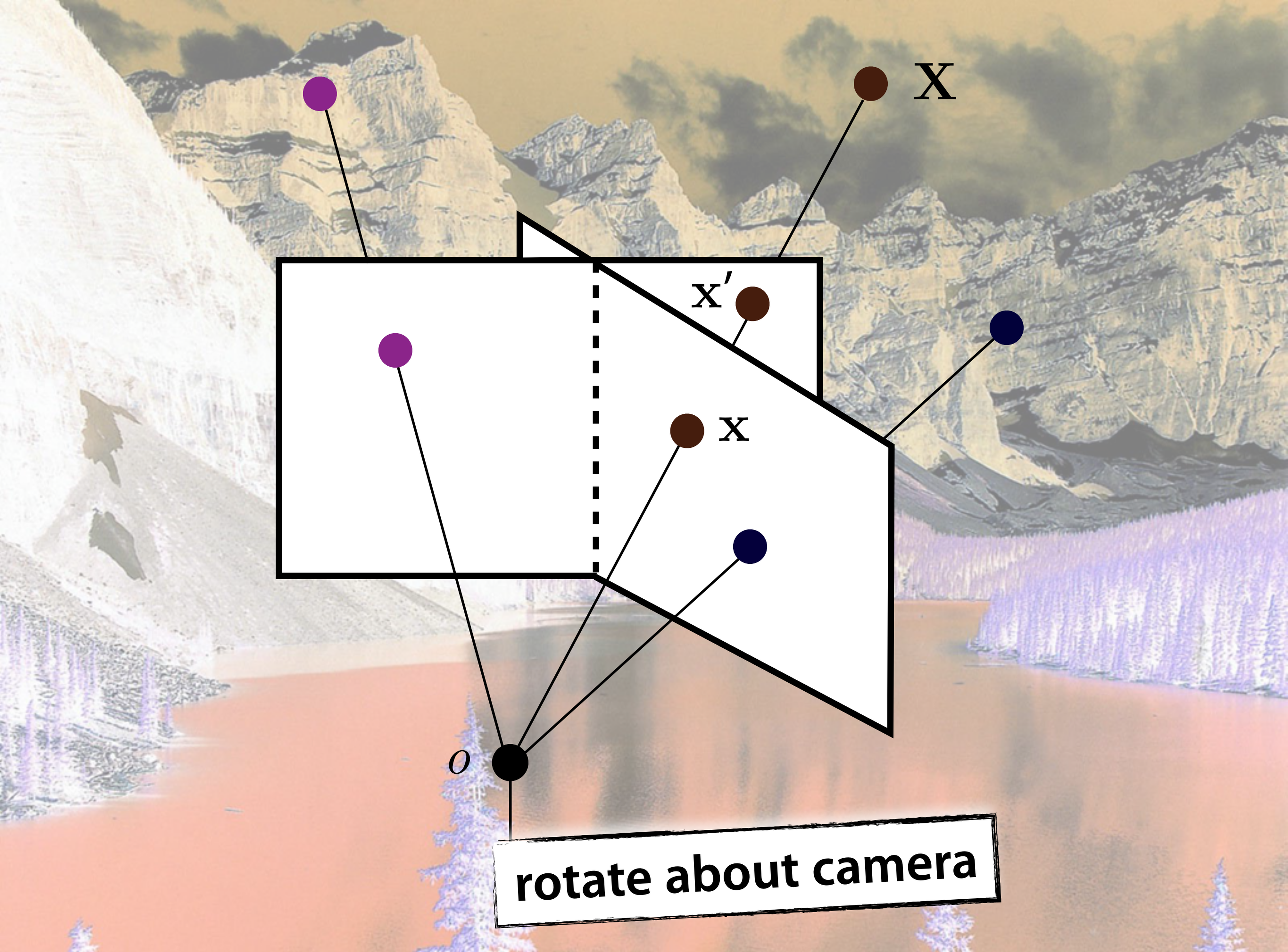
Leon Keer, Ruben Poncia, Remko van Schaik and Peter Westerink

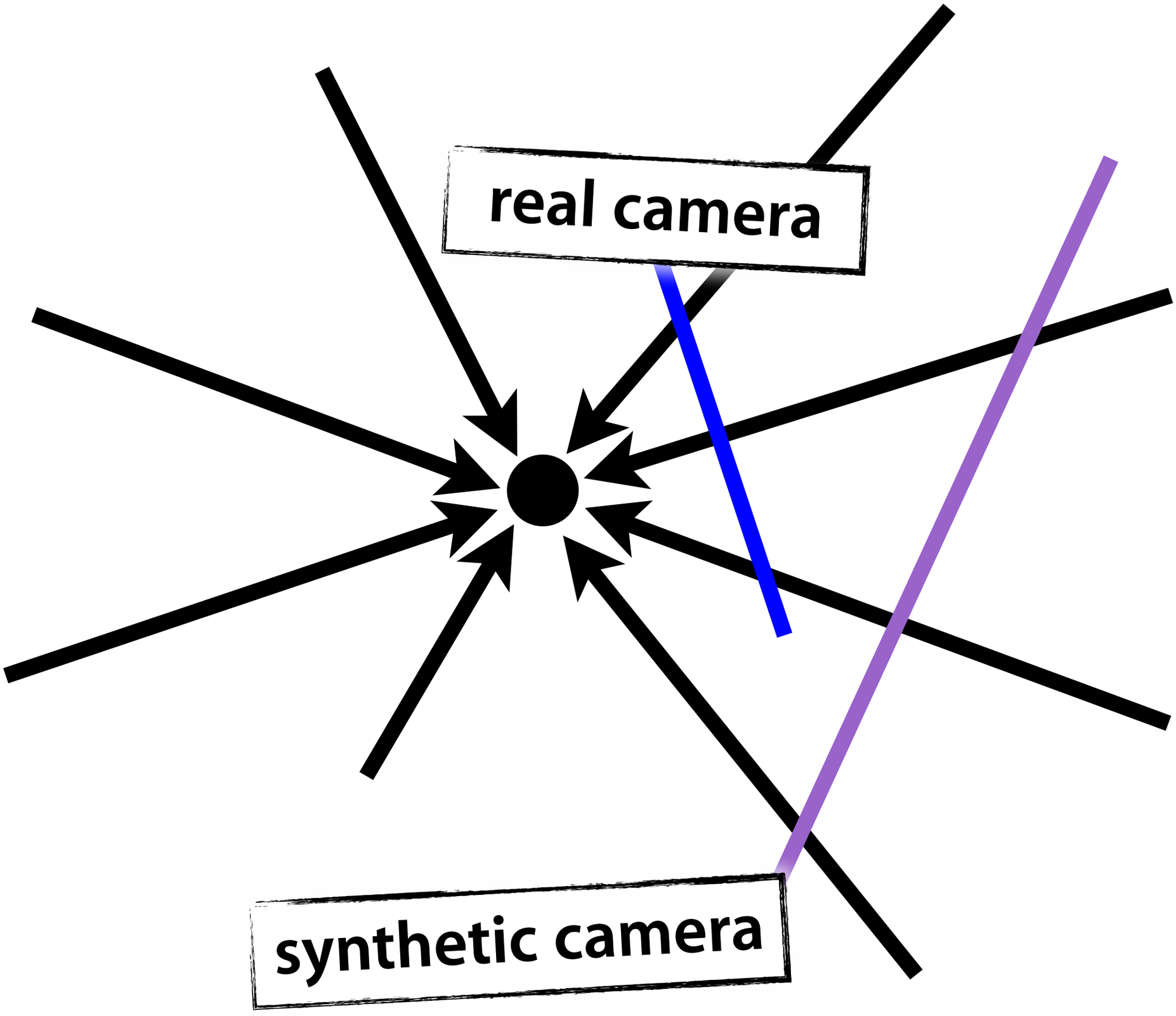




Perspective Warp

ADOBE CREATIVE CLOUD


X
x′
x
o
rotate about camera


real camera
synthetic camera

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$= \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$= \mathbf{H}_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{H}_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$w_2 \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \mathbf{H}_2 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = w_2 \mathbf{H}_1 \mathbf{H}_2^{-1} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

rotating camera

$$w_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = w_2 \mathbf{H}_1 \mathbf{H}_2^{-1} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

