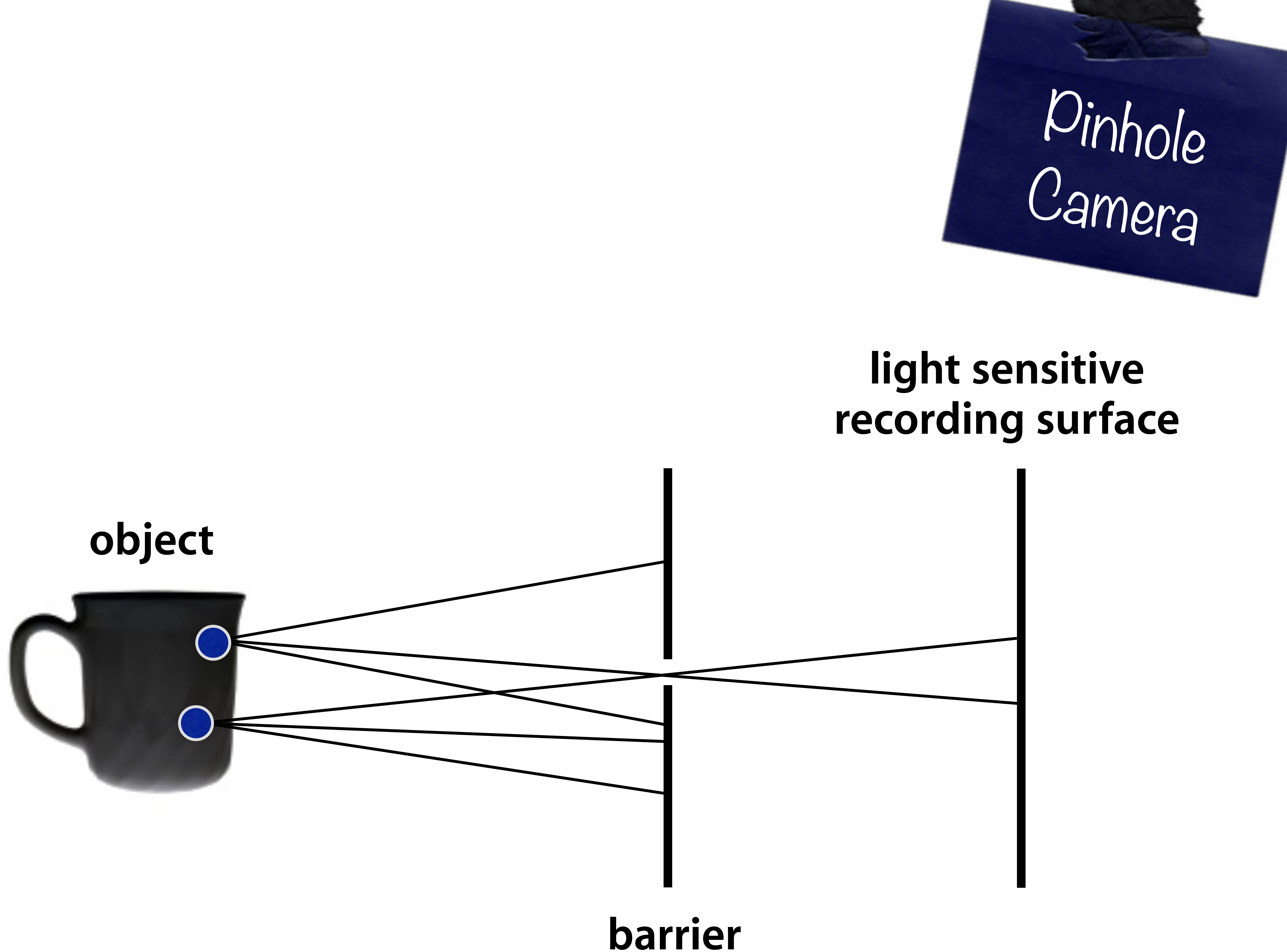
Intro to

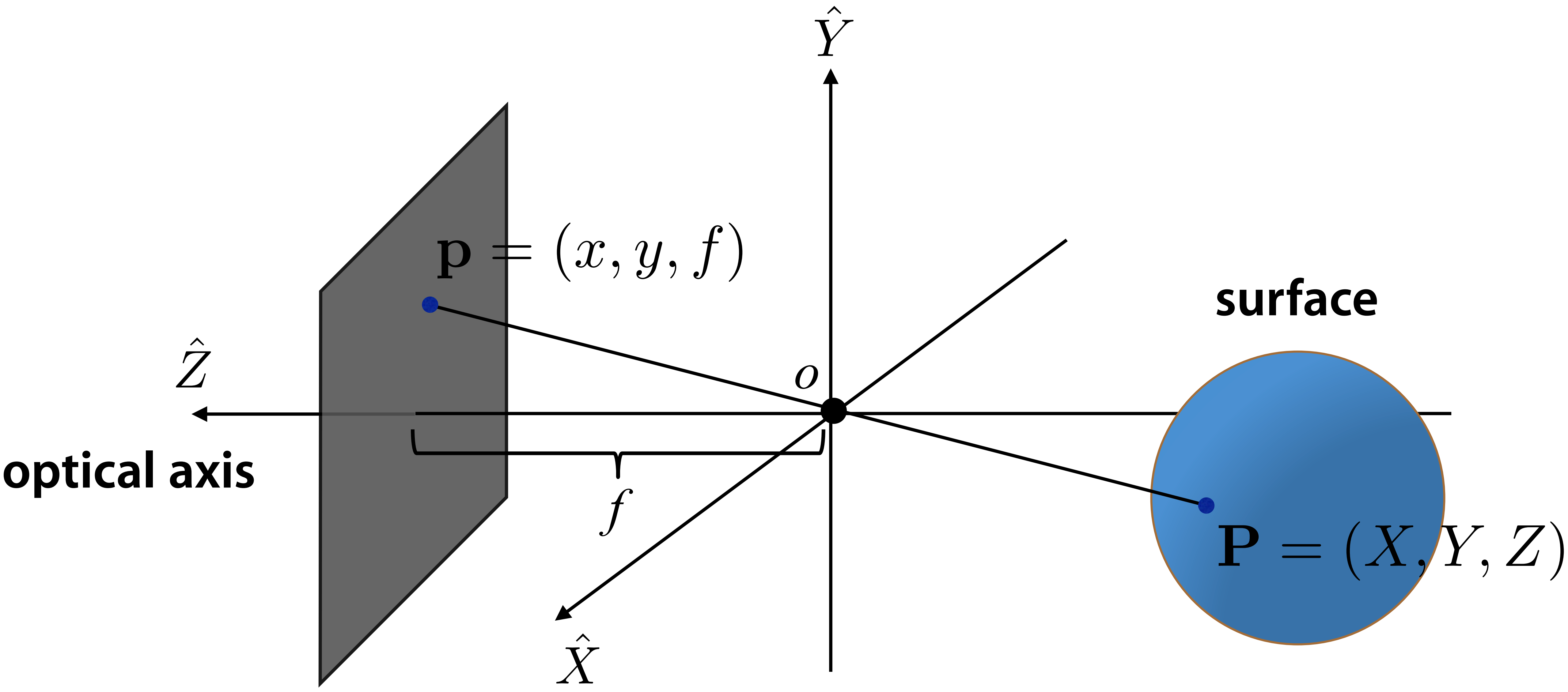
Computer Vision

with Prof. Kosta Derpanis

Image Formation

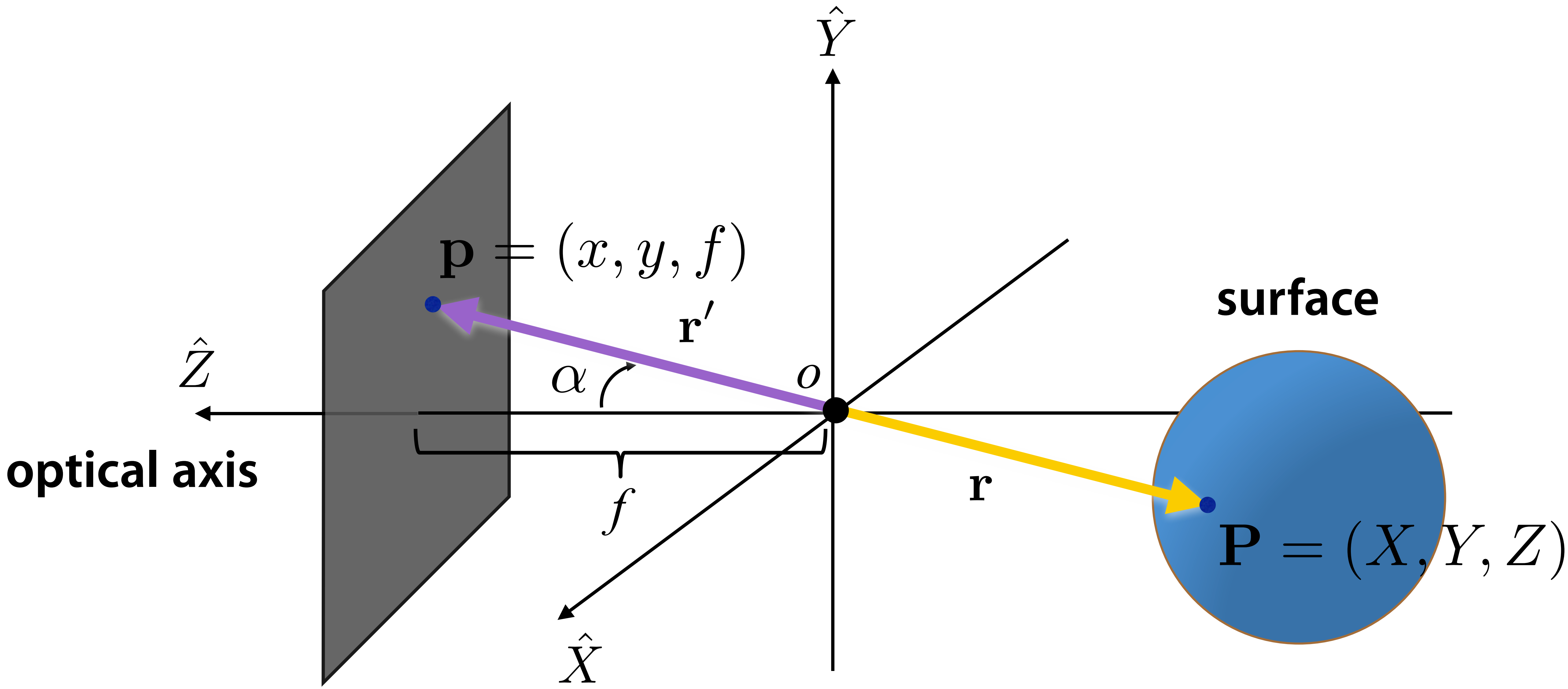
Part 1


Pinhole
Camera
light sensitive
recording surface
object
barrier



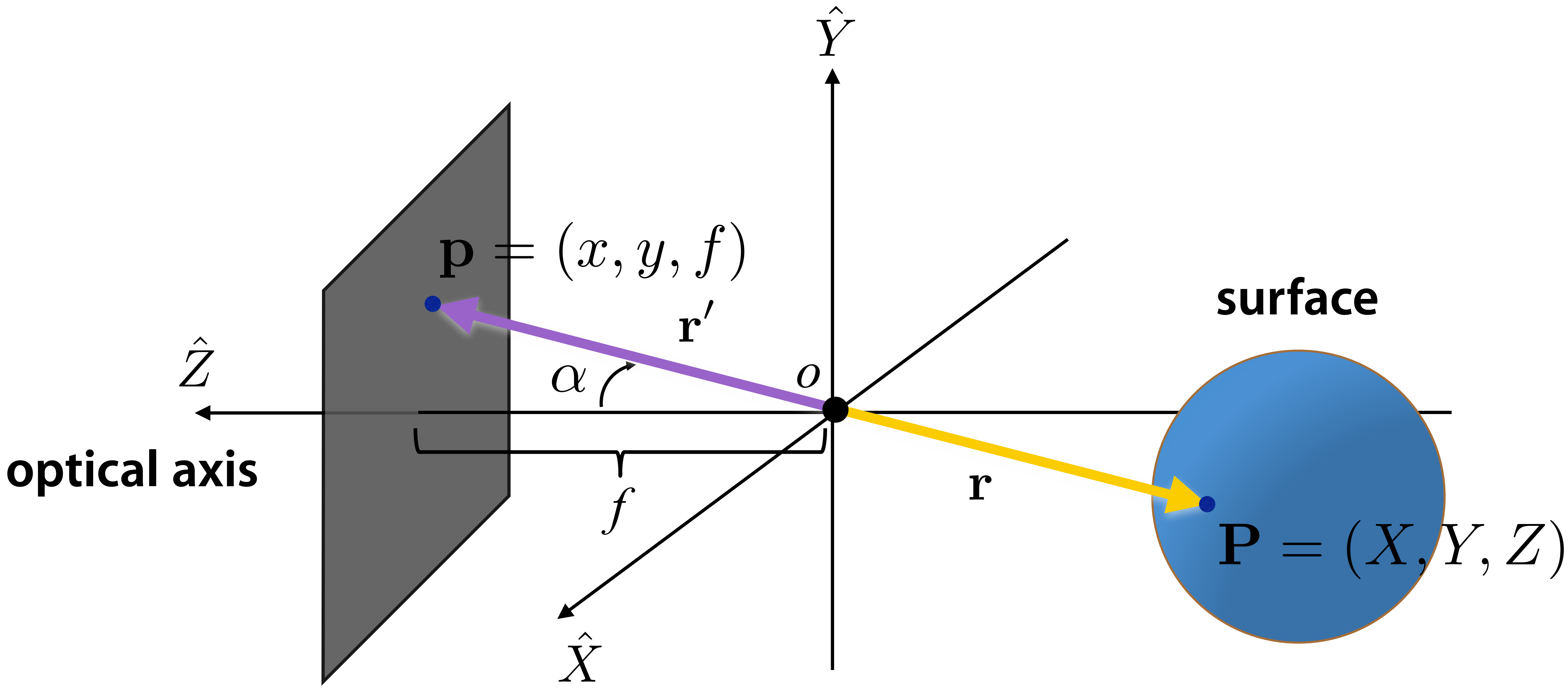
Goal: Relate positions of 3D scene points to their 2D image

Assumption: P and p are collinear



How are $\mathbf{r}$ and $\mathbf{r}'$ related?

