Example 7.15 Construct a PDA to accept the following language L=$\{a^{2n}b^n$,where $n > 0\}$.

Solution: The language consists of 2n number of ‘a’ and n number of ‘b’ belong to the language set. The PDA can be designed as follows—at the time of traversing the fi rst ‘a’, two z_1 are added to the stack with a state change from q_0 to q_1 . At the time of traversing the second ‘a’, one z_1 is popped from the stack with a state change from q_1 to q_0 . By this process after traversing 2n number of ‘a’, only n number of z_1 exist in the stack. At the time of traversing the first ‘b’, the stack top is z_1 and the state is q_0. Those z_1 are popped at the time of traversing n number of ‘b’. The transitional functions are

$\delta(q_0, a, z_0) \rightarrow (q_1, z_1 z_1 z_0)$
$\delta(q_1, a, z_1) \rightarrow (q_0, \lambda)$
$\delta(q_0, a, z_1) \rightarrow (q_1, z_1 z_1 z_0)$
$\delta(q_0, b, z_1) \rightarrow (q_2, \lambda)$
$\delta(q_2, b, z_1) \rightarrow (q_2, \lambda)$
$\delta(q_2, \lambda, z_0) \rightarrow (q_f, z_0)$// Accepted by final state
$\delta(q_2, \lambda, z_0) \rightarrow (q_2, \lambda)$// Accepted by empty stack

Example 7.16 Construct a PDA to accept the language L $=\{a^n b^n c^m$,where n, $m \geq 1\}$ by the empty stack and by the fi nal state.

Solution: The language is in the form $a^n b^n c^m$, where n, $m \geq 1$. In the language set, the number of 'a' and the number of 'b' are the same, but the number of 'c' is different. All the strings in the language set start with n number of 'a's followed by n number of 'b's and ends with m number of 'c's. Here, m and n are both ≥ 1, and the null string does not belong to the language set. The PDA for the language can be designed in the following way. When traversing the 'a's from the input tape, the z_1's are pushed in the stack one by one. At the time of traversing the 'b's, all the z_1's which were pushed into the stack are popped one by one. When the first 'c' will be traversed, at that time the machine is in state q_1 and stack top z_0. At the time of traversal of m number of 'c's, no operation is done on the stack. The transition function for constructing the PDA for the string $a^n b^n c^m$ where m, $n \geq 1$ are

$\delta(q_0, a, z_0) \rightarrow (q_0, z_1 z_0)$
$\delta(q_0, a, z_1) \rightarrow (q_0, z_1 z_1)$
$\delta(q_0, b, z_1) \rightarrow (q_1, \lambda)$
$\delta(q_1, b, z_1) \rightarrow (q_1, \lambda)$
$\delta(q_1, c, z_0) \rightarrow (q_1, z_0)$
$\delta(q_1, \lambda, z_0) \rightarrow (q_1, \lambda)$ accepted by the empty stack
$\delta(q_1, \lambda, z_0) \rightarrow (q_f, z_0)$ accepted by the fi nal state