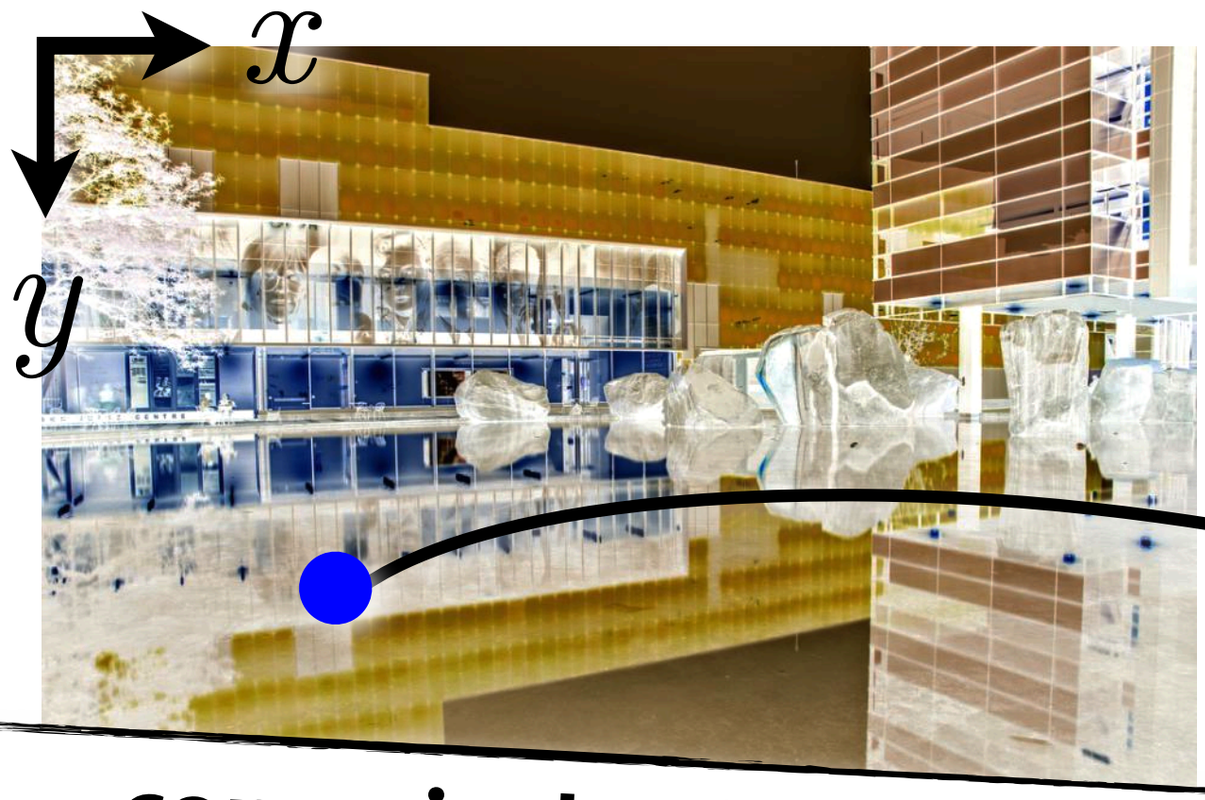
images from a rotating camera are related by a homography