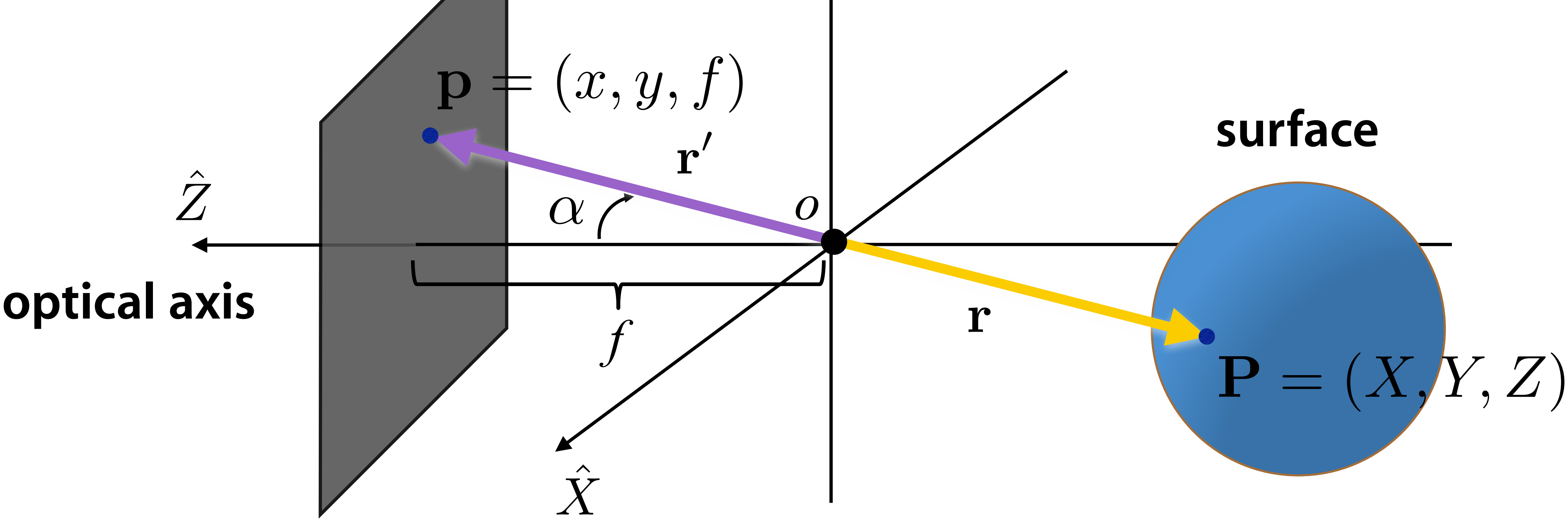
$$\frac{\mathbf{r}'}{\|\mathbf{r}'\|} = -\frac{\mathbf{r}}{\|\mathbf{r}\|}$$



Length of $\mathbf{r}$? $\cos\alpha = -\dfrac{Z}{\|\mathbf{r}\|}$

$$\|\mathbf{r}\| = -\frac{Z}{\cos\alpha} = -Z\sec\alpha$$

Length of $\mathbf{r}'$? $\|\mathbf{r}'\| = f\sec\alpha$

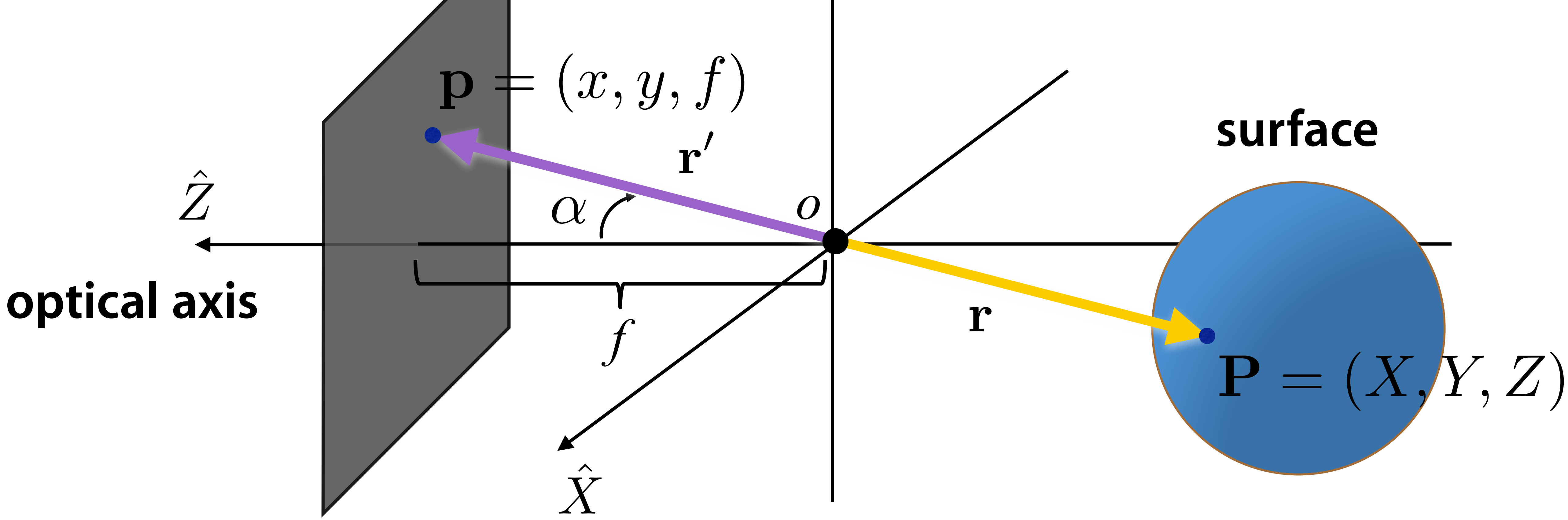


Length of $\mathbf{r}$? $\|\mathbf{r}\| = -Z \sec \alpha$ **Length of $\mathbf{r}'$?** $\|\mathbf{r}'\| = f \sec \alpha$

Recall: $$\frac{\mathbf{r}'}{\|\mathbf{r}'\|} = -\frac{\mathbf{r}}{\|\mathbf{r}\|}$$

$$\frac{1}{f}\mathbf{r}' = \frac{1}{Z}\mathbf{r}$$

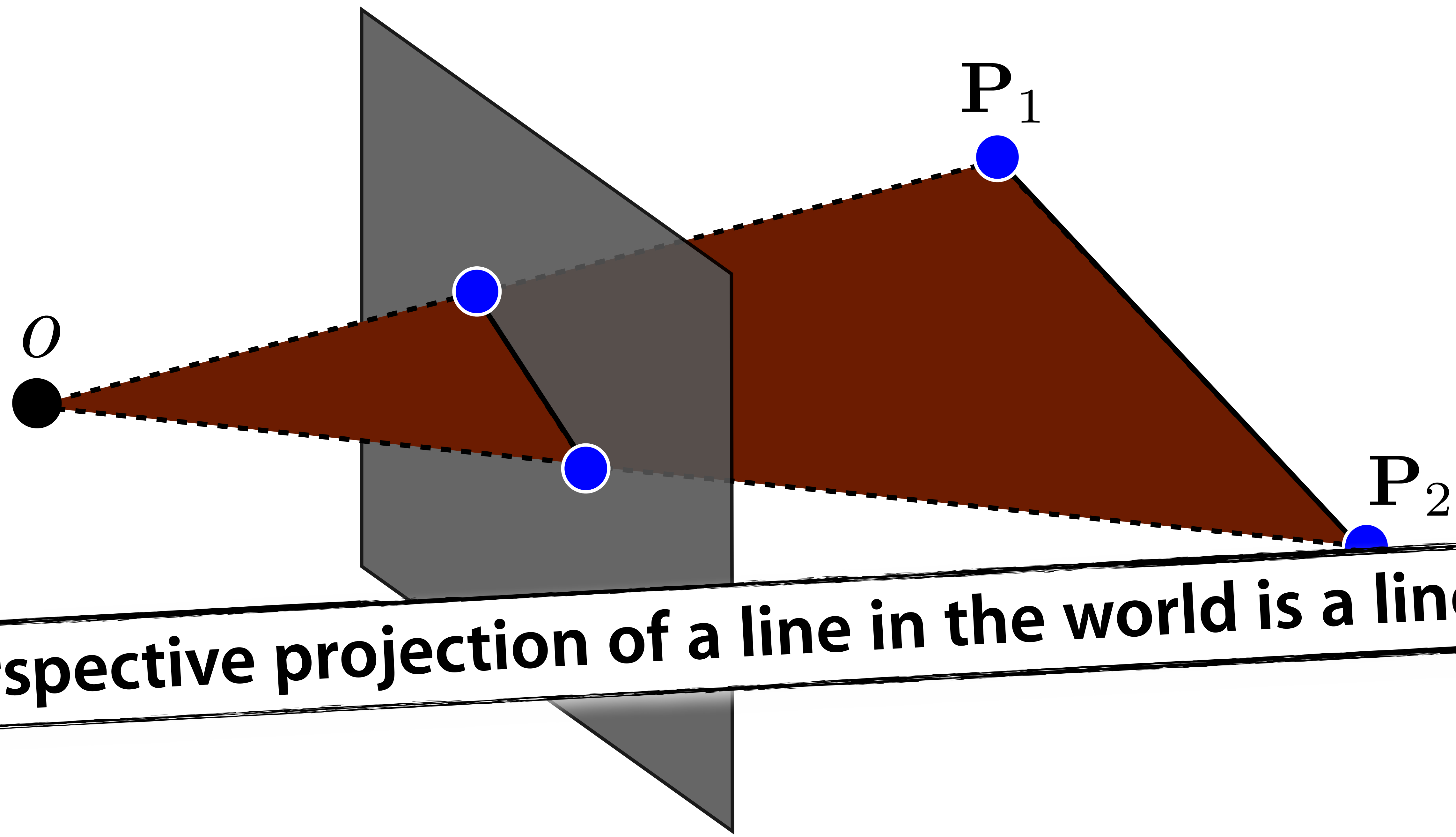
$$\frac{x}{f} = \frac{X}{Z}, \quad \frac{y}{f} = \frac{Y}{Z}$$



$$\frac{x}{f} = \frac{X}{Z}, \quad \frac{y}{f} = \frac{Y}{Z}$$

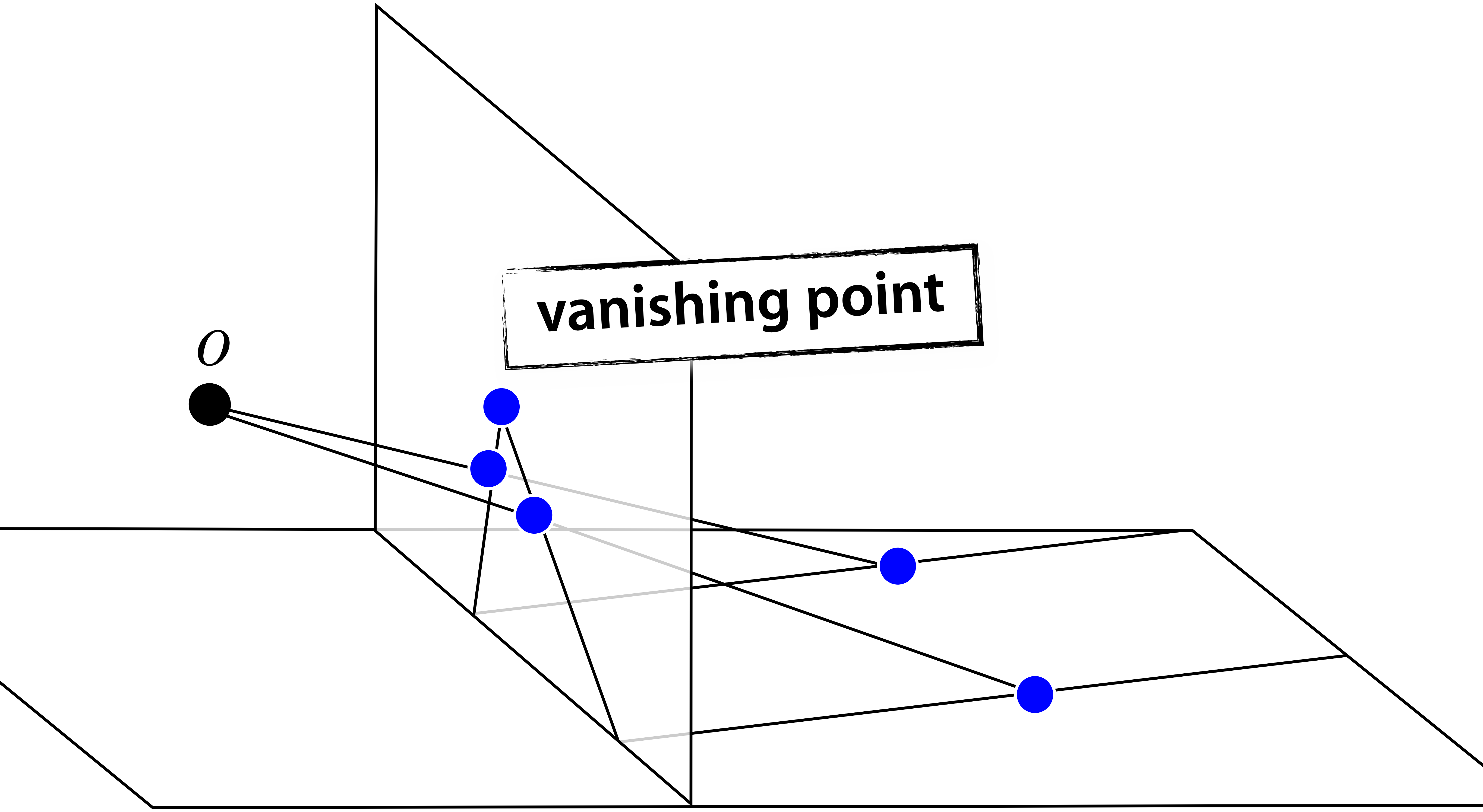
Perspective Projection	$x = f\dfrac{X}{Z}, \quad y = f\dfrac{Y}{Z}$

