NFC Institute of Engineering & Fertilizer Research Faisalabad

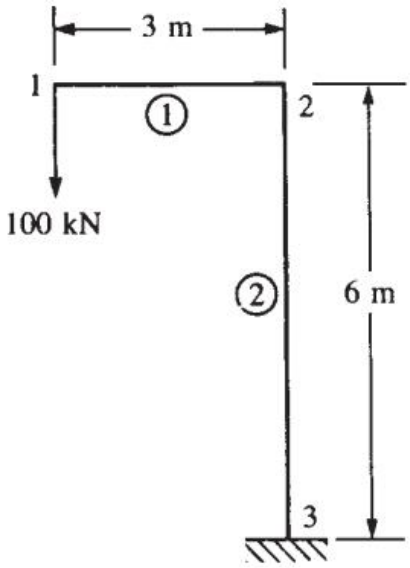
Department of Mechanical Engineering

Course Name: Finite Element Analysis

Examination	Course Code	Session	Semester	Total Marks	Maximum Time
CEP	ME-451	2018	7th	20	1 week

Q. #	Statement	CLO	PLO	Bloom Taxonomy	Marks	KPI
Q. 1		2	5	C5	10	50%

Evaluate the solution by using MATLAB code for given problem. Evaluate nodal dispacements, elemental forces and reaction forces



$E = 210$ GPa
$A = 8 \times 10^{-2}$ m^2
$I = 1.2 \times 10^{-4}$ m^4

Figure P5–12

A rigid plane frame is defined here as a series of beam elements rigidly connected to each other; that is, the original angles made between elements at their joints remain unchanged after the deformation due to applied loads or applied displacements.

K matrix for rigid plane frame

$$[k] = \frac{E}{L} \times \begin{bmatrix} AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S & -\left(AC^2 + \frac{12I}{L^2}S^2\right) & -\left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S \\ & AS^2 + \frac{12I}{L^2}C^2 & \frac{6I}{L}C & -\left(A - \frac{12I}{L^2}\right)CS & -\left(AS^2 + \frac{12I}{L^2}C^2\right) & \frac{6I}{L}C \\ & & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ & & & AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & \frac{6I}{L}S \\ & & & & AS^2 + \frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ \text{Symmetry} & & & & & 4I \end{bmatrix}$$

T matrix for Rigid plane frame

$$[T] = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

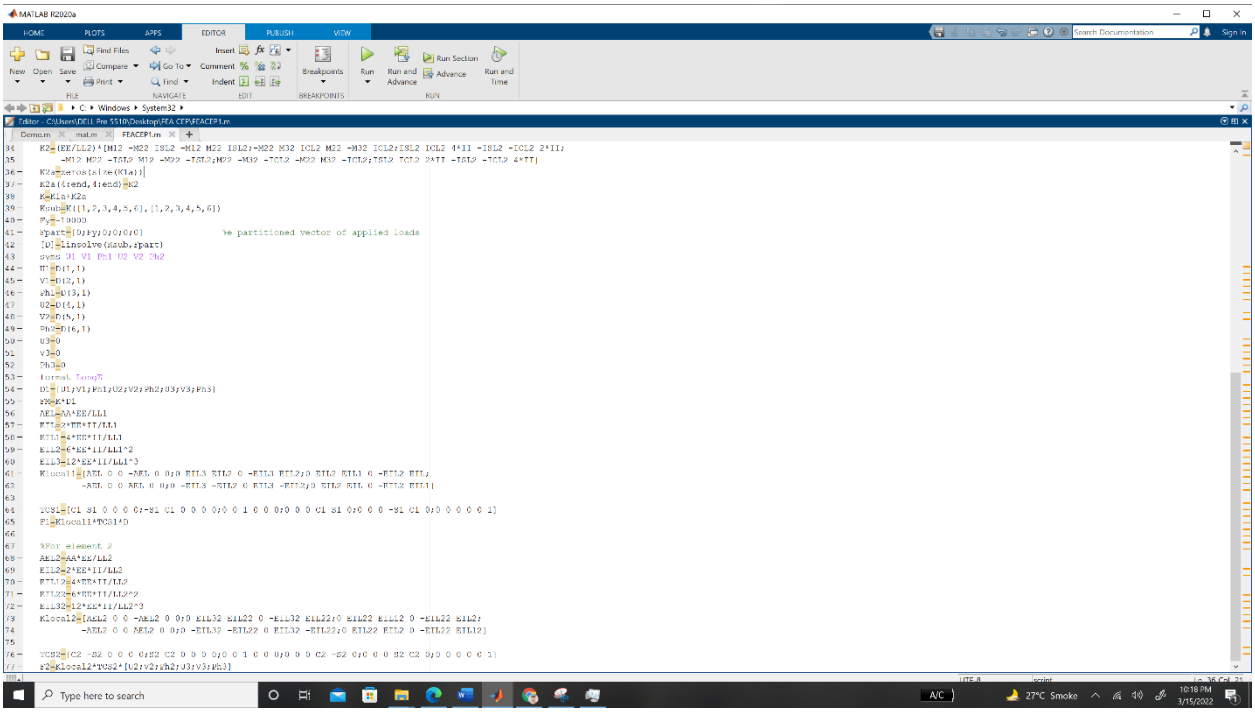
1.1 MATLAB Editor Code for Above Question