FORWARD v INVERSE WARP

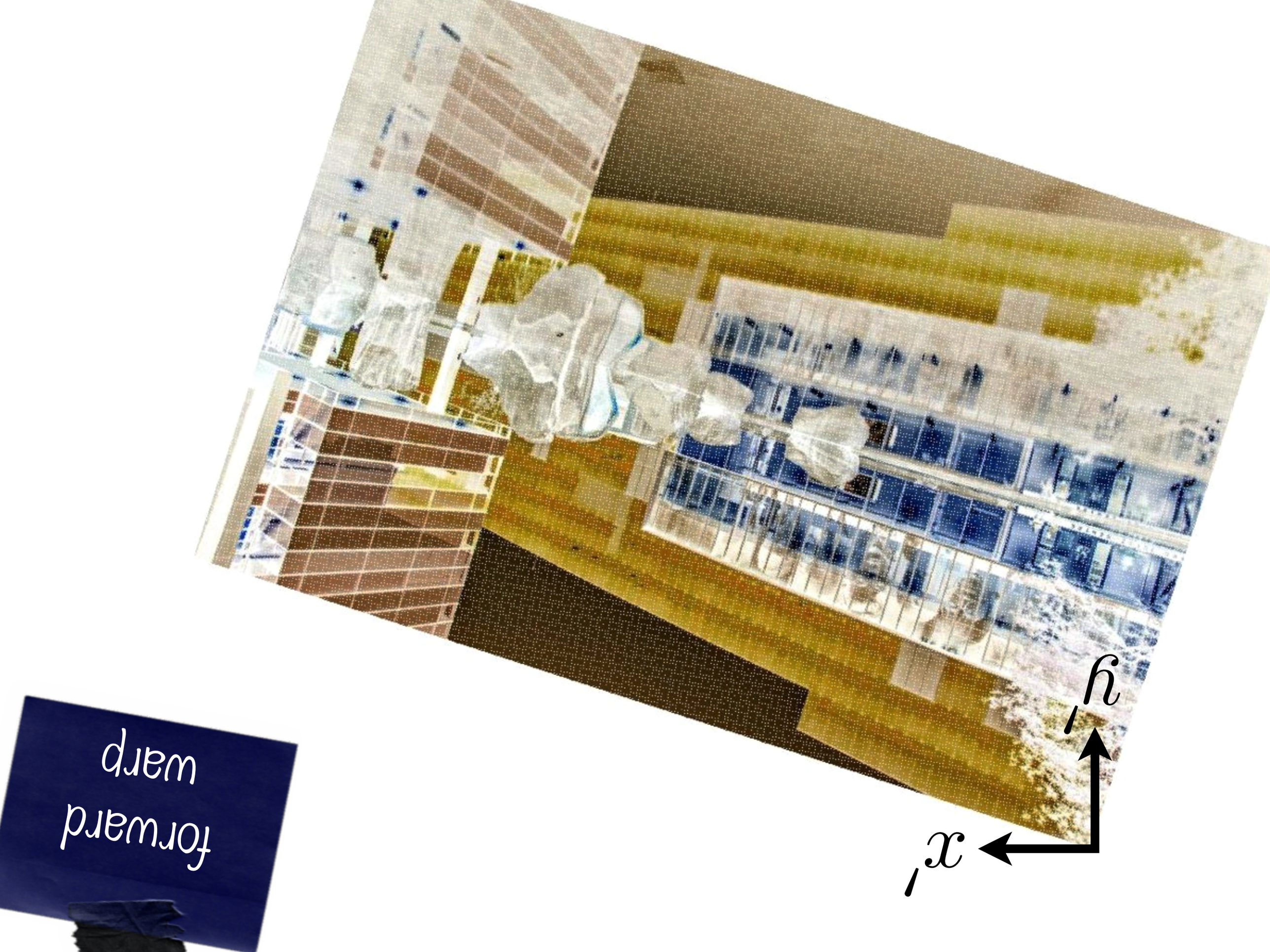
practical issues when warping images

forward warp

$$\mathbf{x}' = \mathbf{T}\mathbf{x}$$



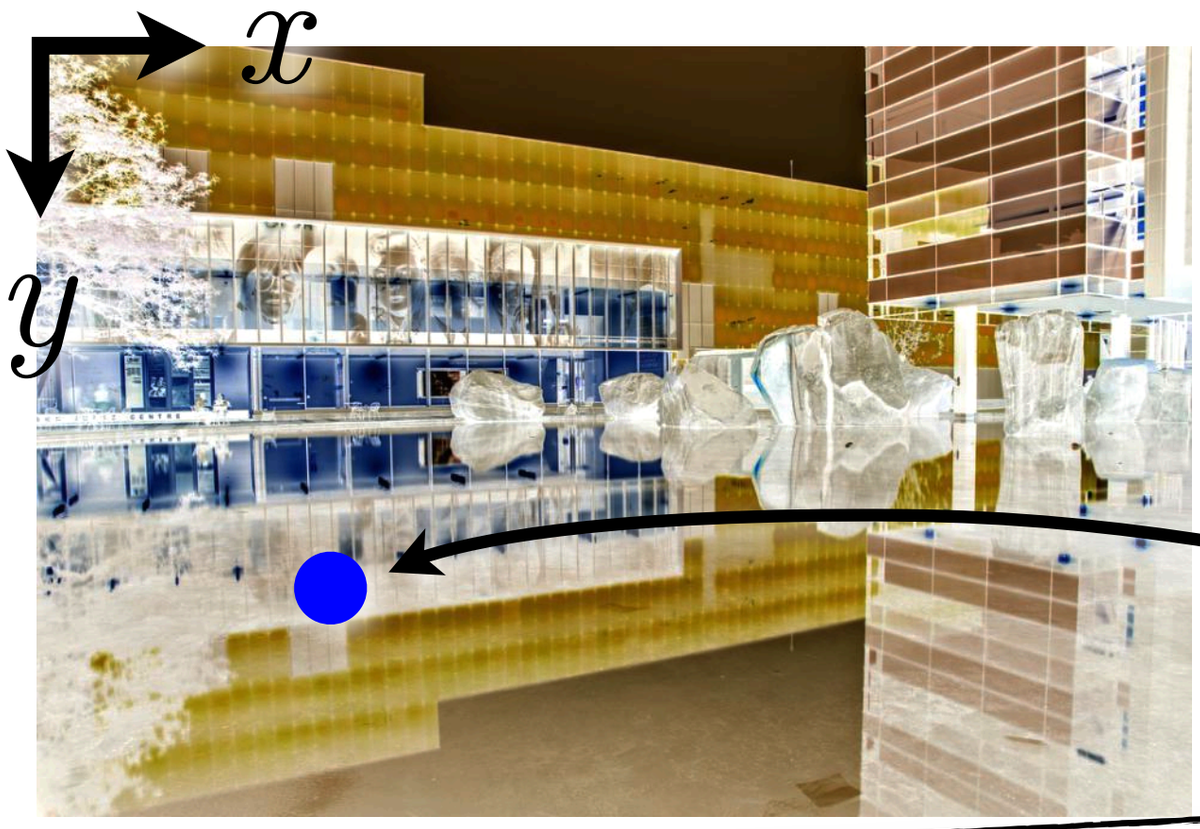
copy pixel value from input to mapped output

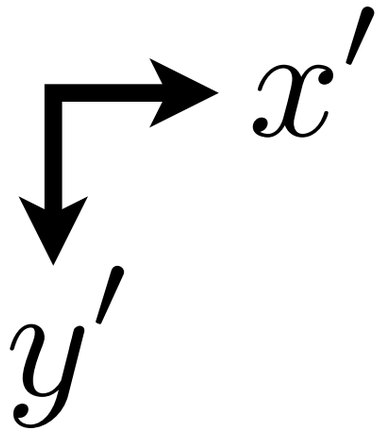

forward warp
x'
y'