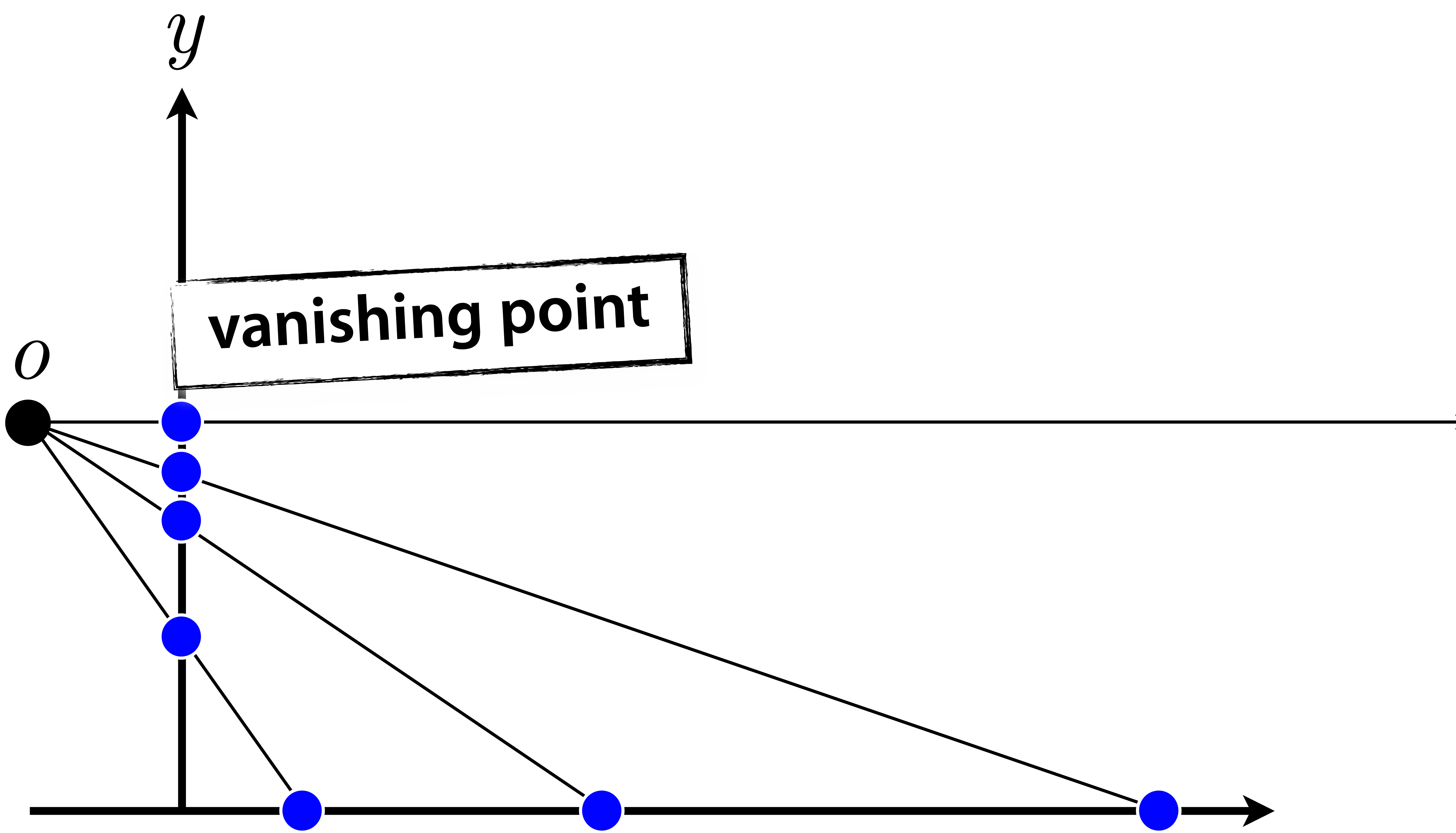
Perspective Projection (flipped)	$x = -f\dfrac{X}{Z}, \quad y = -f\dfrac{Y}{Z}$



Perspective projection of a line in the world is a line

Linear Perspective


vanishing point
O



y
o
vanishing point

Vanishing point derivation

Definitions:

arbitrary point on line

$$\mathbf{P} = (X_1, X_2, X_3)^\top$$

direction vector

$$\mathbf{d} = (d_1, d_2, d_3)^\top$$

line in world space

$$\mathbf{X}(\lambda) = \mathbf{P} + \lambda \mathbf{d}$$

Vanishing point derivation

$$\mathbf{X}(\lambda) = \mathbf{P} + \lambda \mathbf{d}$$

project into image

$$\mathbf{x}(\lambda) = \left(f\frac{X_1 + \lambda d_1}{X_3 + \lambda d_3}, f\frac{X_2 + \lambda d_2}{X_3 + \lambda d_3} \right)$$

$$\mathbf{v} = \lim_{\lambda \to \infty} \left(f\frac{X_1 + \lambda d_1}{X_3 + \lambda d_3}, f\frac{X_2 + \lambda d_2}{X_3 + \lambda d_3} \right)$$

$$v_1 = \lim_{\lambda \to \infty} f \left(\cancel{\frac{X_1}{X_3 + \lambda d_3}} + \frac{\cancel{\lambda} d_1}{\cancel{X_3} + \cancel{\lambda} d_3} \right)$$

Vanishing Point $\quad v_1 = f\dfrac{d_1}{d_3}, \quad v_2 = f\dfrac{d_2}{d_3}$

Vanishing Point

$$v_1 = f\frac{d_1}{d_3}, \quad v_2 = f\frac{d_2}{d_3}$$

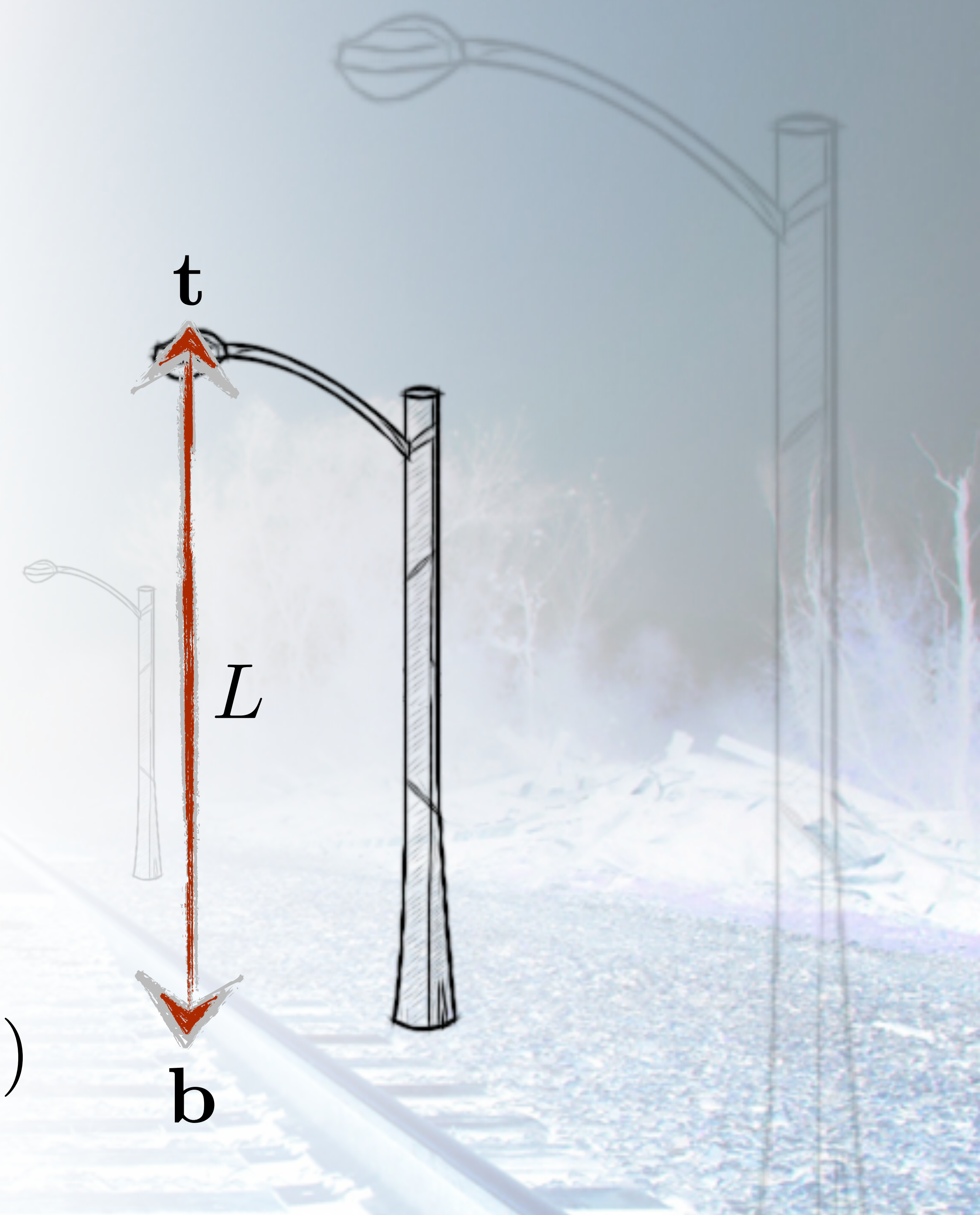
vanishing point depends only on direction vector