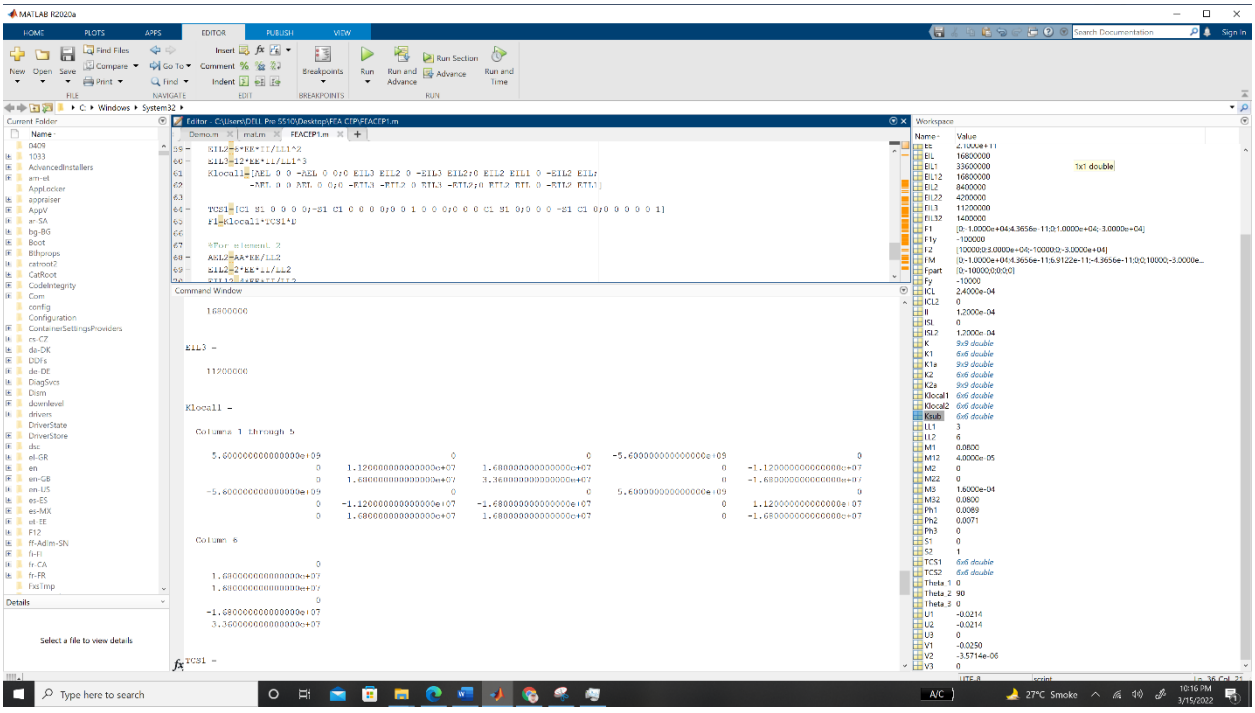
```
EE=210e9;                 %Modulus of elasticity

AA=80e-3;                 %Area of cross section for all elements m2

II=1.2e-4;                %Area moment of inertia for all elements m4

LL1=3;                    %Length of element 1 m

LL2=6;                    %Length of element 2 m

Theta_1=0;                %Angle in degree

Theta_2=90;               %Angle in degree

F1y=-1e5;                 %Applied load(down at node 1)

d3x=0;                    %x-displacement 3 is zero

d3y=0;                    %y-displacement at node 3 is zero

Theta_3=0;                %angular displacement at node 3 is zero

C1=cosd(Theta_1);         %Cosine of angle between local x and global x for element 1

S1=sind(Theta_1);         %Sine of angle between local x and global x for element 1

C2=cosd(Theta_2);         %Cosine of angle between local x and global x for element 2

S2=sind(Theta_2);         %Sine of angle between local x and global x for element 2
```

```
M1=(AA*C1^2)+((12*II*S1^2)/LL1^2)
M2=(AA-((12*II)/LL1^2))*C1*S1
M3=(AA*S1^2)+((12*II*C1^2)/LL1^2)
ISL=6*II*S1/LL1
ICL=6*II*C1/LL1

K1=(EE/LL1)*[M1 M2 -ISL -M1 -M2 -ISL;M2 M3 ICL -M2 -M3 ICL;-ISL ICL 4*II ISL -
ICL 2*II;
   -M1 -M2 ISL M1 M2 ISL;-M2 -M3 -ICL M2 M3 -ICL;-ISL ICL 2*II ISL -ICL 4*II]
```

```
K1a=K1
K1a(9,9)=0


M12=(AA*C2^2)+((12*II*S2^2)/LL2^2)
M22=(AA-((12*II)/LL2^2))*(C2*S2)