forward
warp

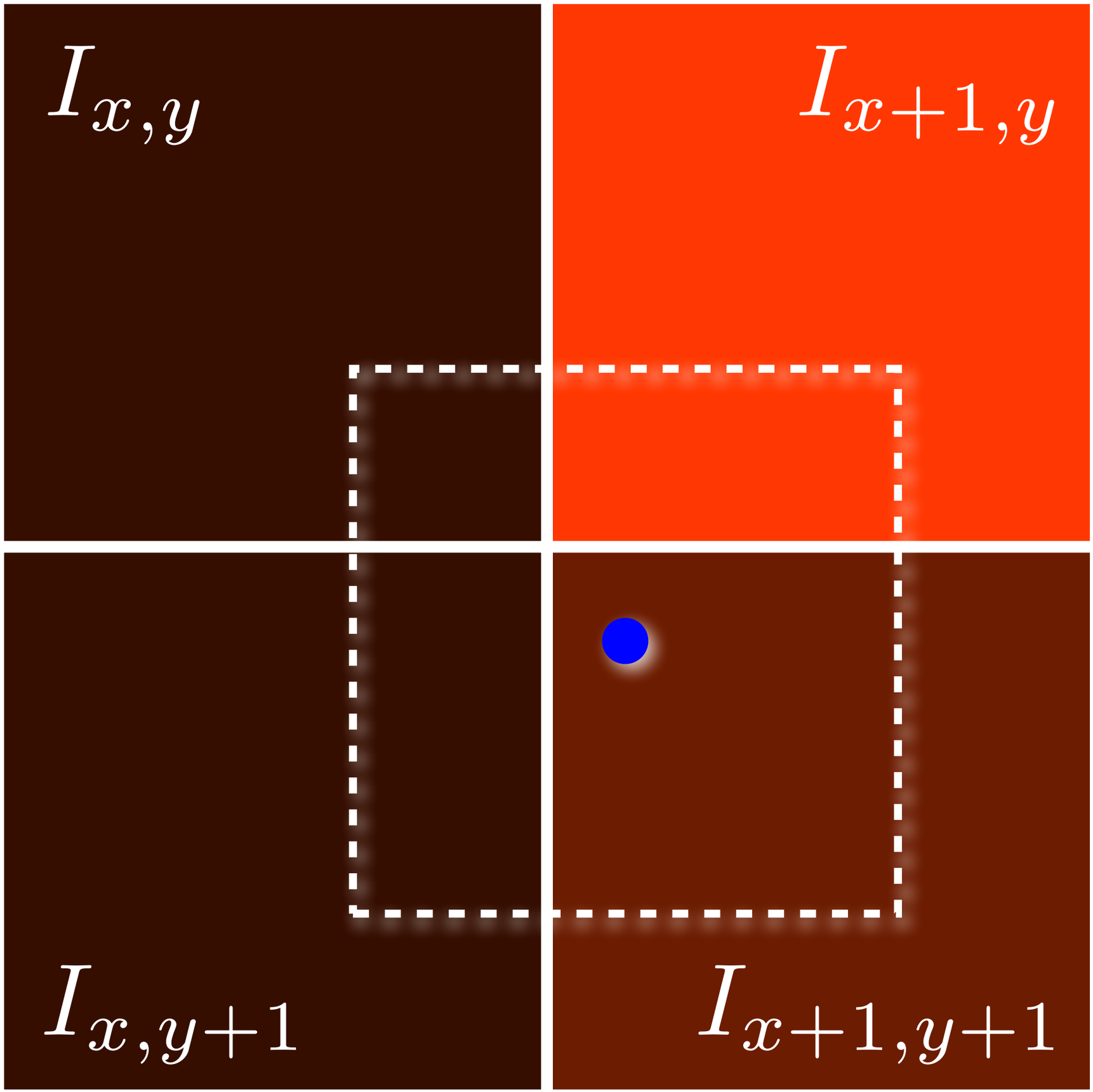
inverse warp

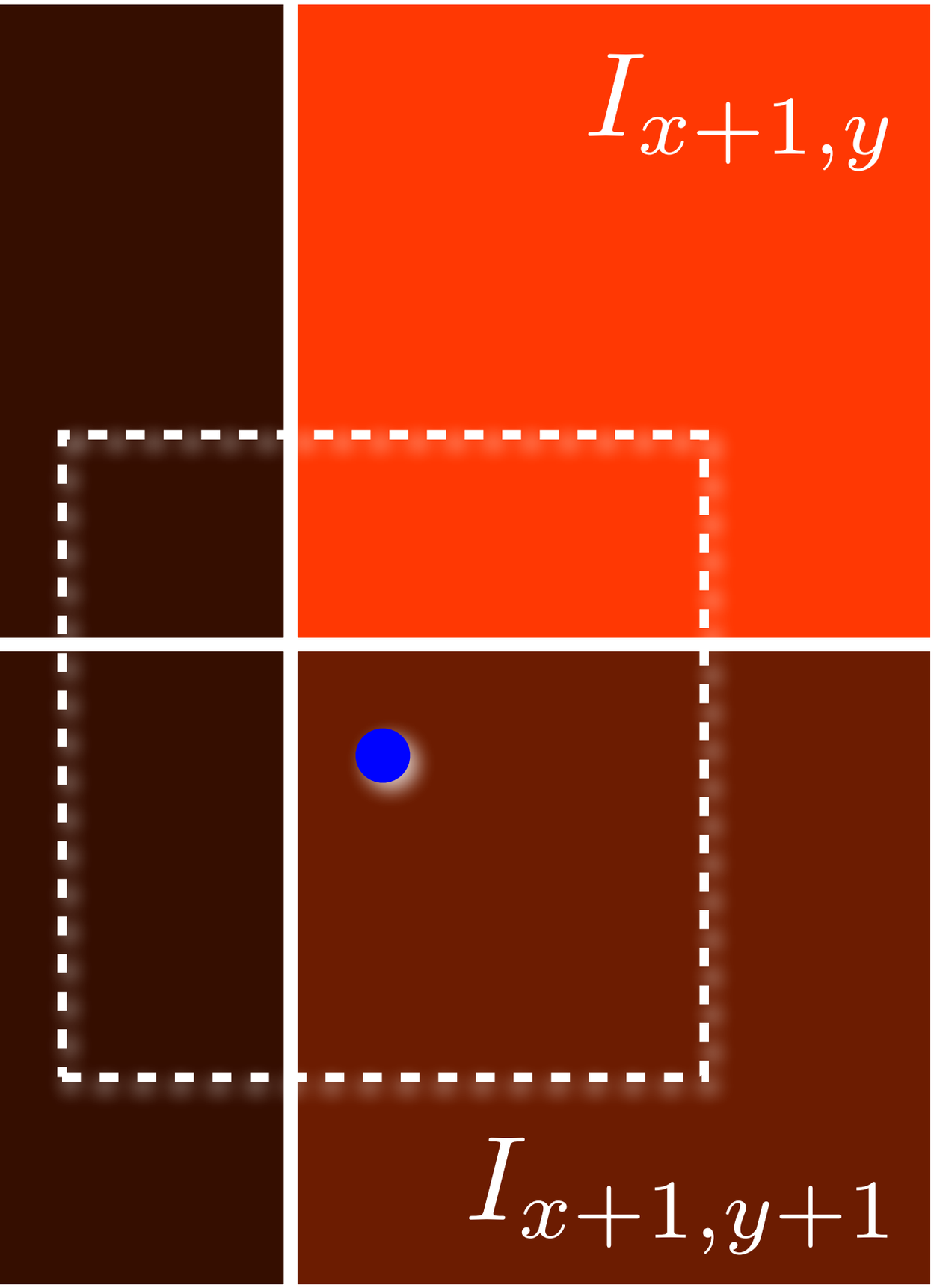
$$\mathbf{x} = \mathbf{T}^{-1}\mathbf{x}'$$