∴ lines with the same direction share the same vanishing point

Nearer objects appear larger

Let L **be the lamp height**

h **the camera height**



Lamp bottom: $\mathrm{b} = (X, -h, Z)$

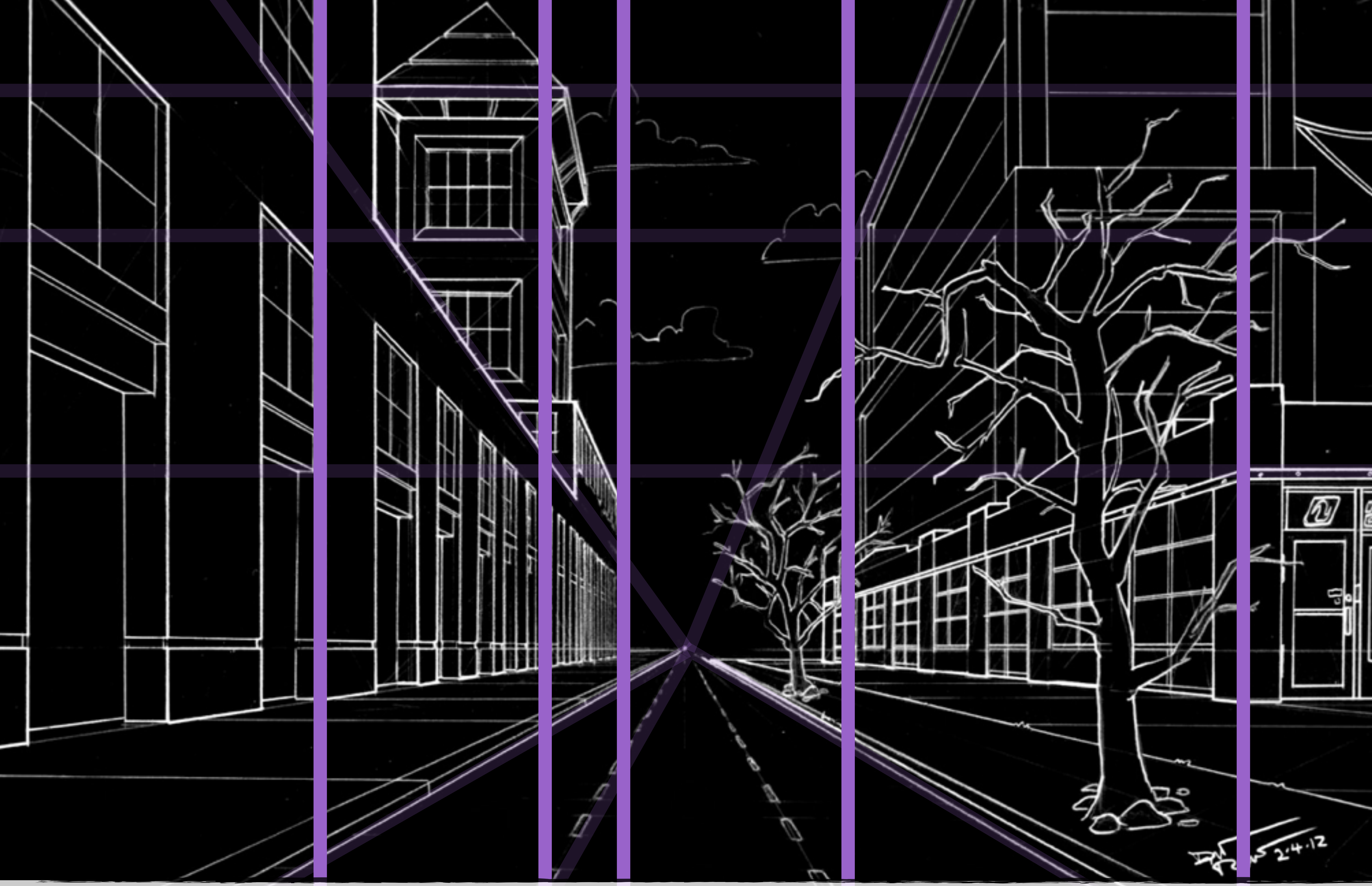
Lamp top: $\mathrm{t} = (X, -h + L, Z)$

Bottom projection: $(fX/Z, -fh/Z)$

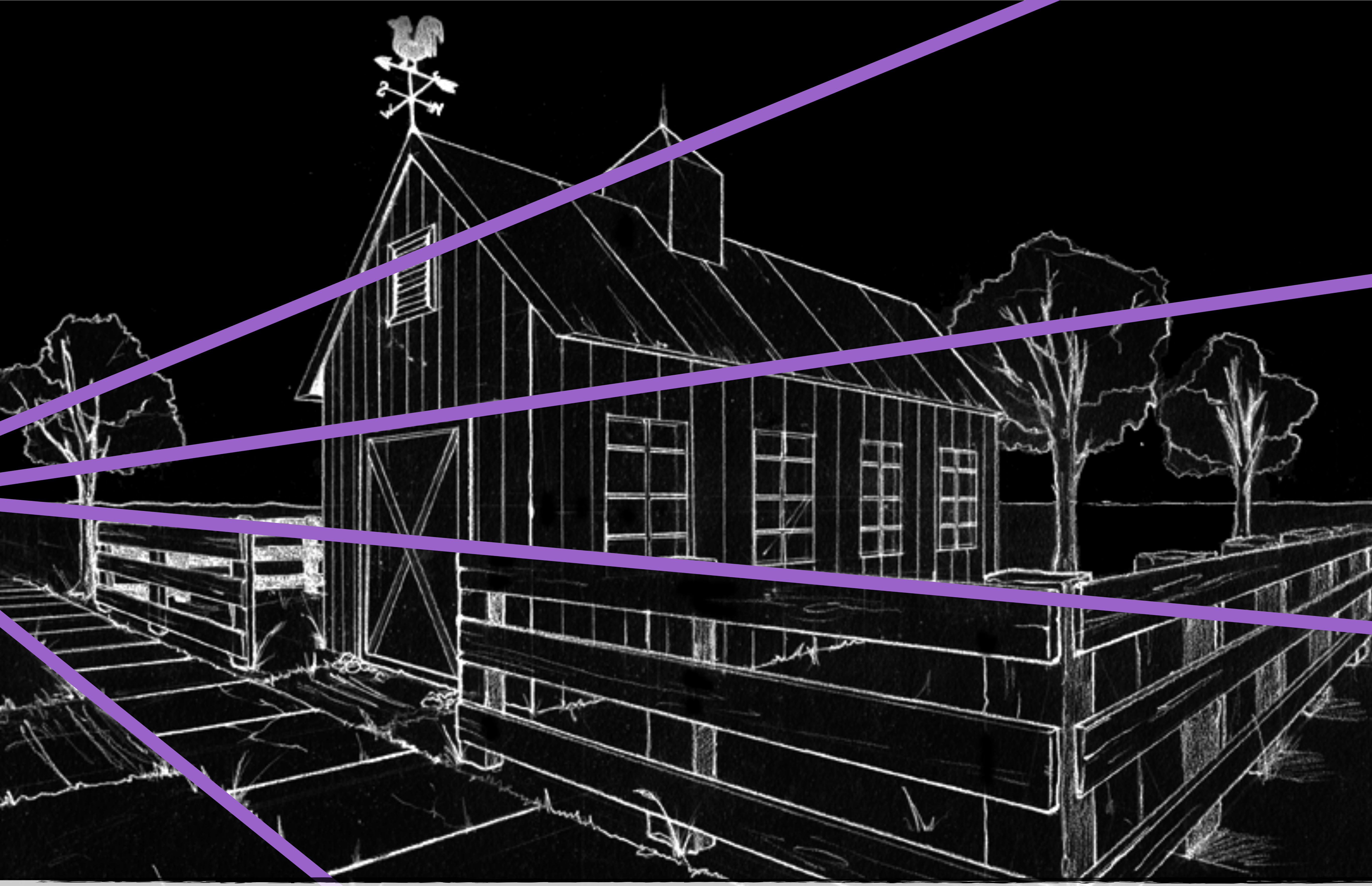
Top projection: $(fX/Z, f(L - h)/Z)$

Image length: $\sqrt{(fX/Z - fX/Z)^2 + (f(L - h)/Z + fh/Z)^2}$

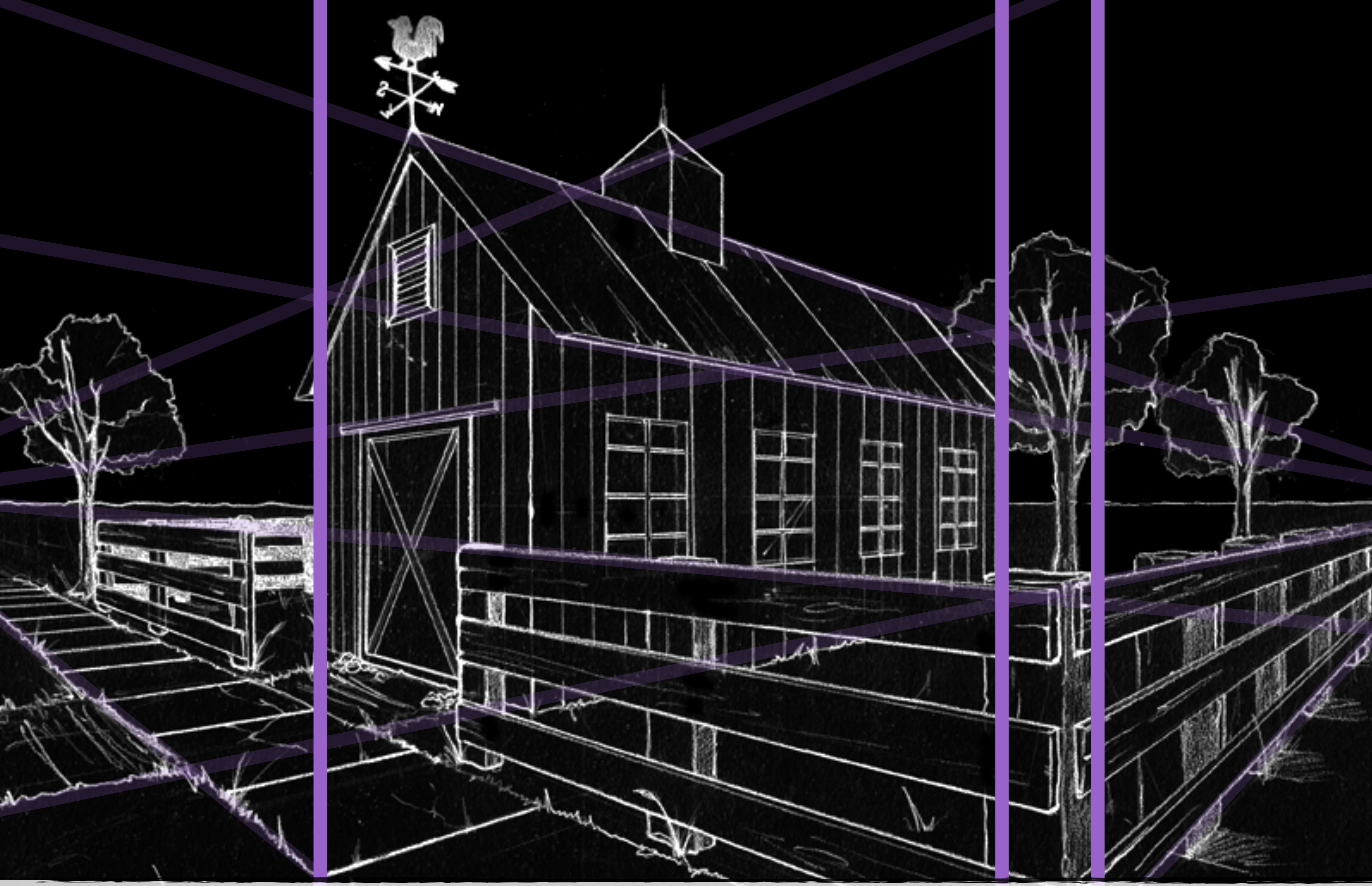
$$\boxed{\textbf{Length in image: } fL/Z}$$