M32=(AA*S2^2)+((12*II*C2^2)/LL2^2)
ISL2=6*II*S2/LL2
ICL2=(6*II*C2)/LL2

K2=(EE/LL2)*[M12 -M22 ISL2 -M12 M22 ISL2;-M22 M32 ICL2 M22 -M32 ICL2;ISL2
ICL2 4*II -ISL2 -ICL2 2*II;
   -M12 M22 -ISL2 M12 -M22 -ISL2;M22 -M32 -ICL2 -M22 M32 -ICL2;ISL2 ICL2 2*II -
ISL2 -ICL2 4*II]

K2a=zeros(size(K1a))

K2a(4:end,4:end)=K2

K=K1a+K2a

Ksub=K([1,2,3,4,5,6],[1,2,3,4,5,6])
Fy=-10000
Fpart=[0;Fy;0;0;0;0]            %e partitioned vector of applied loads
[D]=linsolve(Ksub,Fpart)
syms U1 V1 Ph1 U2 V2 Ph2
U1=D(1,1)
V1=D(2,1)
Ph1=D(3,1)
U2=D(4,1)
V2=D(5,1)
Ph2=D(6,1)
U3=0
V3=0
```

```
Ph3=0
format LongE

D1=[U1;V1;Ph1;U2;V2;Ph2;U3;V3;Ph3]

FM=K*D1
%For element 1

AEL=AA*EE/LL1
EIL=2*EE*II/LL1
EIL1=4*EE*II/LL1
EIL2=6*EE*II/LL1^2
EIL3=12*EE*II/LL1^3
Klocal1=[AEL 0 0 -AEL 0 0;0 EIL3 EIL2 0 -EIL3 EIL2;0 EIL2 EIL1 0 -EIL2 EIL;
    -AEL 0 0 AEL 0 0;0 -EIL3 -EIL2 0 EIL3 -EIL2;0 EIL2 EIL 0 -EIL2 EIL1]

TCS1=[C1 S1 0 0 0 0;-S1 C1 0 0 0 0;0 0 1 0 0 0;0 0 0 C1 S1 0;0 0 0 -S1 C1 0;0 0 0 0 0 1]
F1=Klocal1*TCS1*D

%For element 2
AEL2=AA*EE/LL2
EIL2=2*EE*II/LL2
EIL12=4*EE*II/LL2
EIL22=6*EE*II/LL2^2
EIL32=12*EE*II/LL2^3

Klocal2=[AEL2 0 0 -AEL2 0 0;0 EIL32 EIL22 0 -EIL32 EIL22;0 EIL22 EIL12 0 -EIL22
EIL2;
    -AEL2 0 0 AEL2 0 0;0 -EIL32 -EIL22 0 EIL32 -EIL22;0 EIL22 EIL2 0 -EIL22 EIL12]

TCS2=[C2 -S2 0 0 0 0;S2 C2 0 0 0 0;0 0 1 0 0 0;0 0 0 C2 -S2 0;0 0 0 S2 C2 0;0 0 0 0 0 1]

F2=Klocal2*TCS2*[U2;V2;Ph2;U3;V3;Ph3]
```