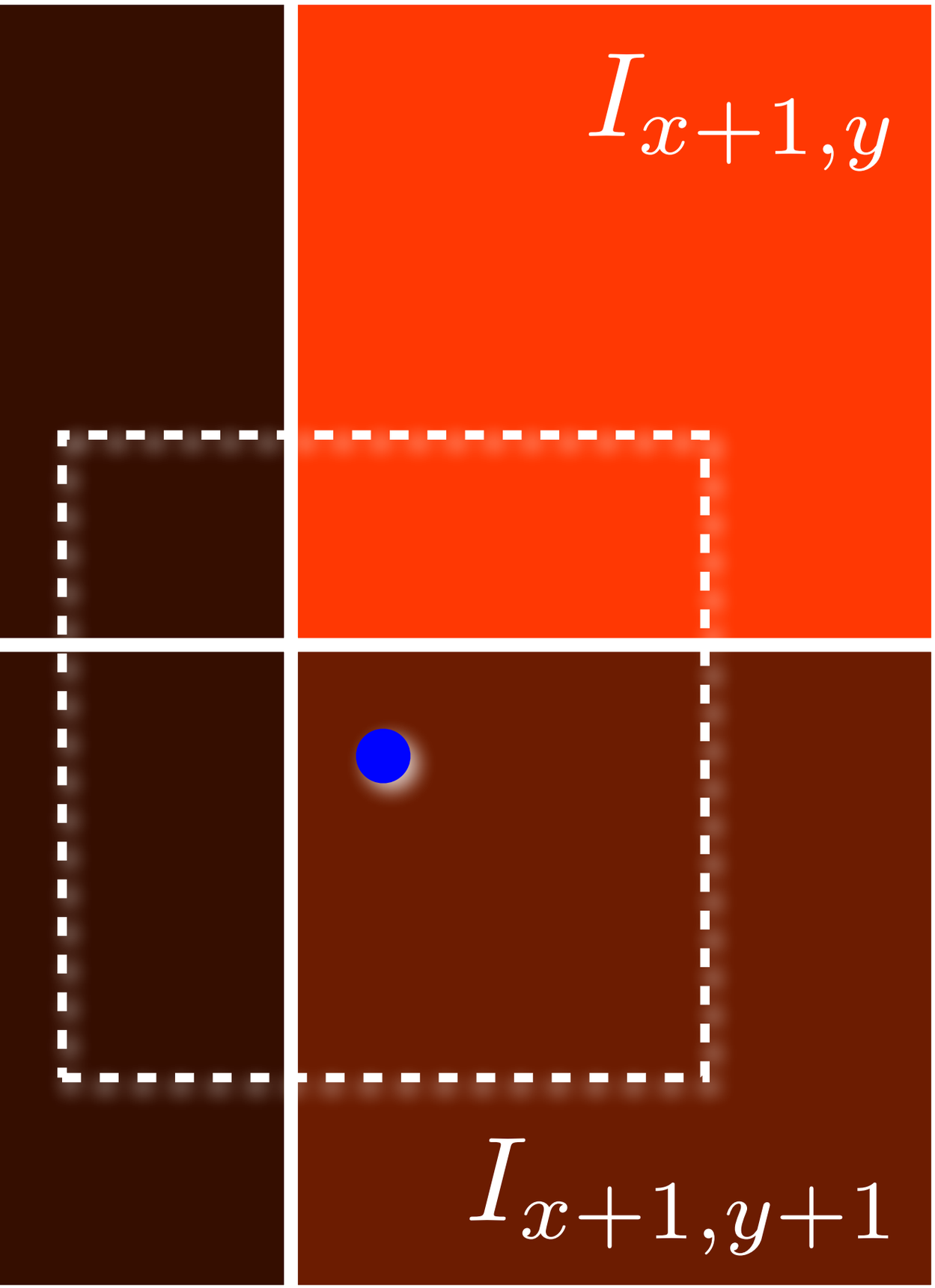


for each output pixel, map to input and “copy” value


$I_{x,y}$
$I_{x+1,y}$
$I_{x,y+1}$
$I_{x+1,y+1}$



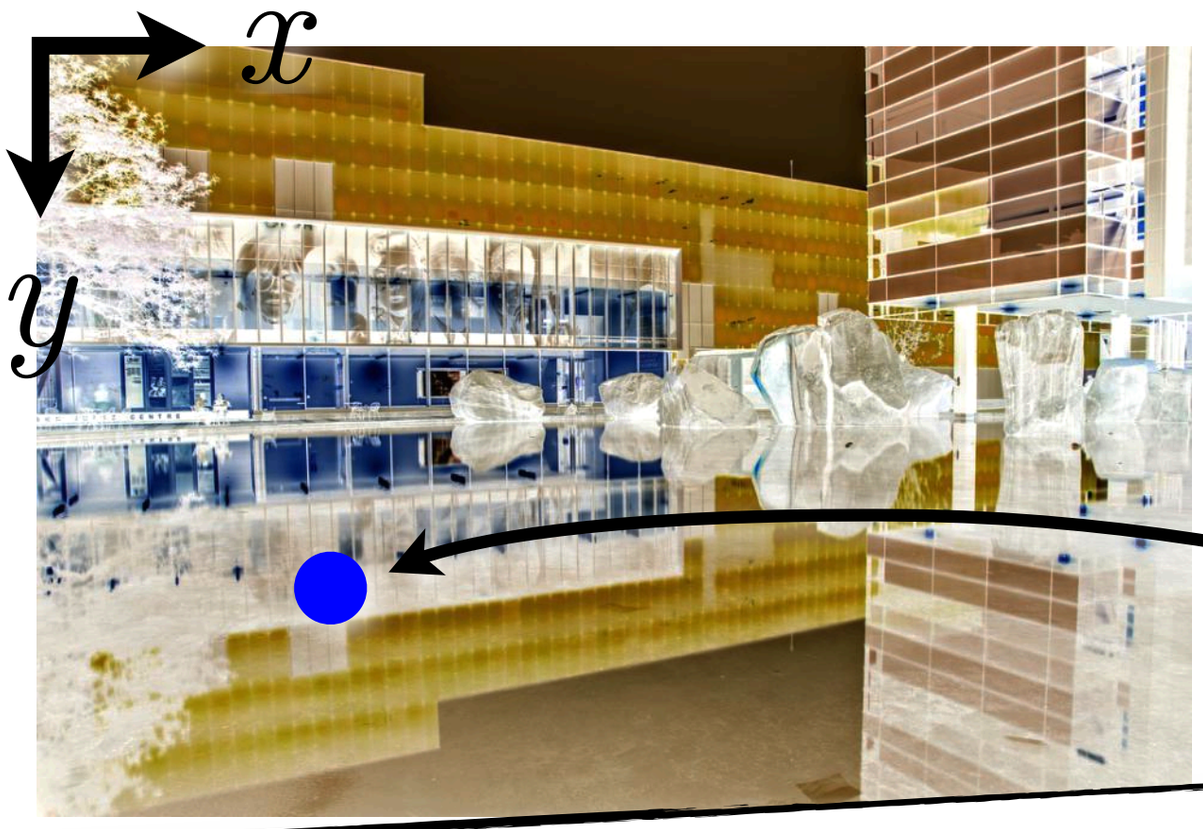
Take colour of closest pixel

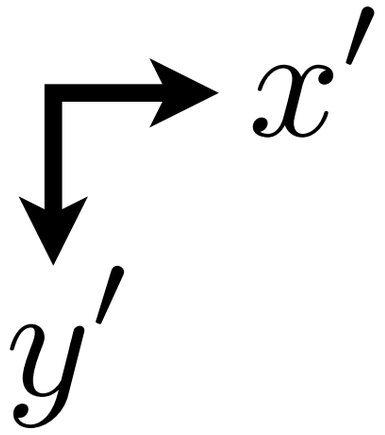


Take weighted average of colours of closest pixels