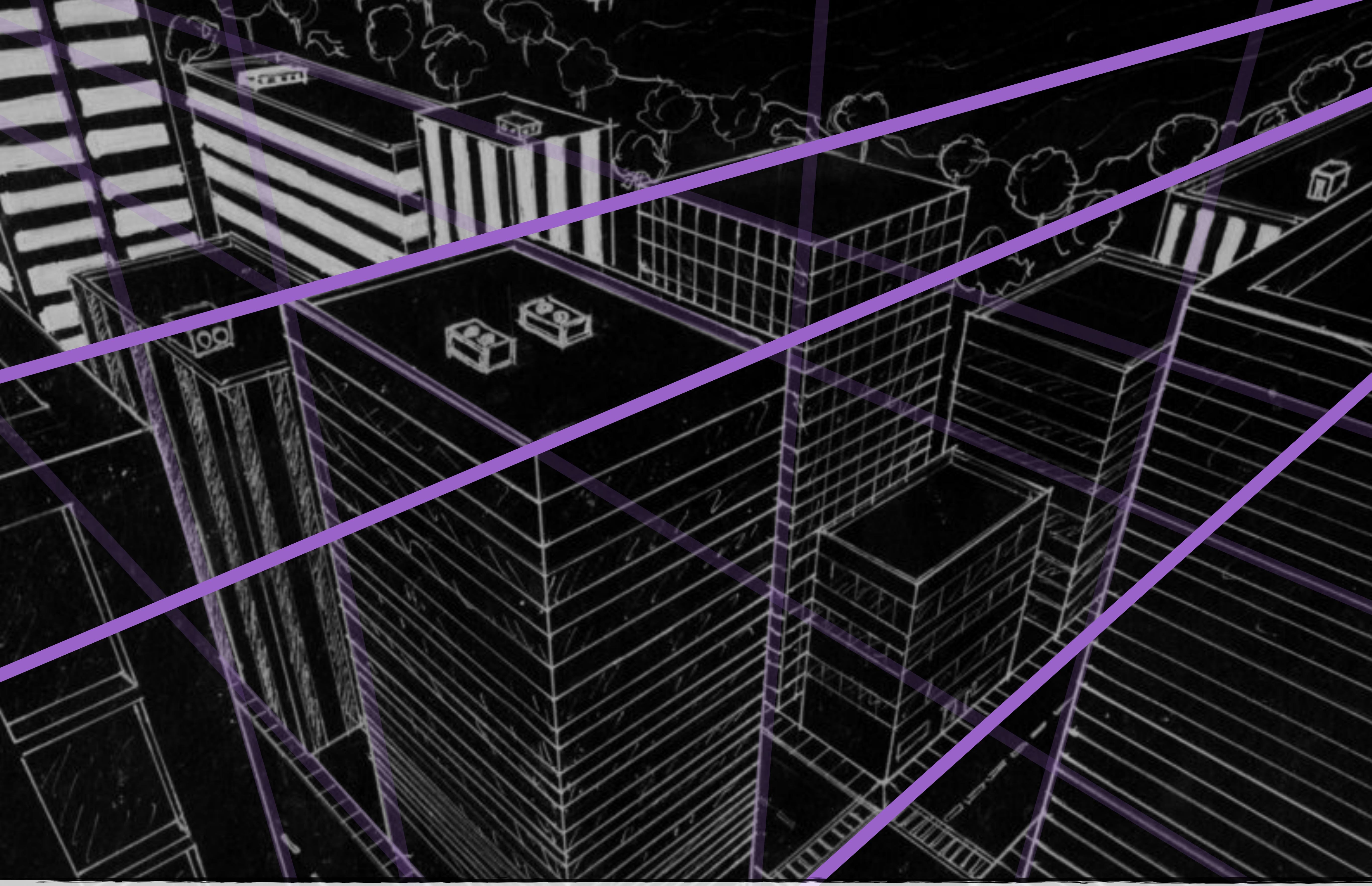
One-point perspective



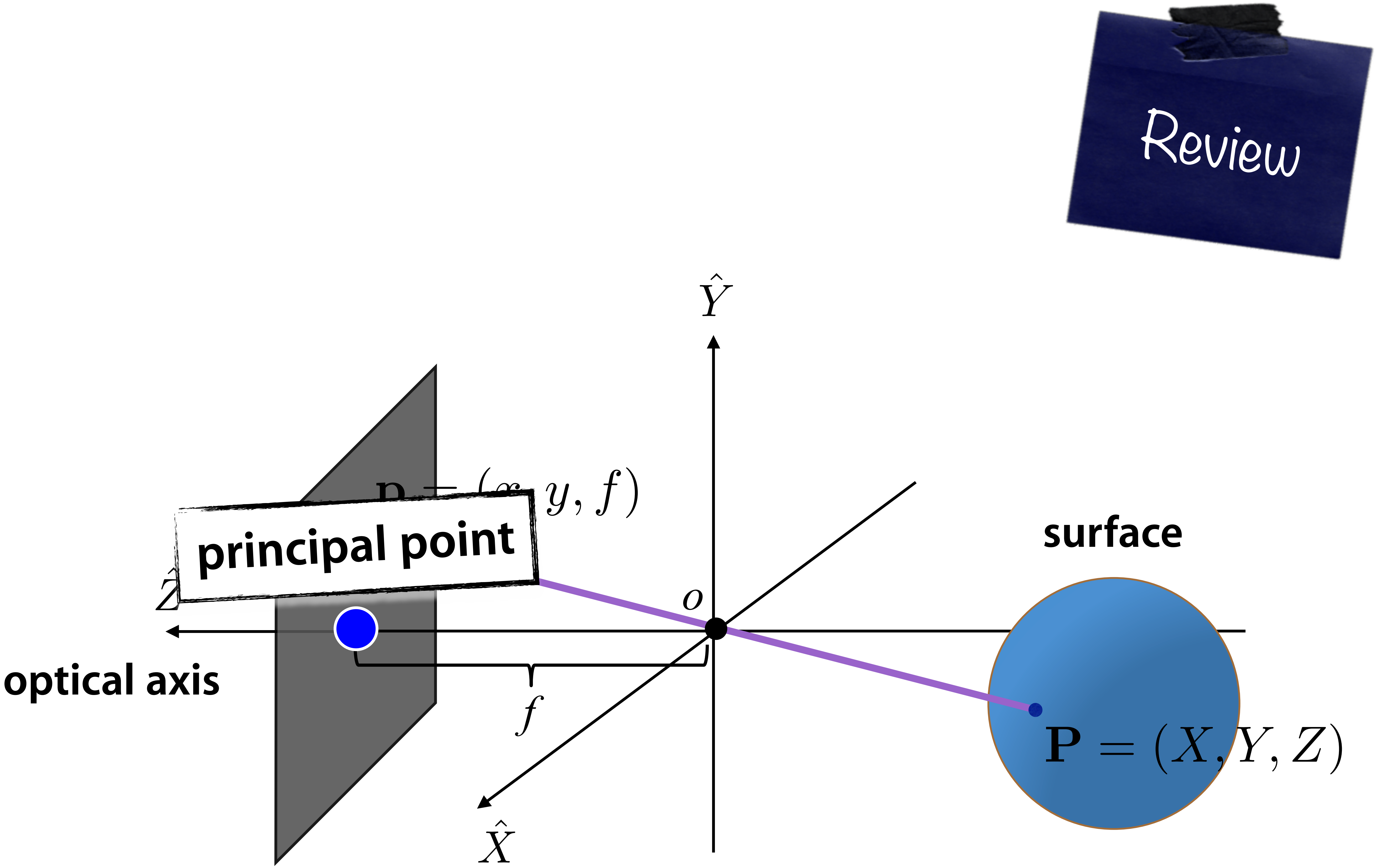
Two-point perspective



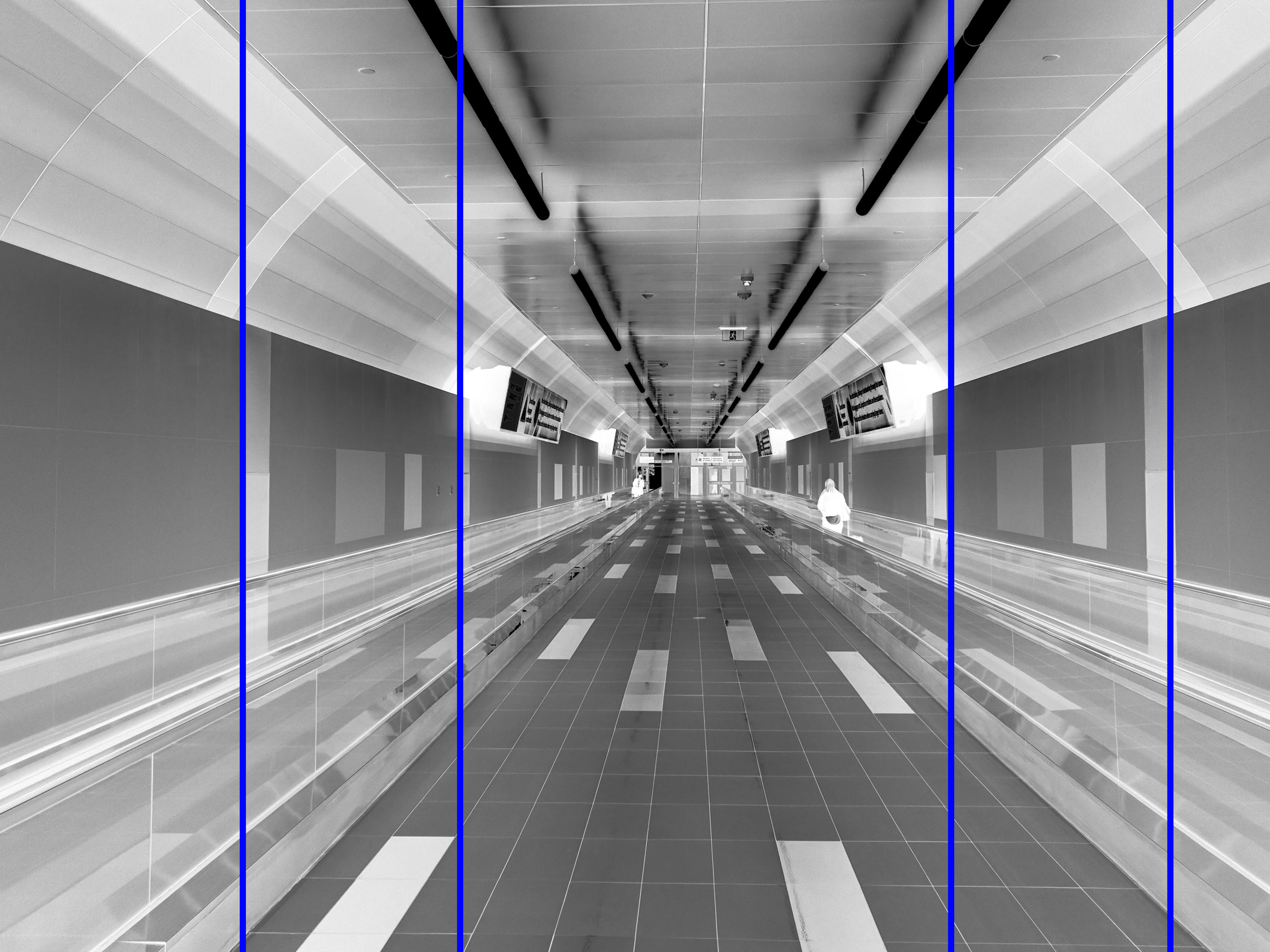
Two-point perspective

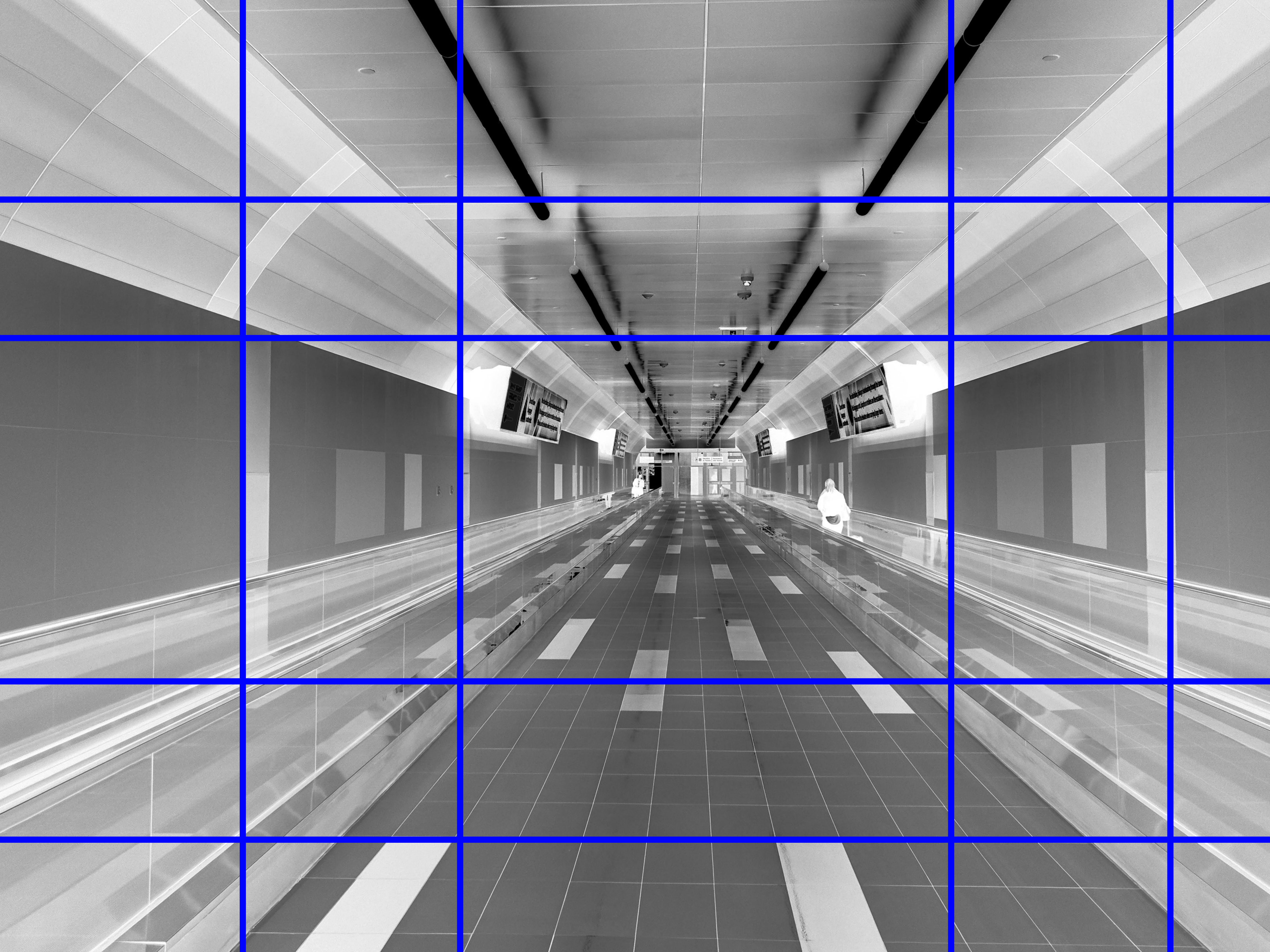


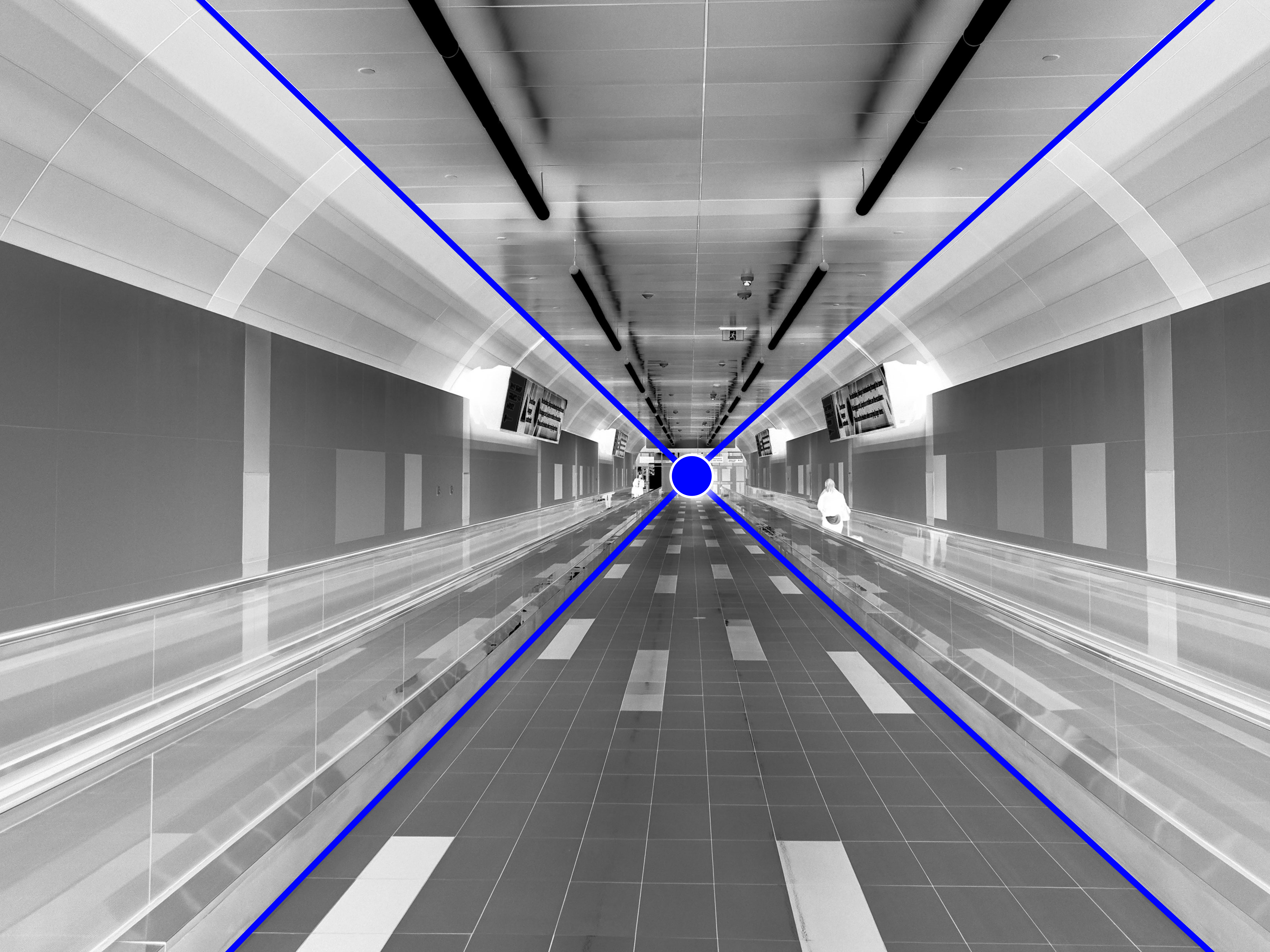