MATLAB R2020a
Editor - C:\Users\DELL Pro 5510\Desktop\FEA CTP\FEACTP1.m
Demo.m
mat.m
FEACTP1.m
K2=(EE/LL2)*[M12 -M22 ISL2 -M12 M22 ISL2;-M22 M32 ICL2 M22 -M32 ICL2;ISL2 ICL2 4*II -ISL2 -ICL2 2*II;
-M12 M22 -ISL2 M12 -M22 -ISL2;M22 -M32 -ICL2 -M22 M32 -ICL2;ISL2 ICL2 2*II -ISL2 -ICL2 4*II]
K2a=zeros(size(K1a))
K2a(11end,4:end)=K2
K=K1a+K2a
Ksub=K([1,2,3,4,5,6],[1,2,3,4,5,6])
Fy=-10000
Fpart=[0;Fy;0;0;0;0] %e partitioned vector of applied loads
[D]=linsolve(Ksub,Fpart)
syms U1 V1 Ph1 U2 V2 Ph2
U1=D(1,1)
V1=D(2,1)
Ph1=D(3,1)
U2=D(4,1)
V2=D(5,1)
Ph2=D(6,1)
U3=0
V3=0
Ph3=0
Normal Tangt
D=[U1;V1;Ph1;U2;V2;Ph2;U3;V3;Ph3]
F=K*D1
AEL=AA*EE/LL1
ETL2=2*EE*II/LL1
ETL1=4*EE*II/LL1
EIL2=6*EE*II/LL1^2
EIL3=12*EE*II/LL1^3
Klocal1=[AEL 0 0 -AEL 0 0;0 EIL3 EIL2 0 -EIL3 EIL2;0 EIL2 EIL1 0 -EIL2 EIL;
-AEL 0 0 AEL 0 0;0 -EIL3 -EIL2 0 EIL3 -EIL2;0 EIL2 EIL 0 -EIL2 EIL1]
TCS1=[C1 S1 0 0 0 0;-S1 C1 0 0 0 0;0 0 1 0 0 0;0 0 0 C1 S1 0;0 0 0 -S1 C1 0;0 0 0 0 0 1]
F1=Klocal1*TCS1*D
%For element 2
AEL2=AA*EE/LL2
EIL2=2*EE*II/LL2
ETL2=4*EE*II/LL2
ETL32=6*EE*II/LL2^2
EIL32=12*EE*II/LL2^3
Klocal2=[AEL2 0 0 -AEL2 0 0;0 EIL32 EIL22 0 -EIL32 EIL22;0 EIL22 EIL2 0 -EIL22 EIL2;
-AEL2 0 0 AEL2 0 0;0 -EIL32 -EIL22 0 EIL32 -EIL22;0 EIL22 EIL2 0 -EIL22 EIL2]
TCS2=[C2 -S2 0 0 0 0;S2 C2 0 0 0 0;0 0 1 0 0 0;0 0 0 C2 -S2 0;0 0 0 S2 C2 0;0 0 0 0 0 1]
F2=Klocal2*TCS2*[U2;V2;Ph2;U3;V3;Ph3]
Type here to search
27°C Smoke
10:18 PM
3/15/2022