inverse warp

$$\mathbf{x} = \mathbf{T}^{-1}\mathbf{x}'$$

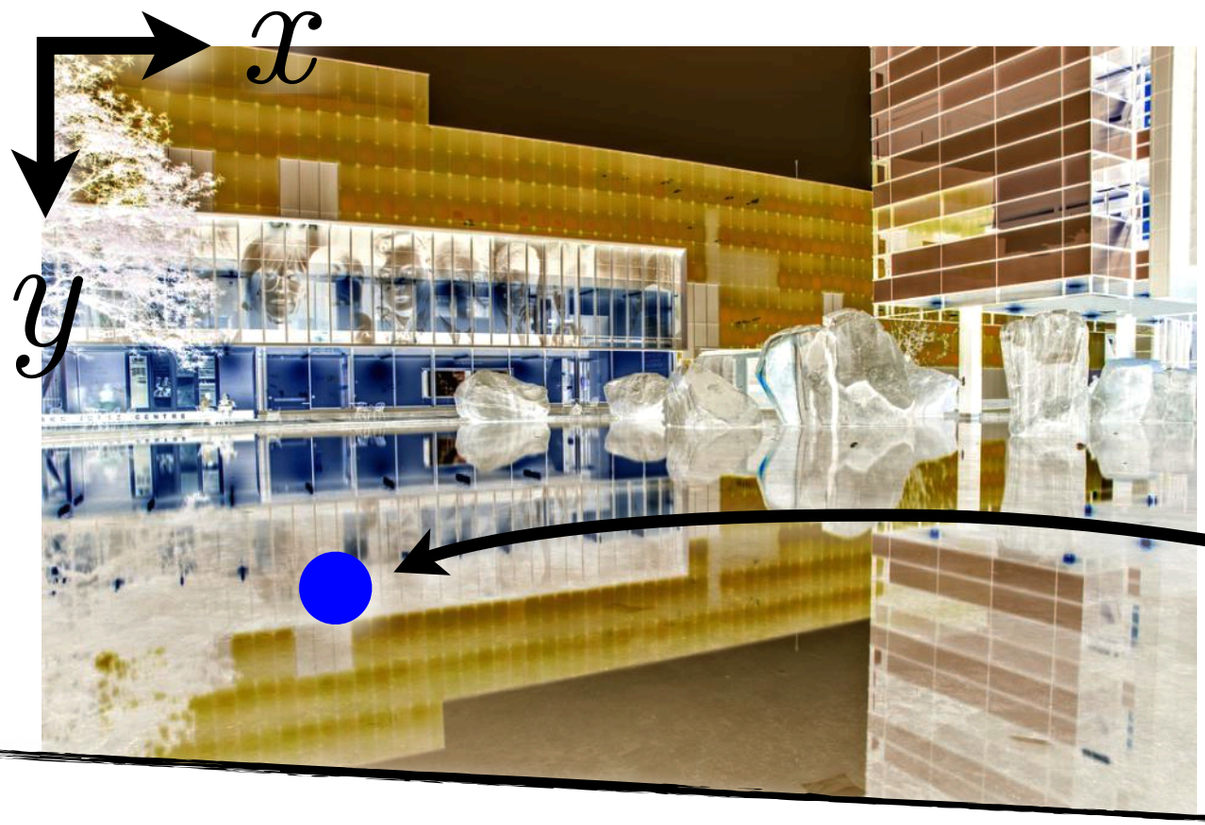




for each output pixel, map to input and “copy” value

inverse warp

$$\mathbf{x} = \mathbf{T}^{-1}\mathbf{x}'$$



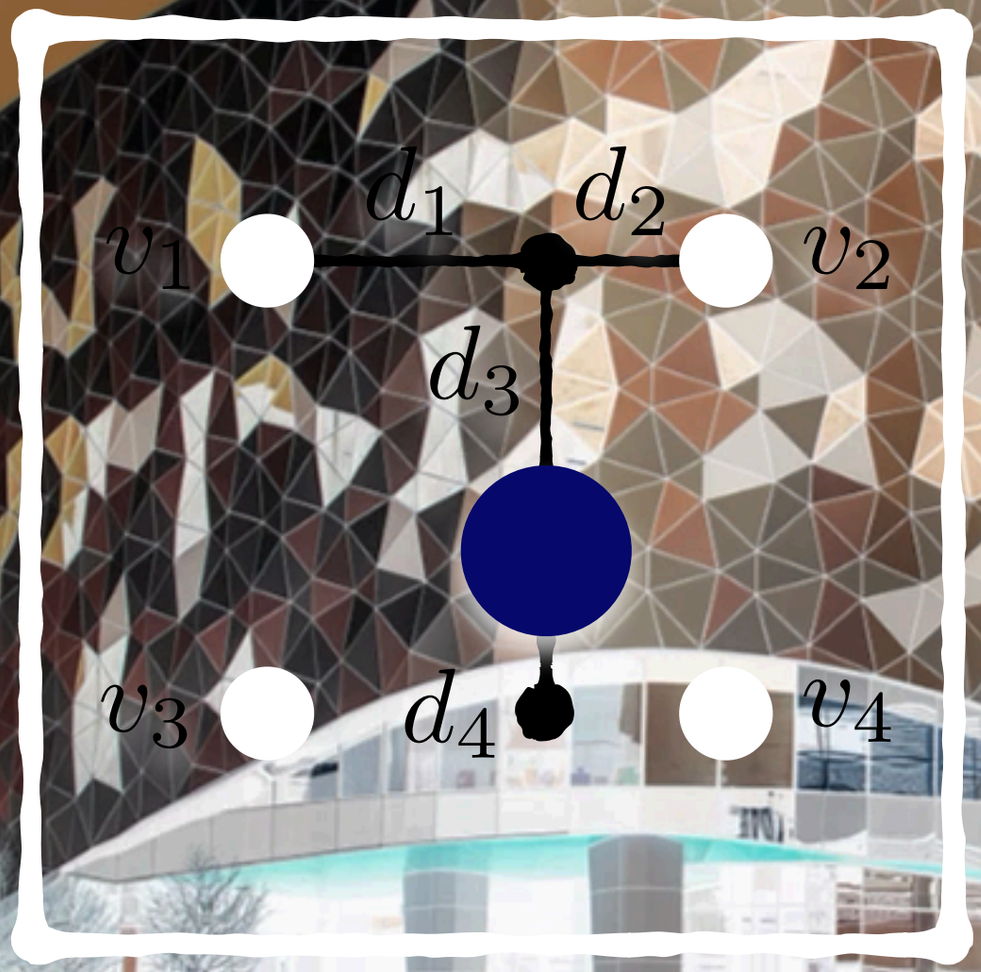
assumes transformation is invertible

x'

y'