Three-point perspective

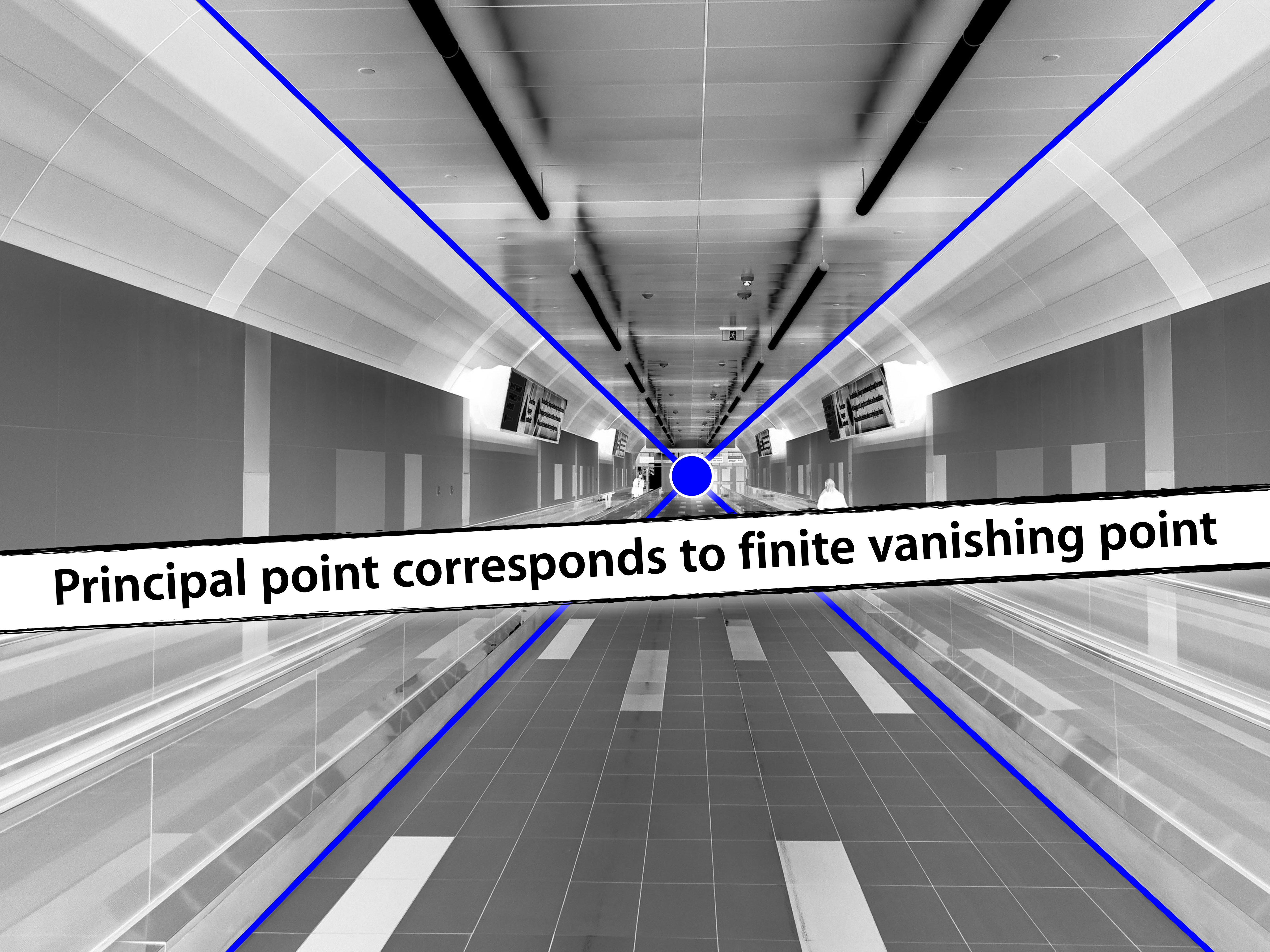


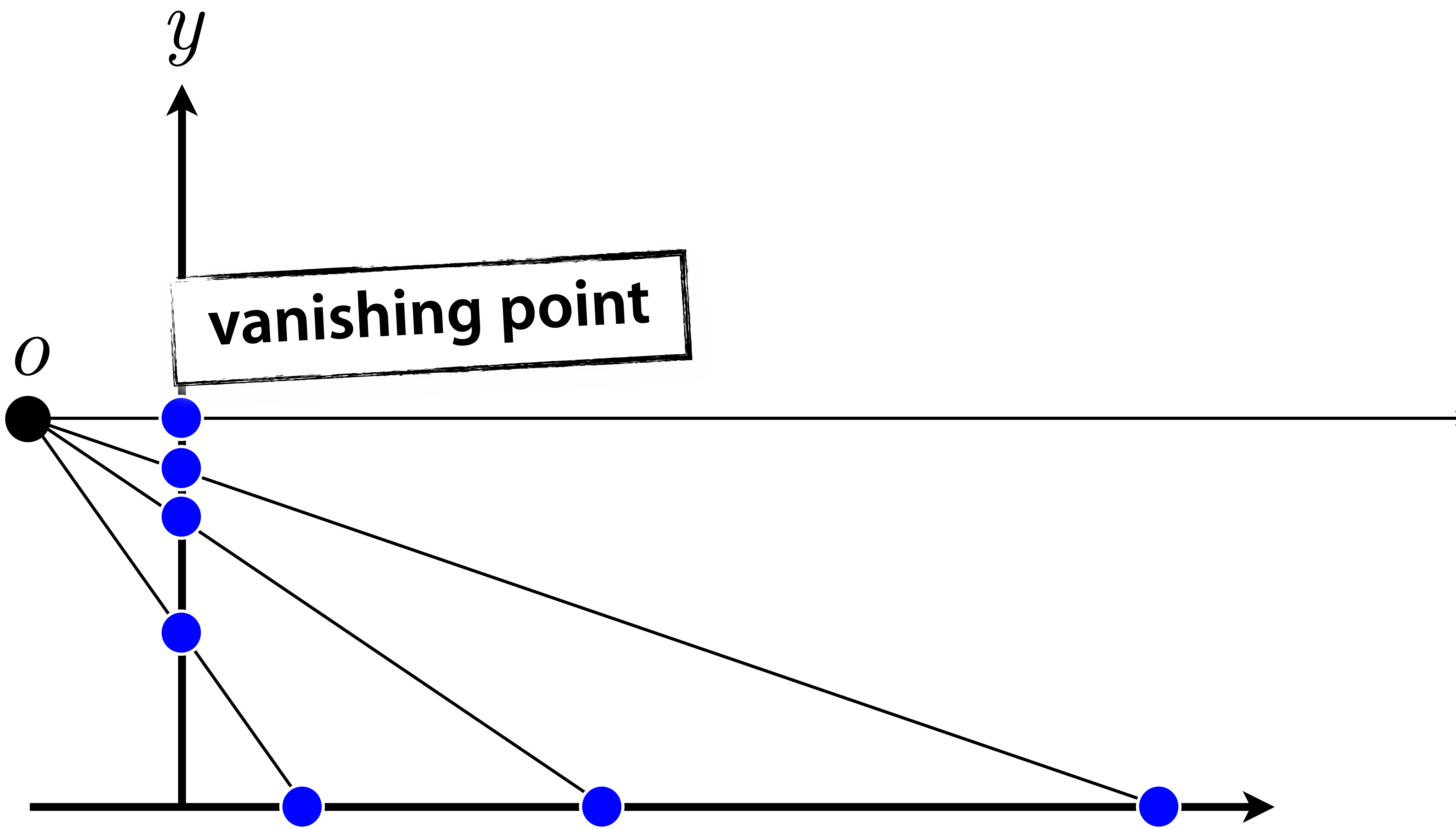
Review
$\hat{Y}$
$\mathbf{p} = (x, y, f)$
principal point
surface
$\hat{Z}$
o
optical axis
f
$\mathbf{P} = (X, Y, Z)$
$\hat{X}$