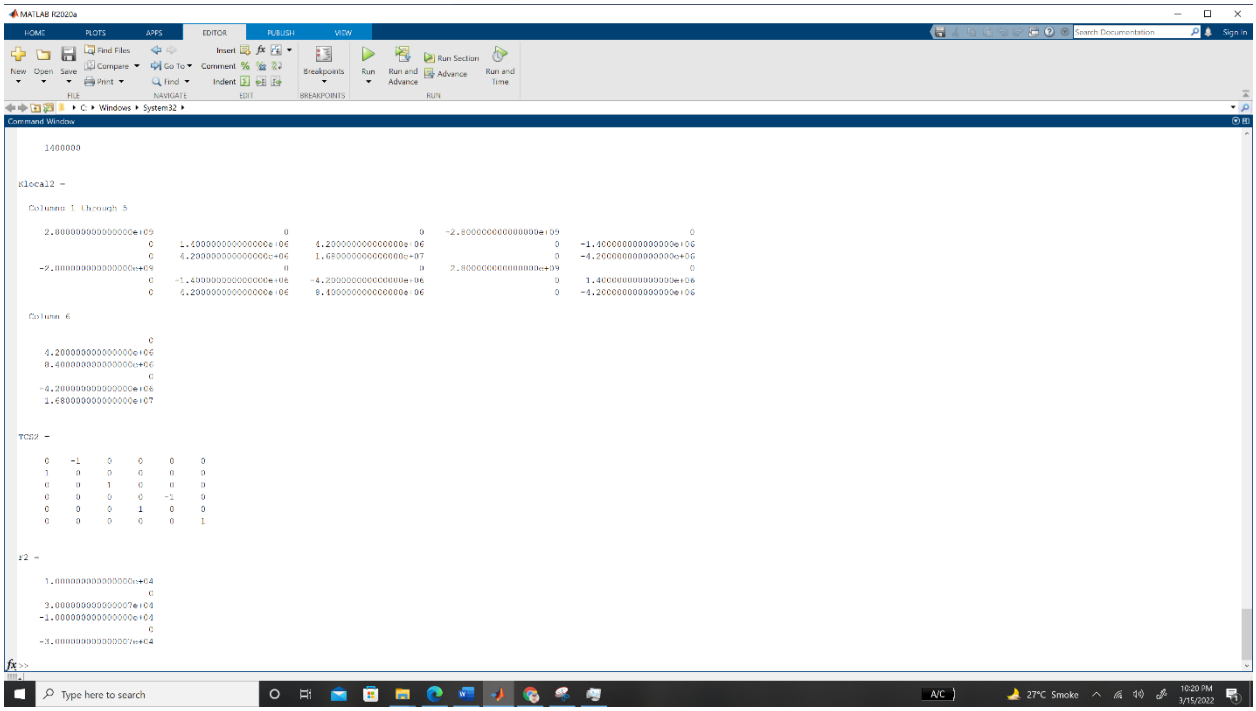

MATLAB R2020a
Editor - C:\Users\DELL Pro 5510\Desktop\FEA CTP\FEACTP1.m
Command Window
Workspace
Name
Value
Type here to search
27°C Smoke
10:19 PM
3/15/2022