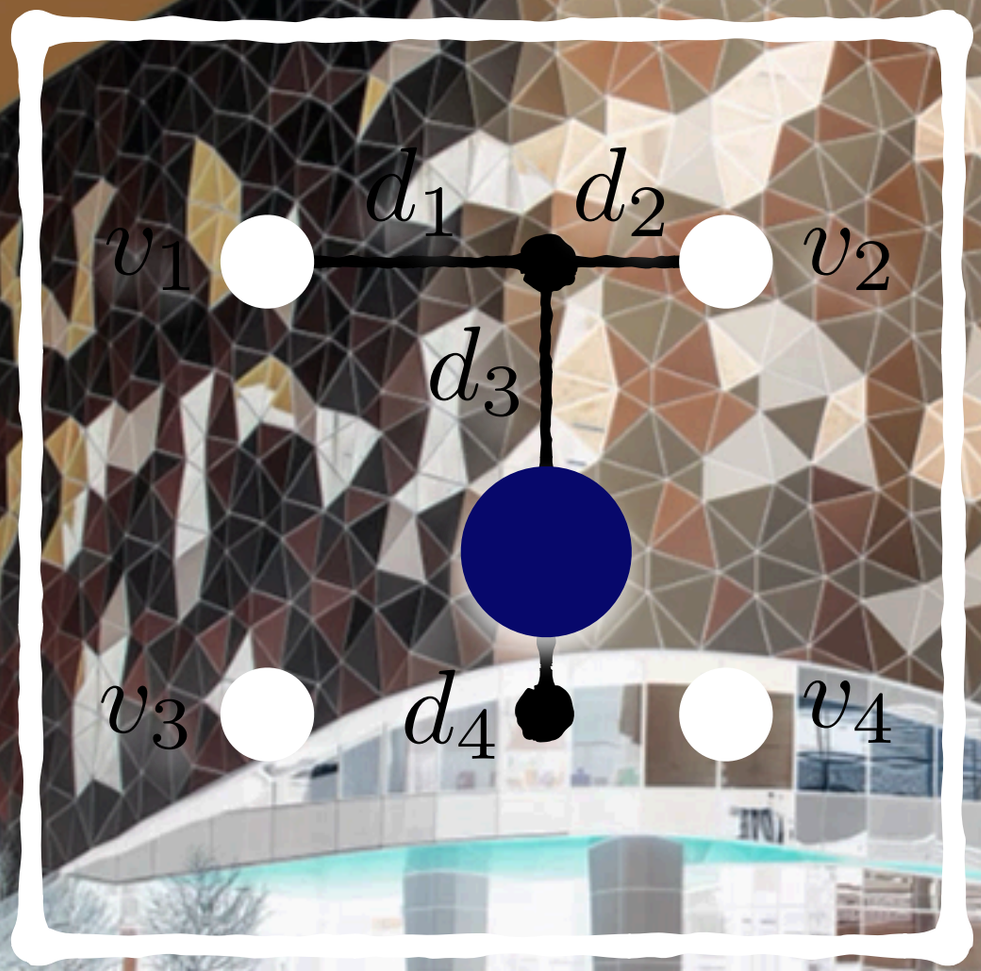
inverse warp

inverse warp



$$q_1 = v_1 d_2 + v_2 d_1$$

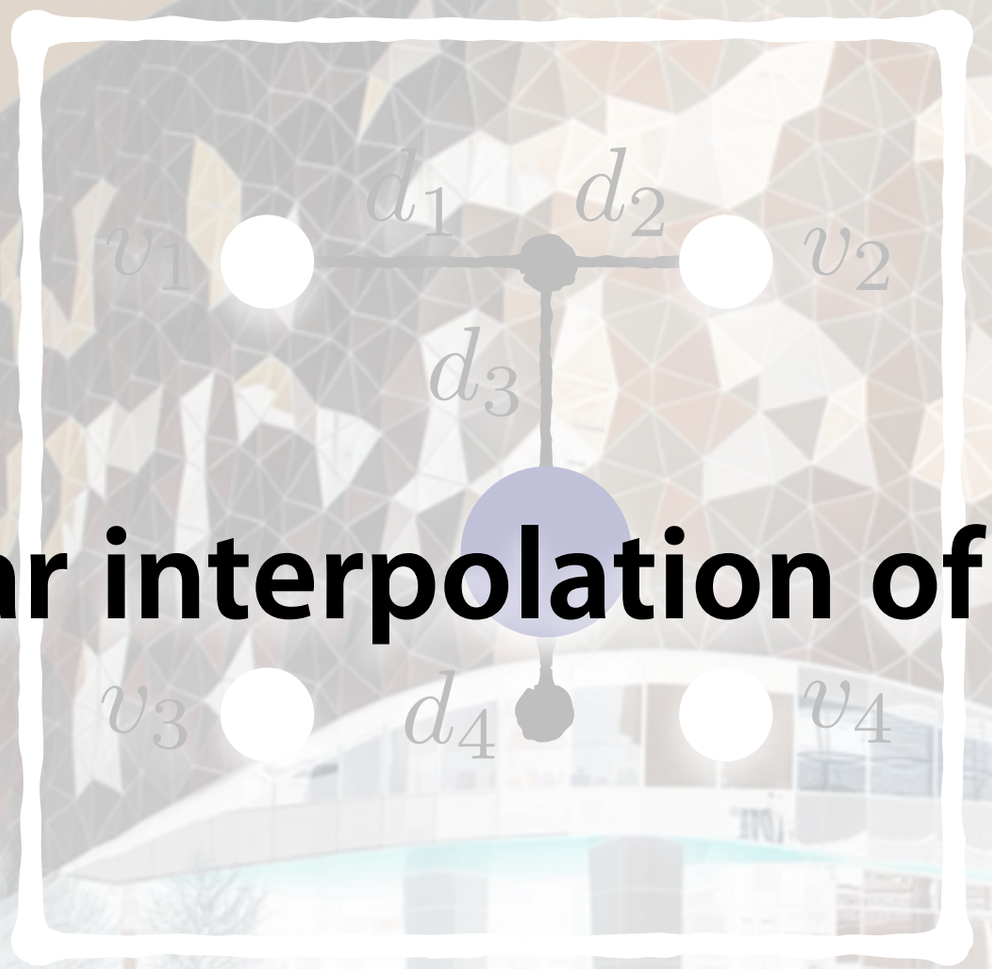
$$q_2 = v_3 d_2 + v_4 d_1$$



$$q_1 = v_1 d_2 + v_2 d_1$$

$$q_2 = v_3 d_2 + v_4 d_1$$

$$q = q_1 d_4 + q_2 d_3$$

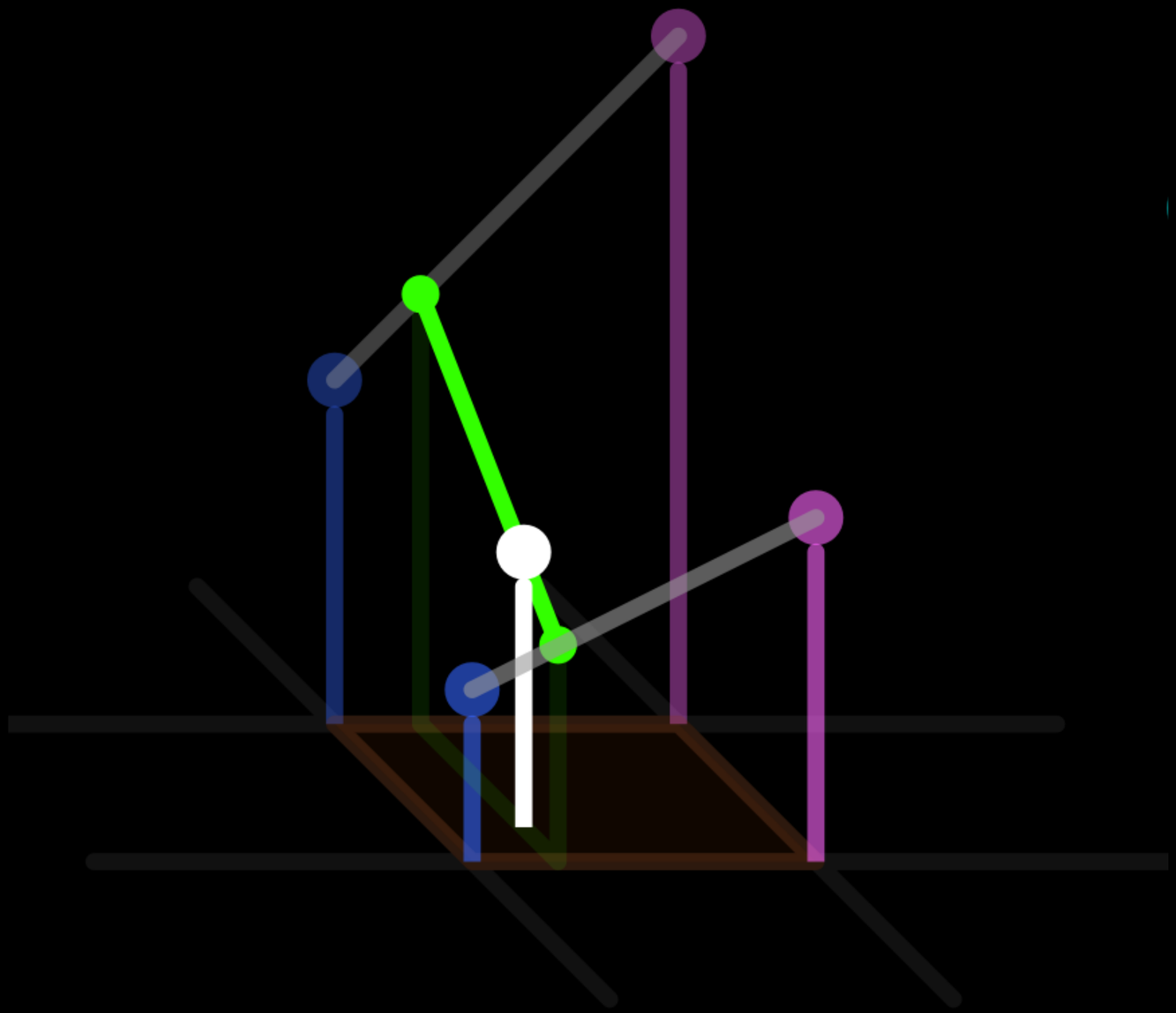


$$q_1 = v_1 d_2 + v_2 d_1$$

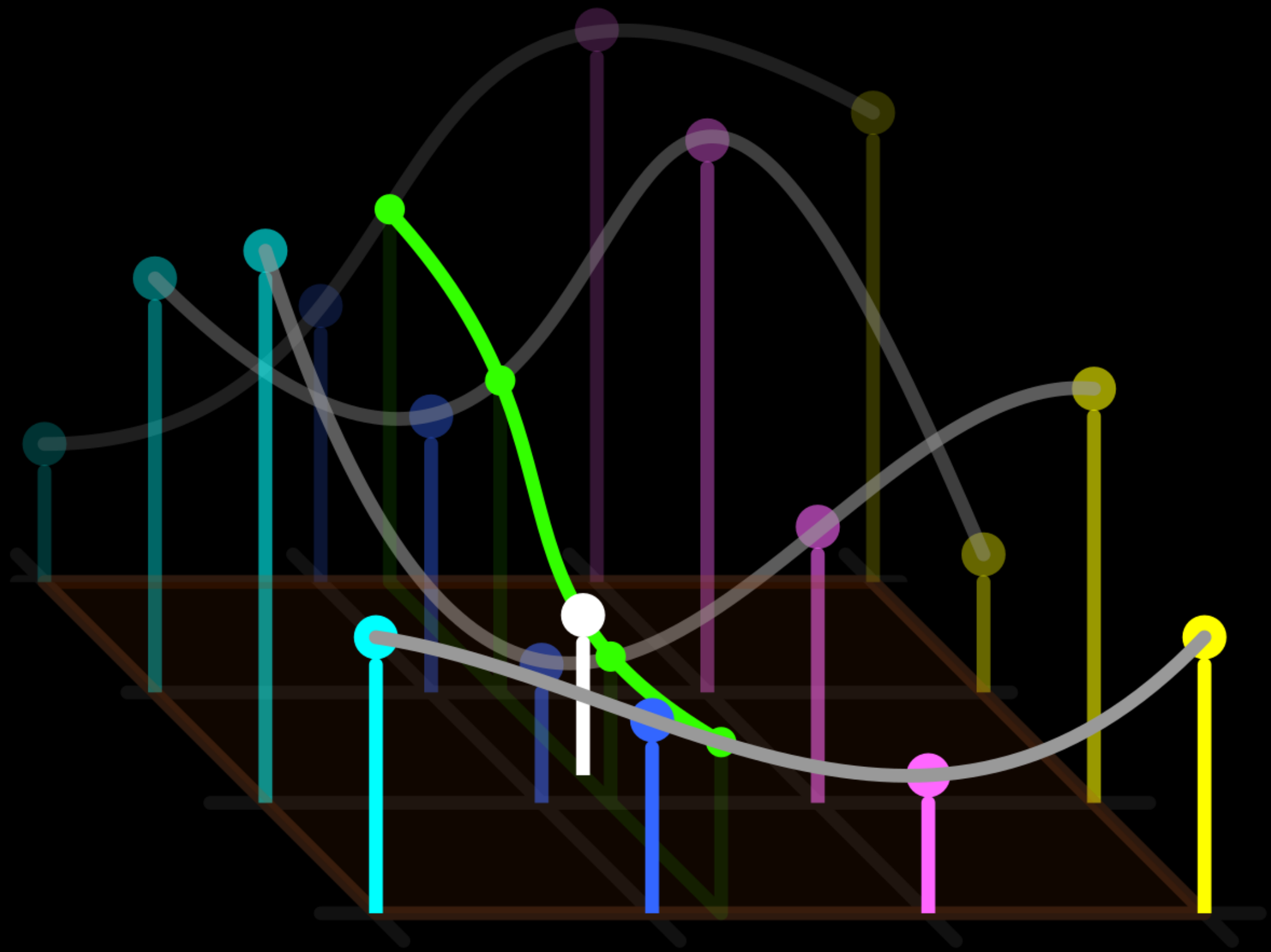
$$q_2 = v_3 d_2 + v_4 d_1$$

linear interpolation of linear interpolations

$$q = q_1 d_4 + q_2 d_3$$



bilinear interpolation



bicubic interpolation

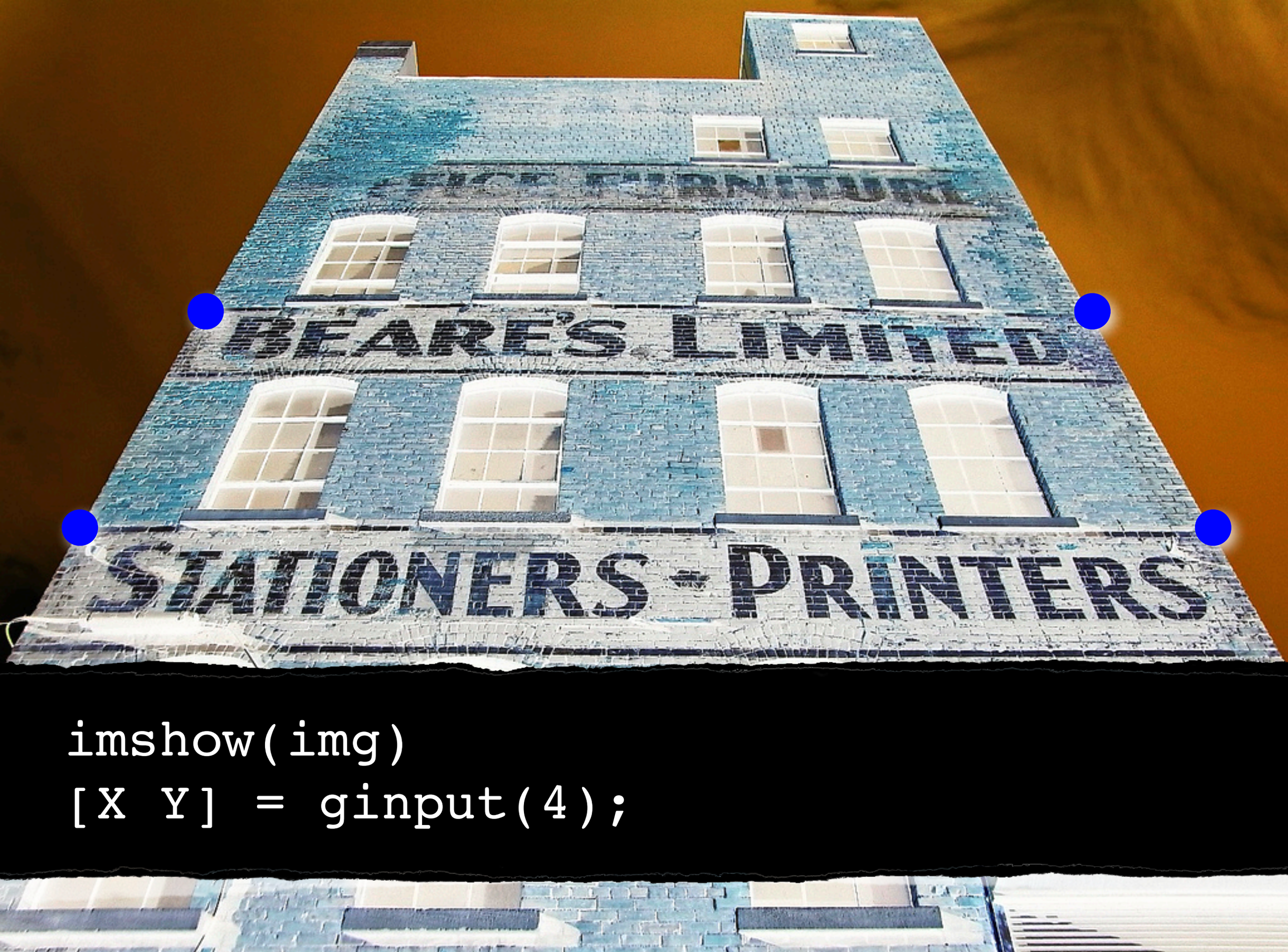
```
img = imread('ghost-building-original.jpg');

imshow(img)
[X Y] = ginput(4);

X2 = [X(1:2); X(1:2)];
Y2 = [Y(1); Y(1); Y(3) + 50; Y(3) + 50];

tform = maketform('projective', [X Y], [X2 Y2]);
img_out = imtransform(img, tform, 'bicubic');

imshow(img_out)
```




```
imshow(img)
[X Y] = ginput(4);
```

```
img = imread('ghost-building-original.jpg');

imshow(img)
[X Y] = ginput(4);

X2 = [X(1:2); X(1:2)];
Y2 = [Y(1); Y(1); Y(3) + 50; Y(3) + 50];
img_out = imtransform(img, tform,
imshow(img_out)
```

set the position of selected points in the output image

```
img = imread('ghost-building-original.jpg');

imshow(img)
[X Y] = ginput(4);

X2 = [X(1:2); X(1:2)];
Y2 = [Y(1); Y(1); Y(3) + 50; Y(3) + 50];

tform = maketform('projective', [X Y], [X2 Y2]);
img_out = imtransform(img, tform, 'bicubic');
```

create homography based on point correspondences

```
img = imread('ghost-building-original.jpg');

imshow(img)
[X Y] = ginput(4);

X2 = [X(1:2); X(1:2)];
Y2 = [Y(1); Y(1); Y(3) + 50; Y(3) + 50];

tform = maketform('projective', [X Y], [X2 Y2]);
img_out = imtransform(img, tform, 'bicubic');
```

warp input image based on transformation

```
img = imread('ghost-building-original.jpg');

imshow(img)
[X Y] = ginput(4);

X2 = [X(1:2); X(1:2)];
Y2 = [Y(1); Y(1); Y(3) + 50; Y(3) + 50];

tform = maketform('projective', [X Y], [X2 Y2]);
img_out = imtransform(img, tform, 'bicubic');

imshow(img_out)
```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