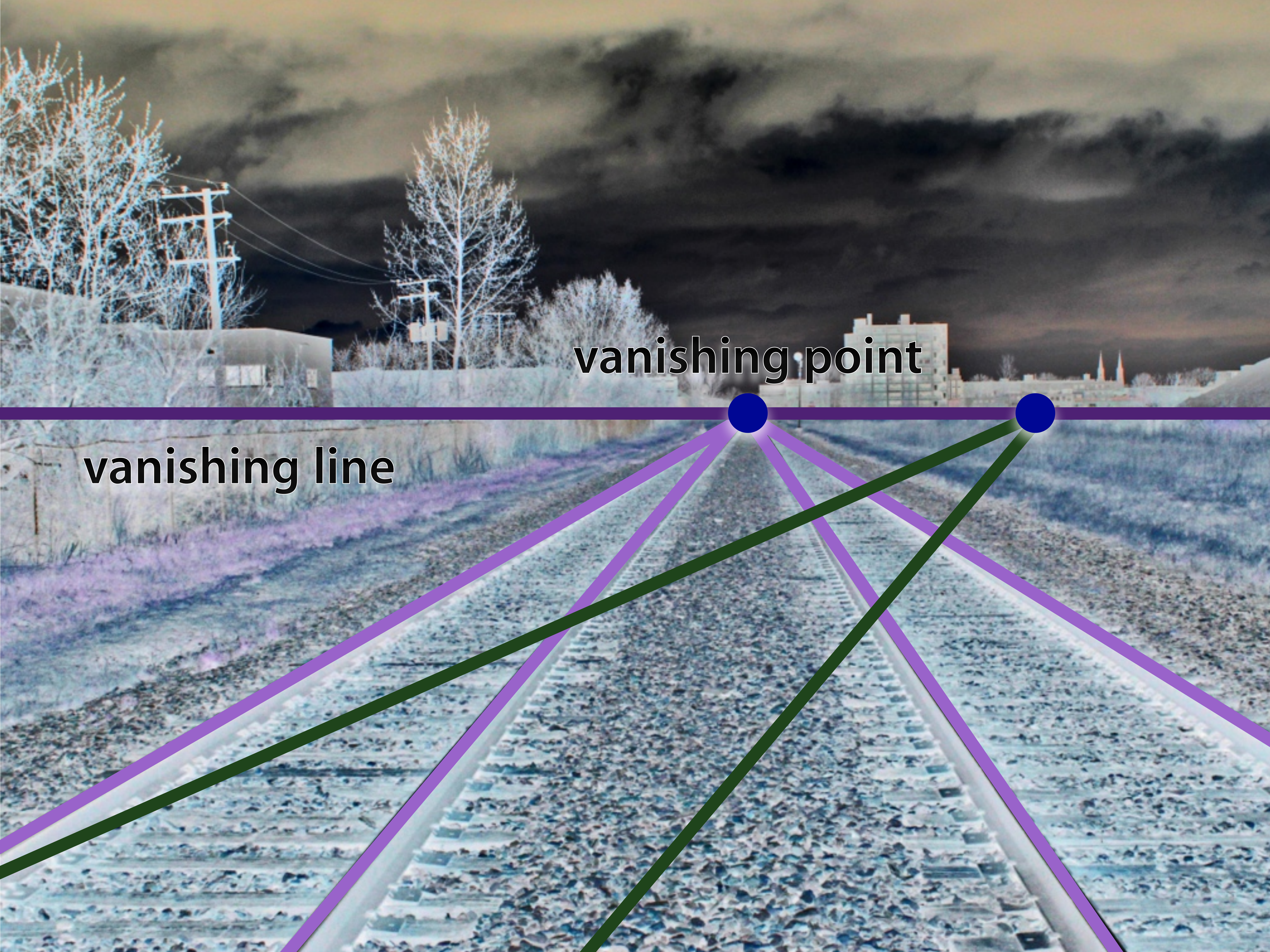


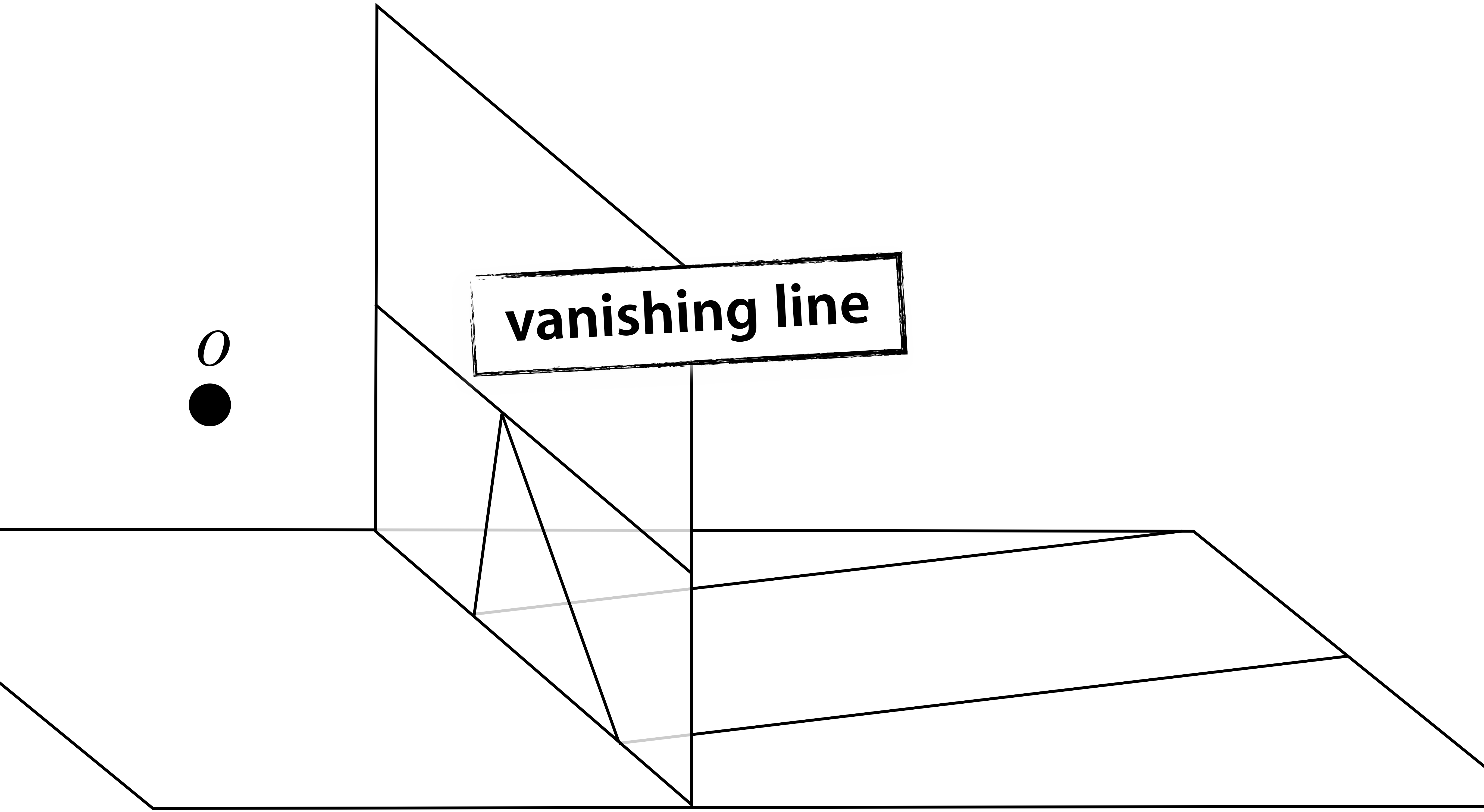



Principal point corresponds to finite vanishing point

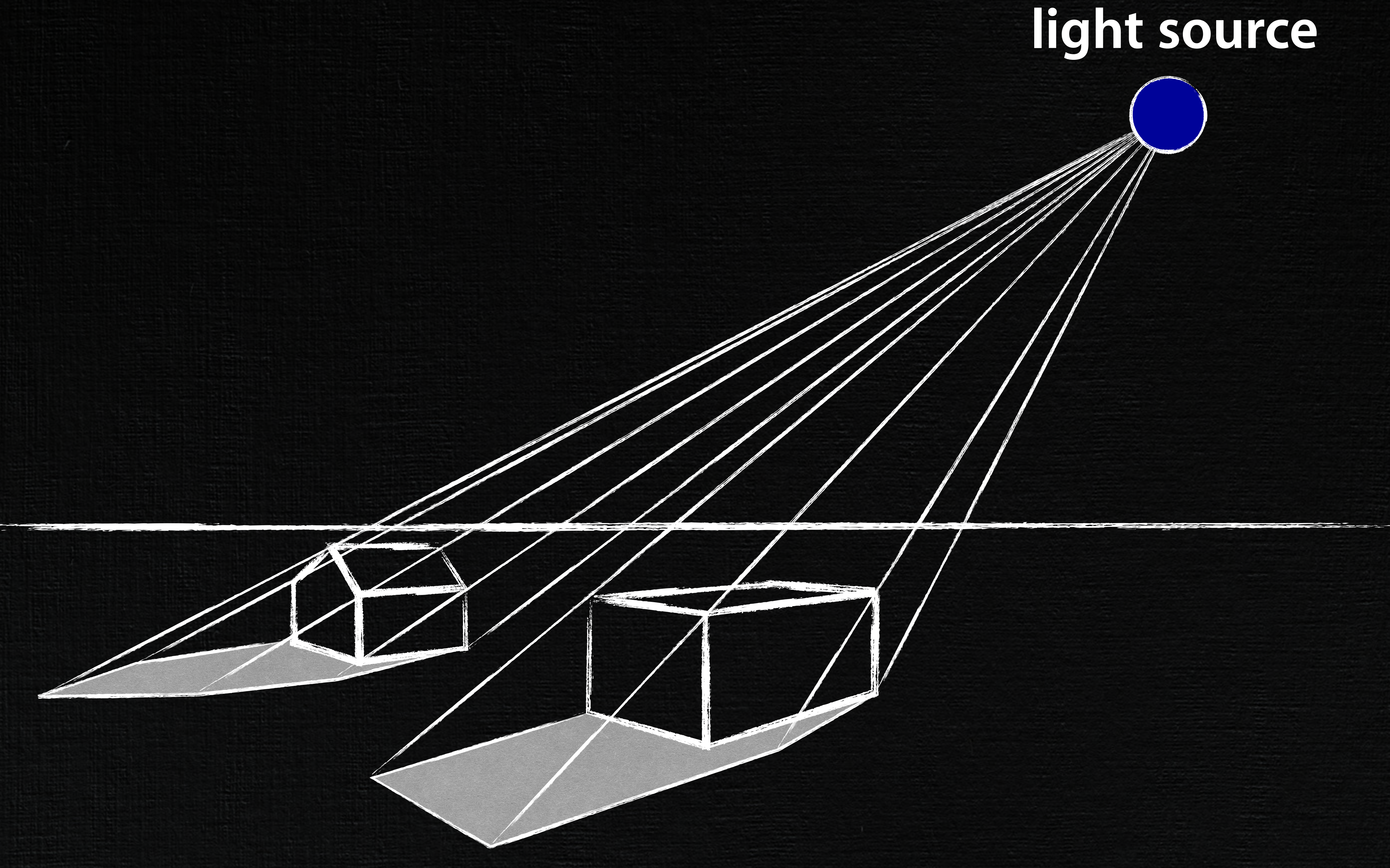


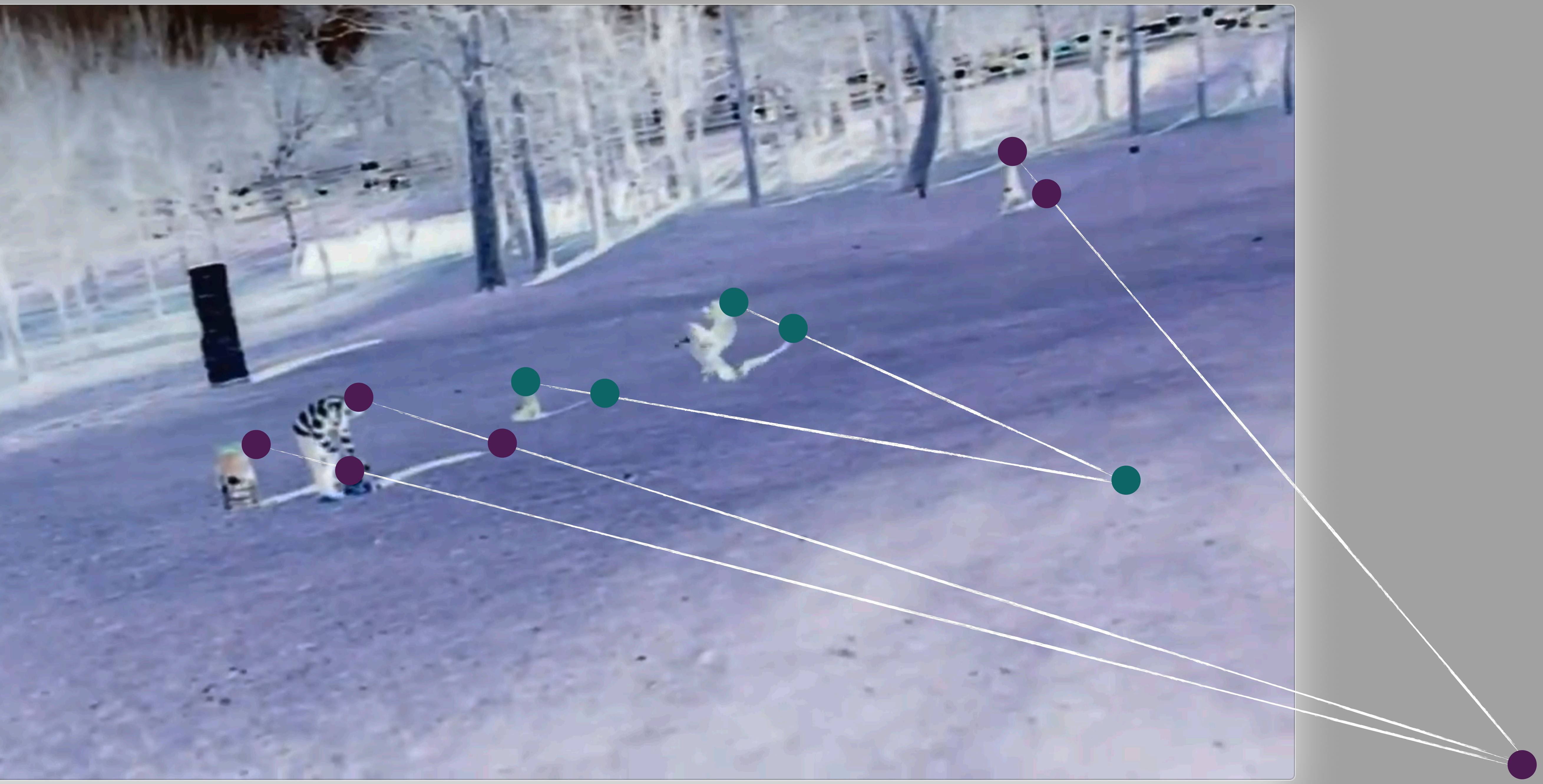
y
vanishing point
o


vanishing point
vanishing line


o
vanishing line

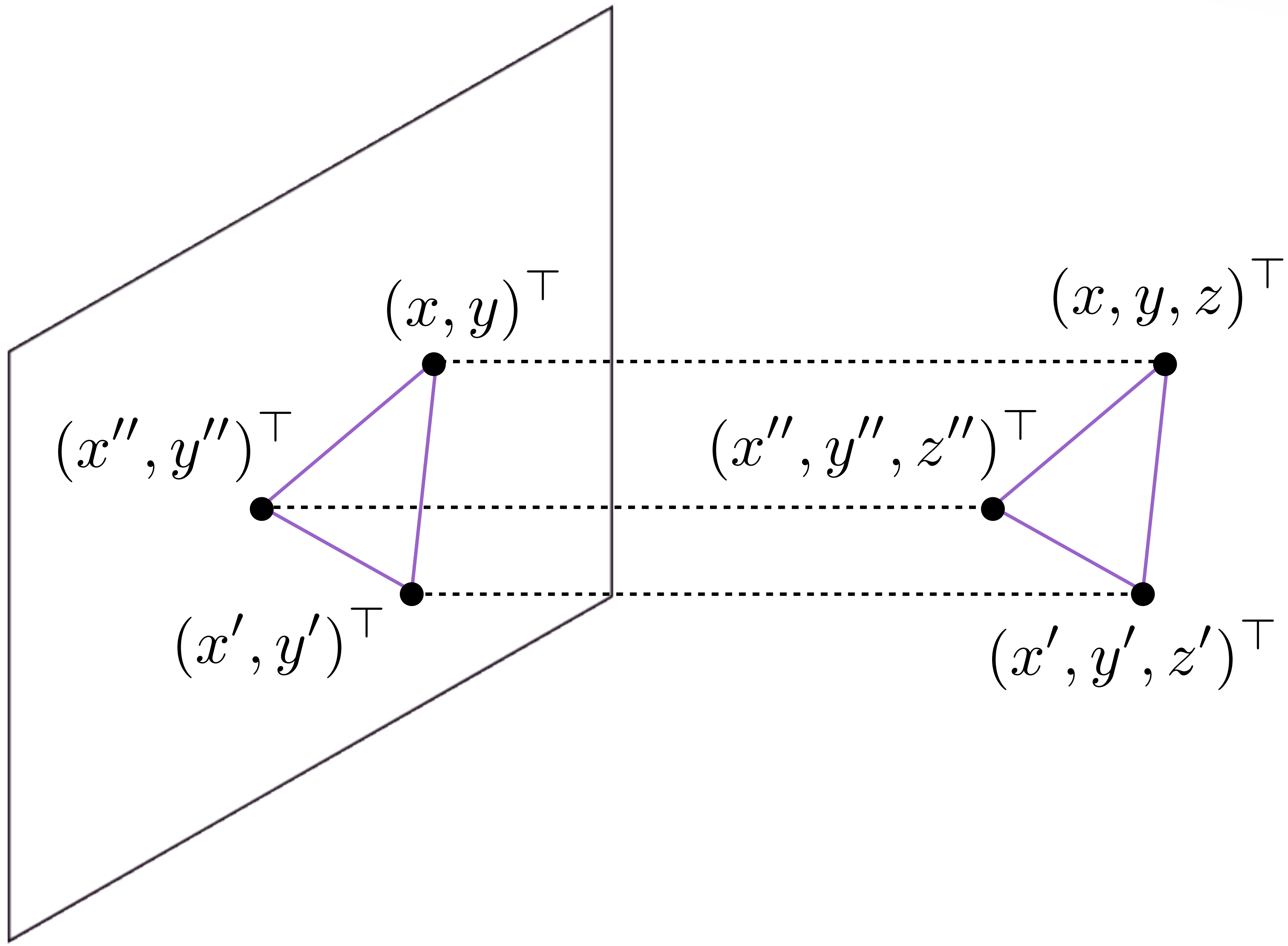

camera is above the balloon gondola

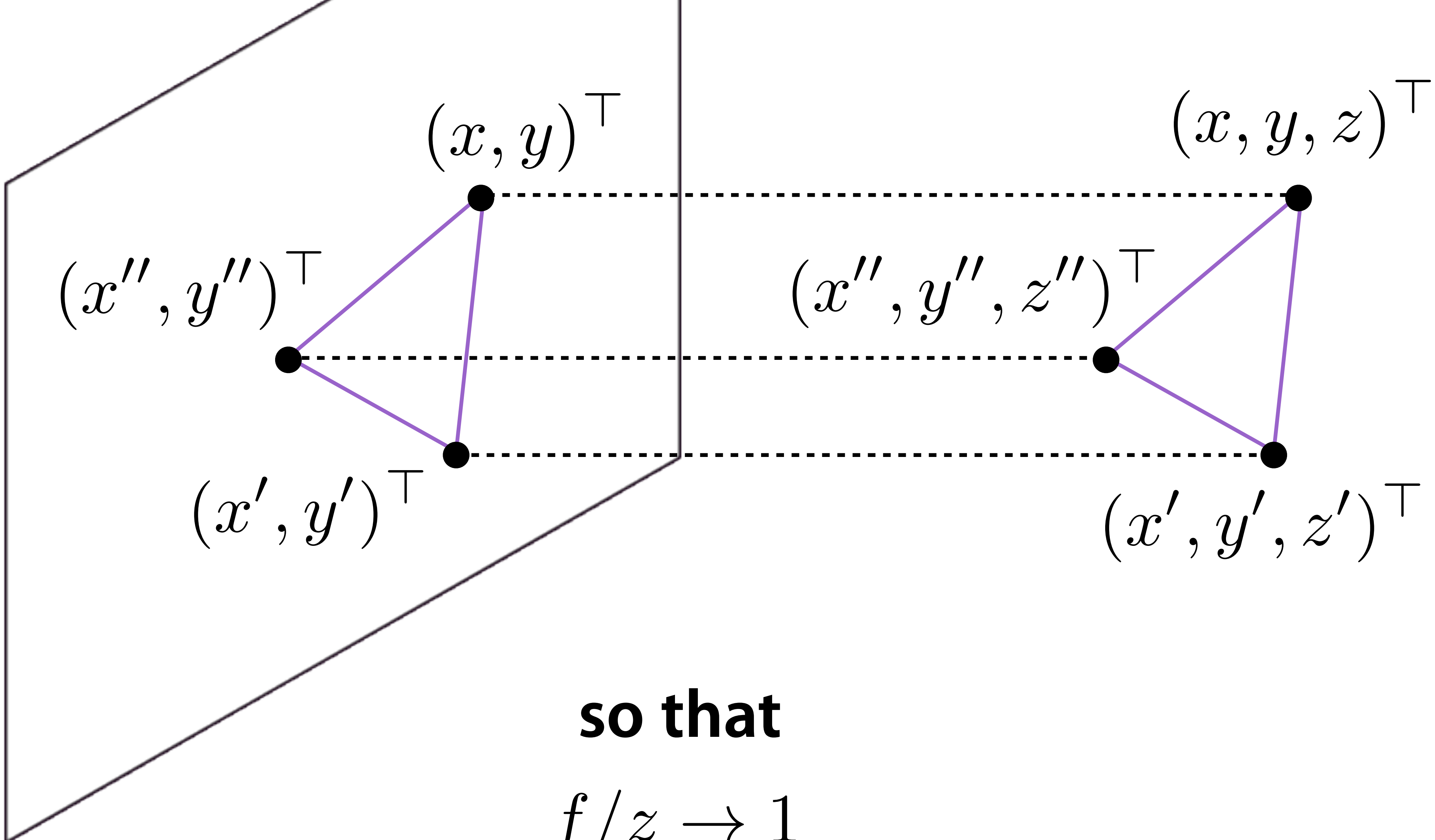

light source



Other projection models

Orthographic





so that

$$f/z \to 1$$

Perspective equations can be approximated as