1.2 Values Saved by MATLAB Workspace

Workspace

Name	Value
AA	0.0800
AEL	5.6000e+09
AEL2	2.8000e+09
C1	1
C2	0
D	[-0.0214;-0.0250;0.0089;-0.0214;-3.5714e-06;0.0071]
D1	[-0.0214;-0.0250;0.0089;-0.0214;-3.5714e-06;0.0071;0;0;0]
d3x	0
d3y	0
EE	2.1000e+11
EIL	16800000
EIL1	33600000
EIL12	16800000
EIL2	8400000
EIL22	4200000
EIL3	11200000
EIL32	1400000
F1	[0;-1.0000e+04;4.3656e-11;0;1.0000e+04;-3.0000e+04]
F1y	-100000
F2	[10000;0;3.0000e+04;-10000;0;-3.0000e+04]
FM	[0;-1.0000e+04;4.3656e-11;6.9122e-11;-4.3656e-11;0;0;10000;-3.0000e...
Fpart	[0;-10000;0;0;0;0]
Fy	-10000
ICL	2.4000e-04
ICL2	0
II	1.2000e-04
ISL	0
ISL2	1.2000e-04
K	*9x9 double*
K1	*6x6 double*
K1a	*9x9 double*
K2	*6x6 double*
K2a	*9x9 double*
Klocal1	*6x6 double*
Klocal2	*6x6 double*
Ksub	*6x6 double*

LL1	3
LL2	6
M1	0.0800
M12	4.0000e-05
M2	0
M22	0
M3	1.6000e-04
M32	0.0800
Ph1	0.0089
Ph2	0.0071
Ph3	0
S1	0
S2	1
TCS1	*6x6 double*
TCS2	*6x6 double*
Theta_1	0
Theta_2	90
Theta_3	0
U1	-0.0214
U2	-0.0214
U3	0
V1	-0.0250
V2	-3.5714e-06
V3	0



1.3 Solution in Command Window

M1 =

8.000000000000000e-02

M2 =

0

M3 =

1.600000000000000e-04

ISL =

0

ICL =

2.400000000000000e-04

K1 =

Columns 1 through 5

5.600000000000000e+09 0 0 -5.600000000000000e+09 0

0 1.120000000000000e+07 1.680000000000000e+07 0 -1.120000000000000e+07

0 1.680000000000000e+07 3.360000000000000e+07 0 -1.680000000000000e+07

-5.600000000000000e+09 0 0 5.600000000000000e+09 0

0 -1.120000000000000e+07 -1.680000000000000e+07 0 1.120000000000000e+07

0 1.680000000000000e+07 1.680000000000000e+07 0 -1.680000000000000e+07

Column 6

0

1.680000000000000e+07

1.680000000000000e+07

0

-1.680000000000000e+07

3.360000000000000e+07

K1a =

Columns 1 through 5

5.600000000000000e+09 0 0 -5.600000000000000e+09 0

0 1.120000000000000e+07 1.680000000000000e+07 0 -1.120000000000000e+07

0 1.680000000000000e+07 3.360000000000000e+07 0 -1.680000000000000e+07

-5.600000000000000e+09 0 0 5.600000000000000e+09 0

0 -1.120000000000000e+07 -1.680000000000000e+07 0 1.120000000000000e+07

0 1.680000000000000e+07 1.680000000000000e+07 0 -1.680000000000000e+07

Column 6

0

1.680000000000000e+07

1.680000000000000e+07

0

-1.680000000000000e+07

3.360000000000000e+07

K1a =

Columns 1 through 5

5.600000000000000e+09 0 0 -5.600000000000000e+09 0

0 1.120000000000000e+07 1.680000000000000e+07 0 -1.120000000000000e+07

0 1.680000000000000e+07 3.360000000000000e+07 0 -1.680000000000000e+07

-5.600000000000000e+09 0 0 5.600000000000000e+09 0

0 -1.120000000000000e+07 -1.680000000000000e+07 0 1.120000000000000e+07

0 1.680000000000000e+07 1.680000000000000e+07 0 -1.680000000000000e+07

0 0 0 0 0

0 0 0 0 0

0 0 0 0 0

Columns 6 through 9

0 0 0 0

1.680000000000000e+07 0 0 0

1.680000000000000e+07 0 0 0

0 0 0 0

-1.680000000000000e+07 0 0 0

3.360000000000000e+07 0 0 0

0 0 0 0

0 0 0 0

0 0 0 0

M12 =

4.000000000000000e-05

M22 =

0

M32 =

8.000000000000000e-02

ISL2 =

1.200000000000000e-04

ICL2 =

0

K2 =

Columns 1 through 5

1.400000000000000e+06 0 4.200000000000000e+06 -1.400000000000000e+06 0

0 2.800000000000000e+09 0 0 -2.800000000000000e+09

4.200000000000000e+06 0 1.680000000000000e+07 -4.200000000000000e+06 0

-1.400000000000000e+06 0 -4.200000000000000e+06 1.400000000000000e+06 0

0 -2.800000000000000e+09 0 0 2.800000000000000e+09

4.200000000000000e+06 0 8.400000000000000e+06 -4.200000000000000e+06 0

Column 6

4.200000000000000e+06

0

8.400000000000000e+06

-4.200000000000000e+06

0

1.680000000000000e+07

K2a =

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0

K2a =

Columns 1 through 5

0 0 0 0 0

0 0 0 0 0

0 0 0 0 0

0 0 0 1.400000000000000e+06 0

0 0 0 0 2.800000000000000e+09

0 0 0 4.200000000000000e+06 0

0 0 0 -1.400000000000000e+06 0

0 0 0 0 -2.800000000000000e+09

0 0 0 4.200000000000000e+06 0

Columns 6 through 9

0 0 0 0

0 0 0 0

0 0 0 0

4.200000000000000e+06 -1.400000000000000e+06 0 4.200000000000000e+06

0 0 -2.800000000000000e+09 0

1.680000000000000e+07 -4.200000000000000e+06 0 8.400000000000000e+06

-4.200000000000000e+06 1.400000000000000e+06 0 -4.200000000000000e+06

0 0 2.800000000000000e+09 0

8.400000000000000e+06 -4.200000000000000e+06 0 1.680000000000000e+07

K =

Columns 1 through 5

5.600000000000000e+09 0 0 -5.600000000000000e+09 0

0 1.120000000000000e+07 1.680000000000000e+07 0 -1.120000000000000e+07

0 1.680000000000000e+07 3.360000000000000e+07 0 -1.680000000000000e+07

-5.600000000000000e+09 0 0 5.601400000000000e+09 0

0 -1.120000000000000e+07 -1.680000000000000e+07 0 2.811200000000000e+09

0 1.680000000000000e+07 1.680000000000000e+07 4.200000000000000e+06 -1.680000000000000e+07

0 0 0 -1.400000000000000e+06 0

0 0 0 0 -2.800000000000000e+09

0 0 0 4.200000000000000e+06 0

Columns 6 through 9

0 0 0 0

1.680000000000000e+07 0 0 0

1.680000000000000e+07 0 0 0

4.200000000000000e+06 -1.400000000000000e+06 0 4.200000000000000e+06

-1.680000000000000e+07 0 -2.800000000000000e+09 0

5.040000000000000e+07 -4.200000000000000e+06 0 8.400000000000000e+06

-4.200000000000000e+06 1.400000000000000e+06 0 -4.200000000000000e+06

0 0 2.800000000000000e+09 0

8.400000000000000e+06 -4.200000000000000e+06 0 1.680000000000000e+07

Ksub =

Columns 1 through 5

5.600000000000000e+09 0 0 -5.600000000000000e+09 0

0 1.120000000000000e+07 1.680000000000000e+07 0 -1.120000000000000e+07

0 1.680000000000000e+07 3.360000000000000e+07 0 -1.680000000000000e+07

-5.600000000000000e+09 0 0 5.601400000000000e+09 0

0 -1.120000000000000e+07 -1.680000000000000e+07 0 2.811200000000000e+09

0 1.680000000000000e+07 1.680000000000000e+07 4.200000000000000e+06 -1.680000000000000e+07