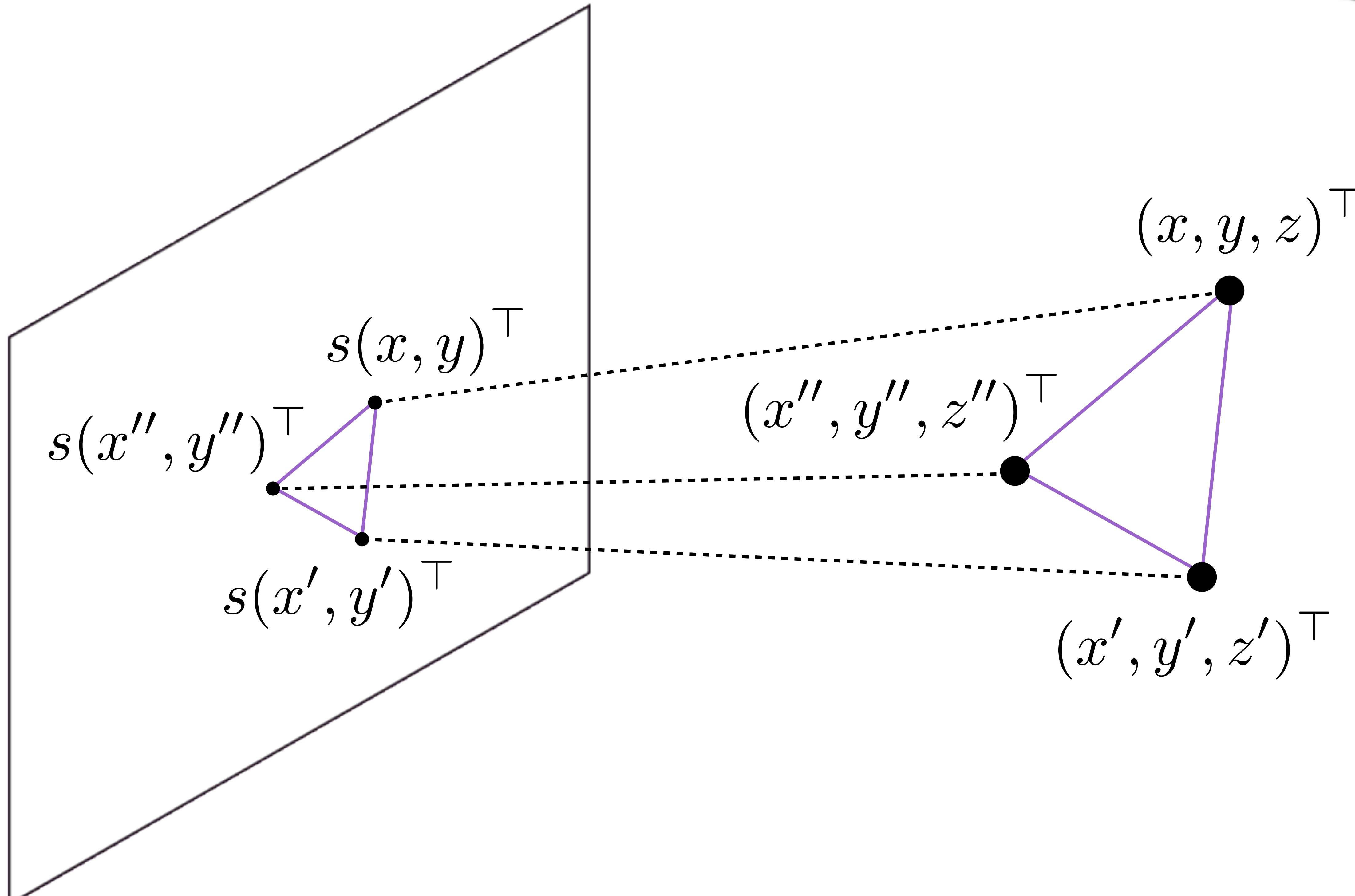
$$x = f\frac{X}{Z} = \frac{f}{Z}X \approx (1)X = X$$

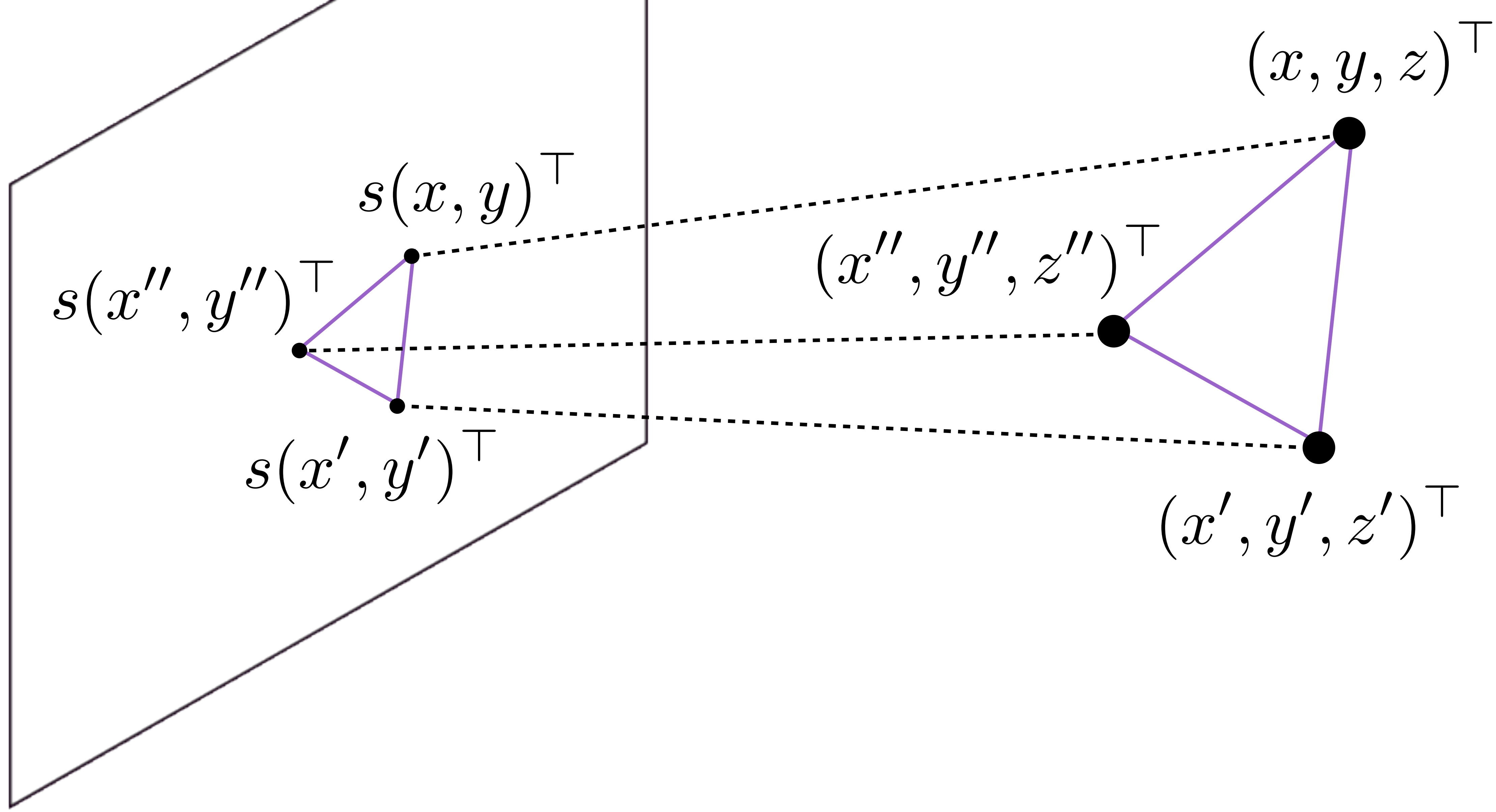
$$y = f\frac{Y}{Z} = \frac{f}{Z}Y \approx (1)Y = Y$$

The Kangxi Emperor’s Inspection Tour (c. 1427-1428)

Wang Hui

Weak Perspective





Assume the distance variation along the optical axis, $d\bar{Z}$, is small compared to the average distance, $\bar{Z}$

Perspective equations can be approximated as

$$x = f\frac{X}{Z} \approx f\frac{X}{\bar{Z}} = sX \qquad y = f\frac{Y}{Z} \approx f\frac{Y}{\bar{Z}} = sY$$