Column 6

0

1.680000000000000e+07

1.680000000000000e+07

4.200000000000000e+06

-1.680000000000000e+07

5.040000000000000e+07

Fy =

-10000

Fpart =

0

-10000

0

0

0

0

D =

-2.142857142857148e-02

-2.500357142857148e-02

8.928571428571449e-03

-2.142857142857148e-02

-3.571428571428571e-06

7.142857142857159e-03

U1 =

-2.142857142857148e-02

V1 =

-2.500357142857148e-02

Ph1 =

8.928571428571449e-03

U2 =

-2.142857142857148e-02

V2 =

-3.571428571428571e-06

Ph2 =

7.142857142857159e-03

U3 =

0

V3 =

0

Ph3 =

0

D1 =

-2.142857142857148e-02

-2.500357142857148e-02

8.928571428571449e-03

-2.142857142857148e-02

-3.571428571428571e-06

7.142857142857159e-03

0

0

0

FM =

0

-9.999999999999956e+03

4.365574568510056e-11

6.912159733474255e-11

-4.365574568510056e-11

0

0

1.000000000000000e+04

-3.000000000000007e+04

AEL =

5.600000000000000e+09

EIL =

16800000

EIL1 =

33600000

EIL2 =

16800000

EIL3 =

11200000

Klocal1 =

Columns 1 through 5

5.600000000000000e+09 0 0 -5.600000000000000e+09 0

0 1.120000000000000e+07 1.680000000000000e+07 0 -1.120000000000000e+07

0 1.680000000000000e+07 3.360000000000000e+07 0 -1.680000000000000e+07

-5.600000000000000e+09 0 0 5.600000000000000e+09 0

0 -1.120000000000000e+07 -1.680000000000000e+07 0 1.120000000000000e+07

0 1.680000000000000e+07 1.680000000000000e+07 0 -1.680000000000000e+07

Column 6

0

1.680000000000000e+07

1.680000000000000e+07

0

-1.680000000000000e+07

3.360000000000000e+07

TCS1 =

1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

F1 =

0

-9.999999999999956e+03

4.365574568510056e-11

0

9.999999999999956e+03

-3.000000000000003e+04

AEL2 =

2.800000000000000e+09

EIL2 =

8400000

EIL12 =

16800000

EIL22 =

4200000

EIL32 =

1400000

Klocal2 =

Columns 1 through 5

2.800000000000000e+09 0 0 -2.800000000000000e+09 0

0 1.400000000000000e+06 4.200000000000000e+06 0 -1.400000000000000e+06

0 4.200000000000000e+06 1.680000000000000e+07 0 -4.200000000000000e+06

-2.800000000000000e+09 0 0 2.800000000000000e+09 0

0 -1.400000000000000e+06 -4.200000000000000e+06 0 1.400000000000000e+06

0 4.200000000000000e+06 8.400000000000000e+06 0 -4.200000000000000e+06

Column 6

0

4.200000000000000e+06

8.400000000000000e+06

0

-4.200000000000000e+06

1.680000000000000e+07

TCS2 =

0	-1	0	0	0	0
1	0	0	0	0	0
0	0	1	0	0	0
0	0	0	0	-1	0
0	0	0	1	0	0
0	0	0	0	0	1

F2 =

1.000000000000000e+04

0

3.000000000000007e+04

-1.000000000000000e+04

0

-3.000000000000007e+04